Quantum Optics:
Summary

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LIGO-Virgo-KAGRA (LVK) Collaboration

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NTHU, Feb. 23rd, 2021

Syllabus :

- Introduction to Quantum Optics:

- **Quantum Mechanics:**
  - qSHO, Uncertainty-relation, Schrodinger/Heisenberg/Interaction pictures

- **Quantum Properties of Light:**
  - Number states, Vacuum States, Coherent States, Squeezed States
  - Phase Space, Quantum State Tomography

- **Simple Optical Instrument:**
  - Beam Splitter, Mach-Zehnder Interferometer
  - HBT, Homedyne Detections
  - Correlation funcitons

Syllabus :

•Photon-atom interaction:

- Rabi oscillation,
- Jaynes-Cummings Hamiltonian,
- Dicke model,
- Cavity-Quantum Electro-Dynamics (c-QED),
- Electromagnetically Induced Transparency (EIT),
- Optical Parametric Oscillator (OPO),
- Dissipative Systems,

• Selected Applications:

- Quantum Sensor: Gravitational Wave Detectors
- Test of Quantum Mechanics: Quantum Zeno effect
- Quantum Communication: QKD
- Quantum Computing: Quantum Photonic Circuit

Photon-atom interaction:

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- Rabi oscillation,
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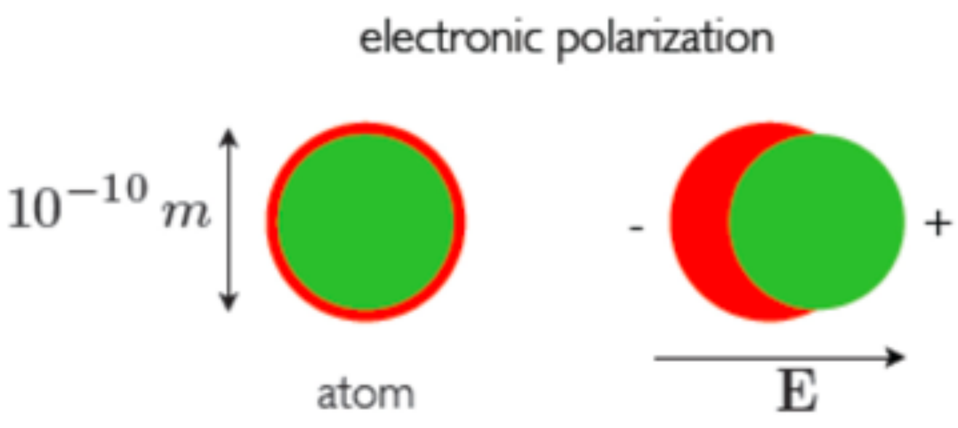
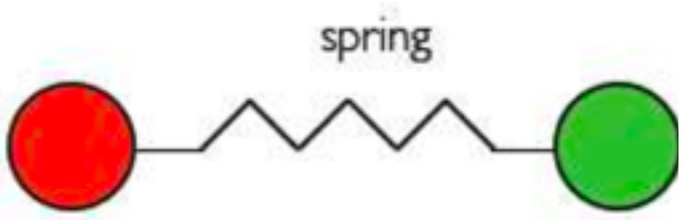
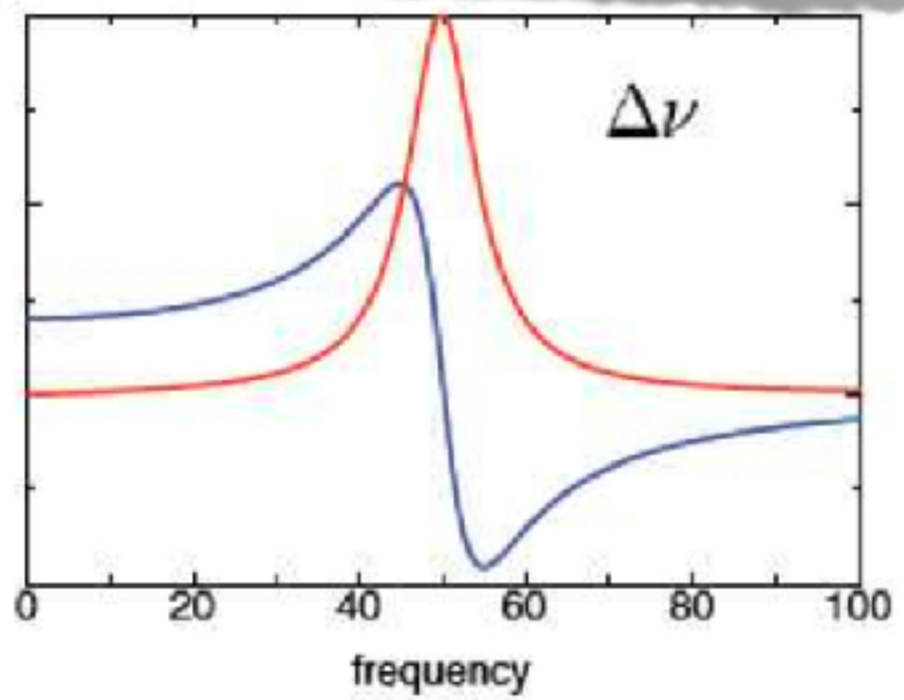
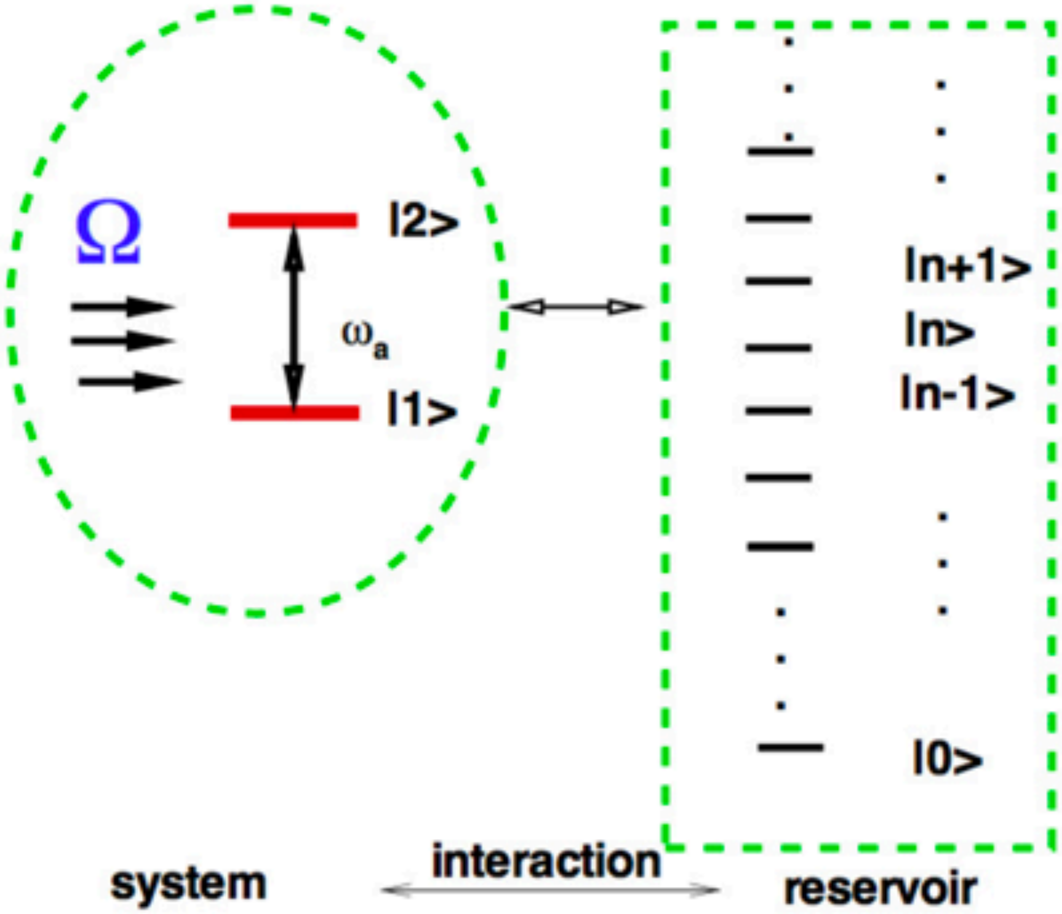
- Dicke model,
- Cavity-Quantum Electro-Dynamics (Cavity-QED),
- Electromagnetically Induced Transparency (EIT),
- Optical Parametric Oscillator (OPO),
- Dissipative Systems,

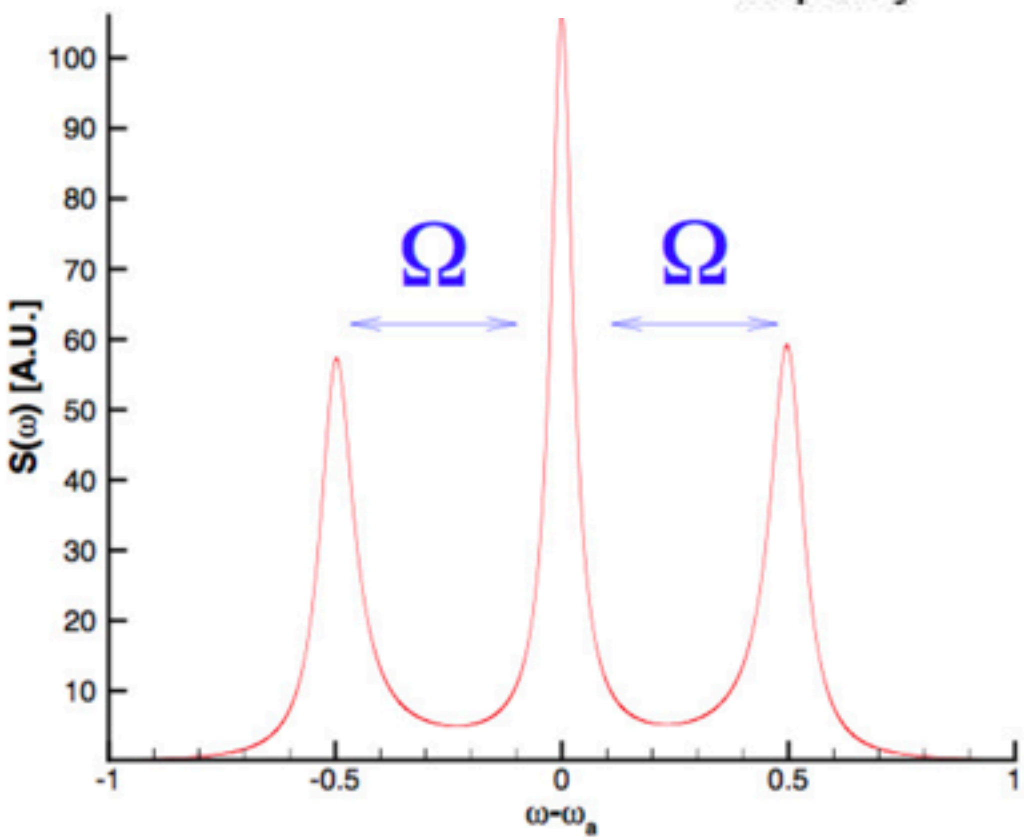
Mollow's triplet:









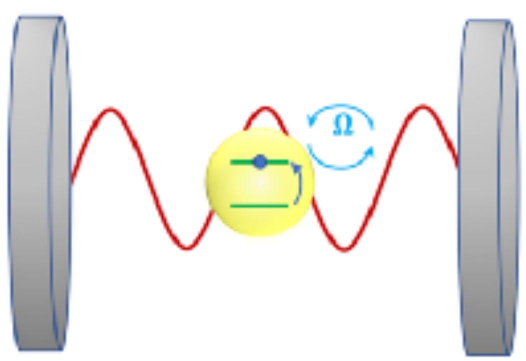


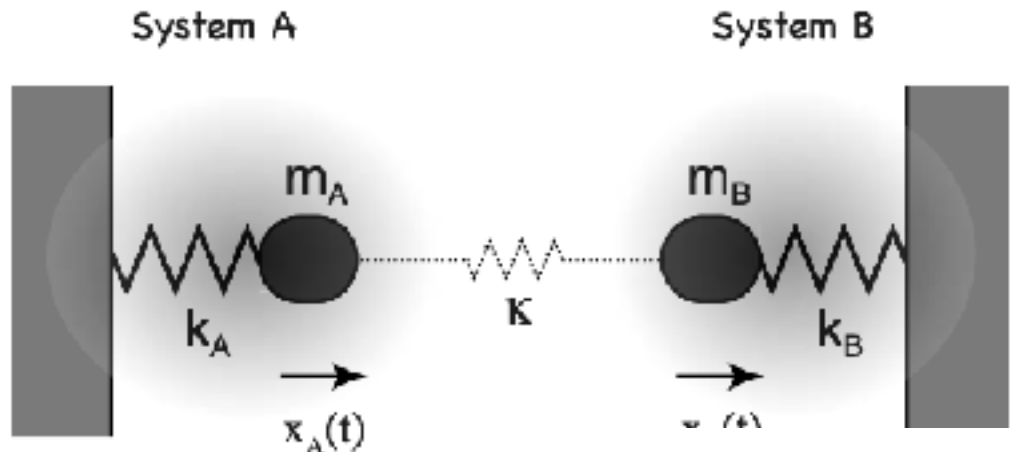
Photon-atom interaction:

•Photon-atom interaction:

- Quantum theory of Fluorescence,
- Cavity Quantum ElectroDynamics, Cavity-QED,
- Quantum theory of Lasers,
- Quantum theory of Nonlinear Optics,
- Quantum Non-demolition Measurement (QND),
- Quantum theory for Nonlinear Pulse Propagation,
- Entangled source generation and Quantum Information,
- Bose-Einstein Condensates (BEC) and Atom Optics,
- Quantum optical test of Complementarity of Quantum Mechanics,
- Quantum optics in Semiconductors,

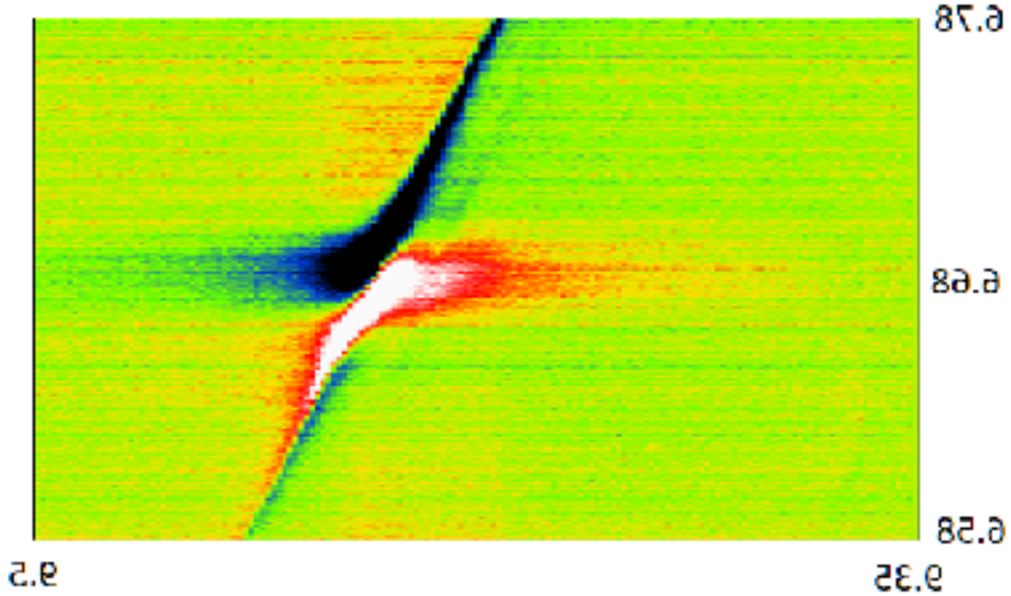
Strong-Coupling:

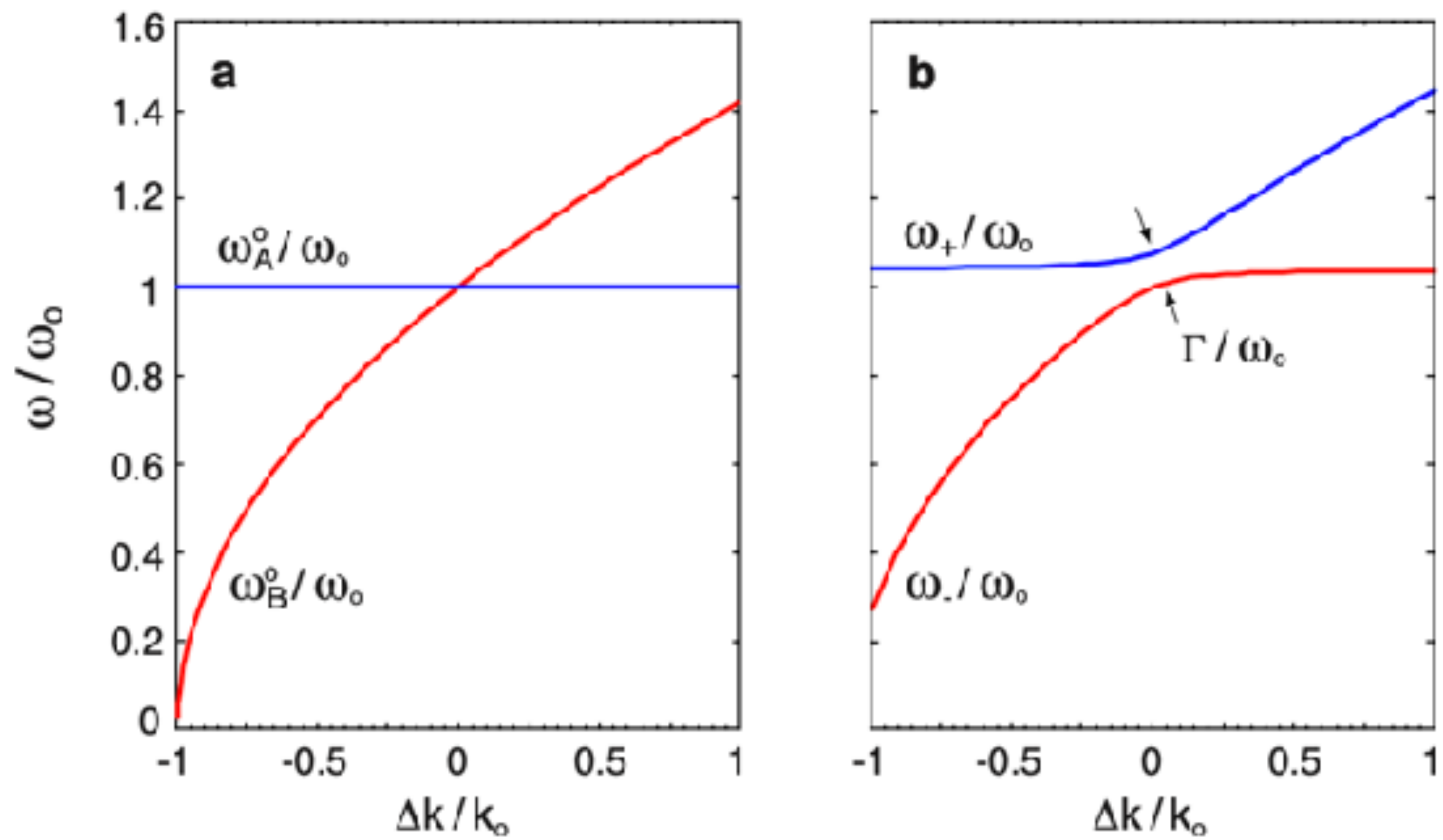


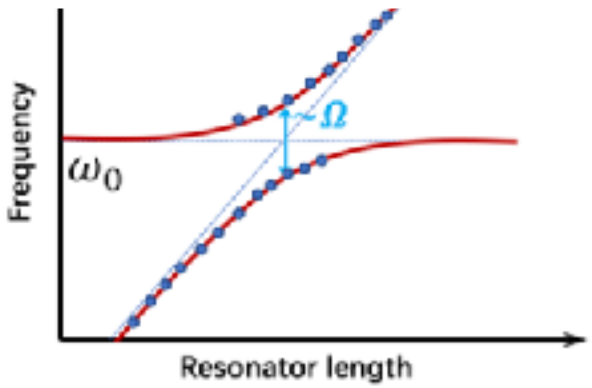


$$m_A\ddot{x}_A + k_A x_A + \kappa(x_A - x_B) = 0$$

$$m_B\ddot{x}_B + k_B x_B - \kappa(x_A - x_B) = 0.$$

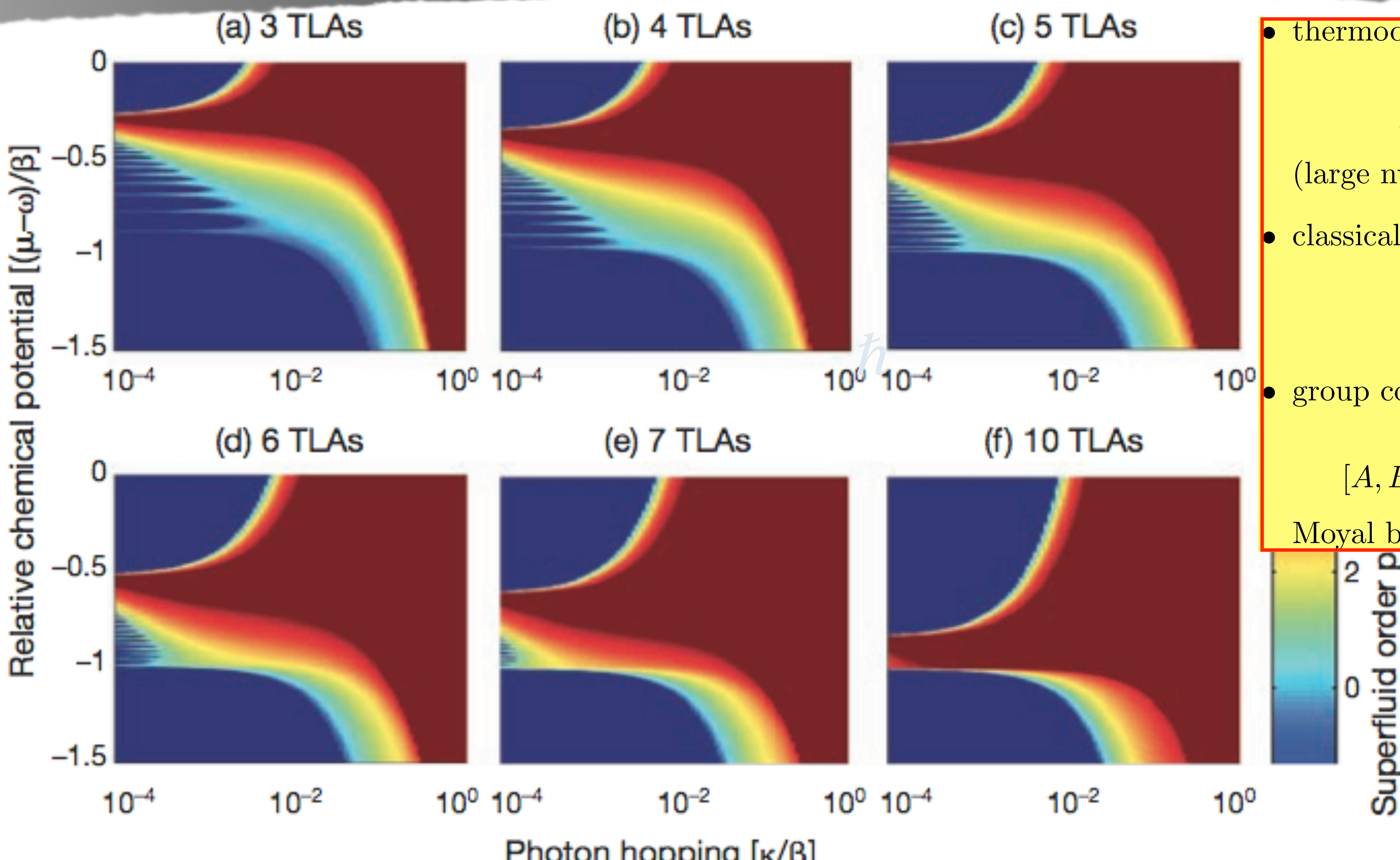






Strong coupling, energy splitting, and level crossings: A classical perspective, Lukas Novotny, American J. Phys. 78, 1199 (2010).

Mott-Superfluid transition: Dicke-Bose-Hubbard Model



- thermodynamical limit

$$N \to \infty$$

(large number limit)

- classical limit:

$$\hbar \to 0$$

- group contraction:

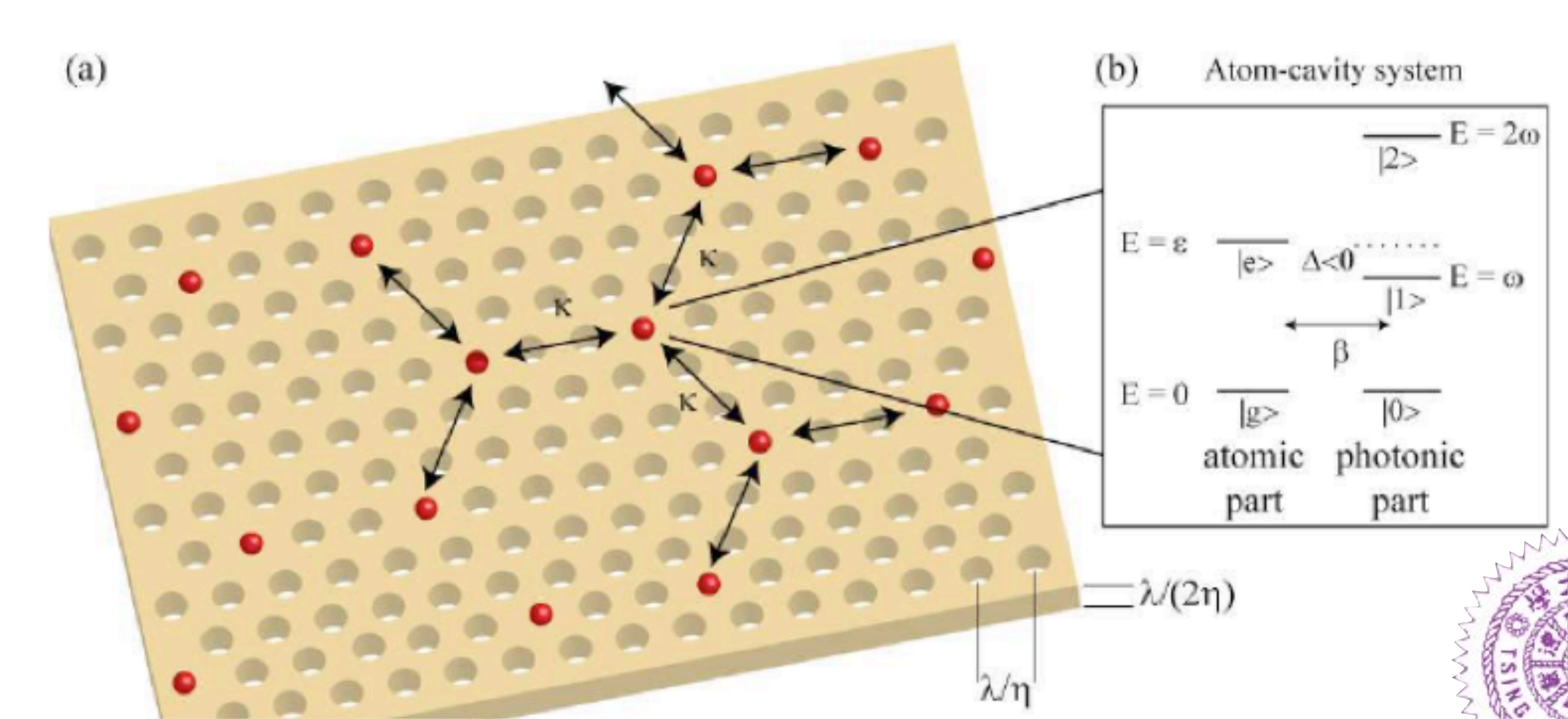
$$[A, B] \to \{A, B\}$$

Moyal bracket $\to$ Poisson bracket

S.-C. Lei and RKL, Phys. Rev. A 77, 033827 (2008).
S.-C. Lei, T.K. Ng, and RKL, Opt. Express 18, 14586 (2010).
S.-C. Lei and RKL, Optics in 2008, Optics & Photonics News, Dec., 44 (2008).

