Instability suppression of clusters of vector-necklace-ring solitons in nonlocal media

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We study the instability suppression of vector-necklace-ring soliton clusters carrying zero, integer, and fractional angular momentums in nonlocal nonlinear media with an arbitrary degree of nonlocality. We show that the combination of nonlocality and mutual trapping of soliton constituent components can completely stabilize the vector-necklace-ring soliton clusters which are otherwise only quasistable in local media. Our results may be useful to studies of the novel soliton states in Bose-Einstein with dipolar long-range interactions.

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I. INTRODUCTION

Recently, propagation and interactions of optical solitons in spatially nonlocal media [1,2] have drawn considerable attention due to their potential applications in optical beam steering and all-optical networks [3,4]. Moreover, the nonlocality appears to play a significant role in the formation of Bose-Einstein condensates with long-range dipolar interaction [5,6]. The nonlocality has profound impacts on soliton dynamics [7] and can support a range of novel solitonic structures, such as discrete solitons [8,9], azimuthons [10], and vortex solitons [11–13]. Stability enhancement of localized structures including fundamental, vortex, and rotating solitons in Bose-Einstein condensates with a nonlocal interaction has already been investigated in a number of works [14,15]. Vector solitons, a localized wave structure consisting of few incoherently coupled components, have also been investigated in nonlocal media [16,17]. It has been also shown that nonlocality plays an important role in the stabilization of vector solitons for both one [18–20] and multidimensional beams [21].

Necklace solitons, azimuthally modulated beams with ringlike configurations, were shown to expand with propagation in nonlinear media due to the repulsive force between the neighboring bright "petals" [22,23]. Their dynamics depend crucially on the nonlinearity of the media [24–26]. However, it was shown that vector interactions (i.e., cross-phase modulations) can drastically affect the propagation of necklace-ring soliton clusters [27]. In nonlocal media only scalar necklacelike solitons have been studied until now [10,13,28,29].

In this paper we study the dynamics of vector-necklace-ring soliton clusters carrying zero, integer, and even fractional angular momentum in nonlocal media with an arbitrary degree of nonlocality. We use the variational approach to derive analytical formulas for the vector-necklace-ring soliton clusters and analyze their stabilities by direct numerical simulations. We show that the combination of nonlocality and vector interactions (i.e., mutual trapping of constituent components of vector beam) can completely stabilize the necklace-ring soliton clusters which are otherwise only quasistable in local media.

II. BASIC MODEL AND VARIATIONAL APPROACH

We consider a vector soliton consisting of N mutually incoherent optical components propagating in a nonlocal nonlinear medium. In cylindrical coordinates the propagation for slowly varying beam envelopes $E_n(x,y,z)$ can be written in the form of following normalized coupled nonlocal nonlinear Schrodinger equations ($n = 1,2,\ldots,N$) [27,30],

$$i\frac{\partial E_n}{\partial z} + \frac{1}{r}\frac{\partial E_n}{\partial r} + \frac{\partial^2 E_n}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 E_n}{\partial \varphi^2} + E_n\delta n(I) = 0, \quad (1)$$

where $I = \sum |E_n|^2$, $r = \sqrt{x^2 + y^2}$, $\varphi = \tan^{-1}(y/x)$, and $\delta n(I) = \int R(\mathbf{r} - \mathbf{r}')I(\mathbf{r}',z)d^2\mathbf{r}'$ represents the nonlocal nonlinear refractive index change. $R(\mathbf{r})$ is so-called nonlocal response function whose width determines the degree of nonlocality. In particular, $R(\mathbf{r}) = \delta(\mathbf{r})$ corresponds to the local Kerr medium. We seek the solution of Eq. (1) in the form of an incoherent superposition of the necklace-type components [27],

$$E_n = U(r)\Phi_n(\varphi)e^{ikz}, \quad (2)$$

satisfying the self-consistency condition,

$$\sum_n |\Phi_n(\varphi)|^2 = 1. \quad (3)$$

Here k denotes the propagation constant. The total intensity of the nonlocal vector solitons is $I = |U(r)|^2$, which depends on the radial coordinate only. Taking into account the condition in Eq. (3), $\Phi_n(\varphi)$ can be expressed as

