Control on the anomalous interactions of Airy beams in nematic liquid crystals

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Abstract: We reveal a controllable manipulation of anomalous interactions between Airy beams in nonlocal nematic liquid crystals numerically. With the help of an in-phase fundamental Gaussian beam, attraction between in-phase Airy beams can be suppressed or become a repulsive one to each other; whereas the attraction can be strengthened when the Gaussian beam is out-of-phase. In contrast to the repulsive interaction in local media, stationary bound states of breathing Airy soliton pairs are found in nematic liquid crystals.

OCIS codes: (190.0190) Nonlinear optics; (190.6135) Spatial solitons; (350.5500) Propagation.

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1. Introduction

During the past several years, self-accelerating Airy beams [1–8] have drawn considerable attention due to their unique characteristics, such as ballistic motion [9, 10], self-healing [11–13], etc. Up to now, a large variety of potential applications of Airy beams have been reported, such as optically mediated particle clearing [14], linear light bullets [15], Airy surface plasmons [16–18], and electron Airy beam [19, 20]. In comparison with the linear regime, it has been illustrated that the nonlinear control of Airy beams becomes a more interesting topic from the physical point of view, due to a host of new phenomena, e.g., nonlinear generation of Airy beams [21, 22], and propagation of Airy beams in nonlinear media [23–30]. In particular, solitons, a localized wavepacket when nonlinear self-focusing and linear diffraction balance each other, can be formed with the Airy beams in nonlinear media. For instance, the dynamics of spatial Airy solitons [31, 32] and spatiotemporal Airy light bullets [33–36] have been studied in different nonlinear physical settings.

The propagation dynamics of Airy beams is also investigated extensively in nonlocal nonlinear media. In optics, nonlocality means that the light-induced refractive index change of a material at a particular location is determined by the light intensity in a certain neighborhood of this location. Nonlocal nonlinearity exists in nematic liquid crystals [37] and thermal media [38]. Many works have shown that nonlocality has profound effects on the solitons propagation [39]. For Airy beams, it has been shown that the boundary conditions of a strongly nonlocal media affect deeply the propagation dynamics of self-accelerating beams [40]. Furthermore, an analytical expression of an Airy beam propagating in a strongly nonlocal nonlinear media was

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derived to show the normalized intensity distribution of the Airy beam is always periodic [41]. We have numerically obtained such periodic intensity distribution of Airy beam in the regime of strong nonlocality [42].

Interactions are important and interesting features of solitons dynamics. They interact like real particles, exhibiting attraction and repulsion on one another [43]. Interactions between Airy pulse and temporal solitons was studied [44, 45]. The interaction of an accelerating Airy beam and a solitary wave was also investigated for integrable and non-integrable equations [46]. In a photorefractive crystal under focusing conditions, the interaction between two incoherent counterpropagating Airy beams leads to light-induced waveguide [47, 48]. Soliton pairs can be generated through interactions of Airy beams [49–51]. It is known that both out-of-phase and in-phase Airy beams are always repulsive in local media with some particular parameters of amplitudes and beam intervals [50, 51]. At the first glance, it seems that it is hard to obtain stationary bound states of Airy solitons in local media [50, 51]. However, in nonlocal media, it has been shown that nonlocality provides a long-range attractive force, leading to the formation of stable bound states of both out-of-phase bright solitons [52–54] and dark solitons [55–58]. Based on this effect of nonlocality, in our previous work, we have obtained stationary bound states (soliton pairs) of in-phase as well as out-of-phase Airy beams in nonlocal nonlinear media [42]. More recently, the evolution of a two-dimensional broad accelerating Airy beam interacting with an intense Gaussian beam was studied numerically and experimentally to demonstrate gravitational dynamics (general relativity) in lead glass with a nonlocal thermal nonlinearity [59].

In this paper, with a fundamental beam, we reveal that the interactions between in-phase Airy beams is controllable. With the long-ranged nonlocal nonlinear interaction, such as nematic liquid crystals, the attractive interaction between in-phase Airy beams can be weakened or switches into a repulsive one through the introduction of another in-phase fundamental Gaussian beam. Moreover, with an out-of-phase fundamental Gaussian beam, one can enhance the attractive interaction between in-phase Airy beams. In contrast to only repulsive force in local media, such a nonlolcal attractive force provided by nematic liquid crystals also leads to the formation of stationary bound states of breathing Airy soliton pairs of in-phase as well as out-of-phase beams.

