Carrier-envelope-phase dependent coherence in double quantum wells

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Abstract: By analyzing the interaction of a few-cycle laser pulse within an asymmetric semiconductor double quantum well structure, we show that the transient coherence thus produced is strongly dependent on the carrier-envelope-phase (CEP) and significantly enhanced due to the Fano-type interference. A method to determine the CEP is proposed by directly mapping the CEP dependent coherence to the quantum beat signals.

OCIS codes: (270.1670) Coherent optical effects; (320.7130) Ultrafast processes in condensed matter, including semiconductors.
There have been significant research activities on quantum coherence and interference phenomena induced by the intersubband transitions (ISBT) of semiconductor quantum wells (QW) in the last decades [1, 2]. A number of fascinating coherence introduced effects have been discovered when lasers are applied to the QW structures, such as tunneling induced transparency [3, 4], electromagnetically induced transparency [5, 6, 7, 8], gain without inversion [9], coherent control of electron population [10], Autler-Townes splitting [11], and terahertz transparency [3, 4], electromagnetically induced transparency [5, 6, 7, 8], gain without inversion [9], coherent control of electron population [10], Autler-Townes splitting [11], and terahertz emission [12]. These studies have considerably modified our understandings of the nature and consequences of quantum coherence on the quantum and nonlinear optical processes in QW systems [13, 14, 15]. Recently, the effects of the carrier-envelope phase (CEP) of few-cycle pulses on the quantum coherence and interference in optical media have drawn lots of attention [16, 17, 18, 19, 20, 21, 22], due to that these investigations can lead to many practical applications in extracting the related information of an ultrashort laser pulse.

In this letter, we theoretically investigate the effects of CEP on the transient coherence produced by an ultrashort laser pulse of a few cycles in an asymmetric double quantum well structures. We demonstrate that the coherent effect is strongly dependent on the CEP, and the magnitude of transient coherence can be enhanced significantly due to the Fano-type interference. We also show that the coherence thus produced can also be mapped into the signal of quantum beats and hence might be used to determine the CEP of few-cycle pulses.

The schematic energy-level diagram of a GaAs/Al\textsubscript{x}Ga\textsubscript{1-x}As coupled quantum well structure are shown in Fig. 1(a): a Al\textsubscript{x}Ga\textsubscript{1-x}As shallow well and a GaAs deep well separated by a thick Al\textsubscript{x}Ga\textsubscript{1-x}As tunnel barrier. This barrier will couple the excited state of deep well with the ground state of shallow well to create a doublet states \( |1\rangle \leftrightarrow |2\rangle \) and \( |1\rangle \leftrightarrow |2\rangle \) simultane-
Tunneling to a continuum of energies takes place from states $|2\rangle$ and $|3\rangle$ through the thin barrier on the right of the deep well. The probability amplitude for the absorption of a photon can be thought as the superposition of two absorption paths, one via level $|2\rangle$ and one via level $|3\rangle$, both decaying by tunneling to the same continuum. Fano-type destructive interference between the two absorption paths may then occur so as to cancel the absorption altogether. Nearly vanishing absorptions due to the Fano effect have already been predicted [23] and observed [3, 4, 24]. As shown in Fig. 1(b), we consider an ultrashort optical pulse of the electric field $E(t) = -\partial A(t)/\partial t$ with the vector potential $A(t) = A_0 e^{-(t-\tau)^2/\tau^2} \sin(\omega t + \phi)$[16, 17, 18, 19], where $A_0$, $\tau$, $\omega$, and $\phi$ are the amplitude, pulse width, carrier-envelope frequency, and the phase of the vector potential, respectively. Let us assume that the electronic wave function in the form of $|\psi\rangle = a_1|1\rangle + a_2|2\rangle + a_3|3\rangle$, then the time evolution equation for $|\psi\rangle$ is governed by the Schrödinger equation, with which we can have the corresponding differential equations for the probability amplitudes $a_j$ as follows:

$$
\dot{a}_1 = i\Omega \xi(t)[a_2(t)e^{-i\Delta t} + qa_3e^{-i(\Delta + \delta)t}],
$$

$$
\dot{a}_2 = -\gamma_2a_2 + i\Omega \xi(t)a_1e^{i\Delta t} + p\sqrt{\Omega \Delta}a_3e^{-i\delta t},
$$

$$
\dot{a}_3 = -\gamma_3a_3 + i\Omega \xi(t)a_1e^{i(\Delta + \delta)t} + p\sqrt{\Omega \Delta}a_2e^{i\delta t},
$$