$$\Phi_n(\varphi) = a_n\cos(m\varphi) + b_n\sin(m\varphi), \quad (4)$$

where m (integer) is the topological charge, a_n and b_n are the complex coefficients satisfying the conditions $\sum_n \mathrm{Re}\{a_n b_n^*\} = 0$ and $\sum_n |a_n|^2 = \sum_n |b_n|^2 = 1$ which define the exact solutions of the nonlocal nonlinear system for any N [27]. In this paper, for convenience and without loss of generality, we consider the two-components model ($N = 2$) with equal powers in each component. Then the complex coefficients a_j and b_j (j = 1,2) satisfy the following relation:

$$a_1 = (1 + p^2)^{-1/2}, \quad b_1 = ipa_1, \quad (5)$$

and

$$a_2 = pa_1, \quad b_2 = \pm i a_1, \tag{6}$$

where $0 \leqslant p \leqslant 1$ is a real parameter [27].

The important characteristic of the vector solitons with nontrivial phase structure is its total angular momentum [23], which can be represented as

$$M = \sum_n M_n = P^{(m)} \frac{m}{2} \sum_n \{a_n^* b_n - a_n b_n^*\}, \tag{7}$$

where $P^{(m)} = 2\pi \int_0^\infty U^2 r dr$ is the power of a scalar vortex with the topological charge m. The ratio of the total angular momentum and the total power, M/P_{tot}, can be regarded as an analog of spin for an optical beam, where

$$P_{\text{tot}} = \sum_n P_n = 1/2 \sum_n (|a_n| + |b_n|) P^{(m)} = P^{(m)}. \tag{8}$$

The value of the spin depends crucially on the parameter p. The spin is zero for $p = 0$ and can be zero or integer when $p = 1$. Moreover, a fractional number of spin exists when $0 < p < 1$ [27].

We first seek analytically the approximated solution of the vector soliton clusters using the variational approach [31]. It is easy to show that the Lagrangian density corresponding to Eq. (1) can be represented as [30]

$$\begin{aligned}\mathcal{L} = \sum_{n=1,2} & \frac{i}{2} r \left(E_n^* \frac{\partial E_n}{\partial z} - E_n \frac{\partial E_n^*}{\partial z} \right) \\ & - r \left(\left| \frac{\partial E_n}{\partial r} \right|^2 + \frac{1}{r^2} \left| \frac{\partial E_n}{\partial \varphi} \right|^2 \right) \\ & + \frac{1}{2} r |E_n|^2 \int R(\mathbf{r} - \mathbf{r}') I(\mathbf{r}', z) d^2\mathbf{r}'. \end{aligned} \tag{9}$$

In subsequent analysis we will assume a Gaussian form of the nonlocal response function $R(r) = (\pi\sigma_0^2)^{-1} \exp(-r^2/\sigma_0^2)$ [13] and an ansatz of a ringlike vortex form for the amplitude of the vector solitons $U(r) = Ar^m \exp\left(-r^2/2\sigma^2\right)$ [30], where σ_0 and σ are the degree of nonlocality and the beam width, respectively. By calculating the effective Lagrangian $L = \int_{-\infty}^{\infty} \mathcal{L} dx dy$ and using the Euler-Lagrange equations [32], we find the amplitude of the vector-necklace solitons as a function of the parameters σ_0 and σ. We then employ this variational solutions as an initial condition to investigate the soliton dynamics by numerically integrating the nonlocal Schrödinger equation Eq. (1) using the split step beam propagation method. The simple solution with $m = 1$ and $p = 0$, represents incoherent superposition of two dipole solitons, whose evolution dynamics was discussed in our previous work [30]. In this work we are interested in the potential instability suppression of vector-necklace-ring soliton clusters consisting of a large number of petals. Therefore we consider here the case of $m = 6$ although our results are applicable to an arbitrary value of m.