2. Model and basic equations

We consider a one-dimensional Airy beam propagating in a nematic liquid crystal in the presence of an applied static electric field. The slowly varying envelope of this Airy beam $\psi(x,z)$ is governed by the dimensionless nonlocal nonlinear Schrödinger equation [46, 60–62],

$$i\frac{\partial \psi}{\partial z}+\frac{1}{2}\frac{\partial^2 \psi}{\partial x^2}+2\theta\psi=0, \tag{1}$$

where the variables x and z are the normalized transverse coordinate and the propagation distance, scaled by the characteristic transverse width x_0 and the corresponding Rayleigh range kx_0^2, respectively [50]. Here, θ describes the change of the director angle from the pretilt state [62], which is related to the nonlinear change to the optical refractive index and can be described by the following diffusive equation [53, 63]:

$$\sigma^2\frac{\partial^2 \theta}{\partial x^2}-\theta+\frac{1}{2}|\psi|^2=0, \tag{2}$$

where σ corresponds to the degree of nonlocality. When $\sigma\to 0$ and $\sigma\to\infty$, It describes for local and strongly nonlocal media, respectively [39]. Using Fourier transformation and the con-

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volution theorem, one can find the solution of Eq. (2) [53, 63]:

$$\theta = \frac{1}{2}\int_{-\infty}^{\infty} R(x-x')|\psi(x')|^2 dx', \tag{3}$$

where $R(x)$ is the normalized nonlocal response function from the liquid crystals [53],

$$R(x) = \frac{1}{2\sigma}\exp\left(-\frac{|x|}{\sigma}\right). \tag{4}$$

In fact, the actual form of the nonlocal response is determined by the details of physical process responsible for the nonlocality. For all diffusion-type nonlinearities, e.g., the re-orientational-type nonlinearities (nematic liquid crystals) [52] and the general quadratic nonlinearity describing parametric interaction [64], the nonlocal response function is an exponential form.

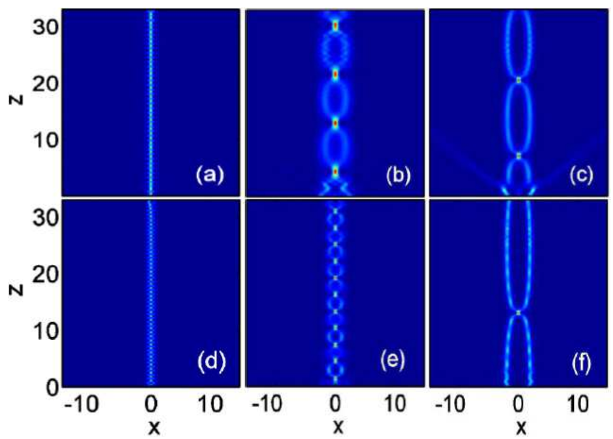

Fig. 1. Trajectories of in-phase interacting Airy beams and Gaussian beam in nematic liquid crystals with the degree of nonlocality $\sigma = 0$ (local case). The amplitude is $A = 3$ in all the plots, and $C = 0$ for (a-c) and $C = 0.3, 0.8, 1$ for (d-f), respectively. The beam intervals are: (a,d) $B = 1$, (b,e) $B = 2$, and (c,f) $B = 3$.

In our previous work [42], we have investigated the interactions of both in-phase and out-of-phase Airy beams in nonlinear media with a Gaussian nonlocal response function. We assumed that the incident beam is composed by a coherent superposition of two shifted counter propagating Airy beams with a relative phase between them [42, 50, 51],

$$\psi(x) = A\{Ai[(x-B)]\exp[a(x-B)] + \exp(i\rho\pi)Ai[-(x+B)]\exp[-a(x+B)]\}, \tag{5}$$

where A is the amplitude, B is the parameter controlling the beam separations, and ρ is the parameter controlling the phase shift with $\rho = 0$ and $\rho = 1$ describing in-phase and out-of-phase Airy beams [42,50,51], respectively. We have also numerically checked (not shown) such interactions of Airy beams in nematic liquid crystals and found that they have similar dynamics with the case of Gaussian nonlocal response function. Of course, study of the interactions between two Airy beams is not our purpose in this paper.