where $\xi(t) = \omega^{-1}\partial [e^{-(t-\tau)^2/\tau^2} \sin(\omega t + \phi)]/\partial t$, the dot overhead means the derivative with respect to time. $q\Omega = \delta\Delta^e = q\mu_1\omega A_0/(2\hbar)$ is the half Rabi frequency for the transition $|1\rangle \leftrightarrow |j\rangle$ ($j = 2, 3$), with $q$ being the ratio of the dipole matrix element between two upper levels. $2\delta$ is the energy splitting due to the tunneling between the upper levels and $\Delta = \omega - \omega_0$ is the detuning between the frequency of the ultrashort pulse and the average transition frequency $\omega_0 = (\omega_2 + \omega_3)/2$, where $\omega_2$ and $\omega_3$ being the transition frequencies corresponding to $|2\rangle \leftrightarrow |1\rangle$ and $|3\rangle \leftrightarrow |1\rangle$, respectively. The decay rates have been added phenomenologically in the above equations (1-3), where $\gamma_{2.3} = \gamma_{2.3l} + \gamma_{2.3d}$ denotes the total decay rate of the upper states including both the population scattering rates $\gamma_{2.3l}$ due to longitudinal optical (LO) phonon emission events at low temperature and the dephasing rates $\gamma_{2.3d}$ due to a combination of quasielastic interface roughness scattering or acoustic phonon scattering. Besides, we have neglected other inhomogeneous broadening effects due to their small influences[25]. Moreover, $p\sqrt{\Omega \Delta} = \sqrt{\Omega \Delta} \gamma_0$ represents the cross-coupling of the two upper states via the LO phonon decay[3, 4], which arises from the tunneling to the continuum through the thin barrier next to the deep well. Here we use $p$ to assess the cross coupling strength[26, 27, 28, 29, 30], where
the limit values $p = 0$ and $p = 1$ correspond, respectively, to no interference and perfect interference.

As an example for the numerical calculations, we consider the structure design of the asymmetric double quantum-well: a 68 Å thick Al$_{0.15}$Ga$_{0.85}$As shallow well and a 70 Å thick GaAs deep well separated by a 20 Å thick Al$_{0.3}$Ga$_{0.7}$As tunnel barrier. The doublet states (|2⟩ and |3⟩) are both coupled to the continuum by a 15 Å thick Al$_{0.3}$Ga$_{0.7}$As barrier, which produces the decay-induced coherence. Note that, for temperature up to 10 K with electron sheet densities smaller than $10^{12}$ cm$^{-2}$, the dephasing rates $\gamma_d$ can be estimated [3] to be $\gamma_d = 4.13$ meV and $\gamma_d = 5.35$ meV. The population-decay rates can be calculated [31]: upon solving the effective mass Schrödinger equation with outgoing waves at infinity, we obtain a set of complex eigenvalues whose real and imaginary parts yield, respectively the quasi-bound state energy levels and resonance widths. For our asymmetric double quantum well structure, the population-decay rates turn out to be $\gamma_2 = 5.6$ meV and $\gamma_3 = 7.0$ meV. In such a scenario, a coupling ultrashort laser can produce the oscillation between the doublet states. Sequentially the induced oscillation is strongly dependent on the CEP of a few-cycle pulse, which produce a CEP dependent transient coherence for $|\rho_{23}| = |a_2(t)a_3^*(t)|$. Direct numerical calculations for the solutions of Eqs. (1-3) demonstrate that the CEP of ultrashort laser pulses with only a few cycles has indeed significant effects on the coherence $\rho_{23}$ in the weak field regime ($\alpha = \Omega/\omega \ll 1$). Figure 2 illustrates this point via some typical examples. The real, imaginary, and absolute values of the transient coherence $\rho_{23}$ is shown with the dependence of the CEP ($\phi$) at the time $t = 4\tau$ for two different pulse widths ($\tau = 9/\omega$, $\tau = 18/\omega$) and for two different Rabi frequencies ($\Omega = \omega/20$, $\Omega = \omega/5$), under the initial conditions $a_1(0) = 1$ and $a_2(0) = a_3(0) = 0$.