Array of Cavity QED

$$\hat{H}^{\mathrm{JC}} = \epsilon\hat{\sigma}_+\hat{\sigma}_- + \omega\hat{a}^\dagger\hat{a} + \beta(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger),$$
$$\hat{H} = \sum_i \hat{H}_i^{\mathrm{JC}} - \sum_{<i,j>} \kappa_{ij}(\hat{a}_i^\dagger\hat{a}_j + \hat{a}_j^\dagger\hat{a}_i) - \sum_i \mu_i\hat{N}_i,$$



A. D. Greentree, C. Tahan, J. H. Cole and L. C. L. Hollenberg, Nature Physics 2, 856 (2006).

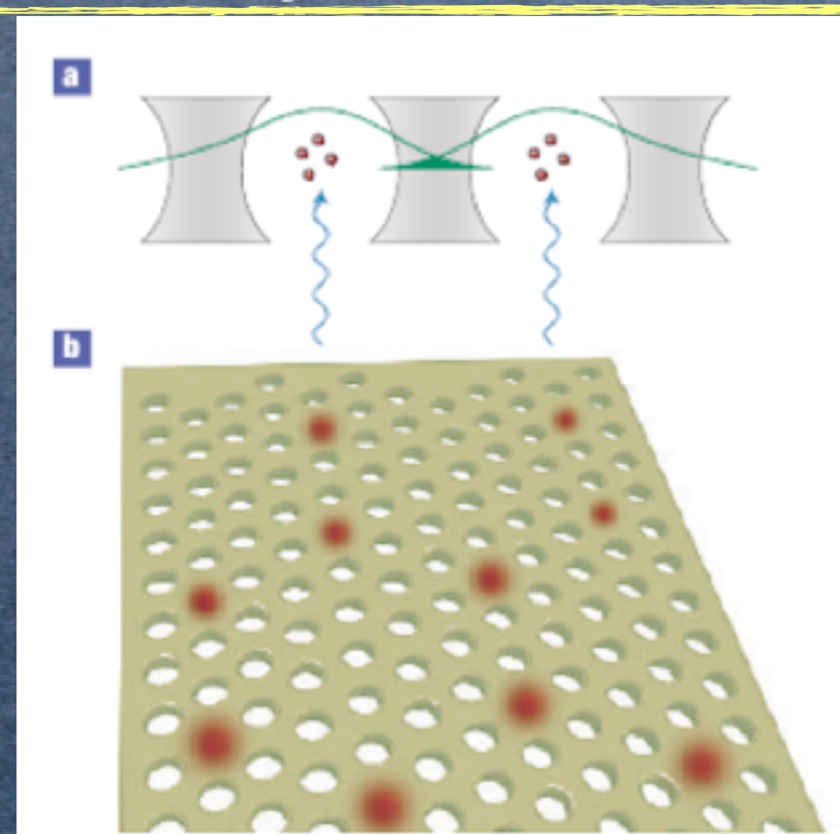
Dicke-Bose-Hubbard Model

$$\hat{H} = \sum_i H_i^{\mathrm{DM}} - \kappa \sum_{ij} a_i^{+} a_j - \mu \sum_i N_i,$$

$$H_i^{DM} = \varepsilon J_i^{+} J_i^{-} + \omega a_i^{+} a_i + \beta (a_i J_i^{+} + a_i^{+} J_i^{-}),$$

where the collective raising and lowering angular momentum operators, and the total number of atomic and photonic excitations N_i are,

$$J_i^{+} = \sum_j \sigma_j^{+}, \quad J_i^{-} = \sum_j \sigma_j^{-}, and \quad N_i = a_i^{+} a_i + J_i^{+} J_i^{-}.$$



Dicke-Bose-Hubbard Model: Infinity size

- SF refers to a superfluid phase with strong interaction of photon hopping.
- MI refers to a Mott insulator phase with equally number of photons in each cavity.
- In the insulator region $|0, 0\rangle$, $|-, 1\rangle$, and $|-, 2\rangle$ denote the negative branches of the dressed-states where the system will change from n to n + 1 excitation per site, simultaneously filling photons in cavities and resulting in a finite gap of spectrum.

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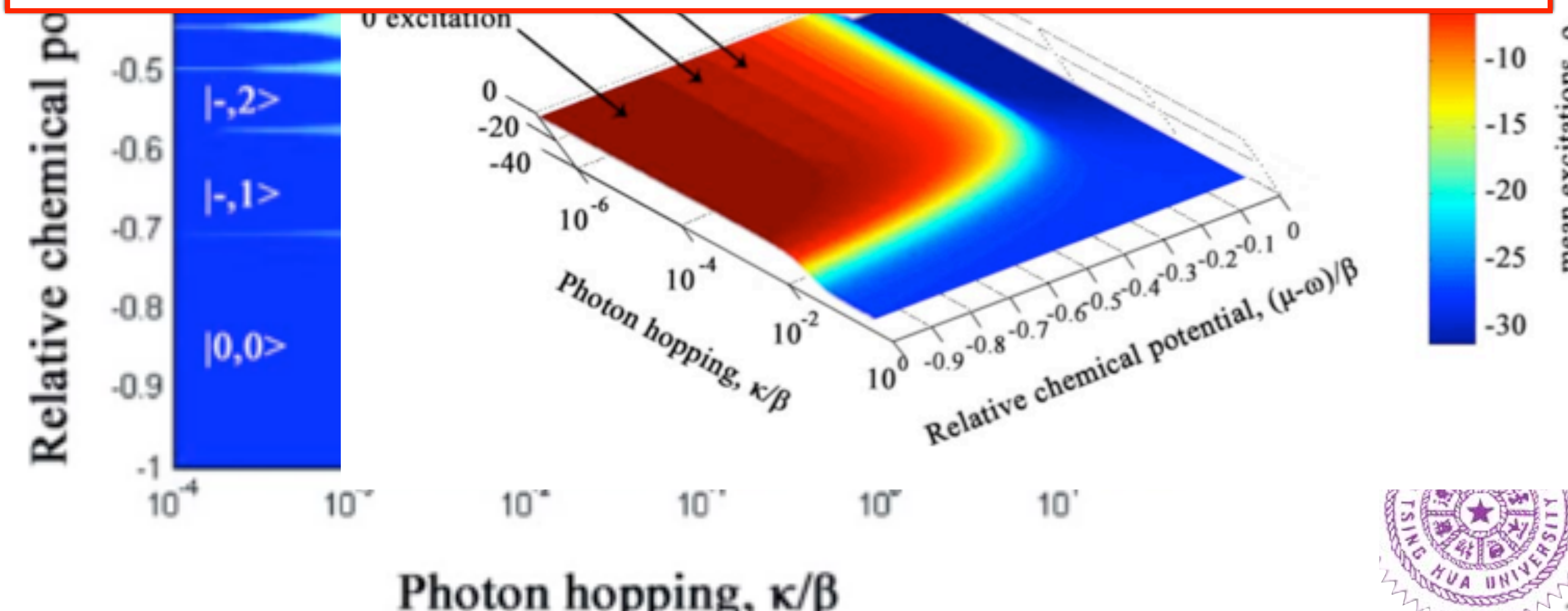
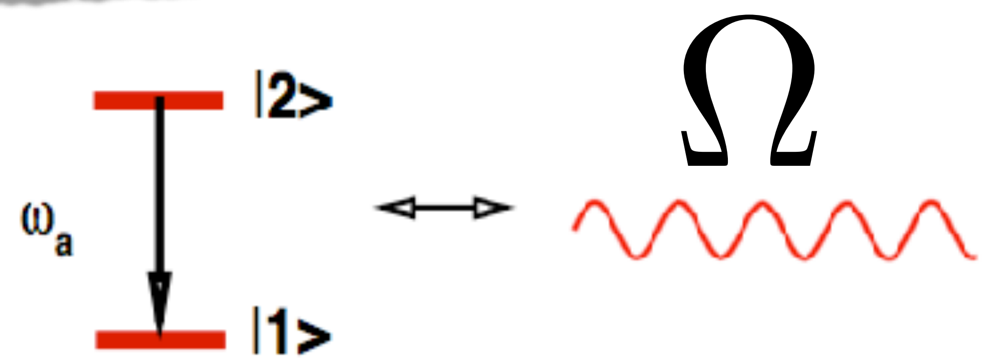
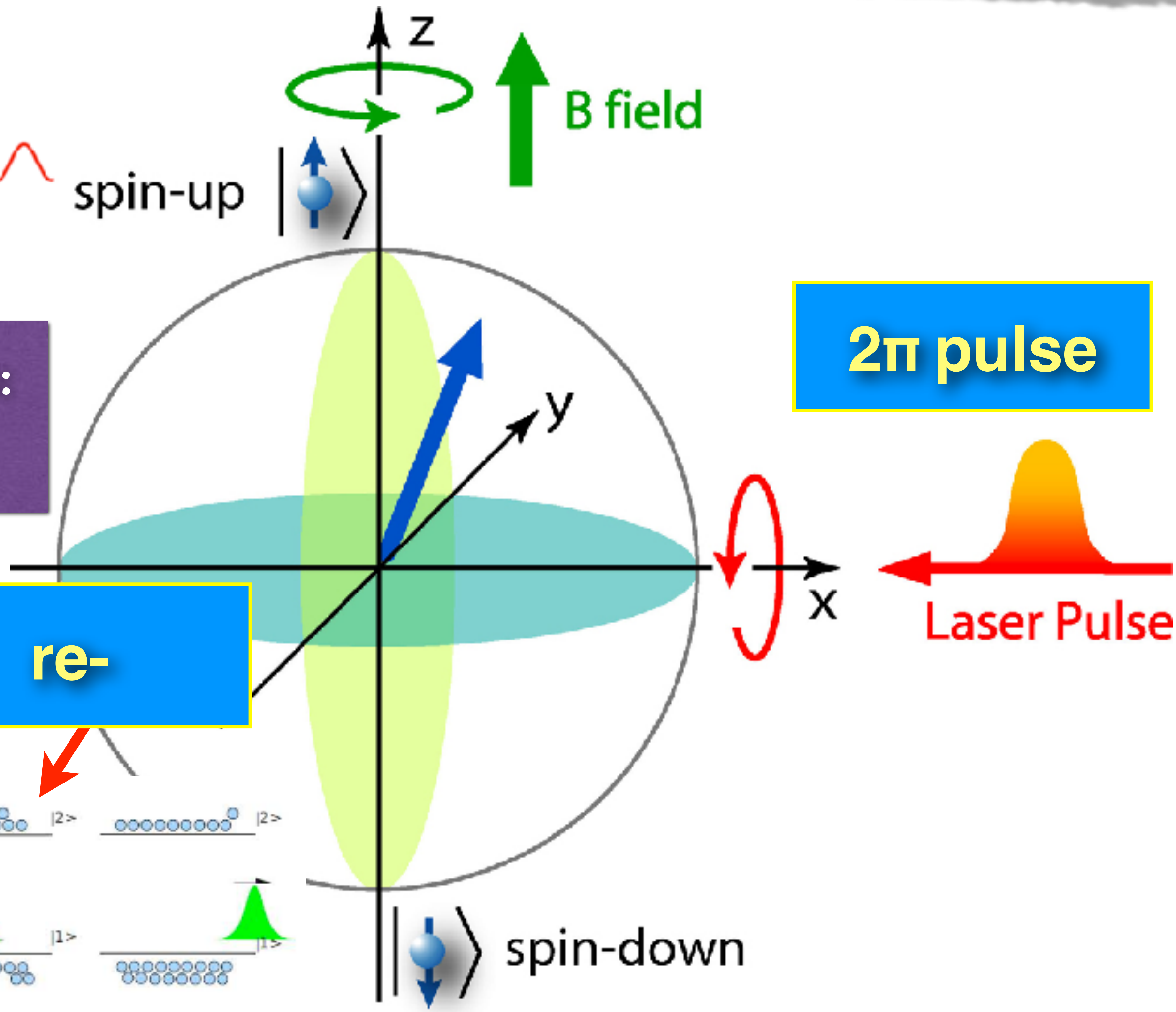


Rabi-Oscillation:





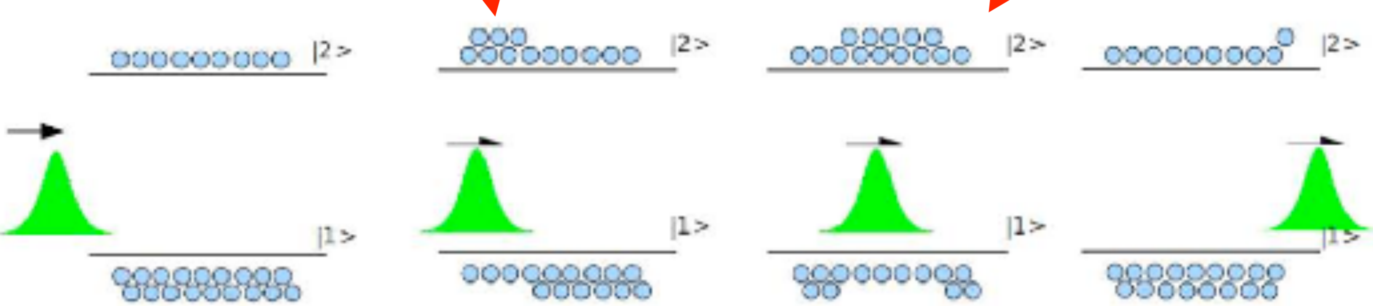
2π pulse

Coherent Interaction:

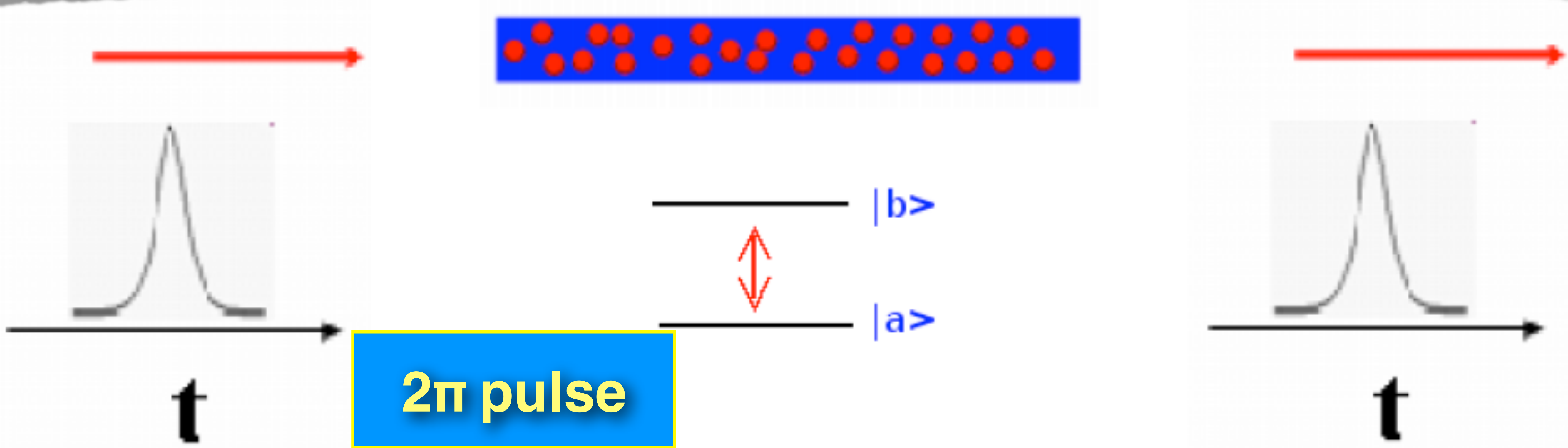
Photon Echo

absorption

re-



Self-Induced-Transparency, SIT



- Two-level atoms become transparent for optical pulses under coherent resonance.
- The quantum mechanical area theorem shows that slowly varying optical pulses with area equal to an integer multiple of 2π.
- This kind of optical pulses is known as *Self-Induced Transparency solitons*.

S. L. McCall and E. L. Hahn, Phys. Rev. Lett. 18, 908 (1967).

SIT solitons: Maxwell-Bloch equations

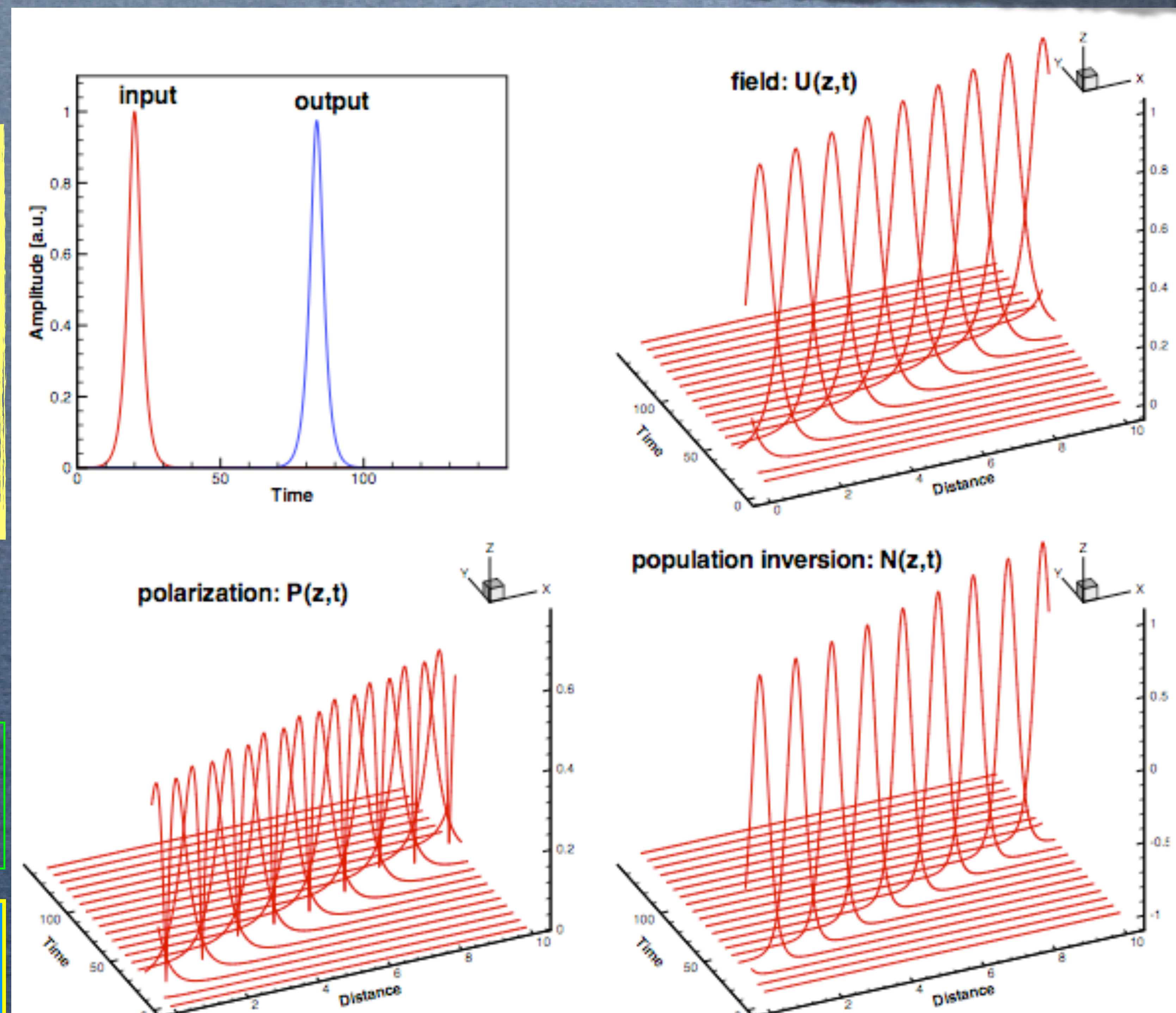
$$\frac{\partial}{\partial t}U(t,z) = -c\frac{\partial}{\partial z}U + \frac{r}{2}P,$$
$$\frac{\partial}{\partial t}P(t,z) = \frac{1}{2}N\,U,$$
$$\frac{\partial}{\partial t}N(t,z) = -(P^*\,U + U^*\,P),$$

The group velocity:

$$v_g = \frac{c}{1+r} = \frac{c}{1+\frac{4n_a}{n_{ph}}},$$

is slowing-down.

slow-light



Quantum SIT Solitons: Maxwell-Bloch eq.

$$\int_z^{z+\Delta z} \hat{P}(z,t)\,dz = \sum_{z \le z_j \le z+\Delta z} \hat{p}_j(t)$$

$$\int_z^{z+\Delta z} \hat{N}(z,t)\,dz = \sum_{z \le z_j \le z+\Delta z} \hat{n}_j(t)$$

U: optical field, $\hat{a}$
P: atomic dipole, $\hat{\sigma}_\pm$
N: population difference, $\hat{\sigma}_z$

$$\frac{\partial \hat{U}(z,t)}{\partial t} = -c\,\frac{\partial \hat{U}(z,t)}{\partial z} + K\,\hat{P}(z,t)$$

$$\frac{\partial \hat{P}(z,t)}{\partial t} = K\,\hat{N}(z,t)\,\hat{U}(z,t)$$

$$\frac{\partial \hat{N}(z,t)}{\partial t} = -2\,K\left\{\hat{P}^\dagger(z,t)\,\hat{U}(z,t) + \hat{U}^\dagger(z,t)\,\hat{P}(z,t)\right\}$$

$$\begin{aligned}
\left[\hat{U}(t,z_1), \hat{U}^\dagger(t,z_2)\right] &= \delta(z_1 - z_2), \\
\left[\hat{P}(t,z_1), \hat{P}^\dagger(t,z_2)\right] &= -\hat{N}(t,z_1)\delta(z_1 - z_2), \\
\left[\hat{P}^\dagger(t,z_1), \hat{N}(t,z_2)\right] &= -2\hat{P}^\dagger(t,z_1)\delta(z_1 - z_2), \\
\left[\hat{P}(t,z_1), \hat{N}(t,z_2)\right] &= 2\hat{P}(t,z_1)\delta(z_1 - z_2),
\end{aligned}$$

Quantum SIT Solitons: Maxwell-Bloch eq.

$$\left[\hat{U}(t,z_1),\hat{U}^\dagger(t,z_2)\right] = \delta(z_1-z_2),$$
$$\left[\hat{P}(t,z_1),\hat{P}^\dagger(t,z_2)\right] = -\hat{N}(t,z_1)\delta(z_1-z_2),$$
$$\left[\hat{P}^\dagger(t,z_1),\hat{N}(t,z_2)\right] = -2\hat{P}^\dagger(t,z_1)\delta(z_1-z_2),$$
$$\left[\hat{P}(t,z_1),\hat{N}(t,z_2)\right] = 2\hat{P}(t,z_1)\delta(z_1-z_2),$$

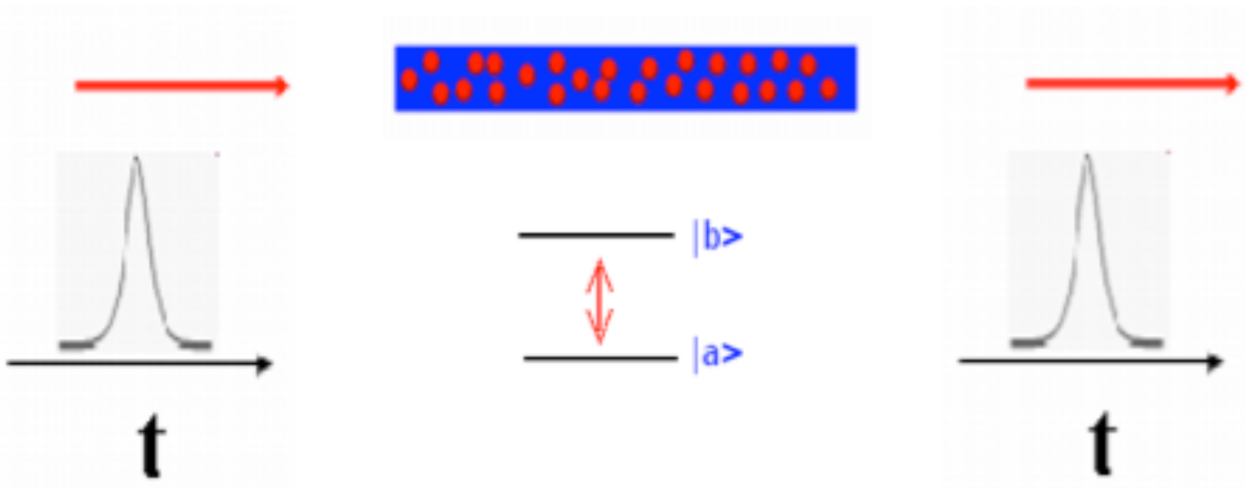
optical field: boson
2-level atom: spin-1/2