In this work, we focus on the manipulation of interactions of Airy beams with the help of a fundamental beam. Similar to Eq. (5), we assume that the incident beam is composed of two shifted counter propagating Airy beams, but with an additional Gaussian beam with a relative phase between them,

$$\begin{aligned}\psi(x) &= A\{Ai[(x-B)]\exp[a(x-B)] + Ai[-(x+B)]\exp[-a(x+B)]\} \\ &+ \exp(i\rho\pi)C\exp\left(-x^2/2\right),\end{aligned} \tag{6}$$

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where C is the amplitude of the Gaussian beam. When $C = 0$, we can obtain the previously studied in-phase Airy beams [42,50,51]. Here, for the Gaussian beam, a coherent superposition is also applied with a relative phase difference, with $\rho = 0$ and $\rho = 1$ describing in-phase and out-of-phase interactions, respectively.

3. Anomalous interactions of Airy beams with in-phase Gaussian beam

First, we consider the interactions between both in-phase Airy beams and Gaussian beam ($\rho = 0$). We fix the amplitudes of Airy beams by setting $A = 3$. In Figs. 1(d)-1(f), in the case of a weak Gaussian beam (C is much smaller than A), we show the interactions for different beam intervals x_0 in nematic liquid crystals with the degree of nonlocality $\sigma = 0$ (local case). As a comparison, we also re-do some previous results with $C = 0$ in Figs. 1(a)-1(c) [42,50,51]. We can see that, with the help of in-phase Gaussian beam, the attractive force between Airy beams becomes stronger, leading to the decrease of the width and breathing period in the bound states when the interval parameter is small, such as $B = 1$ [Figs. 1(a) and 1(d)] and $B = 2$ [Figs. 1(b) and 1(e)], respectively. However, for a larger beam interval $B = 3$ [Figs. 1(c) and 1(f)], an anomalous and interesting phenomena happens. Now, the interaction between Airy beams weakens with its breathing period of the bound state becomes larger obviously.

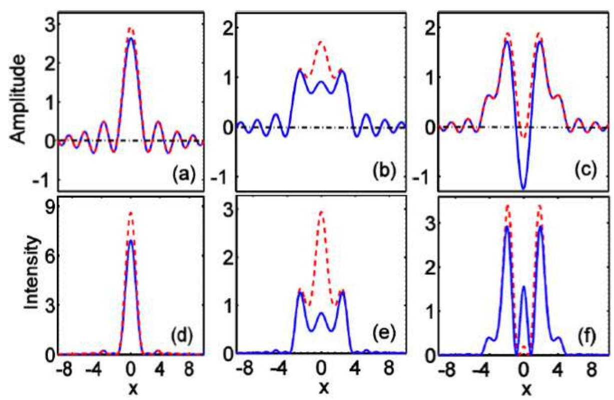

Fig. 2. Amplitude and intensity of the in-phase input beam. The amplitude is $A = 3$ in all the plots. All the solid lines represent $C = 0$, and dashed lines in (a,d), (b,e), and (c,f) represent $C = 0.3, 0.8, 1$, respectively. The beam intervals are: (a,d) $B = 1$, (b,e) $B = 2$, and (c,f) $B = 3$, respectively.

In order to illustrate the physical mechanism of these interactions, we show in Fig. (2) the amplitude and intensity of input beams by calculating Eq. (6) directly. The solid and dashed lines describe the case of $C = 0$ and $C \neq 0$, respectively. It is obviously that, for smaller intervals $B = 1$ [Figs. 2(a) and 2(d)] and $B = 2$ [Figs. 2(b) and 2(e)], the Gaussian beam effectively increases the amplitude and intensity in the center region between the main lobes of two Airy beams, which leads to an increase in the refractive index in this region. This in turn attracts more light to the center, moving the main lobes of each Airy beam toward it [43], and hence the attraction between Airy beams is enhanced naturally. On the contrary, for a larger interval $B = 3$, the amplitude in the center region between two main lobes is negative when $C = 0$ [solid line in Fig. 2(c)]. When a in-phase positive Gaussian beam ($C \neq 0$) overlaps with the Airy beams, the absolute of amplitude in this region will decrease [dashed line in Fig. 2(c)], then the intensity [dashed line in Fig. 2(f)] and the corresponding refractive index in this region will decrease subsequently. Therefore, the attraction between the Airy beams will be weakened [Figs. 1(c) and 1(f)].