III. NUMERICAL RESULTS AND DISCUSSION

Results of our numerical simulations are presented in Figs. 1–7. In these plots the label "scalar" refers to the spatial intensity distribution of the single-component beam or soliton. In case of vector soliton the labels $|E_1|^2 + |E_2|^2$, $|E_1|^2$, and $|E_2|^2$ denote its total intensity distribution as well as those of its constituent components, respectively.

In Figs. 1–3 we demonstrate the instability suppression of vector-necklace-ring soliton clusters with zero total angular momentum ($p = 0$) in nonlocal media with various degrees of the nonlocality (i.e., $\sigma_0 = 0.7$, 1.8, and 10, respectively). In these graphs we plot the spatial intensity distribution of the corresponding scalar and vector beams for a comparison. In all of our simulations, we set the initial beam width at $\sigma = 1$. The dynamics of the vector-necklace solitons is also compared with the expansion of a scalar-necklace beam with 12 lobes and the breakup of a scalar vortex. It is obvious that the nonlocality can improve the stability of both the vector solitons and the scalar beams. However, it will not prevent expansion of the scalar-necklace beam, if the degree of nonlocality is too weak, as shown in Figs. 1(a) and 2(a), and it will ensure stability only in the strongly nonlocal regime, as shown in Fig. 3(a). For instance, the scalar vortex beam breaks up into eight filament beams for a weak nonlocality with $\sigma_0 = 0.7$ in Fig. 1(b). This dynamics is similar to that of local nonlinear media [27]. When the degree of nonlocality (σ_0) increases, the stability of the vortex beam is enhanced as shown in Fig. 2 for the case of $\sigma_0 = 1.8$. In Fig. 2(b), the vortex only breaks up into four filament beams with a moderate nonlocality at the

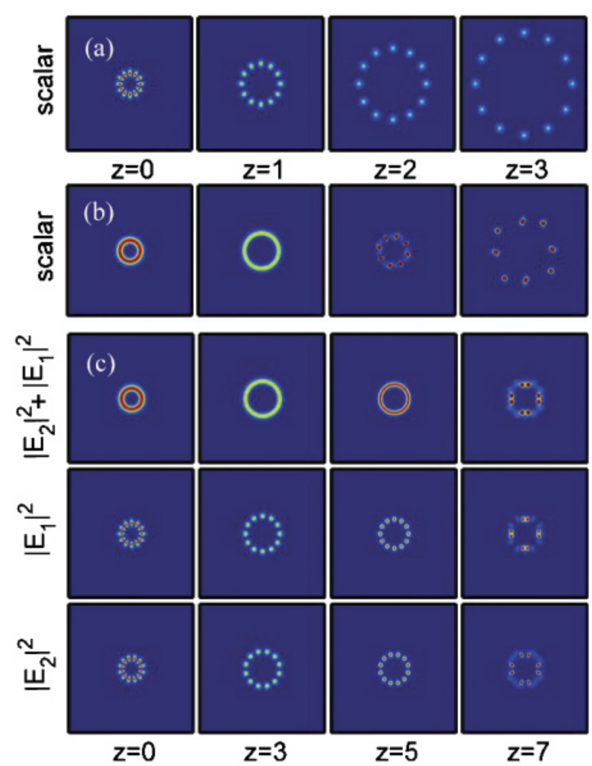

FIG. 1. (Color online) Dynamics and symmetry-breaking instability of (a) a scalar-necklace-ring soliton $E(z=0) = U(r)\cos(6\varphi)$, (b) a scalar vortex $E(z=0) = U(r)\exp(i6\varphi)$, and (c) a vector-necklace-ring soliton in the weakly nolocal regime ($\sigma_0 = 0.7$). The plots labeled "scalar" depict the spatial intensity distribution of the single-component beam; labels $|E_1|^2 + |E_2|^2$, $|E_1|^2$, and $|E_2|^2$ denote the total intensity distribution of the vector beam and those of its constituent components, respectively.