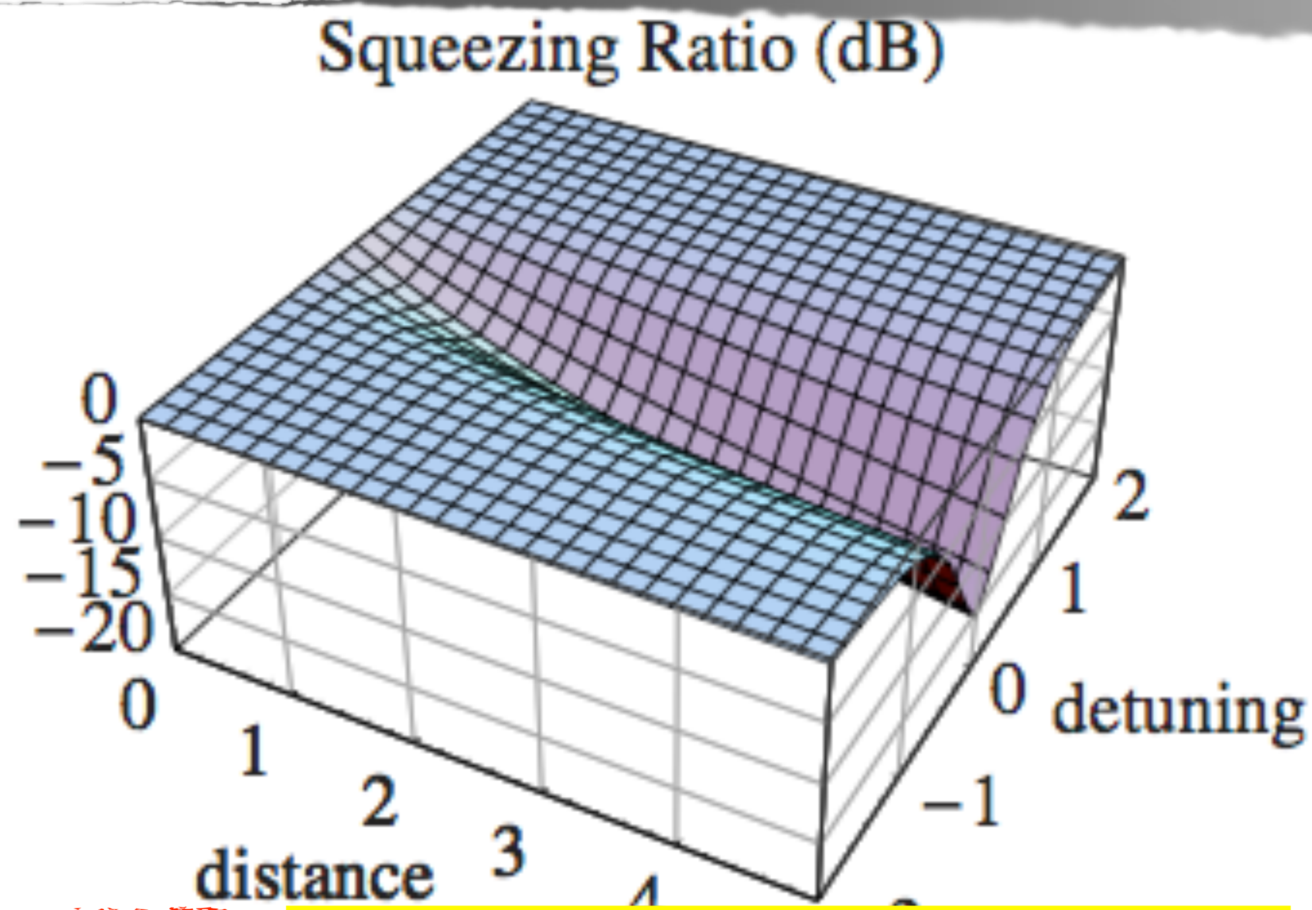
Assume all the atoms are in the ground state, *low-excitation limit*, $\langle\hat{N}\rangle = -1$, then

$$\left[\hat{p}(z_1,t_b),\hat{p}^\dagger(z_2,t_b)\right] = \delta(z_1-z_2).$$

Bosonic Commutation relation

Squeezing of SIT Solitons





$$S = \frac{\mathrm{Var}[\hat{M}(t_e)]}{\mathrm{Var}[\hat{M}(t_b)]} = \frac{\int_{-\infty}^{\infty}\left[|u^A(z,t_b)|^2 + \frac{n_p}{n_a}|p^A(z,t_b)|^2\right]dz}{\int_{-\infty}^{\infty}|f_L(z)|^2 dz}.$$

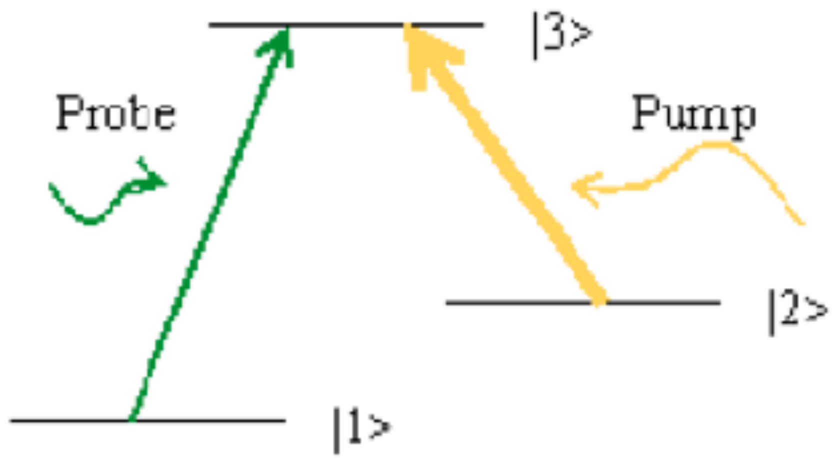
atomic fluctuation becomes less important in a large ensemble

RKL and Y. Lai, Phys. Rev. A 80, 033839 (2009).

3-level systems

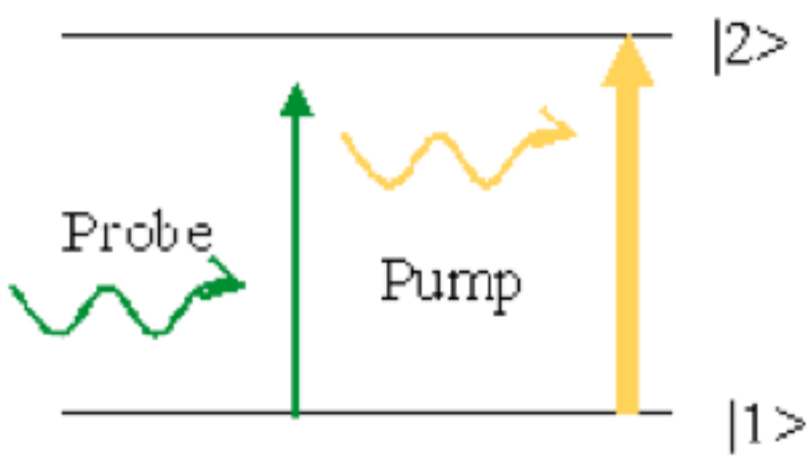
- **Electromagnetically Induced Transparency (EIT)**

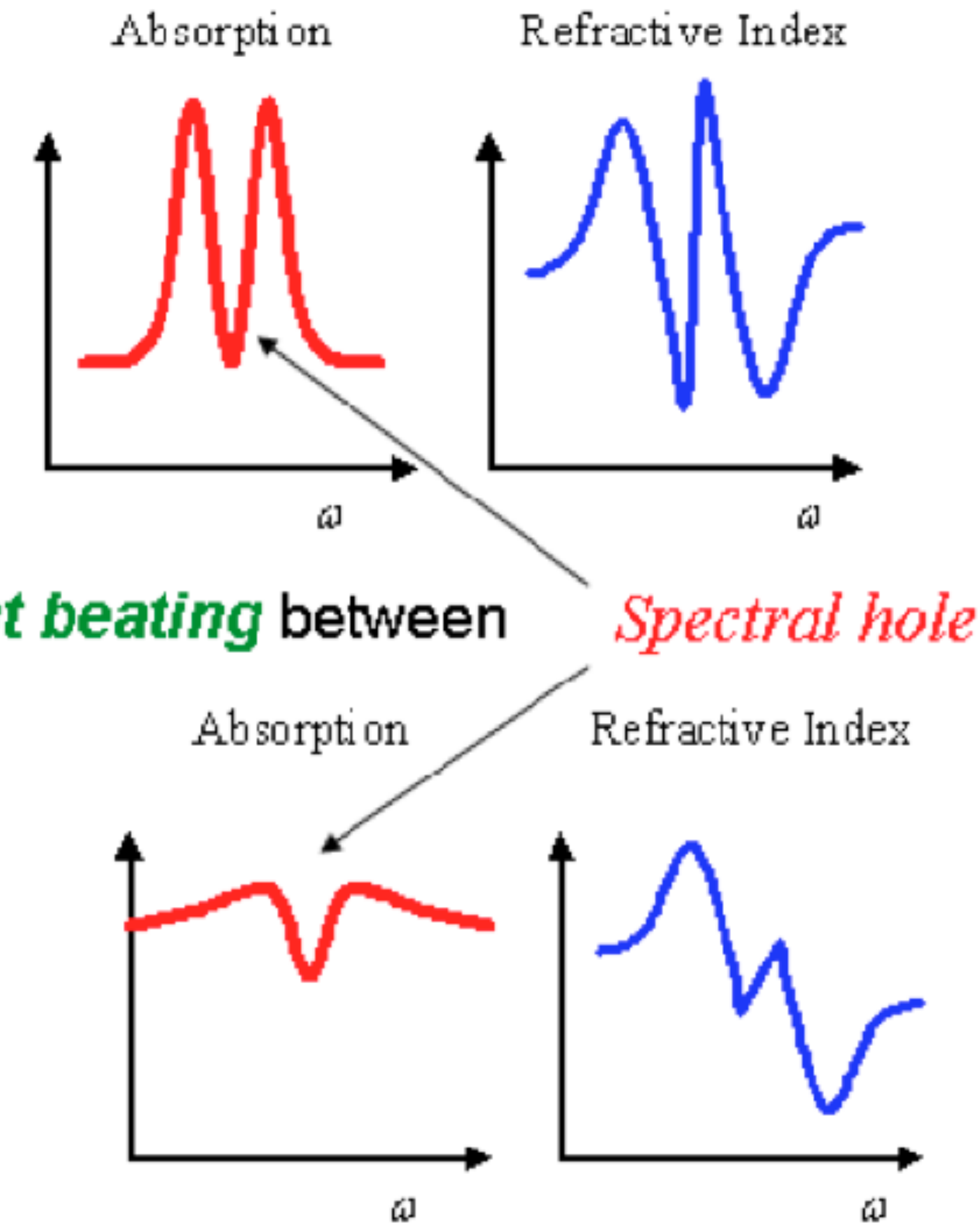
  — Formation of ***dark state*** by intense pump in a three-level system



- **Population Oscillation**

  — Absorption dip generated by ***coherent beating*** between pump and probe in a two-level system

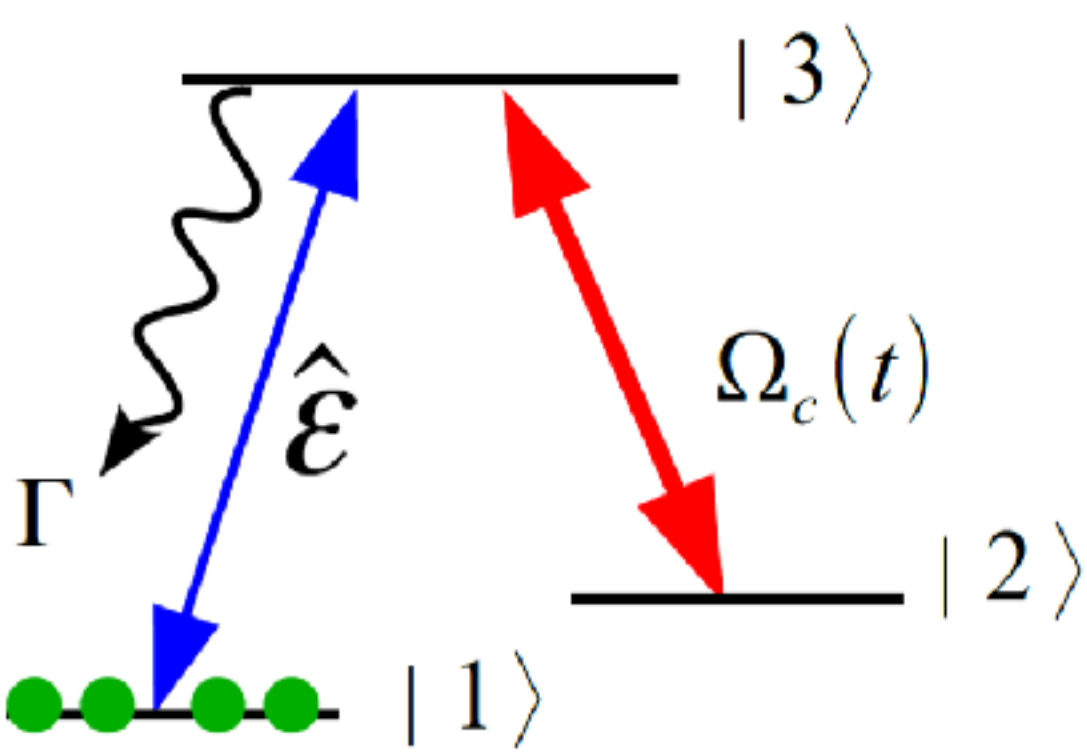


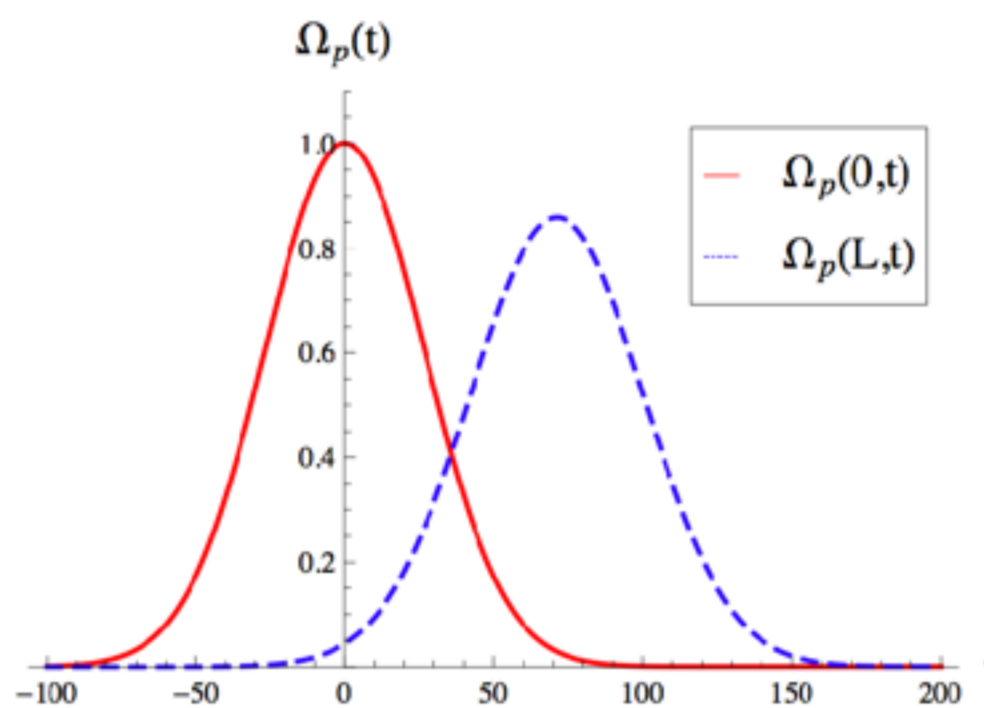


$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}} = igN\hat{\sigma}_{13},$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{13} = -\gamma_{13}\hat{\sigma}_{13} + ig\hat{\mathcal{E}} + i\Omega_c\hat{\sigma}_{12} + \hat{F}_{13},$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{12} = -\gamma_{12}\hat{\sigma}_{12} + i\Omega_c^*\hat{\sigma}_{13} + \hat{F}_{12}$$





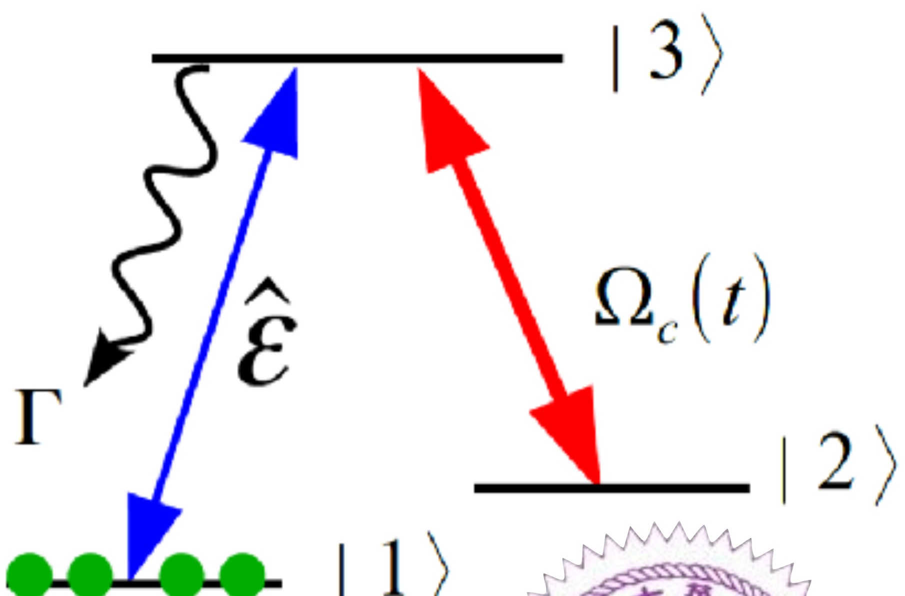
- Since the control field is stronger than the probe field, *low excitation* is assumed the atomic population is almost in the ground state.
- Then, the corresponding dipole transition $\hat{\sigma}_{13}$ changes slowly compared to the excited state decay rate.

$$\hat{\sigma}_{13} = \frac{1}{\gamma_{13}} \left(ig\hat{\mathcal{E}} + i\Omega_c \hat{\sigma}_{12} + \hat{F}_{13} \right).$$

- Typically, the adiabatic condition is applied by assuming

$$\hat{\sigma}_{12} = -g\,\hat{\mathcal{E}}/\Omega_c.$$

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} \right) \hat{\mathcal{E}} = \frac{gN}{\Omega_c} \frac{\partial}{\partial t} \hat{\sigma}_{12}$$



- Dark-State Polariton operator: $\hat{\Psi}$ in EIT system

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{\mathcal{E}}(z,t) - \sqrt{N}\sin\theta(t)\hat{\sigma}_{12}(z,t)$$

- Dark-State Polariton operator: $\hat{\Psi}$ in EIT system

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{\mathcal{E}}(z,t) - \sqrt{N}\sin\theta(t)\hat{\sigma}_{12}(z,t)$$

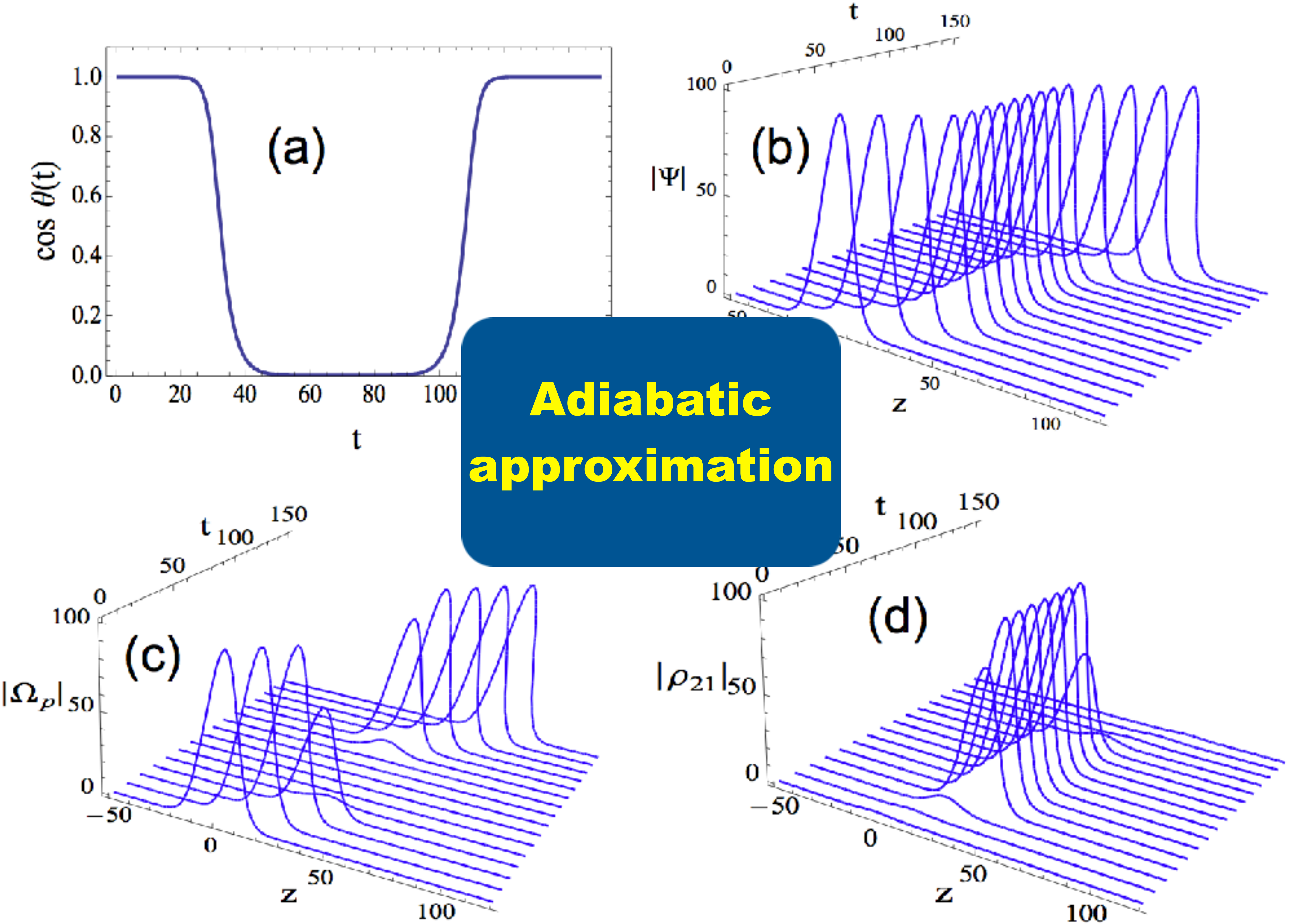
- equation of motion:

$$\left(\frac{\partial}{\partial t} + c\cos^2\theta(t)\frac{\partial}{\partial z}\right)\Psi(z,t) = 0$$

with the two defined angles

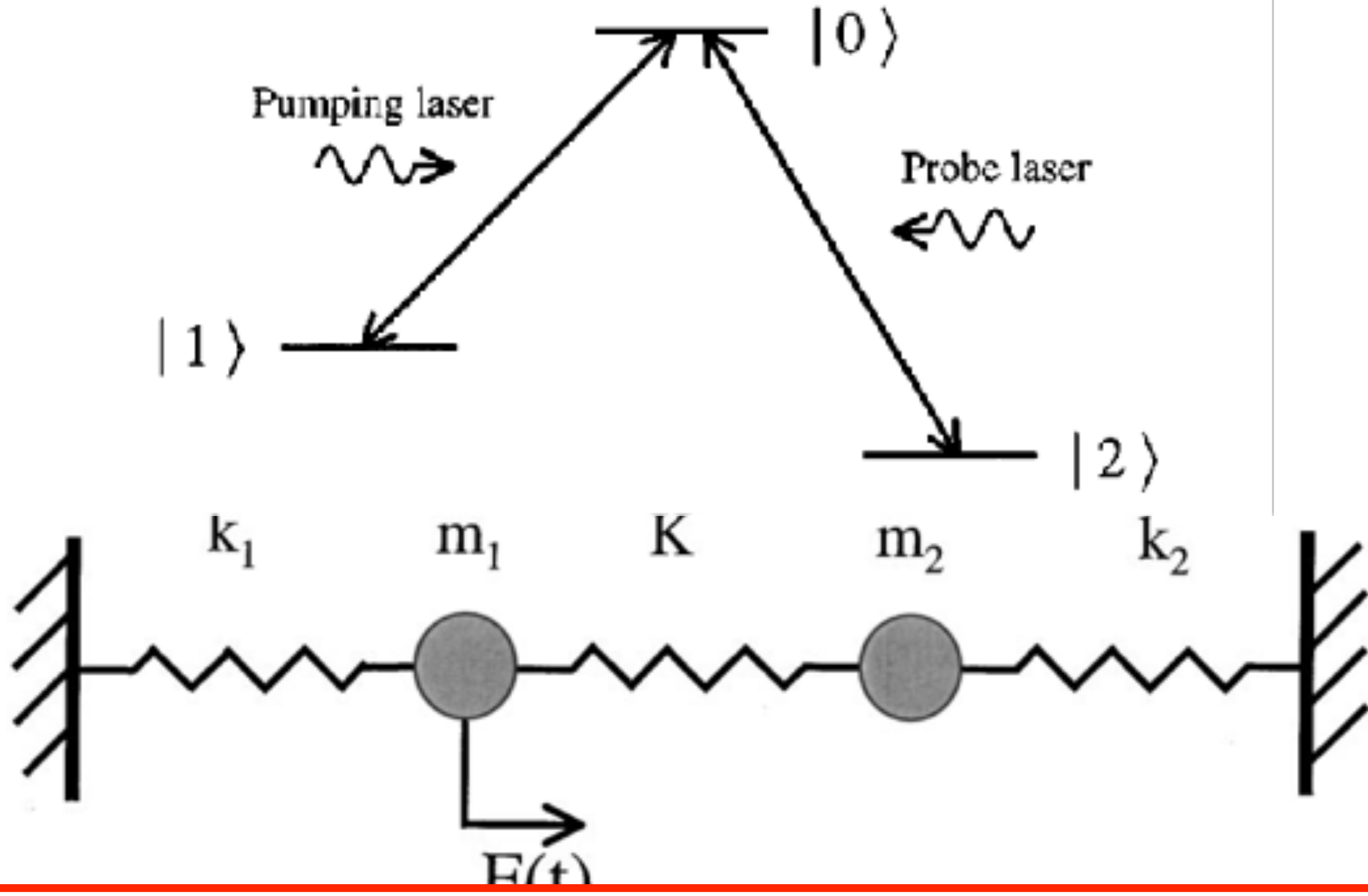
$$\cos\theta = \frac{\Omega_c}{\sqrt{\Omega_c^2 + 4g^2N}} \quad , \quad \sin\theta = \frac{\sqrt{4g^2N}}{\sqrt{\Omega_c^2 + 4g^2N}}$$

M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. 84, 5094 (2000).



