Few-cycle self-induced-transparency solitons

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We reveal the existence of an optical self-induced-transparency soliton family in a two-level absorbing system down to the few-cycle limit. Based on the few-cycle envelope approximation, we introduce a systematical method of reducing the Maxwell-Bloch equations beyond the slowly varying envelope approximation and characterize the parameter space to achieve slow and fast lights. Verified by direct numerical simulations with full vectorial Maxwell-Bloch equations, we also give the relationships for a number of optical cycles on area theory and pulse group velocity, which demonstrate possible coherent photon-matter interactions.

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I. INTRODUCTION

Self-induced-transparency (SIT) soliton phenomenon in two-level atomic systems is one of the most well-known coherent pulse propagation phenomena that have broaden our knowledge in nonlinear optics [1–3]. An incident optical field with a 2π pulse area can propagate in a shape-preserving way which totally inverts and returns the atomic population back corresponding to the integral of Rabi frequency over time [4]. The *area theorem* for SIT systems is a unique constant of motion in spite of changes in pulse length and group velocity. As a member of an optical soliton family, SIT solitons have also been suggested to be potentially capable of playing an important role in pulsed squeezed state generation [5,6], quantum nondemolition measurement [7], and quantum information storage and retrieval [8]. In contrast to the electromagnetically induced-transparency (EIT) phenomena that have been widely utilized for applications like slow lights and quantum memories [9], SIT phenomena are intrinsic nonlinear coherent pulse propagation effects that may have more advantages for short optical pulse applications.

With the recent advances in the production of ultrashort optical pulses of a few optical cycles, subfemtosecond and attosecond pulses are now available in several laboratories [10]. For this area of extreme nonlinear optics [11], the validity of area theorem is re-studied in the few-cycle regime both theoretically [12] and experimentally [13]. When the optical pulse is as short as a few optical cycles, new spectral components from carrier-wave Rabi flopping during pulse propagation are created. Cubic-polynominal featured population difference [12,14], spectral modification [15], subcycle solitary wave formation [16], single-cycle gap soliton [17], and the pulse area evolution [18] are also demonstrated by solving fully vectorial Maxwell-Bloch equations (MBE) through direct numerical simulations.

In addition to solving the fully vectorial Maxwell equations, a generalized few-cycle pulse propagation equation (FPE) was developed to go beyond the slowly varying envelope approximation (SVEA) [19,20]. With the consideration of a transparent medium, a shape-preserving few-cycle soliton propagation is shown to exist down to a single-cycle regime [20,21]. When a resonance excitation field drives in specific resonant absorption two-level media, Maxwell-Drude-Bloch equations and non-SVEA model are proposed to support few-cycle optical solitons [22,23]. For possible coherent light-matter interactions in the regime of extremely short time scale, the existence of *shape preserving* SIT solitons of the few-cycle duration is still unanswered.

In this work we investigate few-cycle optical solitons in a resonance absorbing medium which is modeled by a two-level atomic system. Our formulation, based on few-cycle nonlinear envelope approximation, differs considerably from earlier studies. Instead of the reduced sine-Gordon equation [24], we derive a cubic-quintic nonlinear differential equation from the modified MBE and reveal the possibility of generating shape-preserving few-cycle SIT soliton solutions. The parameter space in terms of atomic density and group velocity for bright solitons is identified from slow-light to superluminal regimes and stability of these solitons are discussed. The possibility of reducing threshold energy to form few-cycle SIT solitons is demonstrated with a wave vector offset.

II. DERIVATION OF MODIFIED MBE BEYOND SVEA

Considered are the dimensionless MBE describing the interaction between a two-level system and an ultrashort electromagnetic wave-packet E,

$$(\partial_z + \partial_t)E = (\partial_z - \partial_t)^{-1}\left(\frac{\Gamma}{2}\partial_t P\right), \tag{1}$$

$$\partial_t P = i\delta P + N E, \tag{2}$$

$$\partial_t N = -\tfrac{1}{2}(P E^* + P^* E). \tag{3}$$

Here P and N are the atomic transition dipole moment and related population difference normalized to total atomic density N_a, respectively. The speed of light is normalized to 1 and the polarizability and resonance detuning frequency are denoted by $\Gamma = N_a\mu^2/(\hbar\epsilon_0\omega_0)$ and $\delta \equiv \omega_0 - \omega_a$, with the dipole coupling coefficient μ, reduced Planck constant, $\hbar$, the vacuum permittivity ϵ_0, the central frequency for the field packet ω_0 and the atomic transition frequency ω_a. To go beyond SVEA we use the polarization $P = P(t,z)\exp(ikz - i\omega_0 t + i\phi_P)$ with the propagation constant expanded by $k = \sum_{m=0}^{m=\infty} k_m(\omega-\omega_0)^m/m!$ in which $k_m = (\partial_\omega)^m k|_{\omega=\omega_0}$ and ϕ_P is an arbitrary phase offset to the polarization. It yields that the right-hand side of Eq. (1) can be approximated by $\frac{\Gamma}{2}[1 + i(2-\sigma)\frac{\partial_t}{\omega_0} - (1-\sigma)^2\frac{\partial_t^2}{\omega_0^2} +$