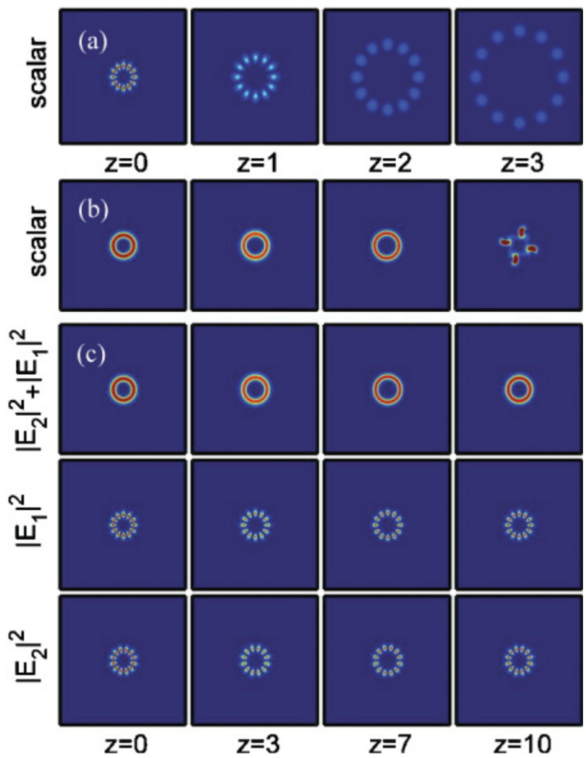

FIG. 2. (Color online) Dynamics of the scalar beams and vector-necklace-ring solitons for the degree of nonlocality $\sigma_0 = 1.8$ with a zero angular momentum ($p = 0$). (a) A scalar-necklace-ring $E(z = 0) = U(r)\cos(6\varphi)$, (b) a scalar vortex $E(z = 0) = U(r)\exp(i6\varphi)$, and (c) a vector-necklace-ring soliton defined by Eqs. (5) and (6).

propagation distance $z = 10$, however, it will break up earlier for a shorter propagation distance, $z = 7$, when the nonlocality is weak $\sigma_0 = 0.7$.

These results confirm the well-known fact that the nonlocality can effectively suppress the azimuthally instability of a vortex. We also find that the vortex is more stable than the necklace beam during the propagation when the nonlocality is weak or moderate by comparing the results shown in both Figs. 1 and 2. Figure 2 also shows that the vector-necklace-ring solitons is stable at the propagation distance $z = 10$, with a moderate degree of nonlocality of $\sigma_0 = 1.8$. On the other hand, the scalar beams are still unstable at the same propagation distance.

Compared to scalar beams, the above results show that the mutual self-trapping of the vector components leads to an enhanced stability for the vector solitons. This property is also seen in a weakly nonlocal case, as depicted in Fig. 1(c). Our simulations also clearly show that the stability of the vector-necklace beams increases with the degree of nonlocality. For examples, the vector beam only breaks up into eight filament beams after a propagation distance $z = 7$ for $\sigma_0 = 0.7$ Fig. 1(c); it remains stable at the propagation distance $z = 10$ in the regime of moderate nonlocality $\sigma_0 = 1.8$ [Fig. 2(c)].

As shown in Fig. 3(d), the strong nonlocality can average out all spatial variations of the beam intensity distribution, leading to the peak of nonlinear refractive index in the center even though there is a singularity in the center of the vector beam. Thus the strong nonlocality induces an effective attractive potential [30], which can completely suppress the repulsion between the neighbor beam "petals" with a π phase flip of the scalar beam [22,27] and prevent the breakup of vector beams, leading to the formation of completely stationary vector soliton clusters, Fig. 3(a)–3(c).