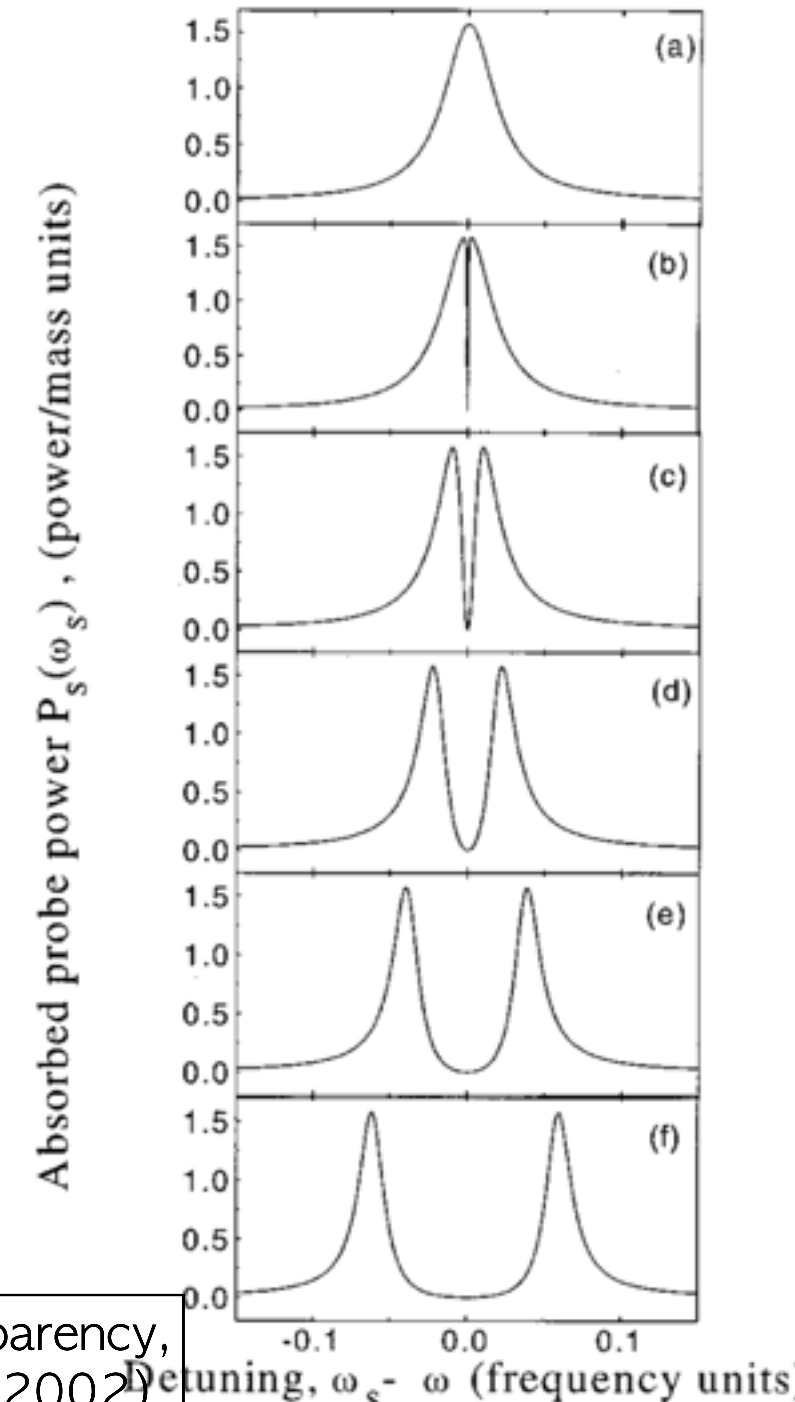
(a)
cos θ(t)
t
(b)
|Ψ|
t
z
(c)
|Ω_p|
t
z
(d)
|ρ21|
t
z
Adiabatic approximation

Classical Analog of EIT



$$\ddot{x}_1(t) + \gamma_1 \dot{x}_1(t) + \omega^2 x_1(t) - \Omega_r^2 x_2(t) = \frac{F}{m} e^{-i\omega_s t},$$

$$\ddot{x}_2(t) + \gamma_2 \dot{x}_2(t) + \omega^2 x_2(t) - \Omega_r^2 x_1(t) = 0.$$



Classical analog of electromagnetically induced transparency,
C. L. Garrido Alzar, M. A. G. Martinez, and P. Nussenzveig, Am. J. Phys. 70, 37 (2002).

Beyond Adiabatic approximation

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- Then, the corresponding dipole transition $\hat{\sigma}_{13}$ changes slowly compared to the excited state decay rate.

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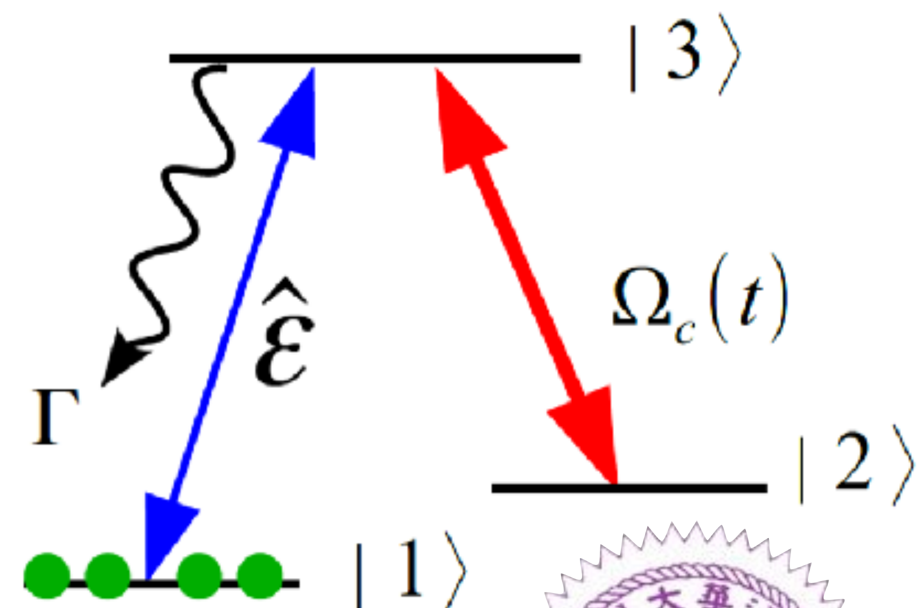


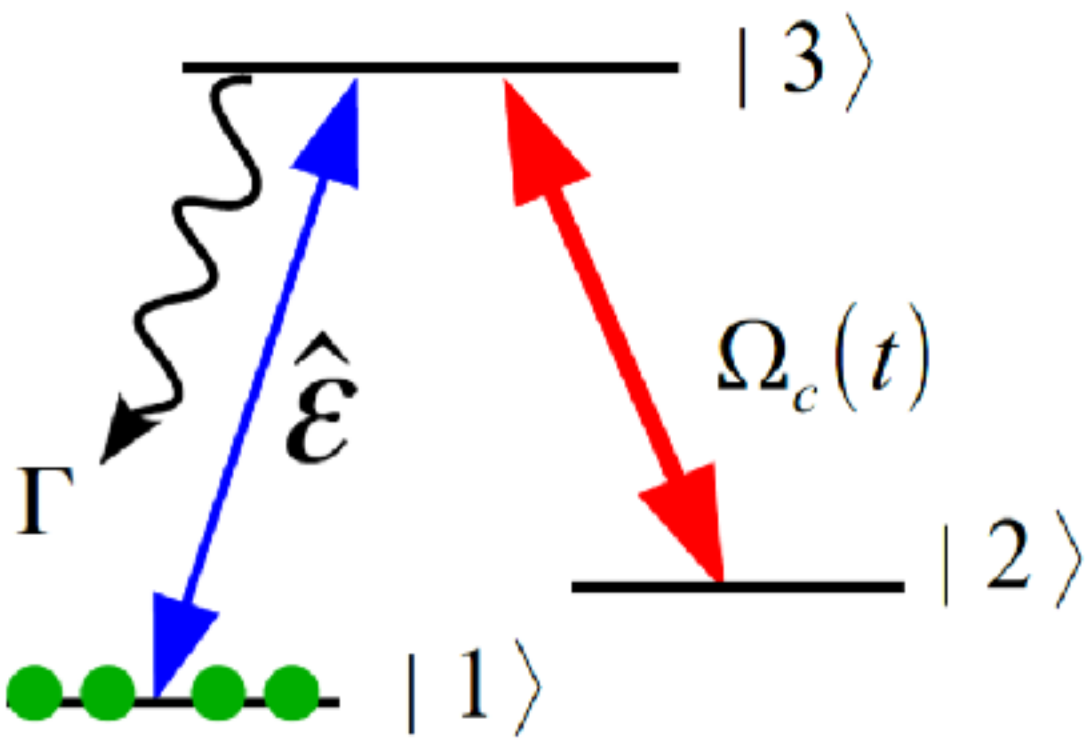
- Dark-State Polariton operator: $\hat{\Psi}$ in EIT system

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{\mathcal{E}}(z,t) - \sqrt{N}\sin\theta(t)\hat{\sigma}_{12}(z,t)$$

The coupled equations of field and atomic polarizations.

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}} = igN\hat{\sigma}_{13},$$
$$\frac{\partial}{\partial t}\hat{\sigma}_{13} = -\gamma\hat{\sigma}_{13} + ig\hat{\mathcal{E}} + i\Omega_c\hat{\sigma}_{12} + \boxed{\hat{F},}$$
$$\frac{\partial}{\partial t}\hat{\sigma}_{12} = i\Omega_c^*\hat{\sigma}_{13}$$

We solve the set of equations **beyond** adiabatic approximation.



Langevin noise operator

By solving the coupled equations:

$$\begin{cases} \dfrac{\partial}{\partial z}\hat{\mathcal{E}} = -\dfrac{g^2 N}{c\gamma}\hat{\mathcal{E}} - \dfrac{gN\Omega_c}{c\gamma}\hat{\sigma} + i\frac{gN}{c\gamma}\hat{F} \\ \dfrac{\partial}{\partial \tau}\hat{\sigma} = -\dfrac{|\Omega_c|^2}{\gamma}\hat{\sigma} - \dfrac{g\sqrt{N}\Omega_c^*}{\gamma}\hat{\mathcal{E}} + i\frac{\Omega_c^*}{\gamma}\hat{F} \end{cases}$$

$$\hat{\mathcal{E}}(z,\tau) = e^{-\alpha}\hat{\mathcal{E}}(0,\tau) + \int_0^\tau d\tau' f(\tau,\tau')\hat{\mathcal{E}}(0,\tau') + \hat{n}.$$

where

$$f(\tau,\tau') \equiv \sqrt{\alpha}e^{-\alpha}\frac{\Omega_c(\tau)\Omega_c^*(\tau')}{\gamma}e^{-\kappa(\tau,\tau')}\frac{I_1\left(\sqrt{4\alpha\kappa(\tau,\tau')}\right)}{\sqrt{\kappa(\tau,\tau')}}$$

The effective Langevin noise operator is introduced by asking the field operator $\hat{\mathcal{E}}$ to satisfy Bosonic commutation relation, *i.e.*:

$$[\hat{\mathcal{E}}(z,\tau), \hat{\mathcal{E}}^\dagger(z,\tau')] = \delta(\tau-\tau'), \qquad \forall z$$

Quantum noise variance through EIT

$$\langle \hat{X}_L^2(\tau)\rangle = 1 - e^{-\mathrm{OD}\left(1+T_p\frac{|\Omega_c|^2}{\Gamma}(\mathrm{OD})\right)}\left(1-\langle \hat{X}_0^2(\tau)\rangle\right) - T_p\int_0^{\tau} d\tau' \,[f(\tau,\tau')]^2\left(1-\langle \hat{X}_0^2(\tau')\rangle\right) + 2\langle \hat{n}^\dagger \hat{n}\rangle.$$

Noise variance from vacuum

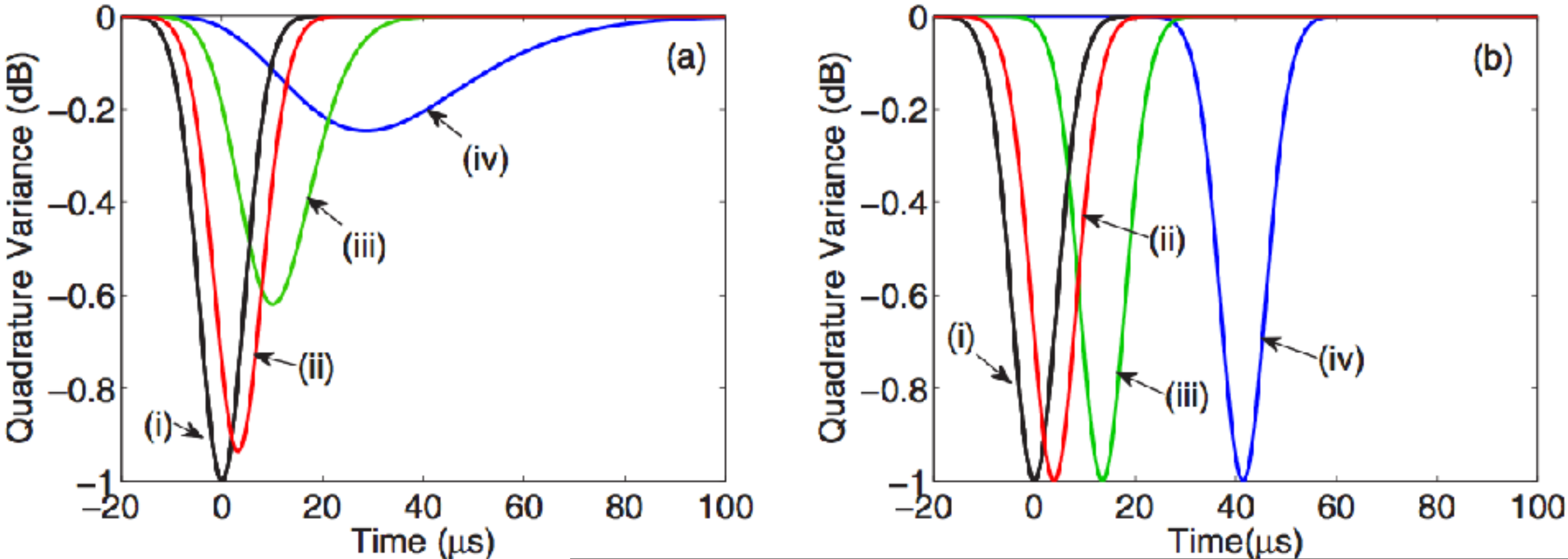
Optical Density (OD)

Noise variance at the input

Response function in EIT

Thermal noises

Beyond Adiabatic approximation | **Adiabatic approximation**

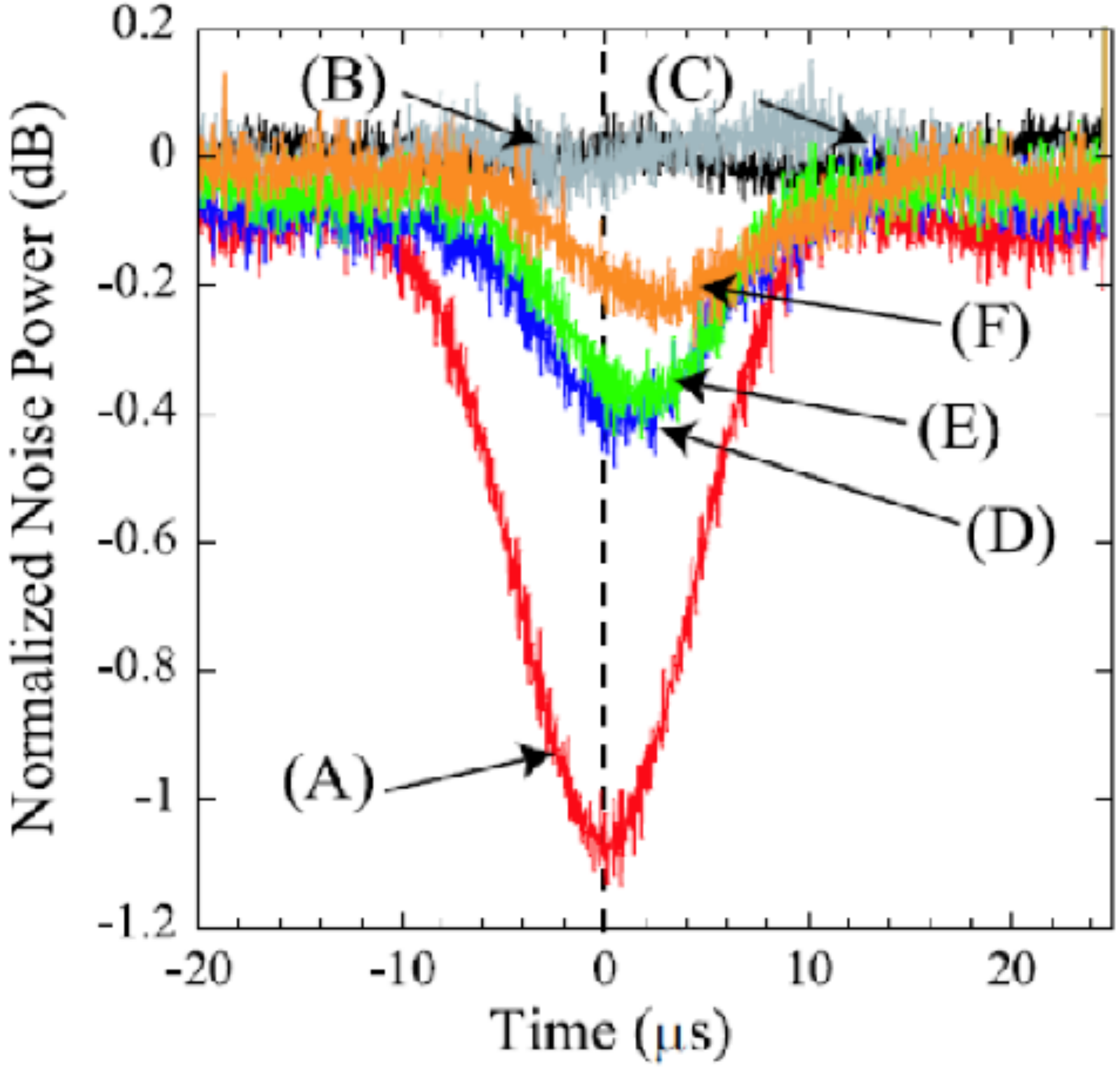


You-Lin Chuang, Ite A. Yu, and RKL, Phys. Rev. A 91, 063818(2015)

PRL 99, 153602 (2007) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2007

Ultraslow Propagation of Squeezed Vacuum Pulses with Electromagnetically Induced Transparency

Daisuke Akamatsu,[1] Yoshihiko Yokoi,[1] Manabu Arikawa,[1] Satoshi Nagatsuka,[1] Takahito Tanimura,[1] Akira Furusawa,[2] and Mikio Kozuma[1,3]



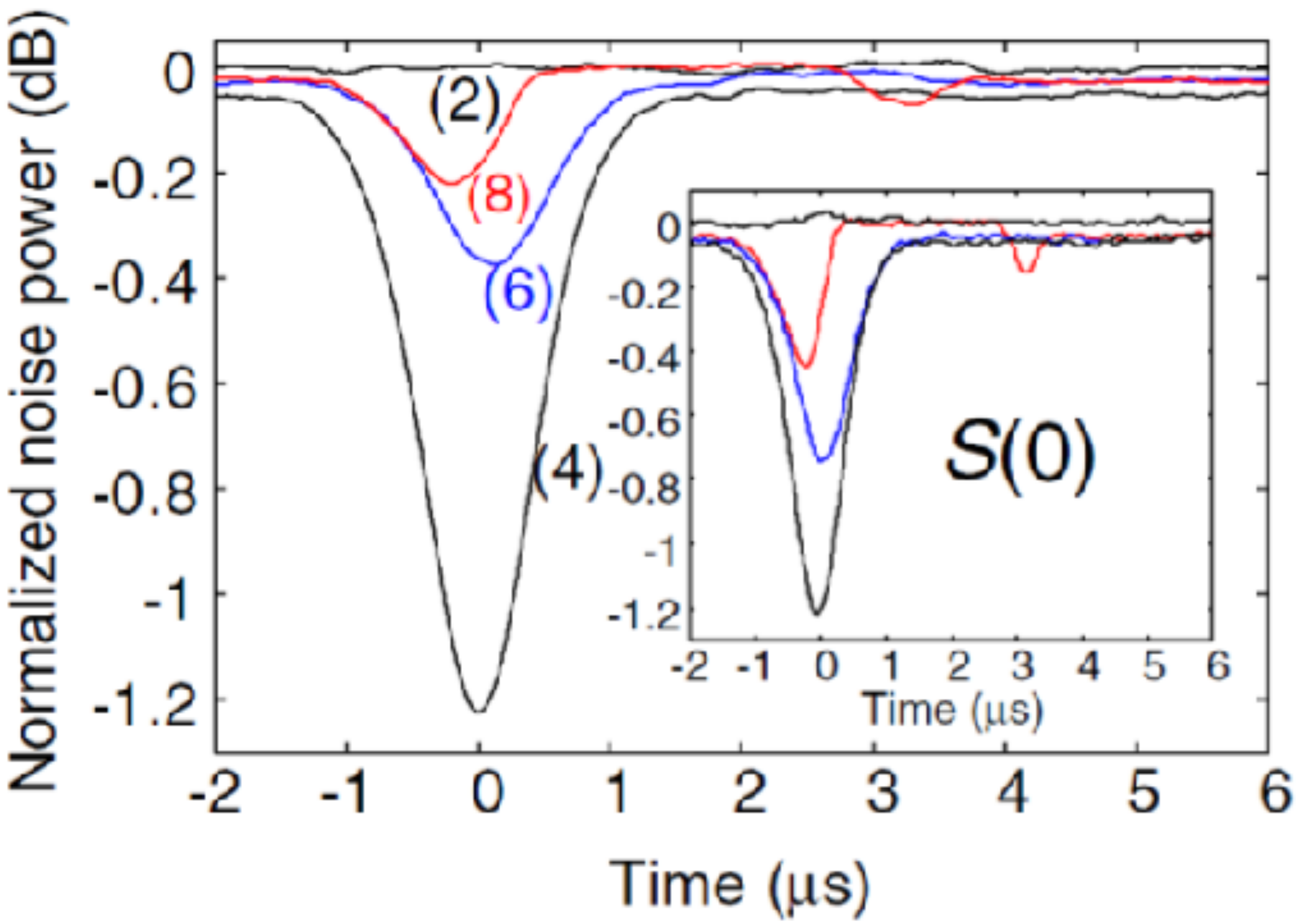
D. Akamatsu et al., Phys. Rev. Lett. 99, 153602 (2007).

You-Lin Chuang, Ite A. Yu, and RKL, Phys. Rev. A 91, 063818(2015).

PRL **100,** 093601 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Storage and Retrieval of a Squeezed Vacuum

Kazuhito Honda,[1] Daisuke Akamatsu,[2] Manabu Arikawa,[2] Yoshihiko Yokoi,[2] Keiichirou Akiba,[2] Satoshi Nagatsuka,[2] Takahito Tanimura,[2] Akira Furusawa,[3] and Mikio Kozuma[1,2,4]



K. Honda et al., Phys. Rev. Lett. 100, 1093601 (2008).

You-Lin Chuang, Ite A. Yu, and RKL, Phys. Rev. A 91, 063818(2015).

Reflecting on pseudo-Hermitian (PT-symmetric) quantum theory, and its analog in (topological) optics

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KAGRA Scientific Congress (KSC) Board
LIGO-Virgo-KAGRA Collaboration

Phys. Rev. Lett. 112, 130404 (2014);
Editors' Suggestion; Physics: Synopsis.
Phys. Rev. A 92, 053815 (2015);
Phys. Rev. A 93, 042122 (2016);
J. Phys. A 51, 414004 (2018);
Phys. Rev. Lett. 123, 080404 (2019);

*http://mx.nthu.edu.tw/~rklee

CSRC, Beijing, Dec. 16th, 2019

Axioms of Quantum Mechanics

1. **State:** The properties of a quantum system are completely defined by specification of its state vector $|\Psi\rangle$. The state vector is an element of a complex Hilbert space $\mathcal{H}$ called the space of states.

2. **Observable:** With every physical property $\hat{A}$ (energy, position, momentum, angular momentum, ...) there exists an associated linear, *Hermitian operator* $\hat{A}$ (usually called observable), which acts in the space of states $\mathcal{H}$. The eigenvalues of the operator are the possible values of the physical properties.

3. **Probability:**

   (a) If $|\Psi\rangle$ is the vector representing the state of a system and if $|\Phi\rangle$represents another physical state, there exists a probability $p(|\Psi\rangle, |\Phi\rangle)$ of finding $|\Psi\rangle$ in state $|\Phi\rangle$, which is given by the squared modulus of the scalar product on $\mathcal{H}$: : $p(|\Psi\rangle, |\Phi\rangle) = |\langle\Psi|\Phi\rangle|^2$ (Born Rule).

   (b) If $\mathcal{A}$ is an observable with eigenvalues a_k and eigenvectors $|k\rangle$, $\hat{A}|k\rangle = a_k|k\rangle$, given a system in the state $|\Psi\rangle$, the probability of obtaining a_k as the outcome of the measurement of $\hat{A}$ is $p(a_k) = |\langle k|\Psi\rangle|$. After the measurement the system is left in the state projected on the subspace of the eigenvalue a_k (Wave function collapse).

4. **Time evolution:** The evolution of a closed system is *unitary.* The state vector $|\Psi(t)\rangle$ at time t is derived from the state vector $|\Psi(t_0)\rangle$ at time t_0 by applying a unitary operator $\hat{U}(t, t_0)$, called the evolution operator: $|\Psi(t)\rangle = \hat{U}(t, t_0)|\Psi(t_0)\rangle$.

Quantum State Tomography

non-Hermitian Quantum Mechanics

Quantum Measurement (weak measurement)

Decoherence

Arrow of Time

Entangled-History

Is Dirac Hermiticity too strong an axiom of quantum mechanics?