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$\mathcal{O}(\frac{\partial_t^3}{\omega_0^3})]P(t,z)$ when $|\partial_z P(t,z)| \ll |k_0 P(t,z)|$ is assumed. The parameter σ which involves the ratio between the phase velocity to group velocity v_g for the nonlinear envelope of the wave packet at a propagation constant near k_0 is denoted by $(1+\omega_0 k_1/k_0)/2 = [1+(\omega_0/k_0)v_g^{-1}]/2$, which stands for the envelope correction terms in the FPE.

We shall address that the Bloch equations in Eqs. (2) and (3) under *dipole approximation* and *rotation wave approximation* [25] are valid when the interacting electric field remains constant over the atomic scale (a few angstroms), which is typically true even for a few-cycle pulse at the wavelength $\sim 1\ \mu$m as long as the electric field amplitude is moderate. Furthermore, the relaxation of the atomic state is also ignored since the the time scale associated with the interaction of resonance radiation (ultrashort pulse) is much shorter then the atomic relaxation time [4]. The Maxwell equation in Eq. (1) is based on the concept of few-cycle envelope equations [19,20], which is justified to be accurate down to a single-cycle pulse duration [26].

III. FEW-CYCLE SIT SOLITONS SOLUTIONS

In order to find the corresponding SIT soliton solution, which retains a constant pulse shape along the propagation direction z, we transform Eqs. (1)–(3) into a moving frame with $\xi = t - z/v_g$ and $\eta = z$. For convenience we use $E = Q\exp(i\Phi)$ and $P = (U+iV)\exp(i\Phi)$ in which the real functions Q, U, V are the amplitudes and Φ is the phase for the complex envelopes. To simplify, the transverse Laplacian and group velocity dispersion are neglected here and and the resonance condition $\delta = 0$ is assumed. In the following only results up to the first-order correction in σ are shown in order to give an explicit mathematical form and a clear physical picture. Similar results can be found by using the same method below for higher order corrections and off-resonance condition. Furthermore, we also assume and apply in calculations that the density of an atom is small and the phase velocity of the optical pulse approaches the speed of light.

The conservation of Bloch vectors implies $U^2+V^2+N^2=1$ [27], which together with the constraint $\partial_\eta Q = 0$ for solitary waves leads to an identity, that is, $[N+(1-{v_g}^{-1})Q^2/\Gamma] = \pm 1 \equiv N_0$. Under these assumptions we can further argue that $\partial_\eta U = \partial_\eta V = \partial_\eta N = 0$ for a shape-preserving SIT soliton solution. Then we assume $\partial_\eta \Phi \equiv \beta$ by implying a change of the propagation constant β due to the field-atom interaction [28]. As a result, the equation of motion for the phase function can be found:

$$\frac{\partial\Phi}{\partial\xi} = -\frac{3}{8}(2-\sigma)Q^2 - \frac{\beta}{2(1-{v_g}^{-1})} + \frac{N_0\Gamma(2-\sigma)}{4(1-{v_g}^{-1})}. \qquad (4)$$

Subsequently, by plugging Eq. (4) into the modified MBE in Eqs. (1)–(3), one readily derives a *cubic-quint* (CQ) nonlinear ordinary differential equation for the SIT soliton envelope function in the few-cycle regime,

$$\frac{\partial^2 Q}{\partial\xi^2} = \left[\frac{N_0\Gamma}{2(1-{v_g}^{-1})} - \Delta^2\right]Q - \left[\frac{1-(2-\sigma)\Delta}{2}\right]Q^3 - \left[\frac{3(2-\sigma)^2}{64}\right]Q^5, \qquad (5)$$

where we introduce a new offset parameter for the propagation constant $\Delta \equiv -\frac{\beta}{2(1-{v_g}^{-1})} + \frac{N_0\Gamma(2-\sigma)}{4(1-{v_g}^{-1})}$. The exact bright soliton solution to Eq. (5) subjected to the condition $Q(\pm\infty) = 0$ is obtained by the well-known form [29]