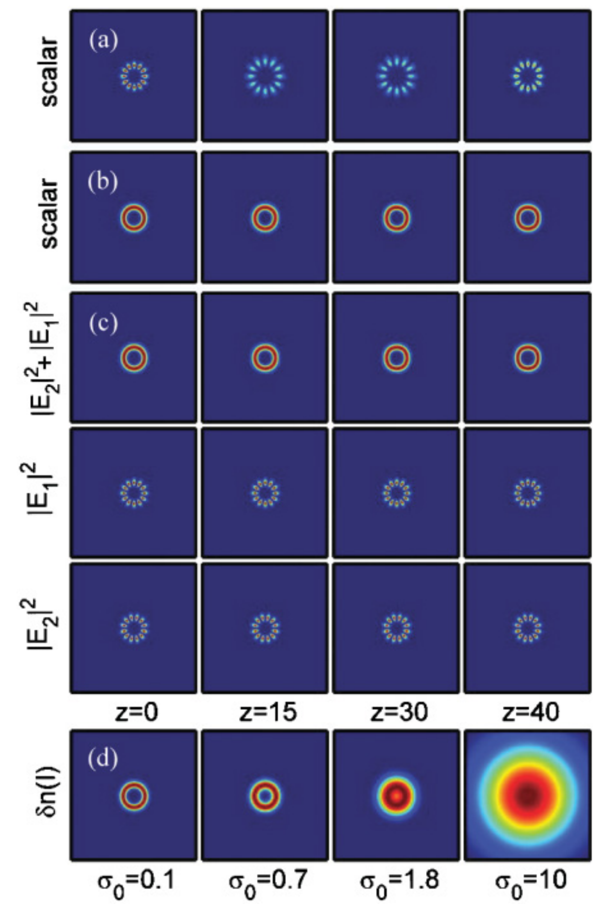

FIG. 3. (Color online) Dynamics and evolution of (a) a scalar-necklace-ring, (b) a scalar vortex, and (c) a vector-necklace soliton with a zero total angular momentum ($p = 0$) in the high nonlocality regime $\sigma_0 = 10$. The initial fields are the same as those in Figs. 1 and 2. The nonlinearity-induced refractive index changes for different values of the degree of nonlocality are shown in (d). Notice that nonlocality averages out the light intensity modulation.

To illustrate the stabilizing effect of nonlocality on solitons in Fig. 4 we plot the stationary propagation distance for both, a scalar vortex soliton with $m = 6$ and a vector soliton (with $p = 0$) as a function of degree of nonlocality. It is evident that the stationary propagation distance increases with a larger nonlocality, resulting in the stabilization of solitons. In addition, it is also clear that the stability of vector solitons is enhanced as a comparison to their scalar counterparts. We also simulated numerically the propagation of the solitons with different topological charges m and found that the stability decreases with a higher value of m. This follows the well-known trend discussed in earlier works on vortex solitons where it has been established that solitons with a higher angular momentum (m) breaks down much faster along propagation.

It should be stressed that even though we only consider here vortex structures with a particular value of charge $m = 6$, our results are applicable to an arbitrary value of m. As an

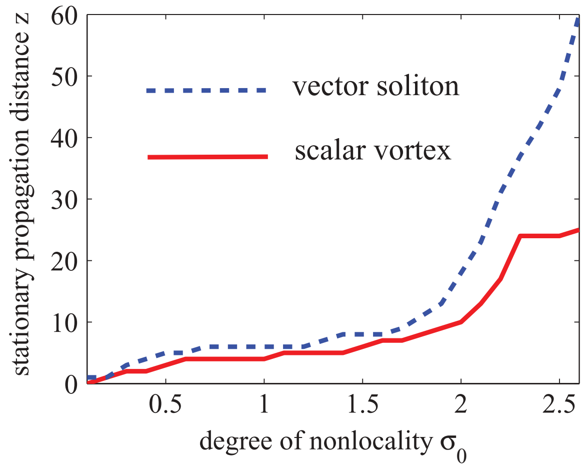

FIG. 4. (Color online) Stationary propagation distance of scalar vortex solitons ($m = 6$) and the vector soliton ($p = 0$) versus degree of nonlocality. The increasing of the stationary propagation distance indicates a better stability.

example, in Fig. 5, we demonstrate the stationary vector-necklace solitons with a topological charge $m = 8$.