This result can be explained physically by the time-dependent perturbation theory with a small parameter $\alpha \ll 1$. Under the initial conditions $a_{2,3}(0) = 0$ and $a_1(0) = 1$, taking $a_j =$
\[ \Sigma_{k} a_{j}^{(k)} \text{ with } a_{j}^{(k)} = \theta(\alpha_{j}^{k}), \quad \text{we can see from Eqs. (1-3) that } a_{2,3}(t) = \theta(\alpha) \text{ and } a_{1}(t) = \theta(\alpha_{0}), \]

\[ \text{thus } \rho_{23} = \theta(\alpha_{2}). \]

Clearly CEP dependence has been produced even for the low Rabi frequency i.e., \( \Omega = \omega \times 20 \). Just as shown in Fig. 2, the dependent amplitude become pronounced as the Rabi frequency increases and the pulse width becomes narrow. The low Rabi frequencies induce less transient coherence and hence are obviously non-favorable from the viewpoint of the experimental measurement. The lower limit for Rabi frequency depends on the precision of the technique in measurement. With state-of-the-art technologies to handle the weak light-QW interaction, relative effects of the QW system considered here can be measured in low temperature (10 K) [2]. Besides, we note that \( \alpha \sim E \) characterizes the electric field \( E(t) \) with the period \( 2\pi \) for the CEP \( \phi \). In such a case, the relation \( \rho_{23} = \theta(\alpha_{2}) \) implies that \( \rho_{23} \) should approximately have the period \( \pi \), as illustrated in Fig. 2.

It should be noted that the interference induced by the resonant tunneling have been included in plotting Fig. 2. According to the decay-rate values \( \gamma_{21} = 5.6 \text{ meV}, \gamma_{21} = 7.0 \text{ meV}, \gamma_{2d} = 4.13 \text{ meV, and } \gamma_{3d} = 5.35 \text{ meV} \), we can obtain the cross coupling strength between [2] and [3] \( \rho = 0.54 \). In order to examine the effect of the interference induced by the resonant tunneling on the CEP dependent coherence, we consider a similar GaAs/AlGaAs asymmetric double quantum well structure consists of two quantum wells (55 Å Al\(_{0.3}\)Ga\(_{0.7}\)As shallow well and 57 Å GaAs deep well) separated by a 35 Å Al\(_{0.4}\)Ga\(_{0.5}\)As tunneling barrier. Aluminum is added to the shallow well in order to reduce the contribution of interface roughness scattering. The energy splitting between the upper levels is calculated to be \( 2\delta = 7.6 \text{ meV} \). For a sheet carrier density of \( 10^{12} \text{ cm}^{-2} \) in the quantum wells, we can obtain the LO-phonon decay rates \( \gamma_{21} = 0.31 \text{ meV} \) and \( \gamma_{21} = 0.26 \text{ meV, and the dephasing rates can be estimated to be } \gamma_{2d} = 0.031 \text{ and } \gamma_{3d} = 0.026 \text{ meV}. \)

Thus, the cross coupling strength is estimated as \( p = 0.90 \). This is close to the ideal value \( p = 1 \) and corresponds to a large tunneling efficiency leading to a strong Fano-type interference effect. With new parameter values of this QW structure, we show in Fig. 3 the

![Fig. 3. The transient coherence \( |p_{23}| \times 10^{3} \) versus the CEP \( \phi \) for the case of (a, c) \( p = 0 \)
and (b, d) \( p = 1 \) at the time \( t = 4\tau \) for different widths, \( \tau \), and Rabi frequencies, \( \Omega \), of the pulse with other parameters \( \hbar \omega = 125 \text{ meV, } q = 1.2, \Delta = 0, 2\delta = 17.6 \text{ meV, } \gamma_{21} = 0.31 \text{ meV, } \gamma_{2d} = 0.26 \text{ meV, } \gamma_{3d} = 0.031 \text{ meV, and } \gamma_{3d} = 0.026 \text{ meV.} \)
transient coherence $|\rho_{23}|$ versus the CEP $\phi$ at the time $t = 4\tau$ under the same initial conditions as in Fig. 2, and it demonstrates that the amplitude of the transient coherence is enhanced. This interesting result is produced from the perfectly interference induced by the resonant tunneling. The large amplitude is obviously favorable from the viewpoint of the experimental measurement in the weak-field regime. More interestingly, the parameters of the electron subbands in QW structures can be engineered to give a desired amplitude of coherence by utilizing so-called structure coherent control in design [2].