C. M. Bender's slides.

$$H = H^{\dagger}$$

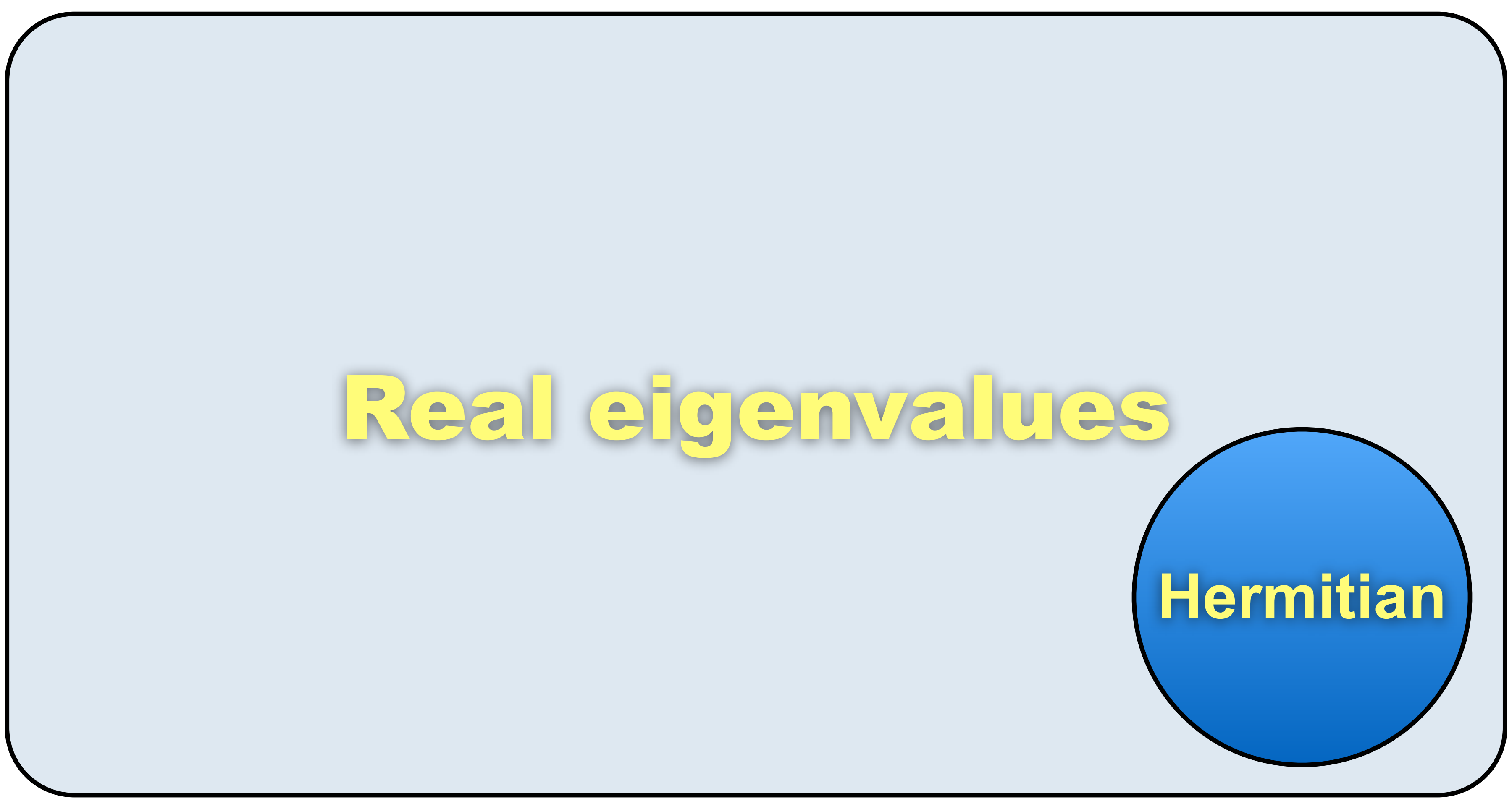
$\dagger$ means transpose + complex conjugate

- guarantees real energy and conserved probability
- but … is a **mathematical** axiom and not a **physical** axiom of quantum mechanics

Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender[1] and Stefan Boettcher[2,3]
[1]Department of Physics, Washington University, St. Louis, Missouri 63130
[2]Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
[3]CTSPS, Clark Atlanta University, Atlanta, Georgia 30314
(Received 1 December 1997; revised manuscript received 9 April 1998)

The condition of self-adjointness ensures that the eigenvalues of a Hamiltonian are real and bounded below. Replacing this condition by the weaker condition of $\mathcal{PT}$ symmetry, one obtains new infinite classes of complex Hamiltonians whose spectra are also real and positive. These $\mathcal{PT}$ symmetric theories may be viewed as analytic continuations of conventional theories from real to complex phase space. This paper describes the unusual classical and quantum properties of these theories. [S0031-9007(98)06371-6]

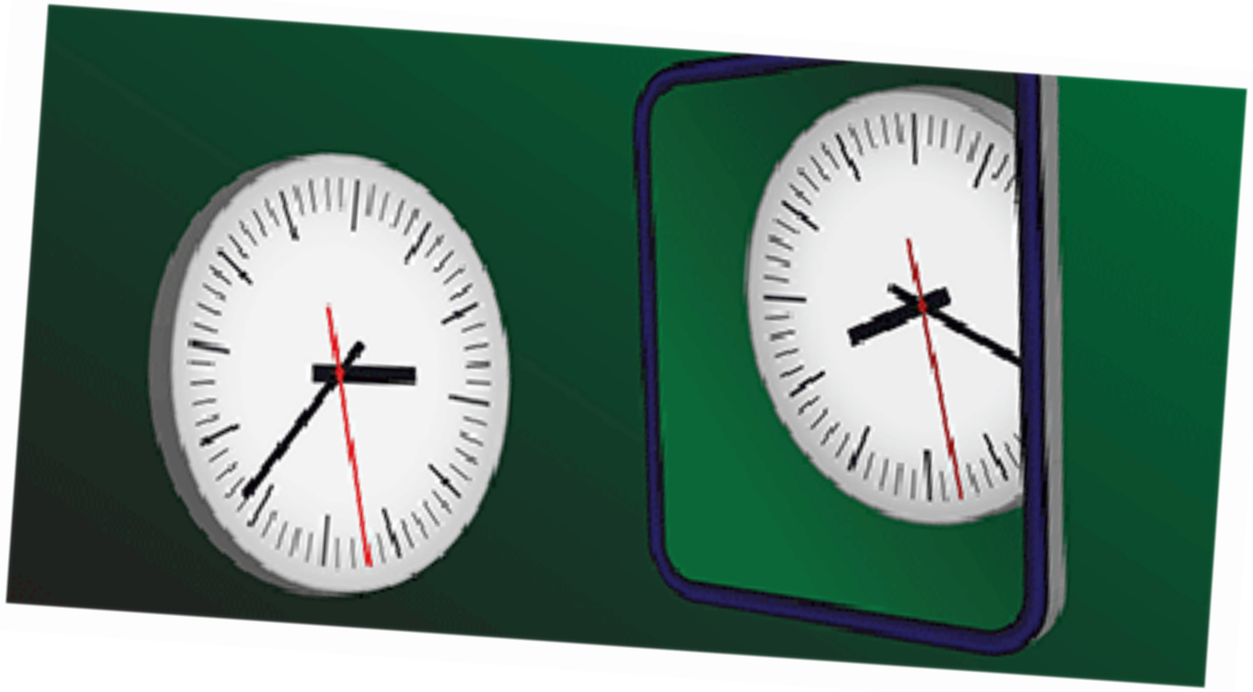


Parity-Time Symmetry

- $\mathcal{P}$: parity operator

$$\hat{p} \to -\hat{p}, \hat{x} \to -\hat{x};$$

- $\mathcal{T}$: time reversal operator



$$\hat{p} \to -\hat{p}, \hat{x} \to \hat{x}, i \to -i.$$

Parity-Time Symmetry-breaking

- $\mathcal{PT}$ symmetric Hamiltonian:

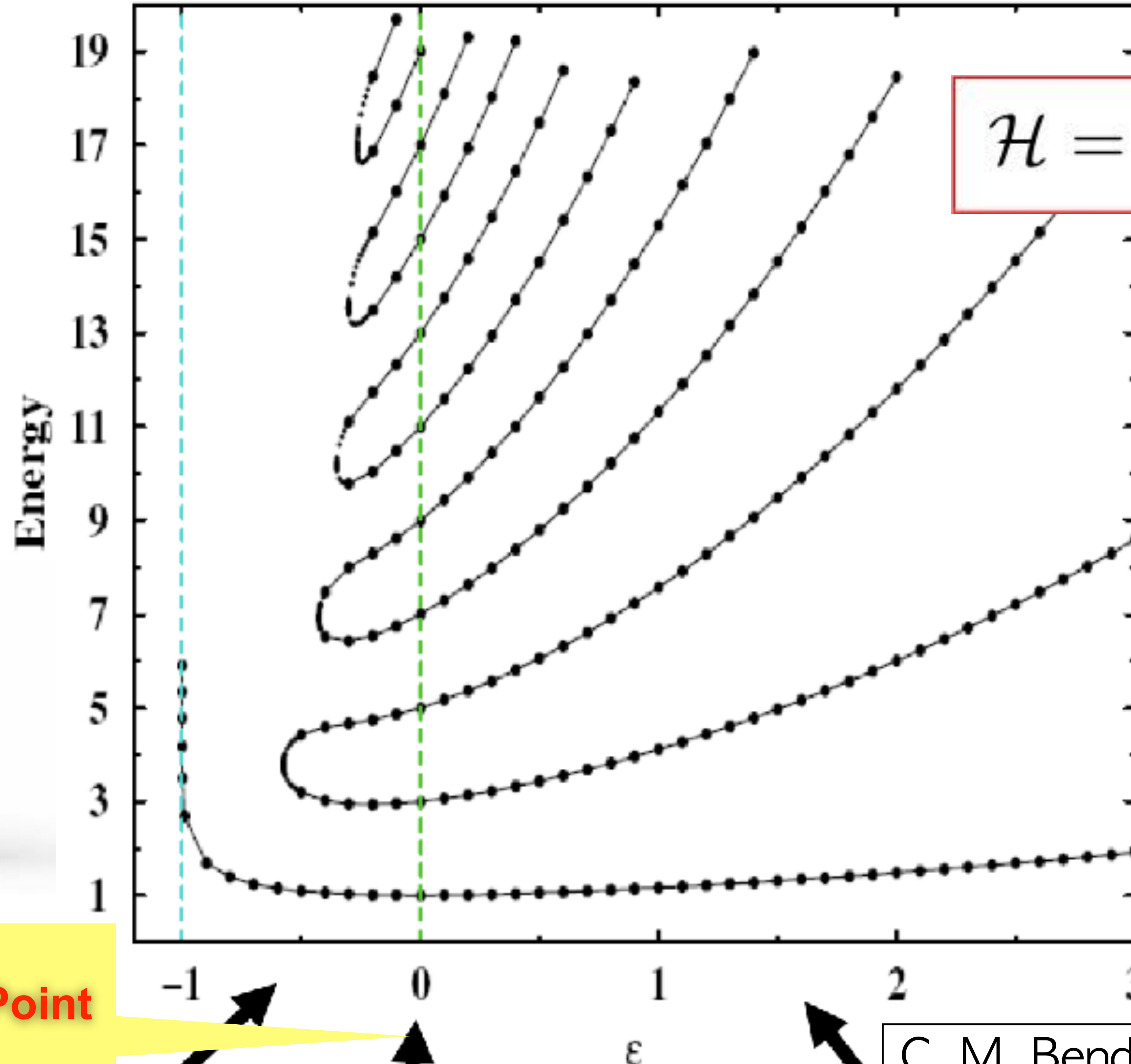
$$\boxed{[\hat{H}, \mathcal{P}\mathcal{T}] = 0,}$$

that is

$$\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle,$$
$$\mathcal{P}\mathcal{T}|\psi_n\rangle = \lambda|\psi_n\rangle, \Rightarrow E_n = E_n^*,$$

- Spontaneous symmetry breaking:

$$\mathcal{P}\mathcal{T}|\psi_n\rangle \neq \lambda|\psi_n\rangle, \Rightarrow E_n \neq E_n^*,$$



$$\mathcal{H} = p^2 + x^2 (ix)^\epsilon$$

- **Exceptional Point**

C. M. Bender and S. Boettcher, PRL 80, 24 (1998).

Region of ***broken*** **PT** symmetry

PT Boundary

Region of ***unbroken*** **PT** symmetry

- Consider a family of differential equations parameterized by a continuous parameter $\epsilon > 0$ in the form:

$$\frac{\partial^2 \psi(x)}{\partial x^2} + V_\epsilon(x)\psi(x) + 2E\psi(x) = 0, \tag{1}$$

- Here, let us specify the definition of $V_\epsilon(x) = -(ix)^\epsilon$, by stating explicitly the branch of logarithm :

$$V_\epsilon(x) = -(ix)^\epsilon = e^{\epsilon \log(ix)} = \begin{cases} -|x|^\epsilon \left[\cos(\epsilon\frac{\pi}{2}) + i\sin(\epsilon\frac{\pi}{2})\right], & \text{for } x > 0; \\ 0, & \text{for } x = 0; \\ -|x|^\epsilon \left[\cos(\epsilon\frac{\pi}{2}) - i\sin(\epsilon\frac{\pi}{2})\right], & \text{for } x < 0\,. \end{cases} \tag{2}$$

- In a Fock state basis, an analytical formula for the matrix element $a_{nm}(\epsilon) = \langle m|\mathrm{H}_\epsilon|n\rangle$ of $\mathrm{H}_\epsilon = \frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{V_\epsilon(x)}{2}$ can be constructed for any natural number n, m and positive ϵ:

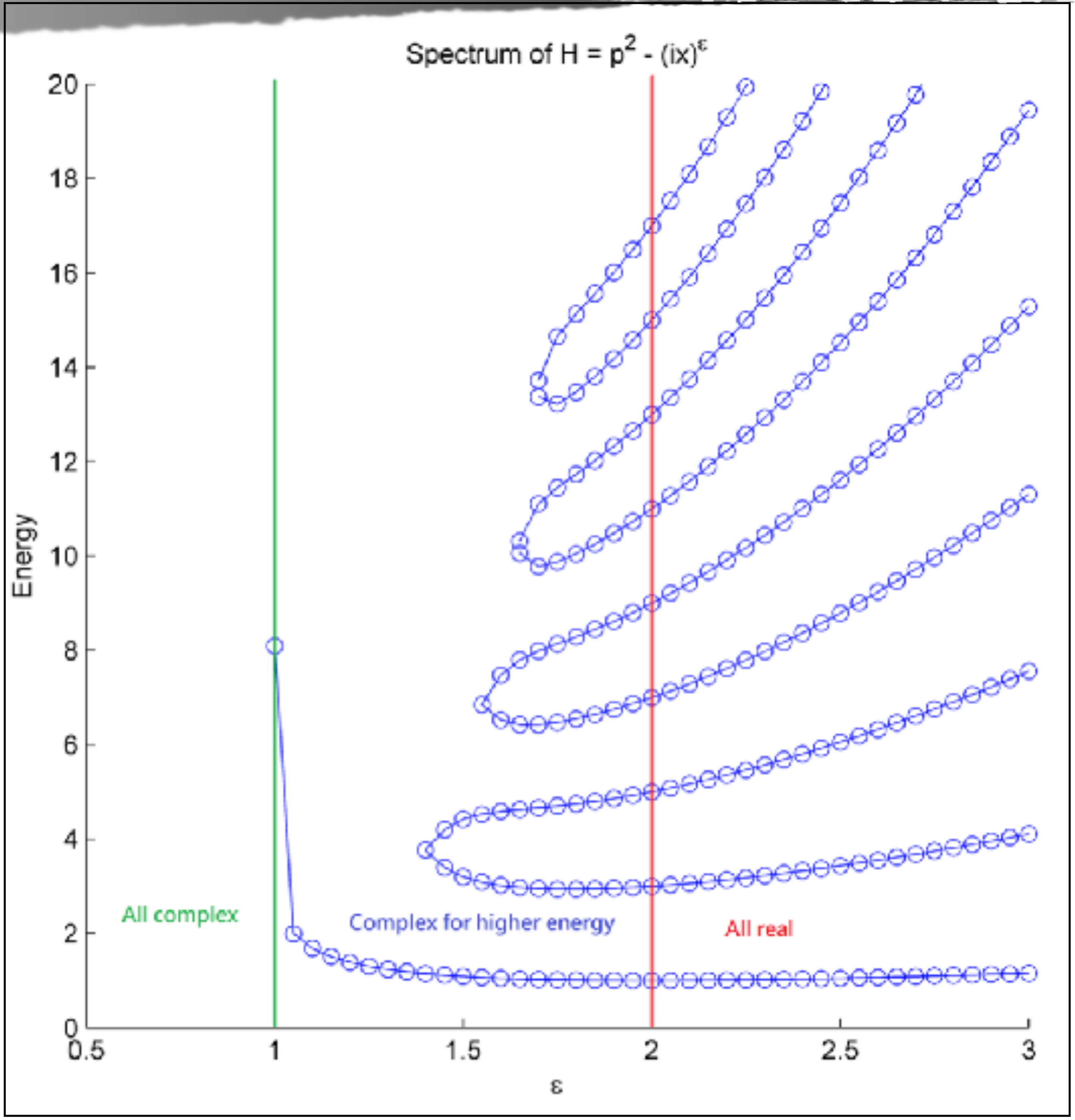
L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).

$$\begin{aligned} a_{nm}(\epsilon) \;&=\; \frac{\sqrt{n(n-1)}}{4}\,\delta_{m,n-2} + \frac{\sqrt{(n+1)(n+2)}}{4}\,\delta_{m,n+2} - \frac{2n+1}{4}\,\delta_{m,n} + \\ &+\; \left[\frac{1-(-1)^{\tilde{n}+\tilde{m}}}{4}\cos(\epsilon\tfrac{\pi}{2}) + \frac{1+(-1)^{\tilde{n}+\tilde{m}}}{4} i\sin(\epsilon\tfrac{\pi}{2})\right] \frac{(-1)^{\lfloor\frac{n}{2}\rfloor+\lfloor\frac{m}{2}\rfloor} 2^{\tilde{n}+\tilde{m}} n!m!}{\lfloor\frac{n}{2}\rfloor!\,\lfloor\frac{m}{2}\rfloor!} \times \\ &\times \Gamma\left(\tfrac{1+\epsilon+\tilde{n}+\tilde{m}}{2}\right) F_A\left(\tfrac{1+\epsilon+\tilde{n}+\tilde{m}}{2}; -\lfloor\tfrac{n}{2}\rfloor, -\lfloor\tfrac{m}{2}\rfloor; \tfrac{2\tilde{n}+1}{2}, \tfrac{2\tilde{m}+1}{2}; 1, 1\right)\delta_{m,n} \end{aligned} \tag{3}$$

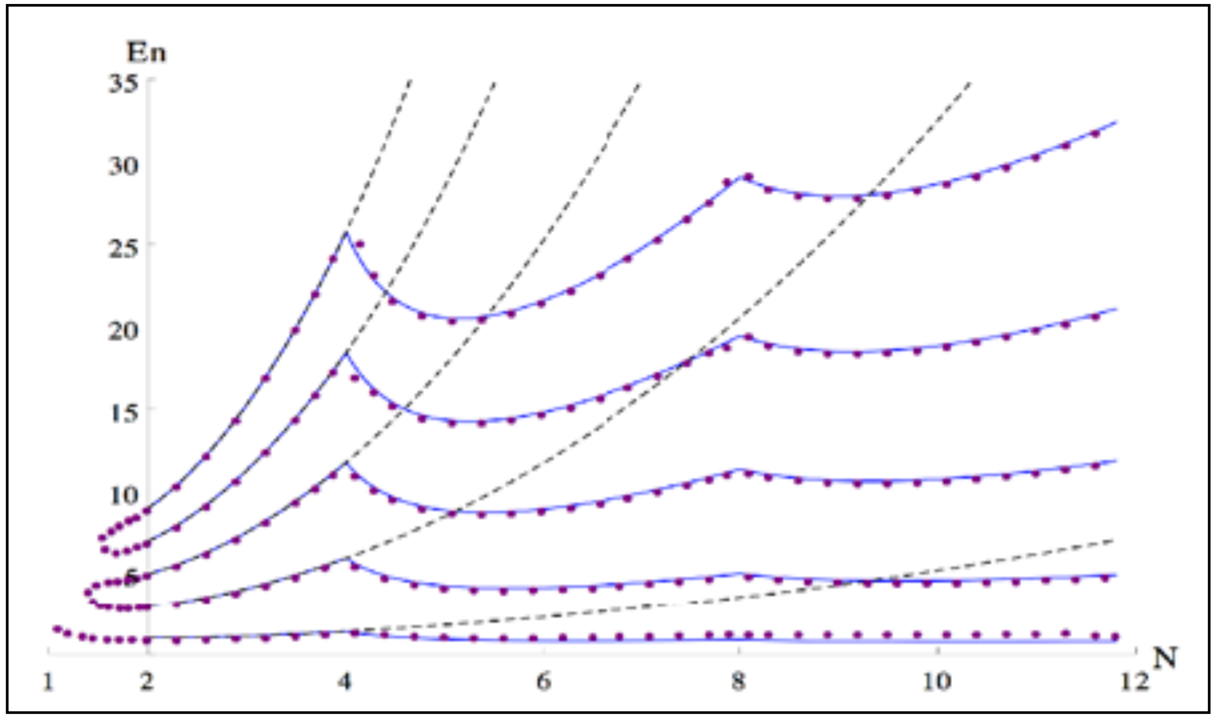
where Γ is an Euler gamma function; F_A is a Lauricella hypergeometric function; symbol $\lfloor\;\rfloor$ denotes a floor function: $\lfloor k \rfloor$ is the largest integer not greater than k; character tilde $\tilde{}$ denotes a binary parity function: $\tilde{k}$ is 0 for an even k and 1 for an odd k.

- By using truncated Fock state basis, we diagonalize the matrix $M_{nn}(\epsilon)$ numerically, having truncated the basis to the first 31, 51, or 71 elements.

Real Spectrum in $\mathcal{PT}$ Hamiltonian



$$\mathcal{H} = p^2 + x^2 (ix)^\epsilon$$



$$\mathcal{H} = p^2 - (ix)^\epsilon$$

REPORTS

OPTICS

Science 346, 972 (2014).

Single-mode laser by parity-time symmetry breaking

Liang Feng,[1*] Zi Jing Wong,[1*] Ren-Min Ma,[1*] Yuan Wang,[1,2] Xiang Zhang[1,2]†

Effective manipulation of cavity resonant modes is crucial for emission control in laser physics and applications. Using the concept of parity-time symmetry to exploit the interplay between gain and loss (i.e., light amplification and absorption), we demonstrate a parity-time symmetry–breaking laser with resonant modes that can be controlled at will. In contrast to conventional ring cavity lasers with multiple competing modes, our parity-time microring laser exhibits intrinsic single-mode lasing regardless of the gain spectral bandwidth. Thresholdless parity-time symmetry breaking due to the rotationally symmetric structure leads to stable single-mode operation with the selective whispering-gallery mode order. Exploration of parity-time symmetry in laser physics may open a door to next-generation optoelectronic devices for optical communications and computing.

PT-Symmetric Quantum Mechanics
Carl M. Bender, Stefan Boettcher, Peter N. Meisinger
Journal of Mathematical Physics, 40 (1999) 2201-2229
10.1063/1.532860

Spectral equivalences, Bethe Ansatz equations, and reality properties in PT-symmetric quantum mechanics
Patrick Dorey, Clare Dunning, Roberto Tateo
Journal of Physics A 34 (2001) 5679-5704.
10.1088/0305-4470/34/28/305

Complex Extension of Quantum Mechanics
Carl M. Bender, Dorje C. Brody, Hugh F. Jones
Physical Review Letters, 89 (2002) 270401 Erratum-ibid 92 (2004) 119902
10.1103/PhysRevLett.89.270401

Faster than Hermitian Quantum Mechanics
Carl M. Bender, Dorje C. Brody, Hugh F. Jones, Bernhard K. Meister
Physical Review Letters, 98 (2007) 040403
10.1103/PhysRevLett.98.040403

The Naimark dilated PT-symmetric brachistochrone
Uwe Guenther, Boris F. Samsonov
Physical Review Letters 101 (2008) 230404
10.1103/PhysRevLett.101.230404

No-ghost theorem for the fourth-order derivative Pais-Uhlenbeck oscillator model
Carl M. Bender, Philip D. Mannheim
Physical Review Letters 100 (2008) 110402
10.1103/PhysRevLett.100.110402

The PT Symmeter

PT Symmetry articles and information

DEC 13, '13
5:57 PM

Local PT symmetry violates the no-signaling principle

AUTHOR **dwh**
CATEGORIES **Tsinghua University, University of Technology Sydney**
TAGS **Min-Hsiu Hsieh, Ray-Kuang Lee, Steven T. Flammia, Yi-Chan Lee**

Yi-Chan Lee, Min-Hsiu Hsieh, Steven T. Flammia, Ray-Kuang Lee

Bender et al. have developed PT-symmetric quantum theory as an extension of quantum theory to non-Hermitian Hamiltonians. We show that when this model has a local PT symmetry acting on composite systems it violates the non-signaling principle of relativity. Since the case of global PT symmetry is known to reduce to standard quantum mechanics, this shows that the PT-symmetric theory is either a trivial extension or likely false as a fundamental theory.

http://arxiv.org/abs/1312.3395
Quantum Physics (quant-ph); High Energy Physics – Theory (hep-th); Mathematical Physics (math-ph)

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COMPLEX CLASSICAL MECHANICS OF A QES

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NONLINEAR MODES AND SYMMETRIES IN

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SYNOPSIS

An Optical System Defies Conventional Band Theory

June 10, 2021 • *Physics* 14, s70

Squeezed wave functions reshape an open quantum system's bulk-boundary properties and generate a new class of parity-time symmetry.