$$Q = \left\{\frac{\tau_p^2}{2}|1-(2-\sigma)\Delta|\left[A_1\cosh\left(\frac{\xi}{\tau_p}\right) \pm 1\right]\right\}^{-1/2}, \qquad (6)$$

where $\tau_p = [2N_0\Gamma/(1-v_g^{-1}) - 4\Delta^2]^{-1/2}$ and $A_1 = [\frac{(2-\sigma)^2}{4\tau_p^2(1-(2-\sigma)\Delta)^2} + 1]^{1/2}$ are real quantities and the sign is chosen for the case of a negative (+) or positive (−) cubic nonlinearity. The corresponding pulse duration (full width half maximum of the envelope function) w_Q, defined in terms of number of optical cycles and the pulse energy $\mathbb{E}$, are calculated with the expressions $\mathsf{w}_Q = 2[\frac{2N_0\Gamma}{(1-{v_g}^{-1})} - 4\Delta^2]^{-1/2}\cosh^{-1}[\frac{\pm 3}{A_1} + 4]$ and $\mathbb{E} = \int_{-\infty}^{\infty} Q^2 d\xi = \frac{16}{|2-\sigma|}\tan^{-1}[(\frac{A_1-1}{A_1+1})^{\pm 1/2}]$, respectively. The induced Bloch vector components which satisfy the conservation of Bloch vectors are obtained by substituting the solution Eqs. (4) and (6) back into the modified MBE in Eqs. (1)–(3).

It is seen in Eq. (5) that the sign of quintic nonlinearity is negative while the sign of cubic nonlinearity alters by varying the polarizability Γ, the group velocity v_g, and the change of propagation constant β, accordingly. The fact that the sign of cubic nonlinearity alters despite negative quintic nonlinearity suggests that our few-cycle SIT soliton family can be categorized by the set of parameters defined by Γ, σ, and β in our system. When $\sigma = 2$, Eq. (6) accurately replicates the chirping-free 2π pulse solutions for a sine-Gorden equation transformed from standard many-cycle MBE [2,27]. It is worth mentioning that the related temporal phase variation for these few-cycle SIT solitons introduces a frequency chirping that is quadratic to electric field amplitude and is given by the expression in Eq. (4). Such a chirping signature depends on the group velocity of the optical pulse (vanishes as $\sigma = 2$) and is very specific for admitting the few-cycle envelope approximation with correction terms.

IV. DISCUSSIONS

First we study SIT soliton in the case $N_0 = -1$ by assuming that all the atoms are initially prepared in the ground state before the emergence of an optical pulse. For τ_p to be real, it is required that $0 < v_1^{(c)} < v_g < v_2^{(c)} < 1$, where $v_1^{(c)}$ and $v_2^{(c)}$ are the cutoff velocities of solitary wave solutions. When the wave packet has the same propagation constant k with respect to the carrier, that is, ($\beta = 0$), in Fig. 1 we demonstrate an example of these few-cycle SIT soliton profiles (a) and its induced atomic Bloch vectors (b), accordingly. Comparisons are illustrated in Figs. 1(c) and 1(d) for SIT soliton solution with and without the envelope correction terms by using finite-difference-time-domain (FDTD) simulation upon the full vectorial Maxwell-Bloch equations [14]. In the absence of the correction term, a severe pulse distortion in the electrical field is observed in addition to the known cubic-polynomial featured population difference [12,14] when the soliton duration goes below 10 optical cycles. Yet employing only the first-order correction, the SIT soliton solutions found in Eq. (6) remain shape

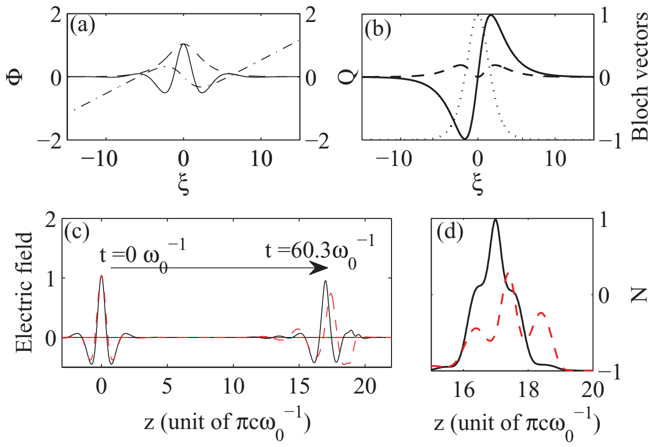

FIG. 1. (Color online) (a) Example of few-cycle SIT soliton solutions for their electrical field amplitude with (solid lines) and without (dashed lines) the carrier and related temporal phase (dashed-dotted lines) in the parameter space marked by A in Fig. 2. (b) The corresponding U, V, and N of Bloch vectors are shown in solid, dashed, and dashed-dotted lines, respectively. (c) FDTD simulations for SIT solitons with (in black) and without (in red) correction terms for which the associate population differences are shown in (d). The parameter used are $\beta = 0$, $\Gamma = 0.06$, and $v_g = 0.9$.

