

Date	Topic	To Know	To Think
Feb. 23rd (Tue.)	Introduction	Scope	☐ Your and My Expectations. ☐ What is the nature of light? ☐ Anything else ?
Feb. 26th (Fri.)	Simple Harmonic Oscillator (SHO)	☐ classical trajectory ☐ analogue to EM waves	☐ Bohmian mechanics ☐ Inverted SHO ☐
week 1 (3/2, 3/9)	Quantum SHO	☐ Fock states, $ n\rangle$ ☐ creation operator, $\hat{a}^\dagger$	☐ single-photon detection ☐ Wave-Particle Duality ☐ photon-number resolving ☐
(3/12, 3/16, 3/19)		☐ Vacuum state ☐ Quantum Fluctuations	☐ Shot Noise Limit ☐ Casimir Force ☐
week 2 (3/23, 3/26)	Quantum Mechanics	☐ Schrödinger picture ☐ Heisenberg picture ☐ Interaction picture	☐ Uncertainty Relation ☐ Probability Interpretation ☐ Measurement problem ☐ Non-locality ☐ Macrorealism ☐
week 3 (3/30, 4/2, 4/6)	Coherent states, $ \alpha\rangle$	☐ photon statistics ☐ bunching ☐ Correlation function	☐ Minimum Uncertainty States ☐ Classical-Quantum boundary ☐
week 4 (4/9, 4/13)	Quantum Phase Space	☐ Wigner function	☐ Quasi-probability ☐ Quantum State Tomography ☐
week 5 (4/20, 4/23)	Squeezed states	☐ $ \xi\rangle$ ☐ OPO	☐ Continuous Variables ☐
week 6 (4/27, 4/29)	Two-mode Squeezed states	☐ EPR pair ☐ Cat states ☐ non-Gaussian states	☐ Quantum Discord ☐ Entanglement ☐ Steering ☐ Bell's inequality ☐
week 7 (5/4, 5/7)	Optical devices	☐ Beam splitter ☐ Mach-Zehnder interferometer	☐ linear optics ☐
week 8 (5/11, 5/14)	Interferometry	☐ Young's Interferometry, $g^{(1)}$ ☐ HBT-Interferometry, $g^{(2)}$	☐ Quantum Phase Estimation ☐ Quantum Fisher Information ☐

Note: Simple Harmonic Oscillators

- ☐ **Simple Harmonic Oscillator, SHO**
- ☐ **Hamiltonian energy**
- ☐ **Phase Space**
- ☐ **EM wave as a SHO**
- ☐ **Sinusoidal waves**
- ☐ **Poynting Energy**
- **More on SHO**
 - ☐ Damped SHO (over-damped, critically-damped, under-damped)
 - ☐ Driven SHO, ☐ Inverted SHO
 - ☐ Pendulum
 - ☐ Nonlinear SHO
 - ☐ Periodic (Orbit) and Chaotic Trajectories
 - ☐ Parity-Time symmetric SHO
 - ☐ Double pendulum/Coupled SHOs
 - ☐ Bose-Einstein Condensates in Double Wells
 - ☐ **EIT and Fano resonance**
 - ☐ Pseudo-Potential

download

[*http://mx.nthu.edu.tw/~rkleee](http://mx.nthu.edu.tw/~rkleee)

From Scratch !!