We now study the quantum beats due to the coherence $\rho_{23}$ produced by a few-cycle ultrafast pulse for the time interval $T > t$ with the initial time $t = t_0 = 4\tau$. The quantum beat note signal $I$ can be given as [32]

$$ I = \langle \psi(T) | \hat{E}_1^{(-)}(T) \hat{E}_2^{(+)}(T) | \psi(T) \rangle + c.c., $$

(4)

with the state of our system $|\psi(T)\rangle$ satisfying $|\psi(T)\rangle = \sum_j a_j |j,0\rangle + b_2 |1,1_{\omega_{21}}\rangle + b_3 |1,1_{\omega_{31}}\rangle$. Here $|n,0\rangle$, $|1,1_{\omega_{ij}}\rangle$ describe the levels $|n\rangle$ ($n = 1,2,3$) with no photon, and ground state $|1\rangle$ with one photon in the field mode $j$ characterizing the transition $|0\rangle \rightarrow |j\rangle$ ($j = 2,3$), respectively. $\hat{E}_1^{(-)}(T) = \delta_2 \hat{a}_1 e^{-i\omega_{21}(T-t)}$ and $\hat{E}_2^{(+)}(T) = \delta_3 \hat{a}_3 e^{i\omega_{31}(T-t)}$ denote the electric field per photon for the mode $j$. Inserting Hamiltonian $H = \hbar \Sigma_j g_j (\hat{a}_j |j\rangle \langle j+1| + \hat{a}_j^\dagger |j+1\rangle \langle j|)$ into the Schrödinger equation, i.e. $\frac{d}{dT} |\psi(T)\rangle / \partial T = -i(H/\hbar) |\psi(T)\rangle$, we obtain

$$ i\left(\frac{d}{dT} + \gamma_j\right) a_j - p\sqrt{\gamma_j}\delta_j (a_3 \delta_{j,2} + a_2 \delta_{j,3}) = g_j b_j, $$

(5)

$$ i\left(\frac{d}{dT} - g_j\right) a_j = 0, j = 2,3, $$

(6)

with $g_j = \mu_0 \varepsilon_j / (2\hbar)$. By solving Eqs. (5,6) under the initial conditions of $b_{1,2}(t) = 0$, the quantum beat signals can be calculated as

$$ I = I_0(\phi) \cos[2\delta(T-t) + \eta(t)], $$

(7)

where $I_0(\phi) = C |\rho_{23}(t)|$ with the coefficient $C$ being determined by $\varepsilon_j$, $g_j$, $\gamma_j$, and $p$. And $\eta(t)$ is an adjustable phase shift of the ultrashort pulse at the time instant $T = t$. Here we have used the assumption of $\gamma_j$, $p\sqrt{\gamma_j} \gg 2g_j$, so that we can neglected the time-dependent term in the coefficient $C$. From Eq. (7), we find that $I_0(\phi)$ depends on the CEP $\phi$ through the CEP dependent coherence $|\rho_{23}(t)|$ as shown in Fig. 2. Thus the CEP of a few-cycle pulse might be determined by measuring the quantum beat signals. By defining the depth of modulation, $M$, in the signal amplitude of quantum beats as $M = 2|I_0(\phi)_{\text{max}} - I_0(\phi)_{\text{min}}|/[I_0(\phi)_{\text{max}} + I_0(\phi)_{\text{min}}]$, with $I_0(\phi)_{\text{max, min}}$ being the maximum (minimum) amplitude. For a certain value of $C$, one can have a much larger value of the modulation depth $M$ in our system than those proposed in the previous schemes [16, 17, 18], as illustrated in Fig. 2.

In conclusion, we have studied the generation of transient coherence induced by few-cycle laser pulses in an asymmetric semiconductor double QW structure, and shown that the coherence thus produced strongly depends on the carrier-envelope phase of the ultrashort laser pulses. Importantly, the amplitude of the CEP dependent transient coherence can be greatly enhanced due to the Fano-type interference. Besides, we also shown that the CEP-dependent coherence can be mapped into the signal of quantum beats, thus one can determine the CEP by measuring the quantum beat signals. We believe that the CEP dependent coherence in our proposed QW structure will also manifest itself in other quantum interference phenomena as well, and hence our study might open up an avenue to explore and utilize the CEP dependent coherent effects and could be exploited in real solid-state devices as high speed optical modulators and switches.
This work is supported by National Natural Science Foundation (NSF) of China under Grants No. 10704017 and No. 10874050, and also partially supported by the National Basic Research Program of China (973 program), Grant No. 2007CB936300 and No. 2005CB724508. We thank Prof. Y. Wu and Ite. Yu for their helpful discussions.