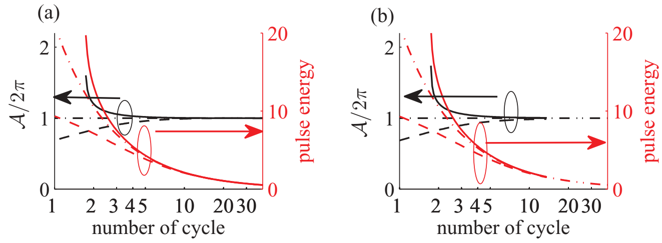

FIG. 3. (Color online) Pulse energy-duration (in red) and pulse area-duration (in black) relationship for slow-light (a) and fast-light (b) few-cycle SIT solitons under the conditions $\beta = 0$ and $\Delta = 0$ are shown in solid and dashed lines, respectively. The solid-dashed line refers to pulse energy duration for SIT solitons under SVEA. The parameters $\Gamma = 0.05$ is used and $N_0 = -1$ and $N_0 = +1$ are used for slow- and fast-light regimes, respectively.

preserved under the fully vectorial propagation environment, down to a pulse duration as short as five optical cycles.

The relationship for the pulse duration of supported few-cycle SIT solitons to the group velocity v_g and the polarizability Γ is plotted in Fig. 2 (black curves) and the dependence of pulse area defined by $\mathcal{A} = \int_{-\infty}^{\infty} Q d\xi$ is plotted in Fig. 2 (red curves) as well. For these SIT solitons their pulse areas exceed 2π and diverge when the group velocity approaches the two cutoffs. Solitons propagating at a velocity near $v_c^{(2)}$ has a diverging pulse area due to the strong quintic nonlinearity as a result of a narrow and intense envelope. Although ultraslow solitons (dashed) at velocity close to $v_c^{(1)}$ have a broad envelope but a divergent quintic nonlinear coefficient, the result is conspicuous because it might fall out of the regions of convergence to admit corrections to the first order. In particular, within this model we confirm the breakdown of area theory near the few-cycle regime and predict its invalidity for a SIT soliton.

We note that the pulse duration of these few-cycle SIT solitons is broadened owing to a nonvanishing offset parameter for the propagation constant Δ. Set $\Delta = 0$ with a change of the propagation constant by choosing $\beta = N_0\Gamma(2-\sigma)/2$, then one obtains a few-cycle SIT soliton solution with a minimal pulse duration at a fixed group velocity. The pulse area of these solitons is found to be less than 2π and their formation energy $\mathbb{E}$ is also lower than those revealed above. In Fig. 3(a) we show the pulse energy-duration relation for the case without a change of propagation constant $\beta = 0$, with a suitable change of propagation constant $\Delta = 0$ and without a correction term. Suppose that $\Delta = 0$, the group velocity cutoffs extend to zero and to the speed of light; meanwhile the pulse duration also goes down with a reduced formation energy.

Besides SIT solitons of slow group velocity, light matter interaction can support other types of solitary waves. In the *inverted media*, where $N_0 = +1$, bright few-cycle SIT solitons are found provided the condition $(1 - v_g{}^{-1}) > 1$ which indicates an exotic propagation effect with $v_g > 1$, namely *superluminal* (fast light) [30,31]. In Fig. 4 we show examples of fast-light SIT soliton given $\Gamma = 0.025$ and

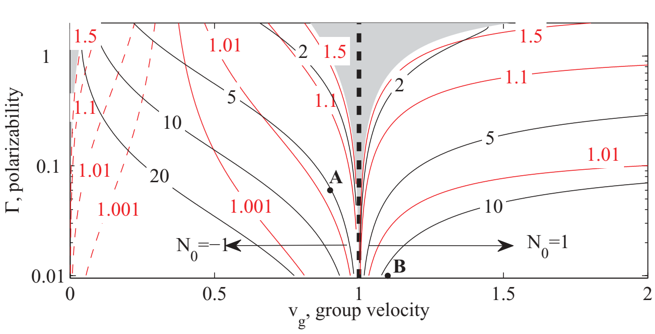

FIG. 2. (Color online) The contour plots for the relationship of pulse duration (in black) and pulse area (in red) (normalized to 2π) for the supported few-cycle SIT solitons in the parameter space defined by the group velocity v_g and the polarizability Γ with the condition $\beta = 0$. The shaded regions define the group velocity cutoffs calculated by the required soliton criterion. To the left of the dashed line $N_0 = -1$ and $v_g < 1$ and to the right $N_0 = 1$ and $v_g > 1$.