Observation of Non-Bloch Parity-Time Symmetry and Exceptional Points
Lei Xiao, Tianshu Deng, Kunkun Wang, Zhong Wang, Wei Yi, and Peng Xue
Phys. Rev. Lett. 126, 230402 (2021)
Published June 10, 2021

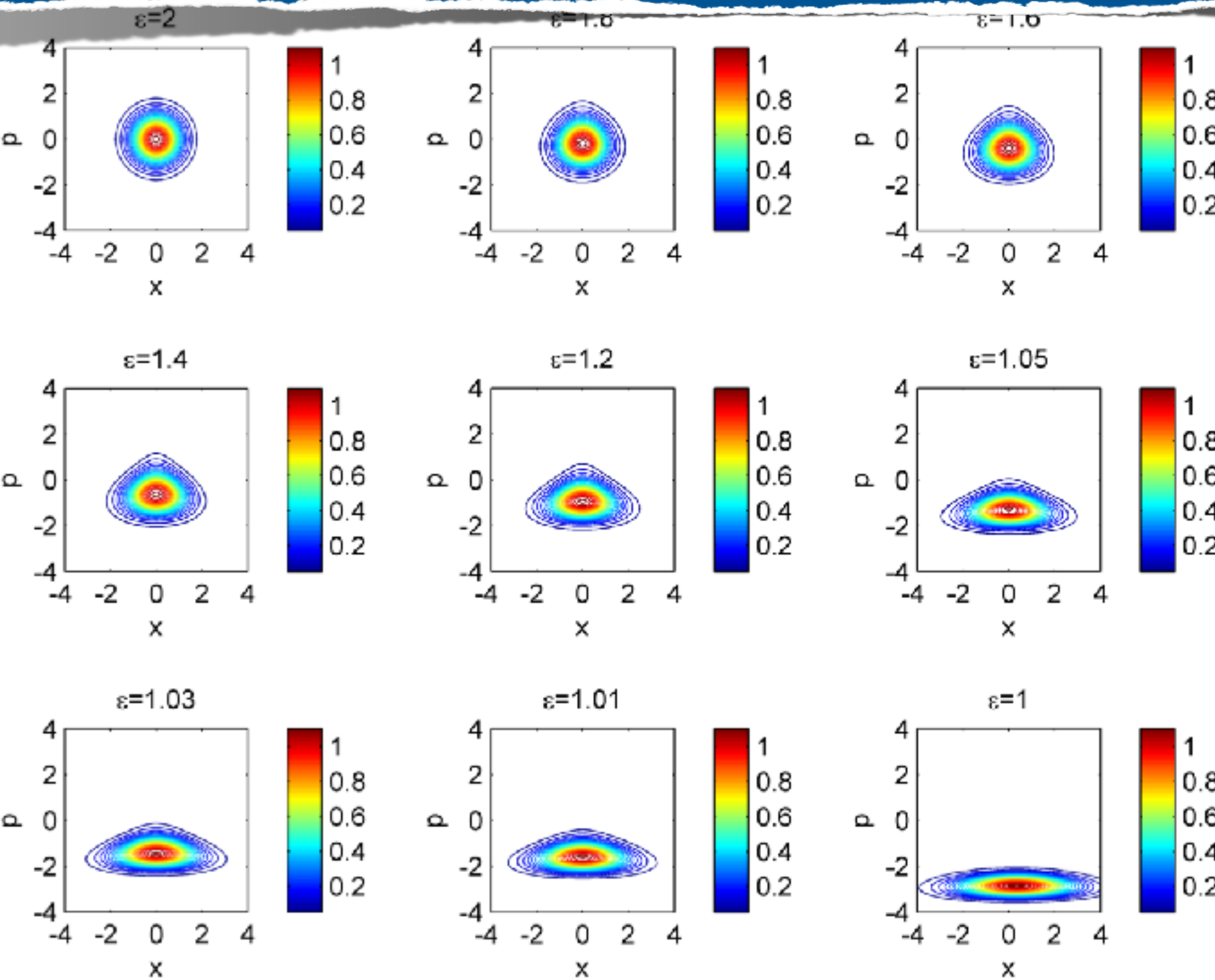
PHYSICAL REVIEW LETTERS **126,** 230402 (2021)

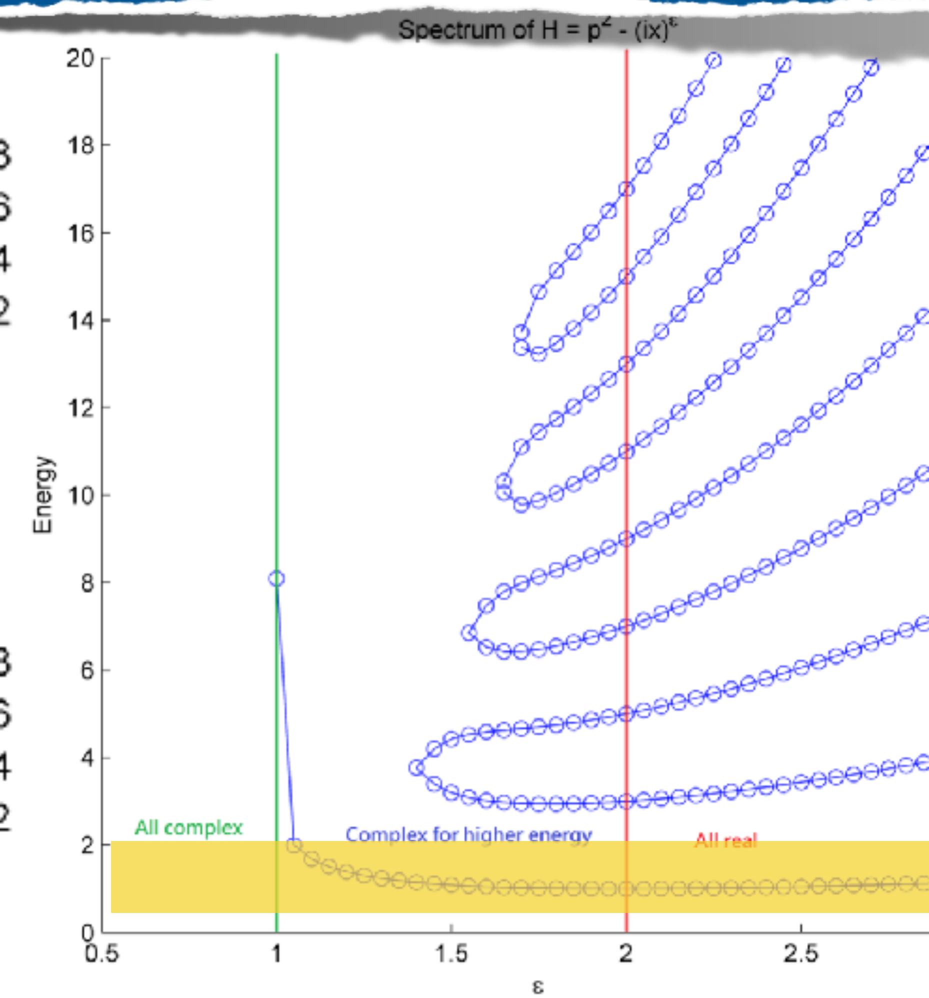
Editors' Suggestion   Featured in Physics

Observation of Non-Bloch Parity-Time Symmetry and Exceptional Points

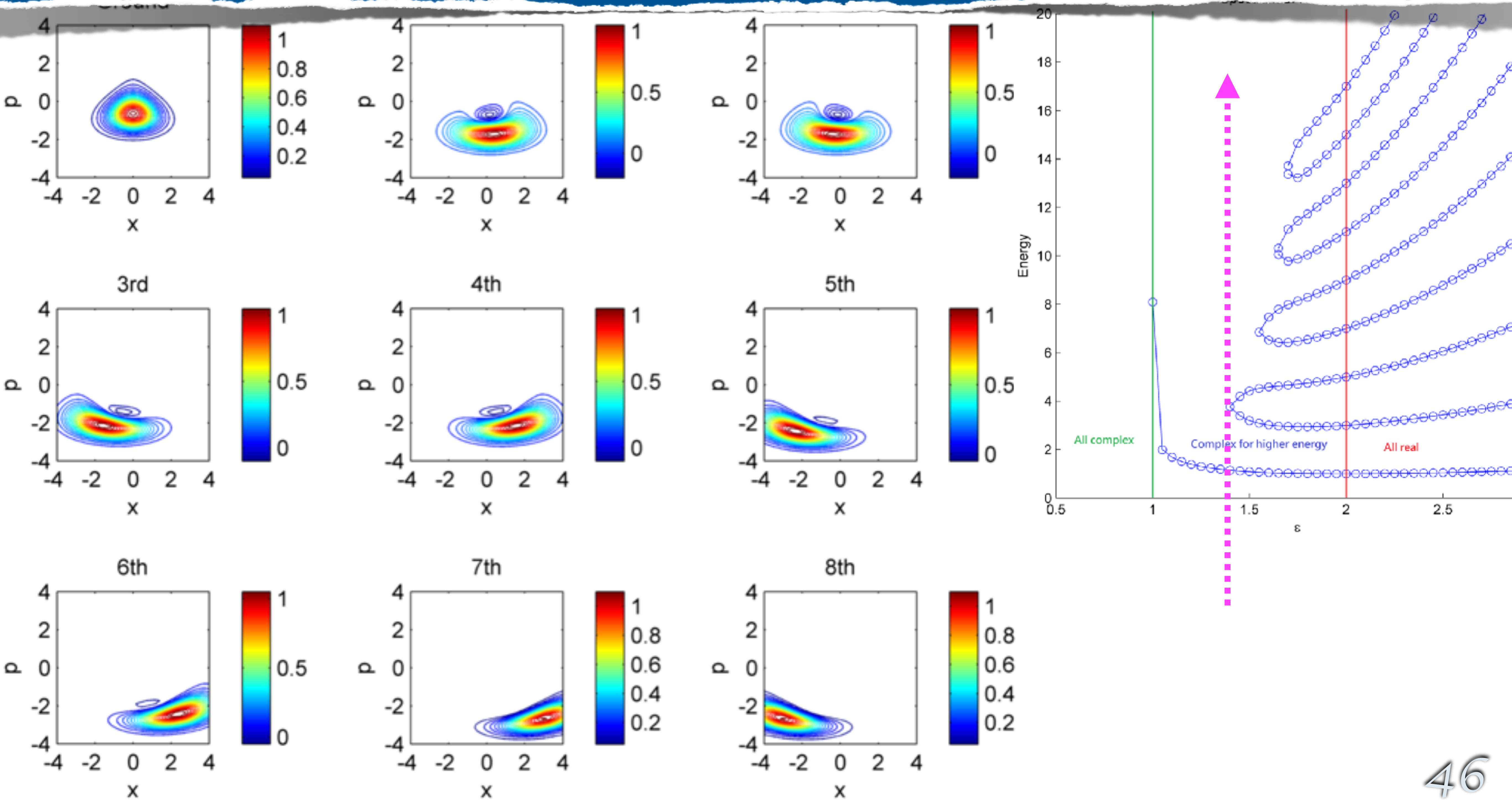
Lei Xiao,[1,*] Tianshu Deng,[2,*] Kunkun Wang,[1] Zhong Wang,[2,†] Wei Yi,[3,4,‡] and Peng Xue[1,§]

$\mathcal{PT}$ in Phase space: Ground states





$\mathcal{PT}$ in Phase space: at the Exceptional Point

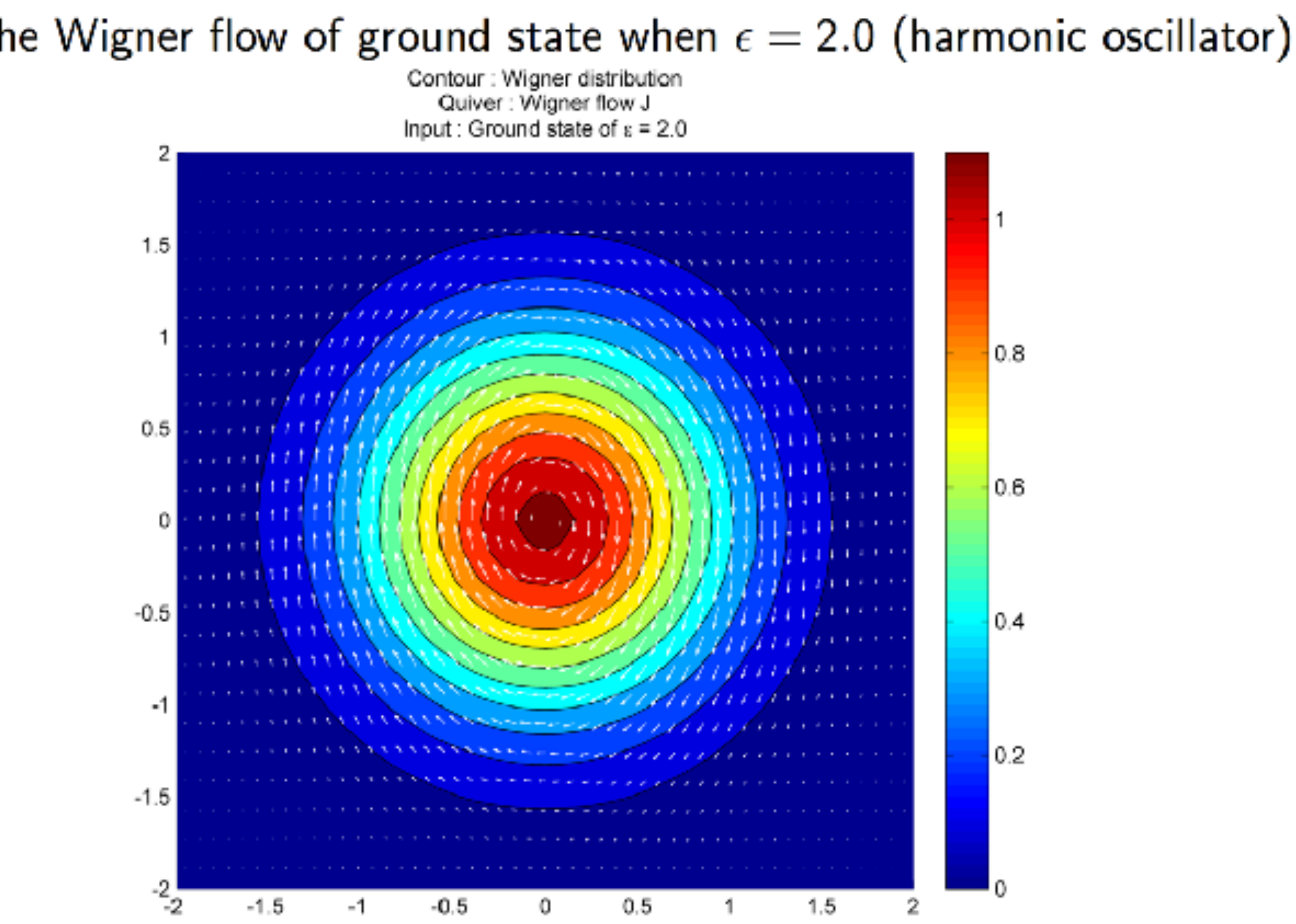


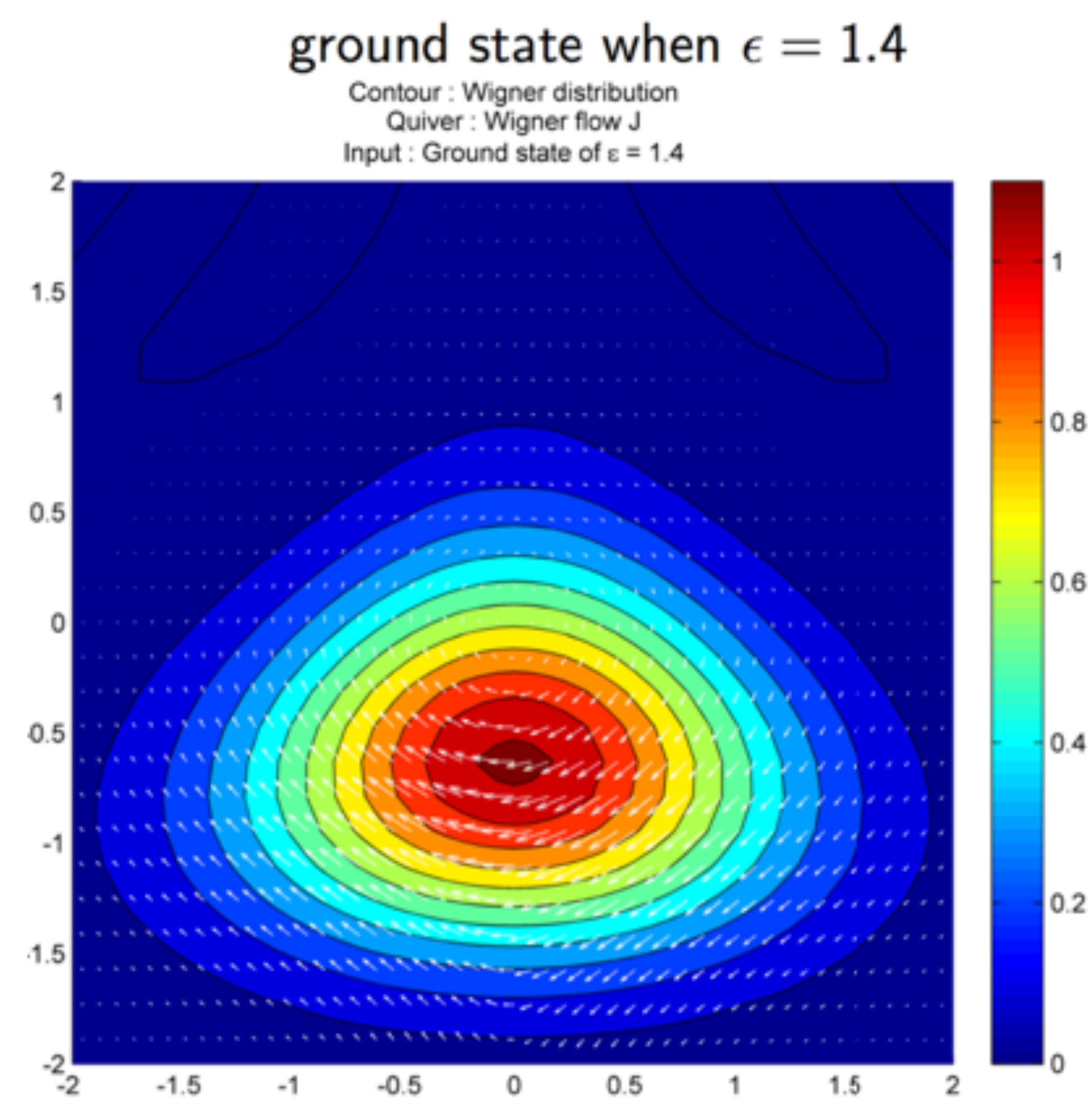
Phase space: Wigner flow

- The time evolution of Wigner distribution can be cast in the form of a flow field $J(x,p;t)$ describes the flow of Wigner's quasiprobability density
  $J_x = \frac{p}{m} W(x,p,t)$
  $J_p = \int d\xi e^{\frac{i\xi p}{\hbar}} \Psi^*(x+\frac{\xi}{2},t)\Psi(x-\frac{\xi}{2},t)[\frac{V(x-\frac{\xi}{2})-V(x)}{\xi} - \frac{V^*(x+\frac{\xi}{2})-V^*(x)}{\xi}]$
- Continuity equation for Hermitian Hamiltonian
  $\frac{\partial}{\partial t} W(x,p;t) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial p} J_p = 0$
- Continuity equation for Hermitian non-Hamiltonian
  $\frac{\partial}{\partial t} W(x,p,t) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial p} J_p = \frac{i}{\hbar}[V^*(x,t) - V(x,t)] W(x,p,t)$

O. Steuernagel, D. Kakofengitis, and G. Ritter, Phys. Rev. Lett. 110, 030401 (2013).

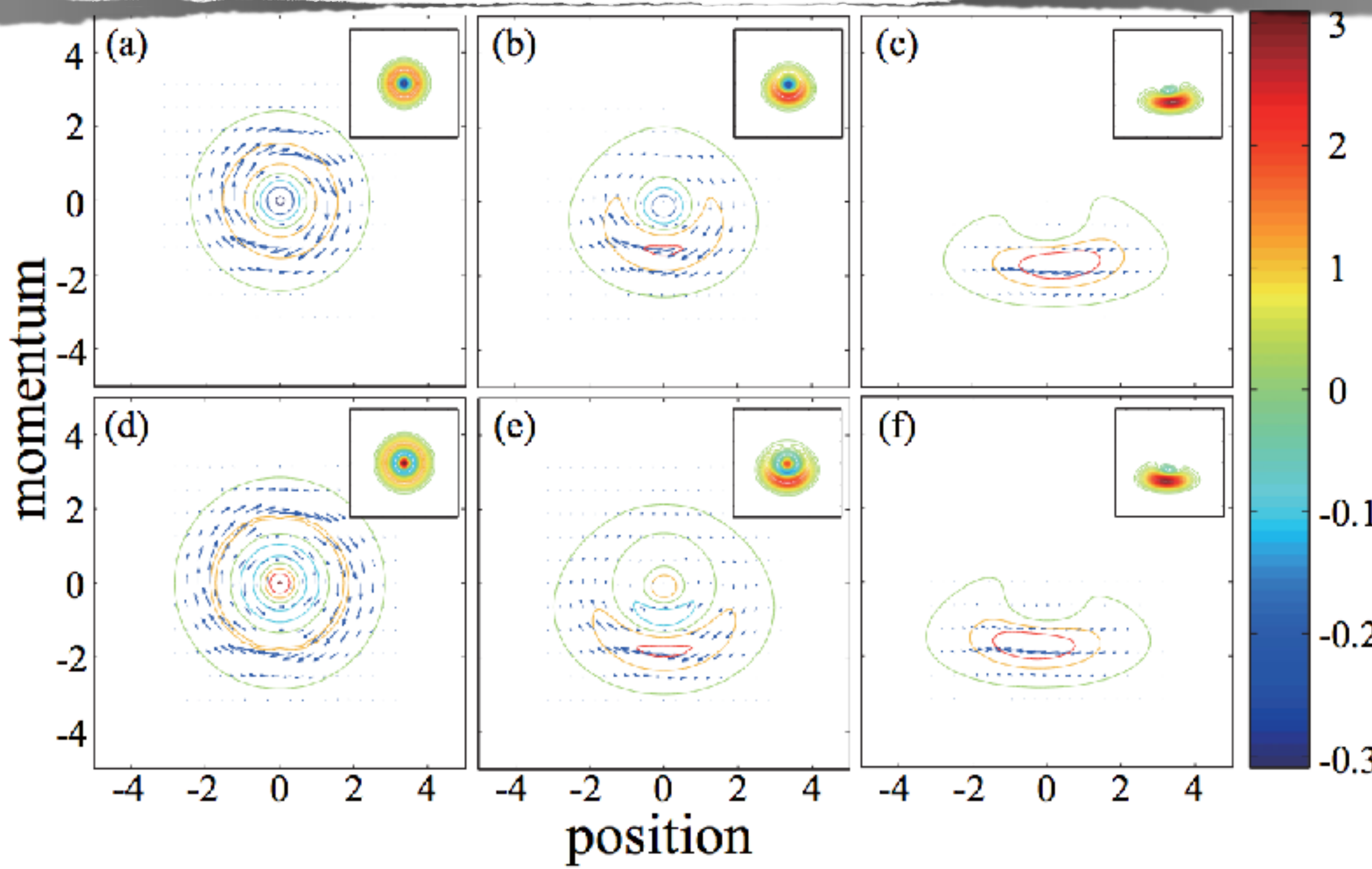
Wigner flow of $\mathcal{PT}$: Ground states



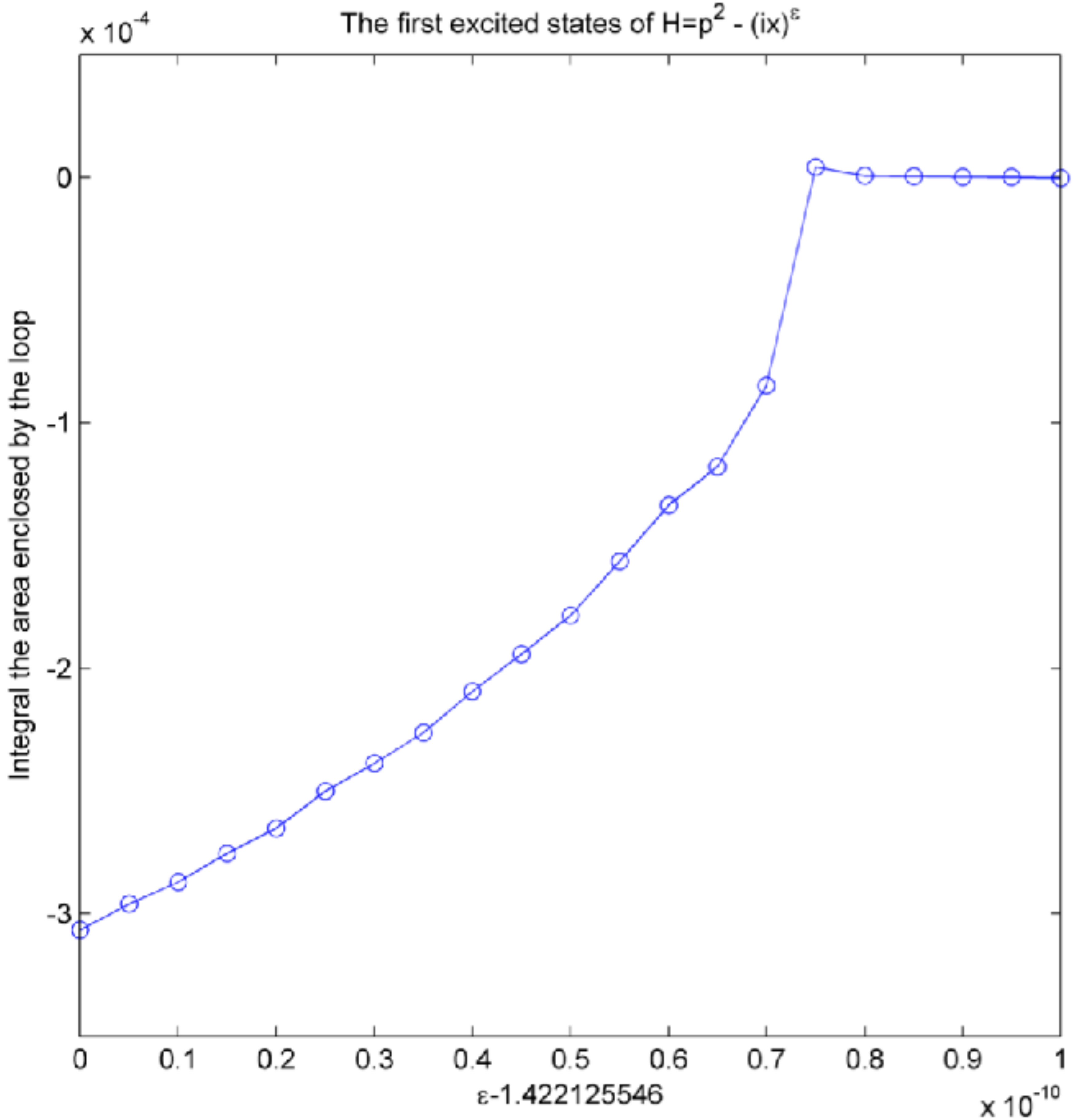


L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).

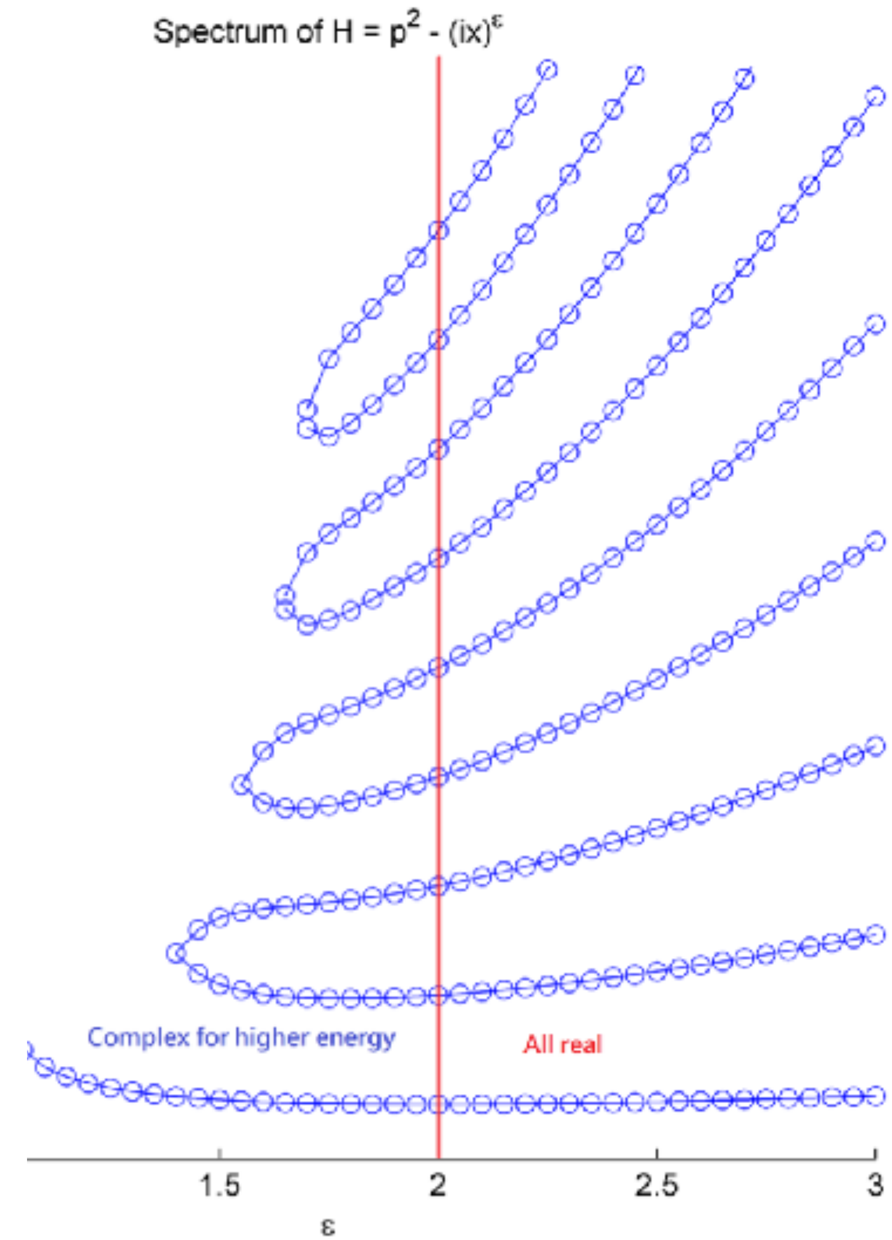
Wigner flow of $\mathcal{PT}$: 1st/2nd Excited states



L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).