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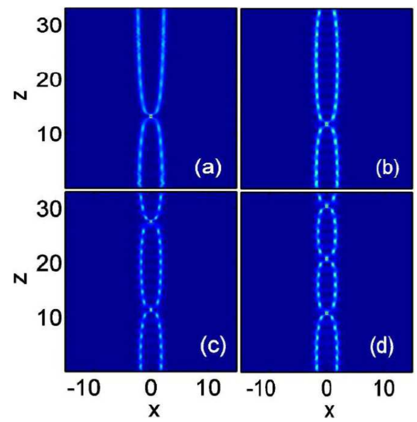

Fig. 3. Trajectories of in-phase interacting Airy beams and Gaussian beam in nematic liquid crystals with the degree of nonlocality $\sigma = 0$ (local case). The amplitude is $A = 3$ and the interval is $B = 3$. The amplitude for the Gaussian beams are $C = 0.9, 1.5, 1.6, 1.7$ for (a-d), respectively.

To demonstrate the interactions of Airy beams with a larger interval controlled by another Gaussian beam, we display in details the dynamics of the interactions in Fig. (3) for $B = 3$. Increasing the amplitude of the fundamental beam C gradually, the attractive force between Airy beams decreases first [Fig. 3(a)], and then increases subsequently [Figs. 3(b)-3(d)]. We also show in Fig. (4) the corresponding intensity distribution of the input beam. We can see that the intensity in the center region between the two main lobes decreases first, and then increases subsequently after it reaches zero. This variation of the intensity or the refractive index will affect the dynamics of Airy beam dramatically, which indicates that the interaction between Airy beams with a larger interval can be flexibly controlled by a fundamental Gaussian beam.

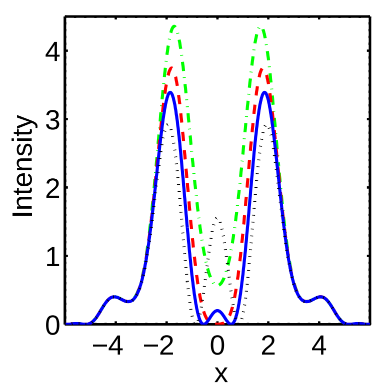

Fig. 4. Intensity distribution of the in-phase input beam with $A = 3$ and $B = 3$. The dotted, solid, dashed, and dash-dotted lines represent the amplitude in the Gaussian beams for $C = 0, 0.8, 1.3$, and 2, respectively.

When the amplitude of Gaussian beam is large enough, i.e., a strong light, we show in Figs. 5(a)-5(c) the interactions with different beam intervals B in local case. Similarly, the corresponding amplitude and intensity of the input beam are displayed in Fig. (6). Interestingly, we find that the repulsion between the Airy beams appears in the in-phase case for all the values

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of interval. However, they may have different physical dynamics. For $B = 1$, the main lobes of the Airy beams have the smallest interval [42, 51]. When a fundamental beam overlaps with the superposed main lobes, the dynamics is similar to the in-phase Airy beams with a larger amplitude. The intensity of superposed main lobes is enhanced [Figs. 6(a) and 6(d)]; while the width is suppressed. Thus, the two solitons generated from the splitting of the superposed main lobes will experience a smaller repulsion force [51]. The refractive index change will make the solitons attract to each other. Moreover the attraction is quite strong over a long distance, but eventually the repulsion will overtake the attraction [Fig. 5(a)].