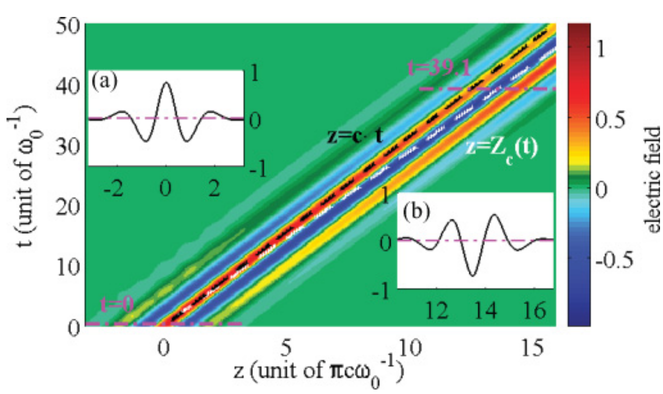

FIG. 4. (Color online) FDTD simulation of a fast-light SIT soliton solution in the parameter space marked by B in Fig. 2. Insets show the electrical field profile at (a) $t = 0$ and (b) $t = 39.1$ (unit of ω_0^{-1}), respectively. Center of mass trajectory (in white) of the soliton and velocity of light (in black) are shown as dashed lines.

$v_g = 1.1$ and demonstrate its superluminal propagation in FDTD simulations by evaluating the the center of mass trajectory of the soliton, defined by $Z_c(t) = (\int_{-\infty}^{\infty} z|E|^2 dz)/(\int_{-\infty}^{\infty} |E|^2 dz)$.

The parameter space for the pulse area and pulse duration of these fast-light SIT solitons without a change of propagation constant ($\beta = 0$) are also plotted in Fig. 2. Similar to the slow-light SIT solitons, their pulse areas exceed 2π and diverge when the group velocity approaches the cutoff described by the shaded area to the right of the dashed line in Fig. 2. The breakdown of area theory near the few-cycle regime is also valid in the superluminal regime. In Fig. 3(b) we also show the pulse area-duration relations and energy-duration relations for the fast-light solitons with and without a change of propagation constant and without correction term, which reveals very similar behavior to those explored in slow-light solitons discussed above.

SIT soliton is proved to be stable against perturbations [27]. Even though the stability of bright solitons in the CQ nonlinear equations has been studied for various combinations of cubic and quintic nonlinearities [32]. In this work the CQ model for SIT soliton is valid with an additional constraint imposed by the conservation of Bloch vectors. To investigate the instability property of these SIT solitons we perform a linear stability scheme numerically with eigenvalue solvers to Eqs. (1)–(3) at the moving frame (ξ,η) not only for the wave solution obtained in Eq. (5) but also for the induced Bloch vector components accordingly. It shows that in the entire spectrum the eigenvalues of perturbed modes are real and therefore these few-cycle SIT solitons are stable provided the initial atomic state is stable. Yet for the fast-light solitons in the inverted media, they are unstable due to the spontaneous instability [31], which is not considered in this paper.

V. CONCLUSIONS

To summarize, by deriving from the modified Maxwell-Bloch equations beyond SVEA we introduce a systematic approach for the studies on the few-cycle SIT solitons. Some families of pulse-area tunable few-cycle SIT optical soliton are categorized in slow-light and fast-light regimes, and verified with FDTD simulations upon the full vectorial Maxwell-Bloch equations. Our work reveals the existence of few-cycle SIT solitons with a pulse area smaller than 2π and the possibility of reducing the formation energy for SIT solitons by choosing a wave vector offset. Supported by the results of this work, it is practically realizable to excite and manipulate few-cycle SIT solitons with a controllable group velocity, pulse energy, and pulse area by a proper adjustment of the wave number offset and temporal chirping behavior. Combining with recent successful synthesis of arbitrary wave forms in the subfemtosecond regimes [33], the prediction in this work paves the way to bring coherent pulse phenomena into the field of extreme nonlinear optics.

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