$\!\!/$), when $\frac{\partial W}{\partial t} = 0$



Parity-Time Hamiltonian

- Generalized Harmonic Oscillator (continuous):

$$\mathcal{H} = p^2 + x^2\,(ix)^\epsilon$$

- Spin-$\frac{1}{2}$ or two-mode coupler (discrete):

$$H = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$

Alice: Local $\mathcal{PT}$ Hamiltonian

- α = 0 (n π), Hermitian
- α ≠ 0 (n π), non-Hermitian
- α = ±π/2, PT-breaking

Hamiltonian: $H = s \begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$

C. M. Bender et al., Phys. Rev. Lett., 98, 040403 (2007)

Eigenstates and eigenvalues:

$$|E_+^R\rangle = \frac{e^{i\alpha/2}}{\sqrt{2\cos\alpha}} \begin{pmatrix} 1 \\ e^{-i\alpha} \end{pmatrix}$$

$$|E_-^R\rangle = \frac{ie^{-i\alpha/2}}{\sqrt{2\cos\alpha}} \begin{pmatrix} 1 \\ -e^{i\alpha} \end{pmatrix}$$

$$E_\pm = \pm s\cos\alpha \equiv \pm\omega_0/2$$

ω_0 **: Energy difference**

Time evolution operator:

$$U(t) = e^{-iHt} = \frac{1}{\cos\alpha} \begin{pmatrix} \cos(t'-\alpha) & -i\sin t' \\ -i\sin t' & \cos(t'+\alpha) \end{pmatrix} \qquad t' \equiv \frac{\omega_0 t}{2}$$

Initial state and final state: $|\psi_I\rangle \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **and** $|\psi_F\rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\psi(t)\rangle \propto \begin{pmatrix} \cos(t'-\alpha) \\ -i\sin t' \end{pmatrix} \qquad \tau = \frac{2(\alpha+\pi/2)}{\omega_0} \qquad \begin{matrix} \alpha \to -\pi/2 \\ \tau \to 0 \end{matrix}$$

Parity-Time Symmetry in Optics

- In optics: propagation replaces time-evolution, i.e., $t \rightarrow z$

$$\hat{H} = p^2/2 + V(x)$$

- $\mathcal{PT}$ Hamiltonian in Optics:

$$\hat{P}\hat{T}\mathcal{H} = \mathcal{H}\hat{P}\hat{T} = p^2/2 + V^*(-x) = p^2/2 + V(x).$$

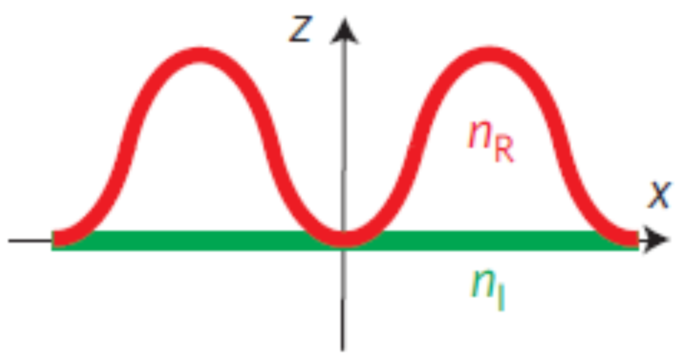
That is, the refractive index profile satisfies

$$V(x) = V^*(-x),$$

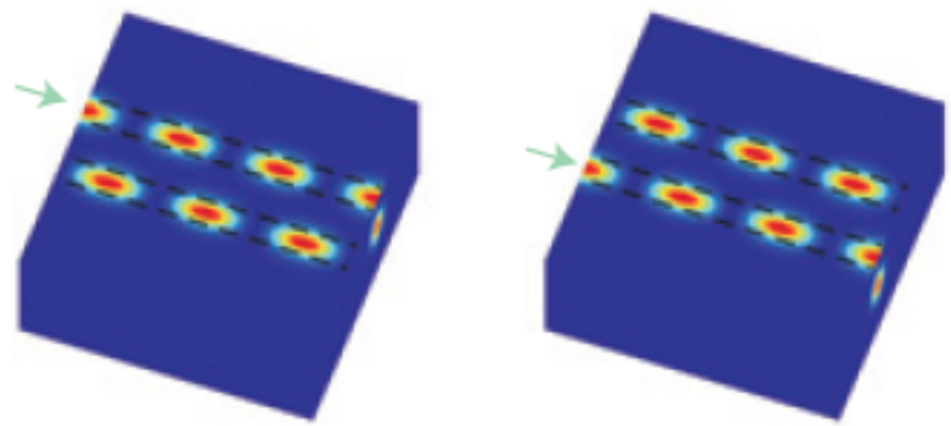
a *even function* for the real part of the index; and a *odd function* for the imaginary part of the index.

Experimental realization in Optical Systems

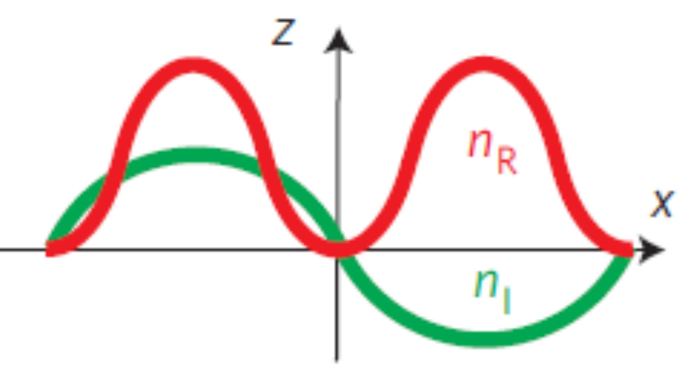
Conventional coupled system



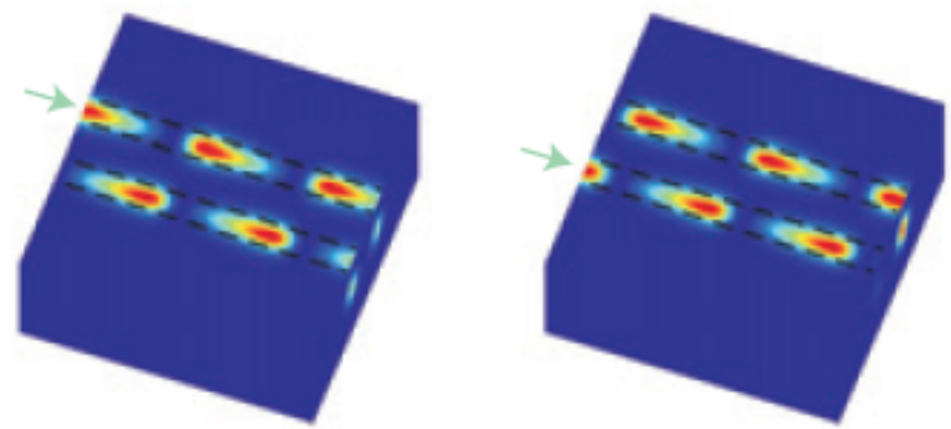
Conventional system



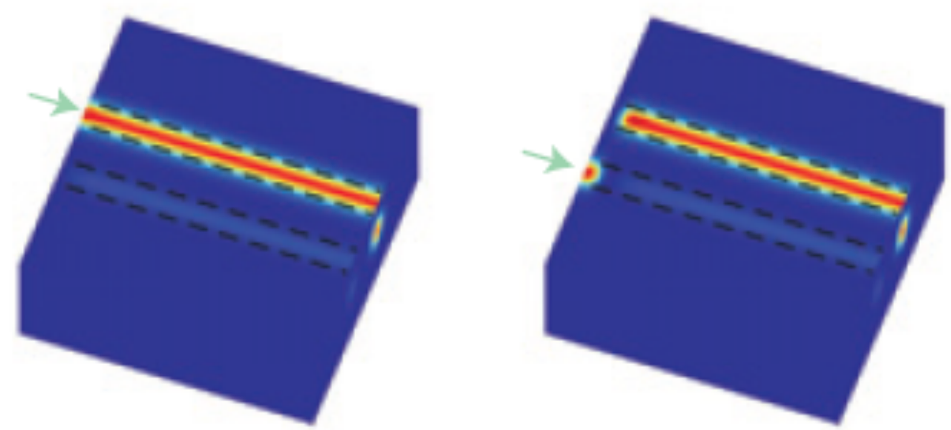
PT-symmetric coupled system



PT-symmetric system below threshold



PT-symmetric system above threshold



Coupled mode approach

$$i\frac{dE_1}{dz} - i\frac{\gamma_G}{2}E_1 + \kappa E_2 = 0$$

$$i\frac{dE_2}{dz} + \kappa E_1 + i\frac{\gamma_L}{2}E_2 = 0$$

Condition of PT symmetry

$$\gamma_G = \gamma_L = \gamma$$

$$H = \begin{pmatrix} i\gamma_G & \kappa \\ \kappa & -i\gamma_L \end{pmatrix}$$

Gain = Loss

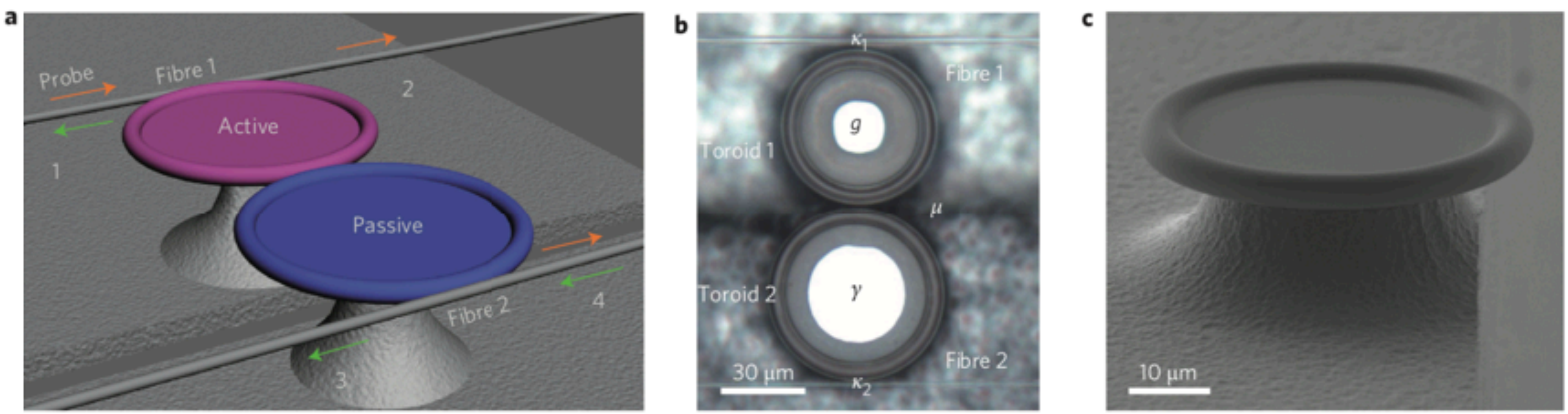
C. E. Rüeter et al., Nature Phys. 6, 192 (2010).

LETTERS
PUBLISHED ONLINE: 22 JUNE 2014 | DOI: 10.1038/NPHOTON.2014.133

nature photonics

Parity–time symmetry and variable optical isolation in active–passive-coupled microresonators

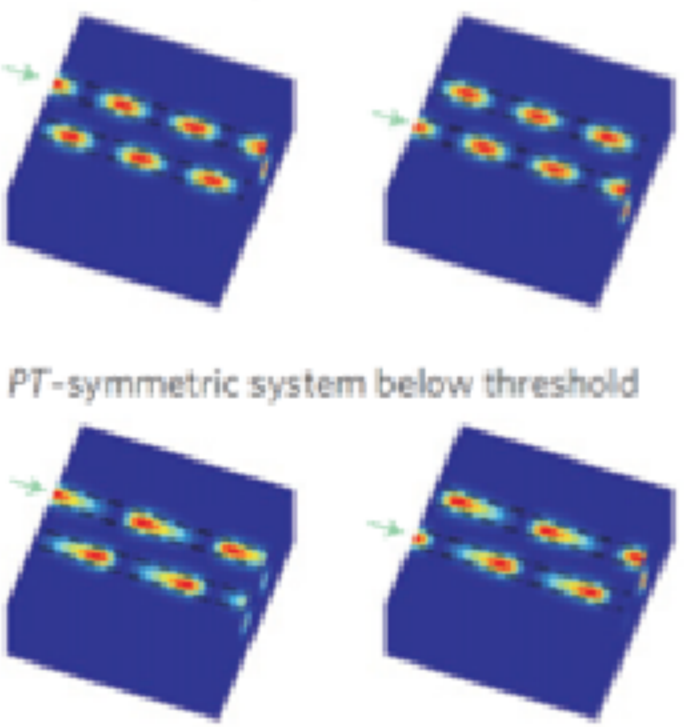
Long Chang[1,2], Xiaoshun Jiang[1,2]*, Shiyue Hua[1,2], Chao Yang[1,2], Jianming Wen[3]*, Liang Jiang[3], Guanyu Li[1,2], Guanzhong Wang[1,2] and Min Xiao[1,2,4]*

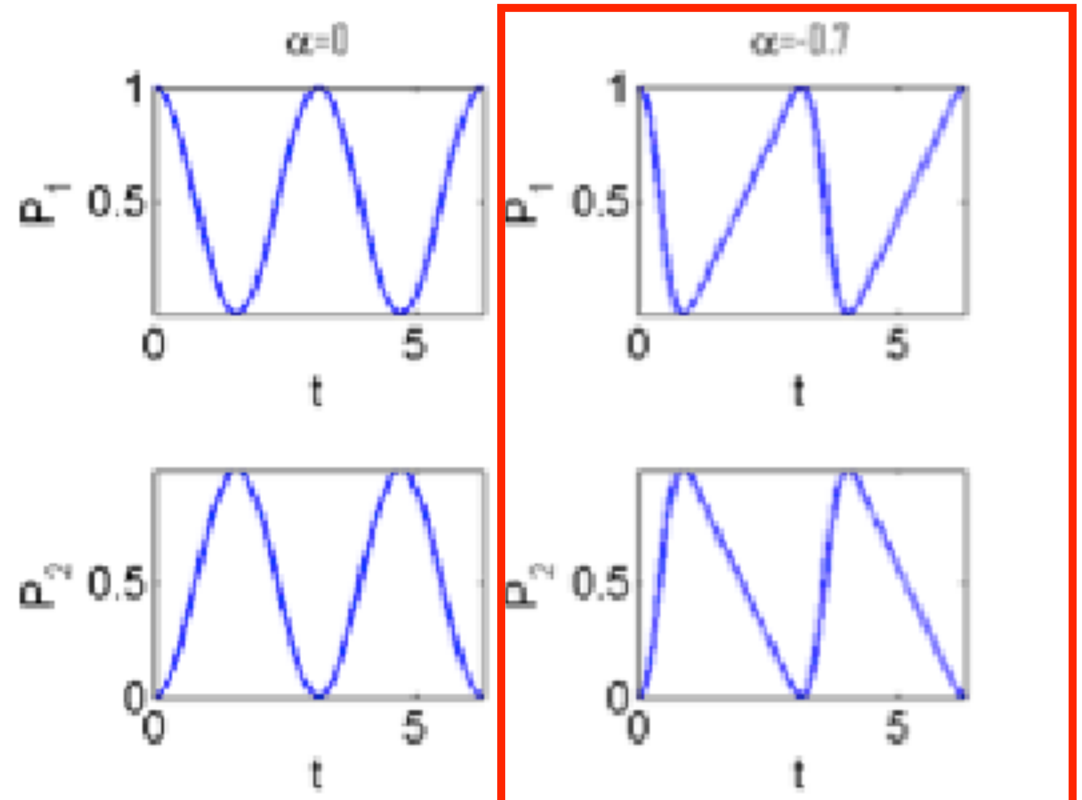


Ultrafast Spin-flip

$$i\frac{\mathrm{d}E_1}{\mathrm{d}z} - i\frac{\gamma G}{2}E_1 + \kappa E_2 = 0$$
$$i\frac{\mathrm{d}E_2}{\mathrm{d}z} + \kappa E_1 + i\frac{\gamma L}{2}E_2 = 0$$

$$H = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$



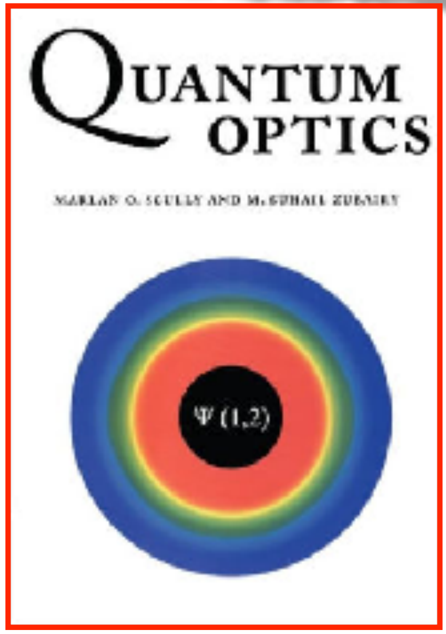


$$|\Psi(t)\rangle \propto \begin{pmatrix} \cos\dfrac{\theta(t)}{2} \\ i\sin\dfrac{\theta(t)}{2} \end{pmatrix}$$

$$P_1 = \cos^2\left(\frac{\theta(t)}{2}\right)$$
$$P_2 = \sin^2\left(\frac{\theta(t)}{2}\right)$$

Quantum Noises with $\mathcal{PT}$ Hamiltonian

$$H = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$



The two output fields:

$$\hat{a}_1(z) = A_1\ \hat{a}_1(0) + B\ \hat{a}_2(0)$$
$$\hat{a}_2(z) = B\ \hat{a}_1(0) + A_2\ \hat{a}_2(0)$$

where

$A_1 = \cos(\zeta z) + \sin(\zeta z)\tan\alpha$, $A_2 = \cos(\zeta z) - \sin(\zeta z)\tan\alpha$, and $B = -i\sin(\zeta z)\sec\alpha$.

The commutation relations are not conserved: $\frac{d}{dz}\left[\hat{a}_j, \hat{a}_j^\dagger\right] \neq 0.$

Quantum Noises with $\mathcal{PT}$ Hamiltonian

$$\hat{a}_1(z) = A_1(z)\ \hat{a}_1(0) + B(z)\ \hat{a}_2(0) + \int_0^z dz'\ \left[A_1(z-z')\hat{n}_1(z') + B(z-z')\hat{n}_2(z')\right]$$

$$\hat{a}_2(z) = B(z)\ \hat{a}_1(0) + A_2(z)\ \hat{a}_2(0) + \int_0^z dz'\ \left[B(z-z')\hat{n}_1(z') + A_2(z-z')\hat{n}_2(z')\right]$$

Obviously, the third terms of each output field in Eq.(4) are the contribution of Langevin noise operators coming from reservoir, which are independent of input fields. The two fields should satisfies bosonic commutation relations. That is

$$\left[\hat{a}_1, \hat{a}_1^\dagger\right] = \left[\hat{a}_2, \hat{a}_2^\dagger\right] = 1$$

$$\left[\hat{a}_1, \hat{a}_2\right] = \left[\hat{a}_1^\dagger, \hat{a}_2^\dagger\right] = \left[\hat{a}_1, \hat{a}_2^\dagger\right] = 0$$

Quantum Noises with $\mathcal{PT}$ Hamiltonian

Renormalize two field operators:

$$\hat{\phi}_1(z) = \frac{A_1}{\sqrt{|A_1|^2 + |B|^2}}\hat{a}_1(0) + \frac{B}{\sqrt{|A_1|^2 + |B|^2}}\hat{a}_2(0)$$

$$\hat{\phi}_2(z) = \frac{B}{\sqrt{|A_2|^2 + |B|^2}}\hat{a}_1(0) + \frac{A_2}{\sqrt{|A_2|^2 + |B|^2}}\hat{a}_2(0)$$

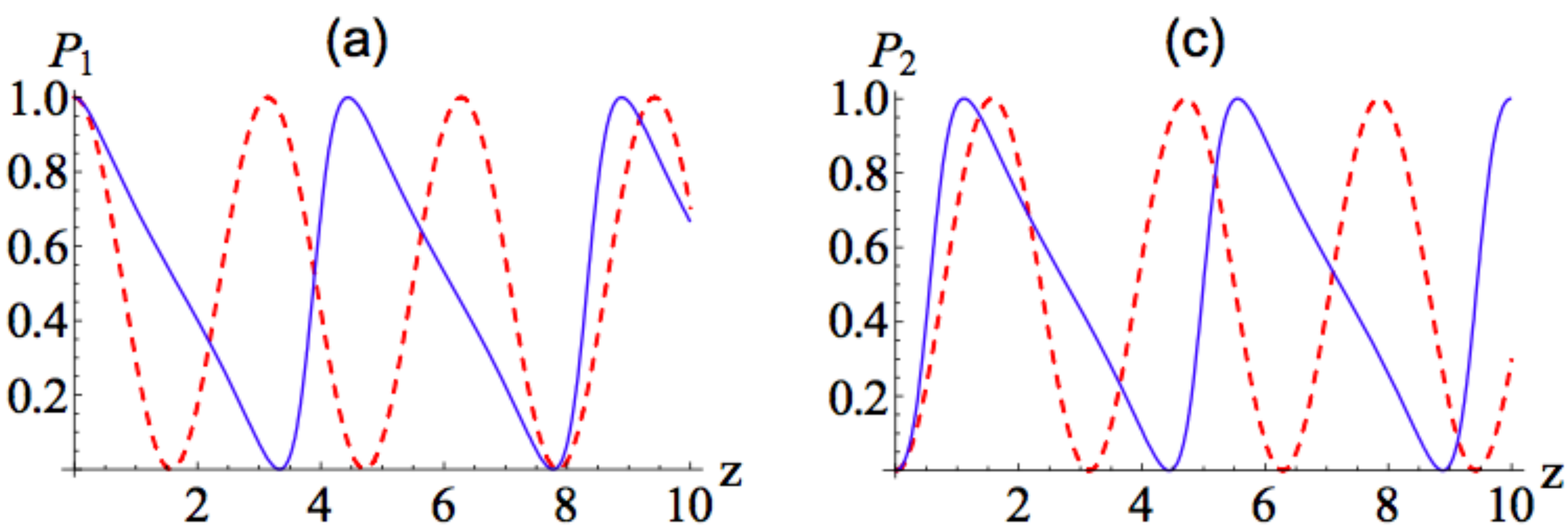
and then $\left[\hat{\phi}_1, \hat{\phi}_1^\dagger\right] = \left[\hat{\phi}_2, \hat{\phi}_2^\dagger\right] = 1$, and $\left[\hat{\phi}_1, \hat{\phi}_2^\dagger\right] \neq 0$.

Coherent state inputs: $|\psi\rangle_{in} = |\alpha\rangle_1 |\alpha\rangle_2$.

$$E_1(z) = \frac{1}{\mathcal{N}_1} \left\{ \left[\cos(\zeta z) + \frac{\gamma}{\zeta} \sin(\zeta z) \right] E_1(0) + \left[-i \frac{\chi}{\zeta} \sin(\zeta z) \right] E_2(0) \right\}$$
$$E_2(z) = \frac{1}{\mathcal{N}_2} \left\{ \left[-i \frac{\chi}{\zeta} \sin(\zeta z) \right] E_1(0) + \left[\cos(\zeta z) - \frac{\gamma}{\zeta} \sin(\zeta z) \right] E_2(0) \right\}$$

Vector form:

$$E_1(z) = \frac{1}{\mathcal{N}_1} \begin{pmatrix} A_1 \\ B \end{pmatrix} \; ; \; E_2(z) = \frac{1}{\mathcal{N}_2} \begin{pmatrix} B \\ A_2 \end{pmatrix}$$



More *or* Less

Non-Hermitian
(Dissipative system)

Hermitian

PT:

Additional
degree of freedom ?

or

Constrains ?

OK, so the eigenvalues are real …
But is this quantum mechanics??

C. M. Bender's slides.

- Probabilistic interpretation??
- Hilbert space with a positive metric??
- Unitarity??

ASK

Alice & Bob

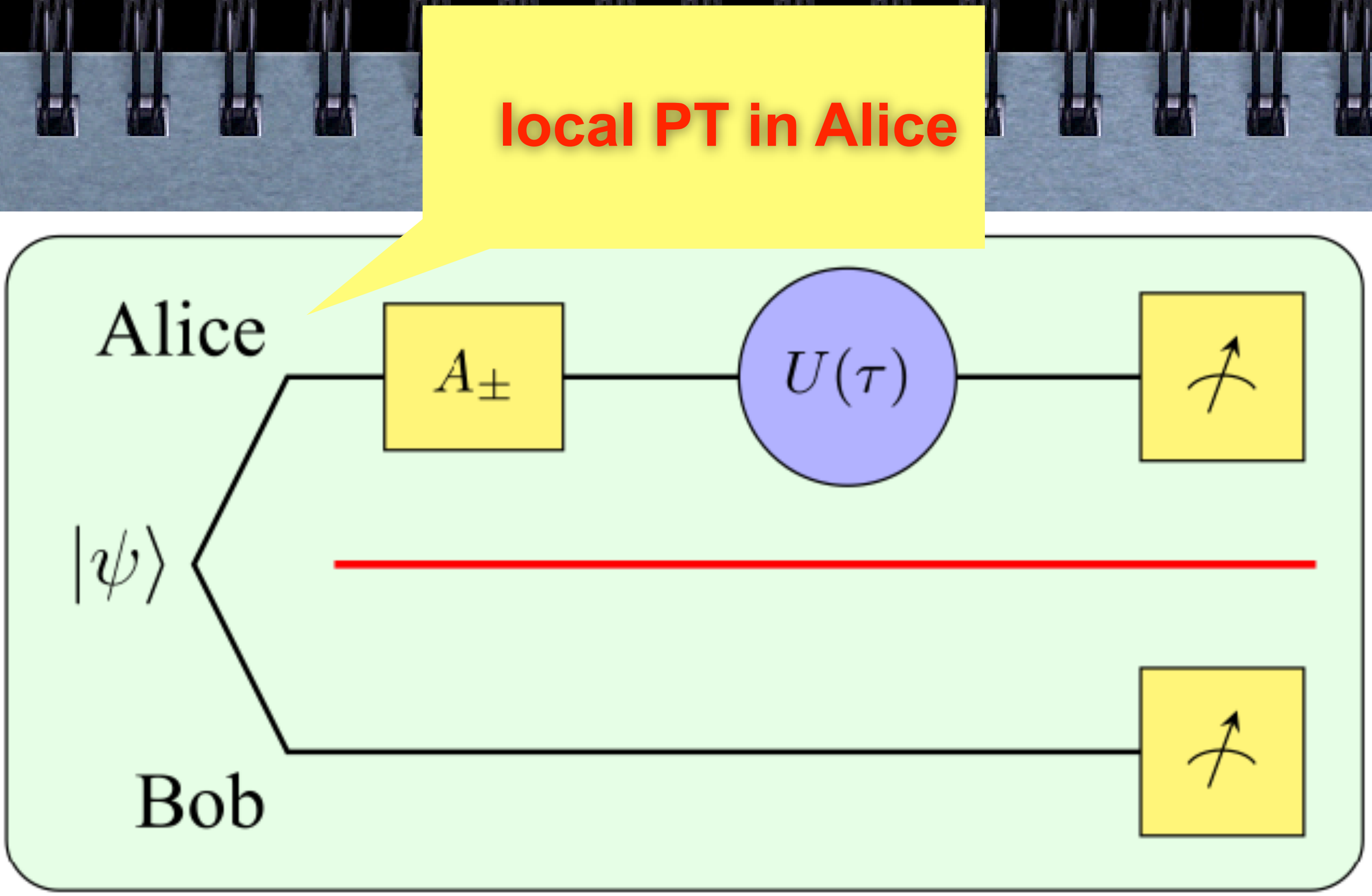


FIG. 1 (color online). Alice and Bob initially share a maximally entangled state $|\psi\rangle$ and are spacelike separated (red line). The circled operators are $\mathcal{PT}$ symmetric, while rectangular ones are conventional operators; the identity gate (a wire) is the same for both theories. Alice's initial choice of $A_\pm$ is followed by $\mathcal{PT}$-symmetric time evolution $U(\tau)$. A projective measurement at the end leads to superluminal signaling.