Now, we consider the dynamics of beams with a nonzero angular momentum. When $p \neq 0$, Eq. (6) describes two types of localized solutions corresponding to two different signs of b_2. While these two solutions have the same spatial intensity distribution and power, they differ in the value of the total angular momentum [23,27]. One of these solutions ($b_2 = -ia_1$) has a zero total angular momentum, while the other solution ($b_2 = ia_1$) is nonzero. Moreover, the total spin of the latter one can even be a fractional number [27]. Figures 6 and 7 demonstrate the dramatically different dynamics of such two solutions when $p = 0.75$. In this case, we have the total spins as zero and 5.76 [27].

It is obvious that, similarly to the local case [27], the vector-necklace beam with a zero total spin is more stable than that with a fractal spin during propagation for both weak and moderate nonlocalities. The zero spin vector beam is stable up to a propagation distance $z = 5$ in case of a weak nonlocality and up to the propagation distance $z = 10$ for a moderate nonlocality. However, at the same propagation distances, the vector-necklace beam with a fractional total spin has already broken up into eight and four filament beams, respectively.

We also find that the dynamics of vector-necklace solitons with a fractional spin is very similar to the evolution of a scalar vortex with $p = 0$ [see, e.g., Figs. 6(a) and 1(b)]. However, the dynamics of solutions with a zero total spin follows a totally different scenario, with behaviors similar to that of a vector-necklace beam with $p = 0$, [see Figs. 6(b) and 1(c)]. It should be noted that stationary vector-necklace beams with zero or fractional total spins can also be realized in strongly nonlocal media, as shown in Fig. 7. One can almost ensure the full stability of solitons in the nonlocal nonlinear system by the fact that in the highly nonlocal limit the nonlocal nonlinear term which describes the nonlinearity of the medium becomes a linear one, which is independent of the light intensity distribution. Then the soliton behaves as a localized structure propagating within the external

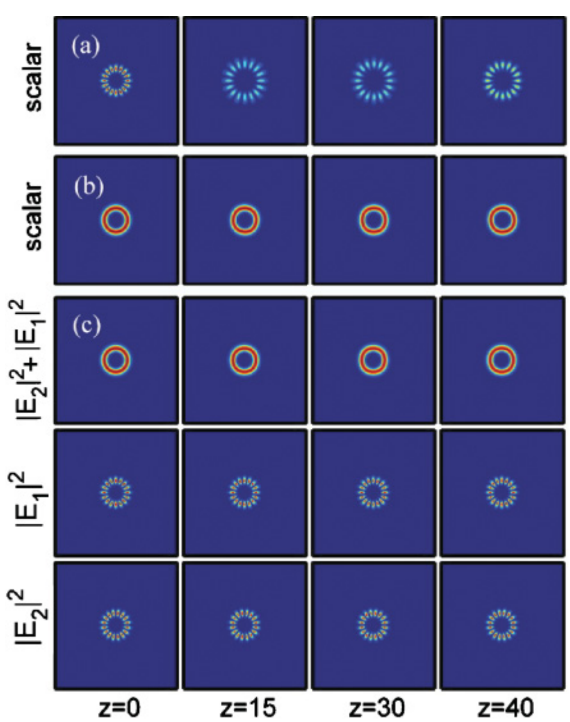

FIG. 5. (Color online) Dynamics and evolution of (a) a scalar-necklace ring, (b) a scalar vortex, and (c) a vector-necklace-ring soliton with zero total angular momentum ($p = 0$) in the high nonlocality regime $\sigma_0 = 10$. The topological charge is $m = 8$.