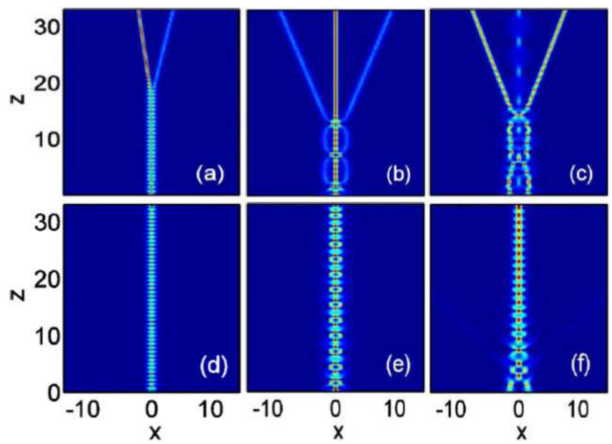

Fig. 5. Trajectories of in-phase interacting Airy beams and Gaussian beam in nematic liquid crystals. The degrees of nonlocality are $\sigma = 0$ (local case) for (a-c), and $\sigma = 0.1, 0.15, 0.42$ for (d-f). The amplitudes are $A = 3$ in all the plots and $C = 0.4, 1.62$, and 2.41 for (a,d), (b,e), and (c,f), respectively. The beam intervals are: (a,d) $B = 1$, (b,e) $B = 2$, and (c,f) $B = 3$.

Although the main lobes have a slightly larger interval for $B = 2$, they will attract to each other in a short distance and then repel when a strong intensity Gaussian beam located in the center between two main lobes [Figs. 6(b) and 6(e)]. This comes from the reason that the main lobes have smaller intervals with the peak of the Gaussian beam, which will provide a repulsive force between the main lobes and the Gaussian beam. Eventually, Airy beams will repel to each other [Fig. 5(b)]. For a larger interval $B = 3$, the propagation of Airy beams also exhibits repulsive effects [Fig. 5(c)]. However, they may have different physical mechanism with Figs. 5(a) and 5(b). As shown in Figs. 6(c) and 6(f), the maximum values of amplitude and intensity are located at the position of main lobes, not in the center between them. The double-peaked profile of the refractive index always attracts a part of the power of Gaussian beam to the main lobes. It is clearly that the major power of the fundamental beam transfers to the Airy beams, propagating as a weak light breathing [Fig. 5(c)].

As mentioned above, for smaller intervals $B = 1$ and $B = 2$, an in-phase Gaussian beam first appears as an interaction enhancer, then as an interaction switch in the interactions of in-phase Airy beams. We also numerically find that the thresholds of amplitude are $C_{th} = 0.32$ and $C_{th} = 1.38$ for $B = 1$ and $B = 2$, respectively. Below the thresholds, the attractions between Airy beams will be enhanced; whereas, above the thresholds, the original attractions will be suppressed and become a repulsive one. For a larger interval $B = 3$, situations become totally different. When $C < 0.8$, the attractions between Airy beams weakens, then enhances when $0.8 < C < 1.9$, and finally becomes a repulsive one when $C > 1.9$.

Stationary bound states of in-phase Airy beams may be obtained when the nonlocality of

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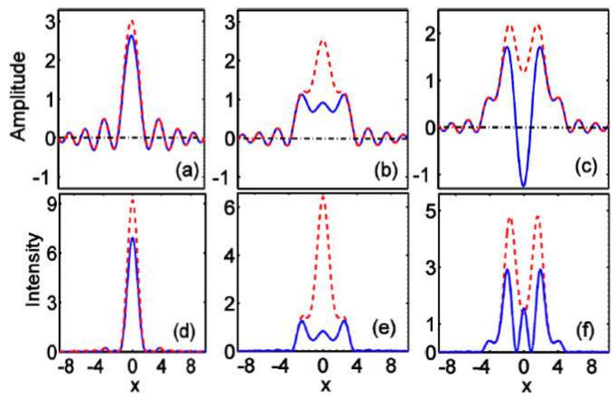

Fig. 6. Amplitudes and intensity of the in-phase input beam. The amplitudes are $A = 3$ for all the plots. All the solid lines represent $C = 0$, and dashed lines in (a,d), (b,e), and (c,f) represent $C = 0.4, 1.62, 2.41$, respectively. The beam intervals are: (a,d) $B = 1$, (b,e) $B = 2$, and (c,f) $B = 3$.

the nematic liquid crystal balances the repulsion, as shown in Figs. 5(d)-5(f). In fact, a stable breathing bound state comes from the balance between nonlocal, nonlinearity, diffraction, and repulsion effects [42]. Here, nonlocality provides a long range attractive force to balance the repulsive interaction between Airy beams [Figs. 5(a)-5(c)], resulting in the formation of bound state.