No-Signaling principle

- *no-signaling* conditions from special relativity:
  $\forall b, B, A_{\pm}$,

$$\sum_a P(a,b|A_+,B) = \sum_a P(a,b|A_-,B) = P(b|B),$$

  where a, b are measurement outcomes of two space-like separated parties Alice and Bob, and $A_{\pm}$ and B are different local measurements done by Alice and Bob on their respective sides.

- The meaning *no-signaling* is that Bob's probability distribution over local measurement outcomes is unaffected by Alice's choice of local measurements.

Alice: Local $\mathcal{PT}$ Hamiltonian

- α = 0 (n π), Hermitian
- α ≠ 0 (n π), non-Hermitian
- α = ±π/2, PT-breaking

C. M. Bender et al.,
Phys. Rev. Lett., 98, 040403 (2007)

Hamiltonian: $H = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$

Eigenstates and eigenvalues:

$$|E_+^R\rangle = \frac{e^{i\alpha/2}}{\sqrt{2\cos\alpha}}\begin{pmatrix} 1 \\ e^{-i\alpha} \end{pmatrix}$$

$$|E_-^R\rangle = \frac{ie^{-i\alpha/2}}{\sqrt{2\cos\alpha}}\begin{pmatrix} 1 \\ -e^{i\alpha} \end{pmatrix}$$

$$E_\pm = \pm s\cos\alpha \equiv \pm\omega_0/2$$

ω_0 **: Energy difference**

Time evolution operator:

$$U(t) = e^{-iHt} = \frac{1}{\cos\alpha}\begin{pmatrix} \cos(t'-\alpha) & -i\sin t' \\ -i\sin t' & \cos(t'+\alpha) \end{pmatrix} \qquad t' \equiv \frac{\omega_0 t}{2}$$

Initial state and final state: $|\psi_I\rangle \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **and** $|\psi_F\rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\psi(t)\rangle \propto \begin{pmatrix} \cos(t'-\alpha) \\ -i\sin t' \end{pmatrix} \qquad \tau = \frac{2(\alpha+\pi/2)}{\omega_0} \qquad \begin{matrix} \alpha \to -\pi/2 \\ \tau \to 0 \end{matrix}$$

- According to the process of the *gedanken experiment* and the previous two assumptions, if Alice first uses the operator $A_+ = I$ or $A_- = \sigma_x$ with respect to the information she wants to send and sets the time of evolution to $\tau = \pi/\Delta E$, the joint final states are

$$
\begin{aligned}
|\psi_f^\pm\rangle &= [U(\tau)A_\pm \otimes e^{-iIt} I]|\psi\rangle \\
&\propto \frac{1}{\sqrt{2}}\left[e^{i\phi_+}\frac{1}{ie^{-i\epsilon}}|+_x\rangle \pm e^{i\phi_-}\frac{1}{ie^{i\epsilon}}|-_x\rangle\right],
\end{aligned}
$$

where $e^{i\phi_\pm} = \frac{\sin\alpha \mp i}{\sqrt{1+\sin^2\alpha}}$ and $e^{i\epsilon} = \frac{-2\sin\alpha + i\cos^2\alpha}{1+\sin^2\alpha}$.

- Here we note that the *normalization constants* have been renormalized in the way of conventional quantum mechanics, since in the end Bob will measure it using conventional quantum mechanics.

- Following the previous protocol, Alice and Bob both measure their systems with the conventional quantum projectors $|\pm_y\rangle\langle\pm_y|$, which gives the joint probabilities

$$P(a,b|A_\pm,B) = \langle\psi_f^\pm|(|a\rangle\langle a|\otimes|b\rangle\langle b|)|\psi_f^\pm\rangle,$$

where the possible outcomes of a and b are $+_y$ or $-_y$.

cos ϵ = 0

if α = 0 (n π)

- After a simple calculation, we have the two marginal probabilities

$$\sum_{a=\pm_y} P(a,+_y|A_\pm,B) = \frac{1}{2}[1 \pm \cos\epsilon\sin(2\phi_+ - \epsilon)].$$

The two equations are the same only when $\cos\epsilon = 0$, which implies that the no-signaling condition is always violated unless $\alpha = n\pi$, i.e. the system used by Alice is Hermitian.

PRL 112, 130404 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

Local $\mathcal{PT}$ Symmetry Violates the No-Signaling Principle

Yi-Chan Lee,[1,2,*] Min-Hsiu Hsieh,[2] Steven T. Flammia,[3] and Ray-Kuang Lee[1,4]

Synopsis: Reflecting on an Alternative Quantum Theory

APS/Alan Stonebraker

Local PT Symmetry Violates the No-Signaling Principle
Yi-Chan Lee, Min-Hsiu Hsieh, Steven T. Flammia, and Ray-Kuang Lee
Phys. Rev. Lett. **112**, 130404 (2014)
Published April 3, 2014

Since 1998, physicists have explored a modified quantum theory based on spacetime reflections. This so-called $\mathcal{PT}$ symmetric theory makes certain predictions—such as time shortcuts in the evolution between two states—that conflict with conventional quantum mechanics, while still being compatible with observations. However, a new assessment of this alternative model suggests that it is fundamentally flawed. In *Physical Review Letters*, the authors show the $\mathcal{PT}$ symmetric theory violates the proscription against faster-than-light communication.

A basic tenet of quantum mechanics is that the Hamiltonian equation describing the energy in a quantum system should have a mathematical property, called Hermiticity, so as to guarantee that the predicted energy values are all real. But one can ensure real energy values another way, by insisting that the Hamiltonian is symmetric to a combination of reflections in space ($\mathcal{P}$) and time ($\mathcal{T}$). In the past, researchers have used the $\mathcal{PT}$ symmetric model to describe certain optical systems.

If $\mathcal{PT}$ symmetry were actually fundamental—replacing the Hermiticity constraint—then radical consequences would arise, such as quantum systems evolving faster than normally predicted. But a challenge to this notion comes from Yi-Chan Lee of National Tsing-Hua University in Hsinchu City, Taiwan, and colleagues. They consider two implicit assumptions in the $\mathcal{PT}$ symmetric theory, involving how it is locally defined and how its predictions are computed. They put these assumptions to the test with a classic thought experiment, in which Alice and Bob share two entangled states. By choosing how she measures her state, the team found that Alice could send information to Bob faster than the speed of light. The authors believe this result rules out $\mathcal{PT}$ symmetry as a fundamental theory, but it still could be useful as an effective theory and an interesting model for open systems in classical optics. – *Michael Schirber*

nature photonics

LETTERS

PUBLISHED ONLINE: 8 AUGUST 2016 | DOI: 10.1038/NPHOTON.2016.144

Experimental investigation of the no-signalling principle in parity–time symmetric theory using an open quantum system

Jian-Shun Tang[1,2†], Yi-Tao Wang[1,2†], Shang Yu[1,2], De-Yong He[1,2], Jin-Shi Xu[1,2], Bi-Heng Liu[1,2], Geng Chen[1,2], Yong-Nan Sun[1,2], Kai Sun[1,2], Yong-Jian Han[1,2*], Chuan-Feng Li[1,2*] and Guang-Can Guo[1,2]

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Naimark Dilation

For a non-Hermitian Hamiltonian with real eigenvalues

$$H|E_\pm^R\rangle = E_\pm|E_\pm^R\rangle \qquad H^\dagger|E_\pm^L\rangle = E_\pm|E_\pm^L\rangle$$

$$\Phi \equiv [|E_+^R\rangle, |E_-^R\rangle] \qquad \Xi \equiv [|E_+^L\rangle, |E_-^L\rangle]$$

Biorthonormality of left and right eigenstates

$$\Xi^\dagger\Phi = \Phi^\dagger\Xi = \mathbf{I}_2$$

$$\widetilde{E} \equiv \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}$$

Dilated Hilbert space

$$\mathbf{V} = \sqrt{\frac{\cos\alpha}{2}}\begin{pmatrix} \Phi & \Xi \\ \Xi & -\Phi \end{pmatrix} \qquad \mathbf{E} \equiv \begin{pmatrix} \widetilde{E} & 0 \\ 0 & \widetilde{E} \end{pmatrix}$$

$$\mathbf{H} = \mathbf{V}\mathbf{E}\mathbf{V}^\dagger$$

U. Günther and B. F. Samsonov,
Phys. Rev. Lett., 101, 230404 (2008).

Dilated Hermitian Hamiltonian

$$\mathbf{H} = \frac{\cos\alpha}{2}\begin{pmatrix} H\eta^{-1} + \eta H & H - H^\dagger \\ H^\dagger - H & H\eta^{-1} + \eta H \end{pmatrix} \qquad \eta \equiv \Xi\Xi^\dagger$$

Dilated time evolution operator

$$\mathbf{U}(t) = \frac{\cos\alpha}{2}\begin{pmatrix} U(t)\eta^{-1} + \eta U(t) & U(t) - \eta U(t)\eta^{-1} \\ \eta U(t)\eta^{-1} - U(t) & U(t)\eta^{-1} + (t)) \end{pmatrix}$$

Dilation matrix

$$\eta = \frac{1}{\cos\alpha}\begin{pmatrix} 1 & -i\sin\alpha \\ i\sin\alpha & 1 \end{pmatrix}$$

Dilated quantum states

$$|\Psi\rangle \equiv \begin{pmatrix} \psi \\ \chi \end{pmatrix} = \begin{pmatrix} \psi \\ \eta\psi \end{pmatrix}$$

$$\mathbf{H}|\Psi\rangle = \begin{pmatrix} H & 0 \\ 0 & H^\dagger \end{pmatrix}\begin{pmatrix} \psi \\ \chi \end{pmatrix} \qquad \mathbf{U}(t)|\Psi\rangle = \begin{pmatrix} U(t) & 0 \\ 0 & \eta U(t)\eta^{-1} \end{pmatrix}\begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

$$|\psi_I\rangle = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$|\Psi_I\rangle = \frac{1}{\sqrt{2(|C_1|^2 + |C_2|^2) + 2i\sin\alpha(C_1C_2^* - C_1 * C_2)}}\begin{pmatrix} \cos\alpha C_1 \\ \cos\alpha C_2 \\ C_1 - i\sin\alpha C_2 \\ i\sin\alpha C_1 + C_2 \end{pmatrix} \begin{matrix} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{matrix}$$

$\mathcal{PT}$ Without Balanced Gain/Loss

General PT symmetric Hamiltonian

$$H_1 = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}$$

$$E_\pm = \pm s\cos\alpha$$

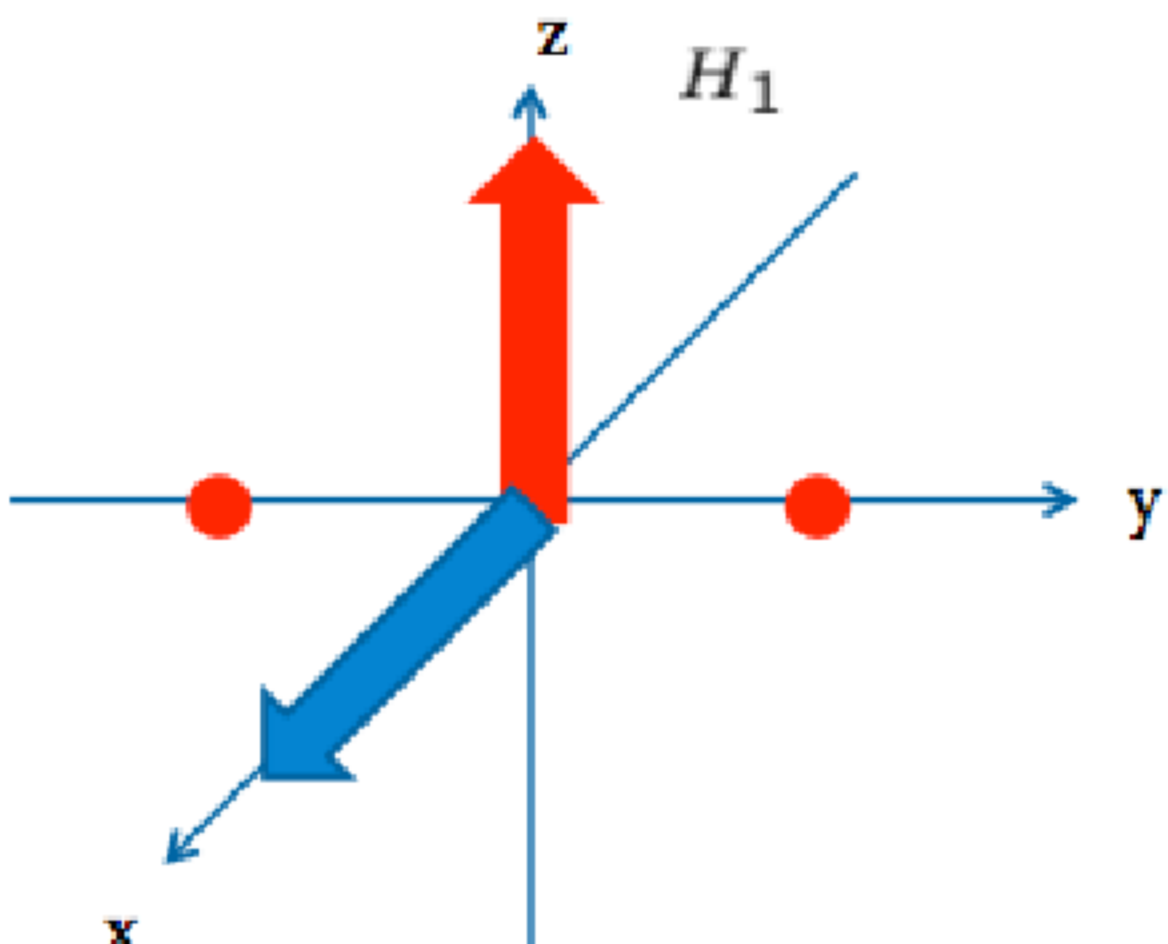
$$H_2 = s\begin{pmatrix} 0 & 1-\sin\alpha \\ 1+\sin\alpha & 0 \end{pmatrix}$$

$$H_3 = s\begin{pmatrix} i\sin\alpha & -i \\ i & -i\sin\alpha \end{pmatrix}$$

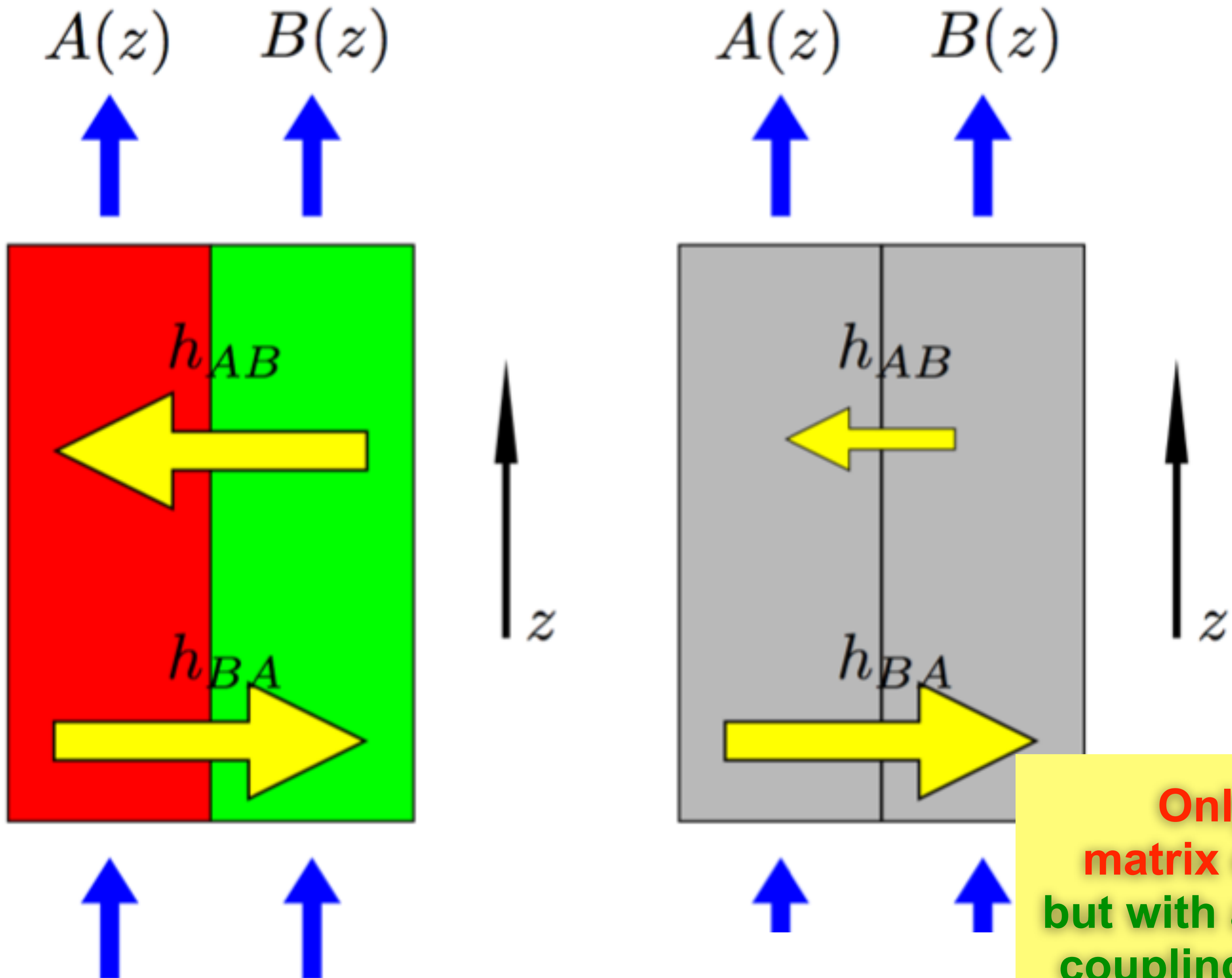
$$H_4 = s\begin{pmatrix} 0 & -i+i\sin\alpha \\ i+i\sin\alpha & 0 \end{pmatrix}$$

$$H_5 = s\begin{pmatrix} 1 & -i\sin\alpha \\ -i\sin\alpha & -1 \end{pmatrix}$$

$$H_6 = s\begin{pmatrix} 1 & -\sin\alpha \\ \sin\alpha & -1 \end{pmatrix}$$



Y-C. Lee, J.B. Liu, Y.-L. Chuang, M.-H. Hsieh, and RKL, Phys. Rev. A 92, 053815 (2015).



Only Real matrix elements, but with asymmetric coupling/tunneling

$$H = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}, \quad H_2 = s\begin{pmatrix} 0 & 1-\sin\alpha \\ 1+\sin\alpha & 0 \end{pmatrix}$$

Y-C. Lee, J.B. Liu, Y.-L. Chuang, M.-H. Hsieh, and RKL, Phys. Rev. A 92, 053815 (2015).

$H^\dagger \eta = \eta H$

Theorem (The structure of $\mathcal{PT}$-symmetric quantum systems)

Let $\mathcal{H}$ be a $\mathcal{PT}$-symmetric operator. η is a metric operator of $\mathcal{H}$ iff there exists an invertible operator Ξ such that

$$\Xi^{-1} H \Xi = J = \begin{bmatrix} J_{n_1}(\lambda_1, \overline{\lambda}_1) & & & & & \\ & \ddots & & & & \\ & & J_{n_p}(\lambda_p, \overline{\lambda}_p) & & & \\ & & & J_{n_{p+1}}(\lambda_{p+1}) & & \\ & & & & \ddots & \\ & & & & & J_{n_r}(\lambda_r) \end{bmatrix}, \quad (1)$$

and

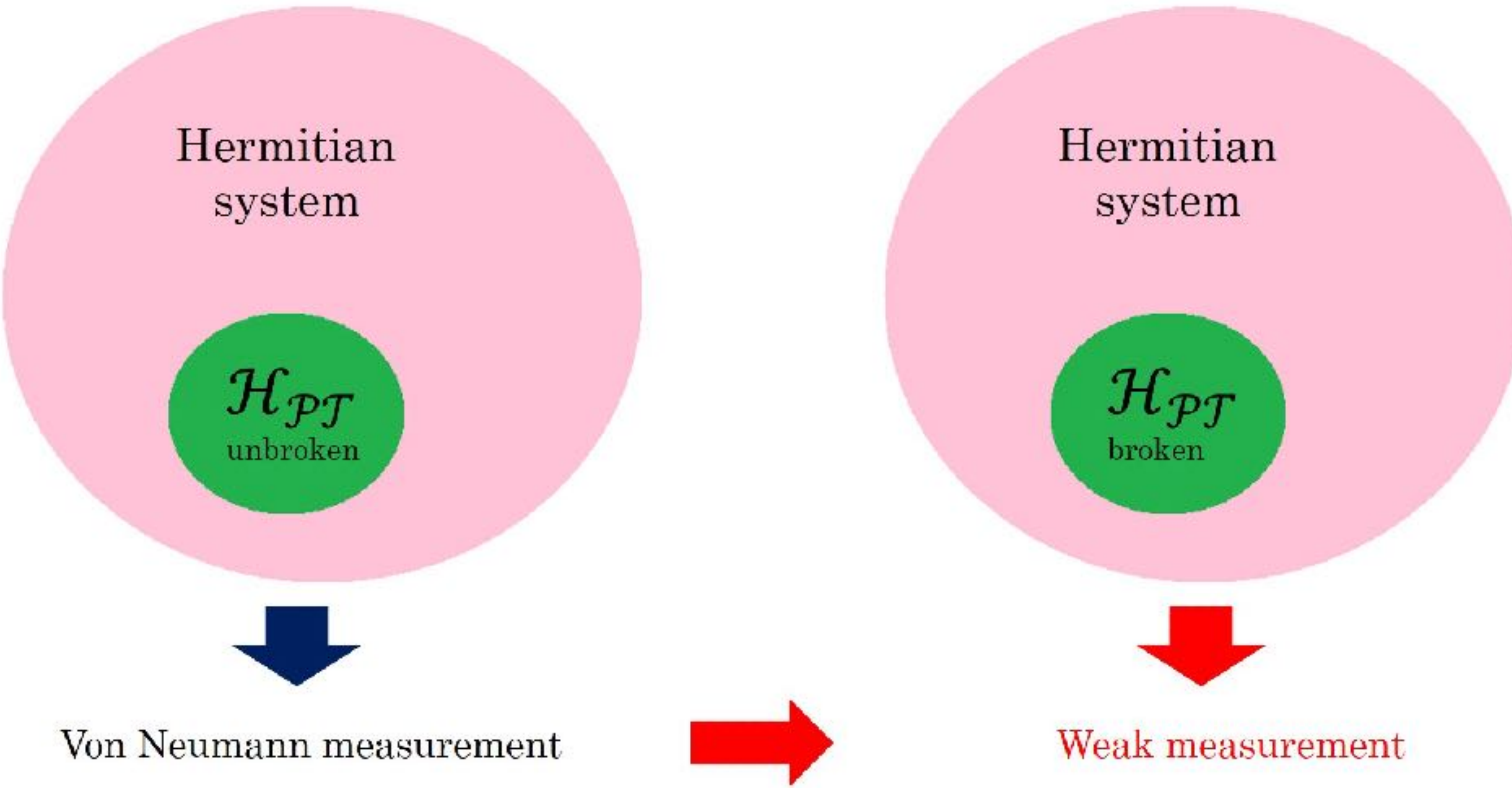
$$\Xi^\dagger \eta \Xi = S = \begin{bmatrix} S_{2n_1} & & & & & \\ & \ddots & & & & \\ & & S_{2n_p} & & & \\ & & & \epsilon_{n_{p+1}} S_{n_{p+1}} & & \\ & & & & \ddots & \\ & & & & & \epsilon_{n_r} S_{n_r} \end{bmatrix}, \quad (2)$$

where $2n_j$ are the orders of Jordan blocks $J_{n_j}(\lambda_j, \overline{\lambda}_j)$, n_l are the orders of Jordan blocks $J_{n_l}(\lambda_l)$,

$$S_k = \begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix}_{k \times k}, \quad \epsilon_j = \pm 1 \text{ is uniquely determined by } \eta.$$

M. Huang, R.-K. Lee, and J. Wu, J. Phys. A: Math. Theor. 51, 414004 (2018).

Unbroken $\mathcal{PT}$-symmetry and broken $\mathcal{PT}$-symmetry



$$H^\dagger \eta = \eta H$$

M. Huang, R.-K. Lee, and J. Wu, J. Phys. A: Math. Theor. 51, 414004 (2018).

PHYSICAL REVIEW LETTERS **123,** 080404 (2019)

Simulating Broken $\mathcal{PT}$-Symmetric Hamiltonian Systems by Weak Measurement

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(Received 27 September 2018; published 23 August 2019)

By embedding a $\mathcal{PT}$-symmetric (pseudo-Hermitian) system into a large Hermitian one, we disclose the relations between $\mathcal{PT}$-symmetric quantum theory and weak measurement theory. We show that the weak measurement can give rise to the inner product structure of $\mathcal{PT}$-symmetric systems, with the preselected state and its postselected state resident in the dilated conventional system. Typically in quantum information theory, by projecting out the irrelevant degrees and projecting onto the subspace, even local broken $\mathcal{PT}$-symmetric Hamiltonian systems can be effectively simulated by this weak measurement paradigm.