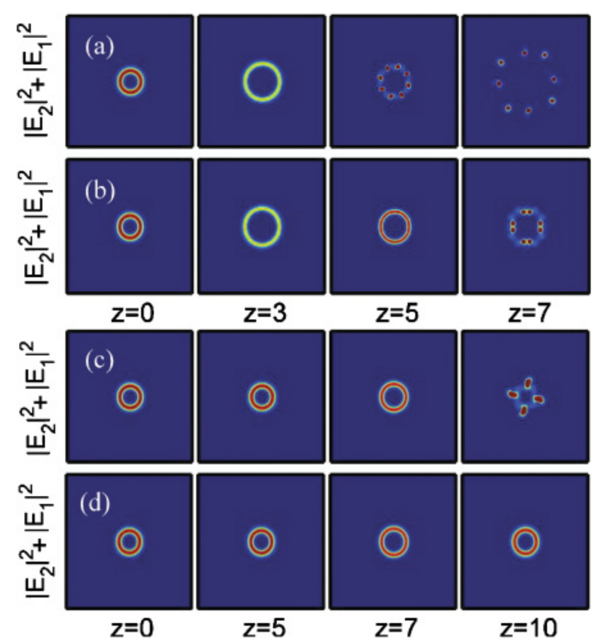

FIG. 6. (Color online) Evolutions of vector-necklace-ring solitons (only the total intensity distributions are shown) with a nonzero spin in each component ($p = 0.75$) when the nonlocality is weak ($\sigma_0 = 0.7$) [(a) and (b)] and moderate ($\sigma_0 = 1.8$) [(c) and (d)]. Graphs (a),(c) and (b),(d) depict intensity distribution of the vector-necklace rings with nonzero ($M = 5.76$) and zero total angular momentum, respectively.

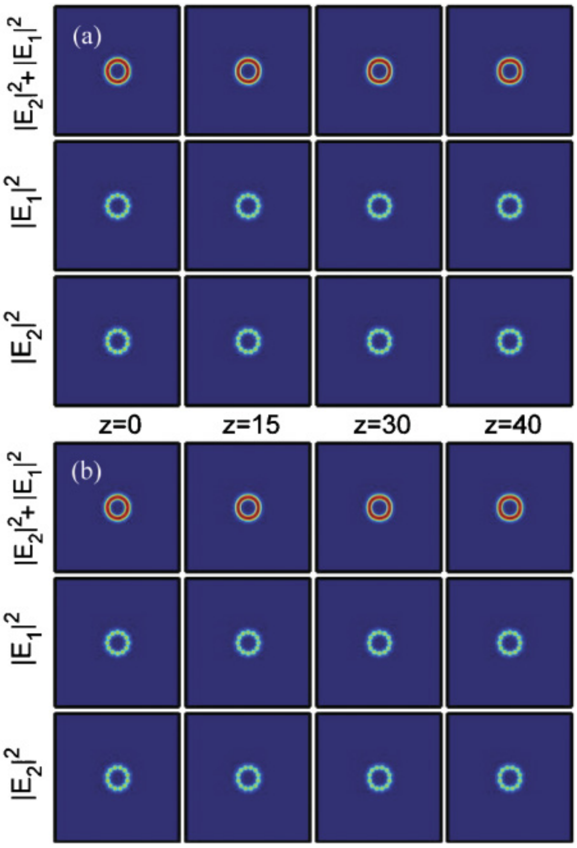

FIG. 7. (Color online) Stable propagation of vector-necklace solitons with a nonzero spin in each component ($p = 0.75$) in a strongly nonlocal regime ($\sigma_0 = 10$). The total values of spin for (a) and (b) are nonzero ($M = 5.76$) and zero, respectively.

waveguide with the spatial profile of the nonlocal response function [11].

For the case of $p = 1$, the solutions for Eqs. (5) and (6) represent a two-component vortex ring with a zero or integer total angular momentum [27]. We numerically investigated their evolution (not shown) and find that their dynamics is similar to that of the vector-necklace beam with a zero or fractional total spin (i.e., as shown in Figs. 6 and 7).

IV. CONCLUSION

In conclusion, we have investigated analytically and numerically the instability suppression of vector-necklace-ring soliton clusters carrying zero, integer, and even fractional angular momentums in nonlocal media with an arbitrary degree of nonlocality. We show that the combination of nonlocality and mutual trapping of constituent components can completely stabilize the vector-necklace-ring soliton clusters. The results presented here may have potential applications for the studies of optical necklace soliton clusters [33] in bilayer and multilayer structures [34] and vector matter waves of multicomponents Bose-Einstein condensations [35] with a dipolar long-range interaction.

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