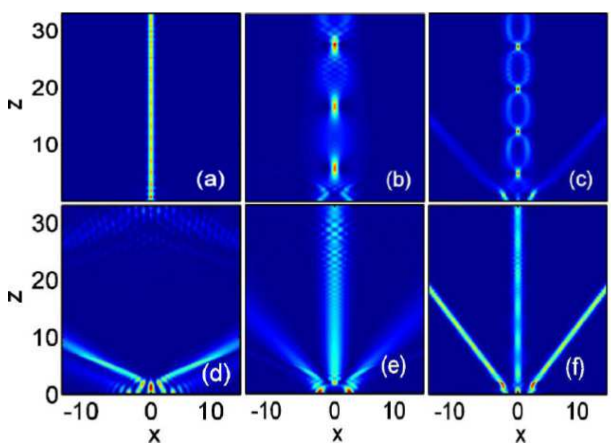

Fig. 7. Trajectories of out-of-phase interacting Airy beams and an Gaussian beam in nematic liquid crystals with the degree of nonlocality $\sigma = 0$ (local case). The amplitude is $A = 3$ in all the plots, and $C = 0.8, 0.3, 0.2, 2.4, 1.1, 0.7$ for (a-f), respectively. The beam intervals are: (a,d) $B = 1$, (b,e) $B = 2$, and (c,f) $B = 3$, respectively.

4. Anomalous interactions of Airy beams with out-of-phase fundamental Gaussian beam

For the out-of-phase interactions ($\rho = 1$), in the case of a weak Gaussian beam (C is much smaller than A), we show in Figs. 7(a)-7(c) the interactions for different beam intervals B in local media. The corresponding amplitude and the intensity of the input beam are demonstrated in Fig. (8). Compared Figs. 7(a)-7(c) with Figs. 1(a)-1(c), we can see that, in the case of out-

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of-phase situation, the interactions between Airy beams become weaker, leading to an increase both of the width and breathing period of bound states for smaller separations, such as $B = 1$ [Fig. 7(a)] and $B = 2$ [Fig. 7(b)]. However, for a larger beam separation $B = 3$ [Fig. 7(c)], surprisingly, the interaction between Airy beams strengthens with its breathing period of the bound state becomes smaller obviously. Similar to the in-phase case, an out-of-phase Gaussian beam also acts as an interaction switch for two in-phase Airy beams.

It has been shown that for solitons interacting with a π phase difference, the refractive index in the central region is lowered by their overlap. Therefore, the centroid of each soliton moves outward and the solitons repel to each other [43]. For smaller intervals $B = 1$ [Figs. 8(a) and 8(d)] and $B = 2$ [Figs. 8(b) and 8(e)], the out-of-phase fundamental beam decreases the amplitude and intensity in the center region between the main lobes of Airy beams, which leads to an decrease in the refractive index in this region. Then, the attraction between Airy beams is suppressed. Whereas, for a larger interval $B = 3$, the amplitude in the center of the region between the main lobes is negative when $C = 0$ [solid line in Fig. 8(c)]. When an out-of-phase Gaussian beam overlaps with the Airy beams, the absolute of amplitude in this region will increase [dashed and dotted lines in Fig. 8(c)], then the intensity [dashed and dotted lines in Fig. 8(f)] and the corresponding refractive index in this region will increase accordingly. Therefore, the attraction between the Airy beams will be strengthened [Fig. 7(c)]. When the amplitude of Gaussian beam increases as a strong light, we show in Figs. 7(d)-7(f) the interactions with different beam intervals B in local case. Similar as the in-phase case, the repulsion between the Airy beams appears for all the intervals.

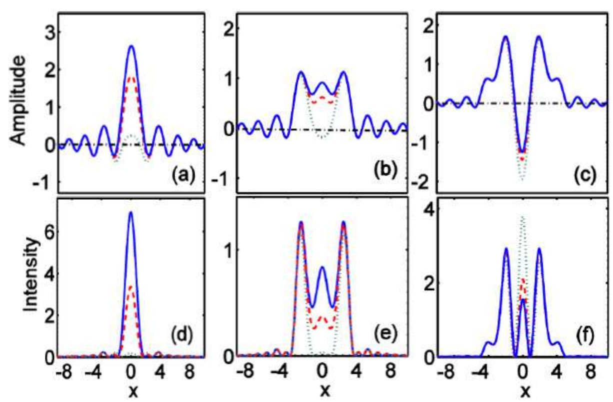

Fig. 8. Amplitudes and intensity of the out-of-phase input beam. The amplitude is $A = 3$ in all the plots. All the solid lines represent $C = 0$, dashed lines in (a,d), (b,e), and (c,f) represent $C = 0.8, 0.3, 0.2$, and dotted lines in (a,d), (b,e), and (c,f) represent $C = 2.4, 1.1$, and 0.7, respectively. The beam intervals are: (a,d) $B = 1$, (b,e) $B = 2$, and (c,f) $B = 3$.

Stationary out-of-phase bound states may be obtained with larger amplitudes of both Airy beams and fundamental beam in nonlocal nematic liquid crystals [42], as shown in Fig. (9). Here, we only consider the interaction of the Airy beams with the interval $B = 3$. The nonlocality can balance the repulsion [Fig. 9(a)], resulting in helping the formation of breathing bound state [Fig. 9(b)]. The intensities of bound breathing solitons at the input $z = 0$ and the output $z = 30$ are also displayed in Figs. 10(a) and 10(b), which indicates that part of power in Airy beams transfer to the Gaussian beam.

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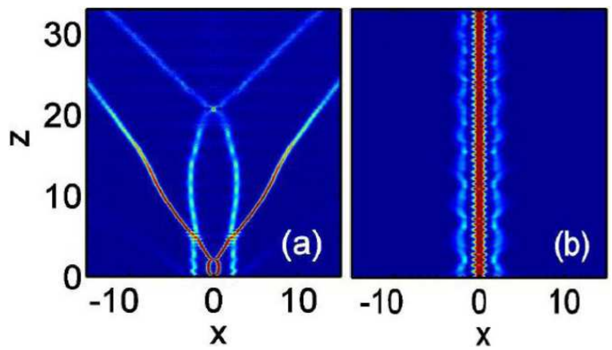

Fig. 9. Trajectories of out-of-phase interacting Airy beams and Gaussian beam in nematic liquid crystals. The degrees of nonlocality are $\sigma = 0$ (local case) for (a), and $\sigma = 0.8$ for (b). The amplitudes are $A = 4.7$ and $C = 3$; while the beam interval is $B = 3$.

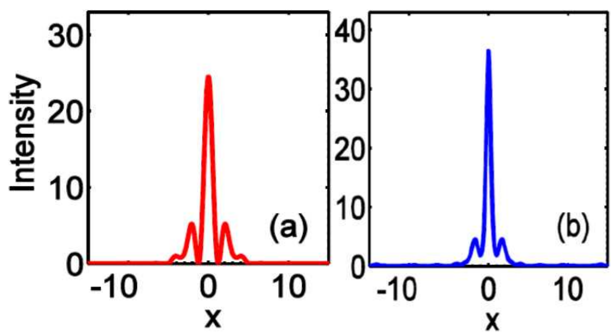

Fig. 10. The intensity distribution of Fig. 9(b) at the propagation distance $z = 0$ (a) and $z = 30$ (b).

5. Conclusions

In conclusion, we have demonstrated that the interactions between in-phase Airy beams can be flexibly controlled by a fundamental Gaussian beam. In particular, we show that the attractive interaction between Airy beams can be suppressed or become a repulsive one with an in-phase fundamental Gaussian beam, or be strengthened with an out-of-phase Gaussian beam. Stationary bound states of breathing solitons of in-phase as well as out-of-phase beams were also revealed with the help of nonlocality. We hope our theoretical results may motivate future experimental observations for the interactions between Airy beams in different physical settings with nonlocal nonlinearity, such as lead glass [38] and nematic liquid crystals [52]. Our results may have potential applications in optical interconnects, beam steering, and all-optical devices.

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