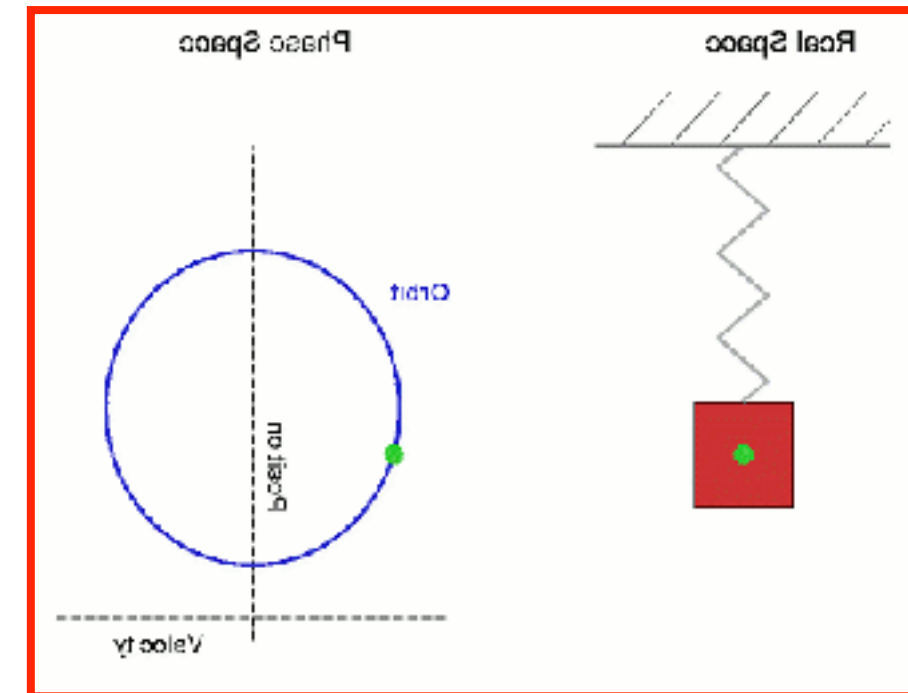
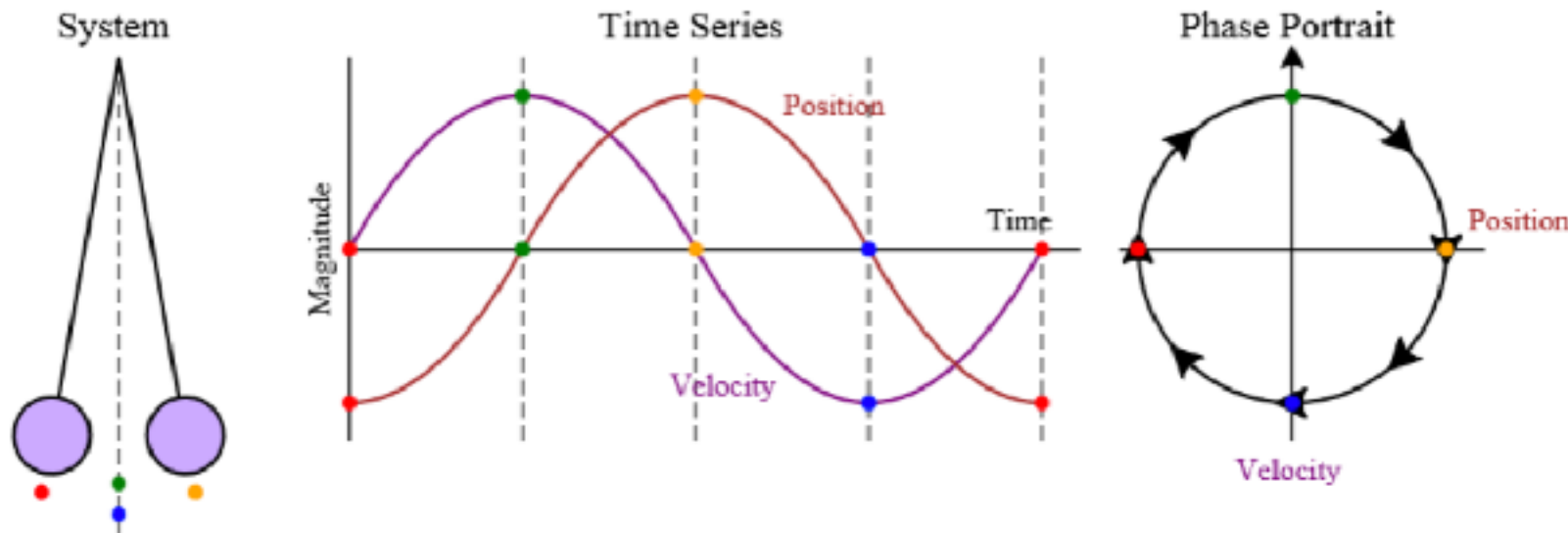
- How much do you known about SHO ?

Possible Goals:

- **Find a possible (but simple) building block for EM wave quantization !**
- **Classical Counterpart !?**
- **Constraints?**
 - **Energy Conservation**
 - **Momentum Conservation**

Simple Harmonic Oscillator, SHO

• Classical Mechanics



$$\vec{F} = m\vec{a} = m \frac{d^2 \vec{x}}{dt^2} = -k \vec{x}.$$

$$\vec{x}(t) = A \cos(\omega_0 t + \phi_0), \quad \omega_0^2 = \frac{k}{m},$$

$$H = \text{K.E.} + \text{P.E.} = \frac{1}{2} \frac{p^2}{m} + \frac{1}{2} k x^2.$$

- Newton's law
- Hooke's law
- Linear force (parabolic potential)
- Hamiltonian (energy) is constant (conserved).
- Equally distributions in KE and PE
- Sinusoidal motion
- Periodic orbit (trajectory) in phase space
- Response bandwidth

Maxwell's Equations



And God Said

$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

and *then* there was
light.



Edinburg, Scotland
Nov. 2018



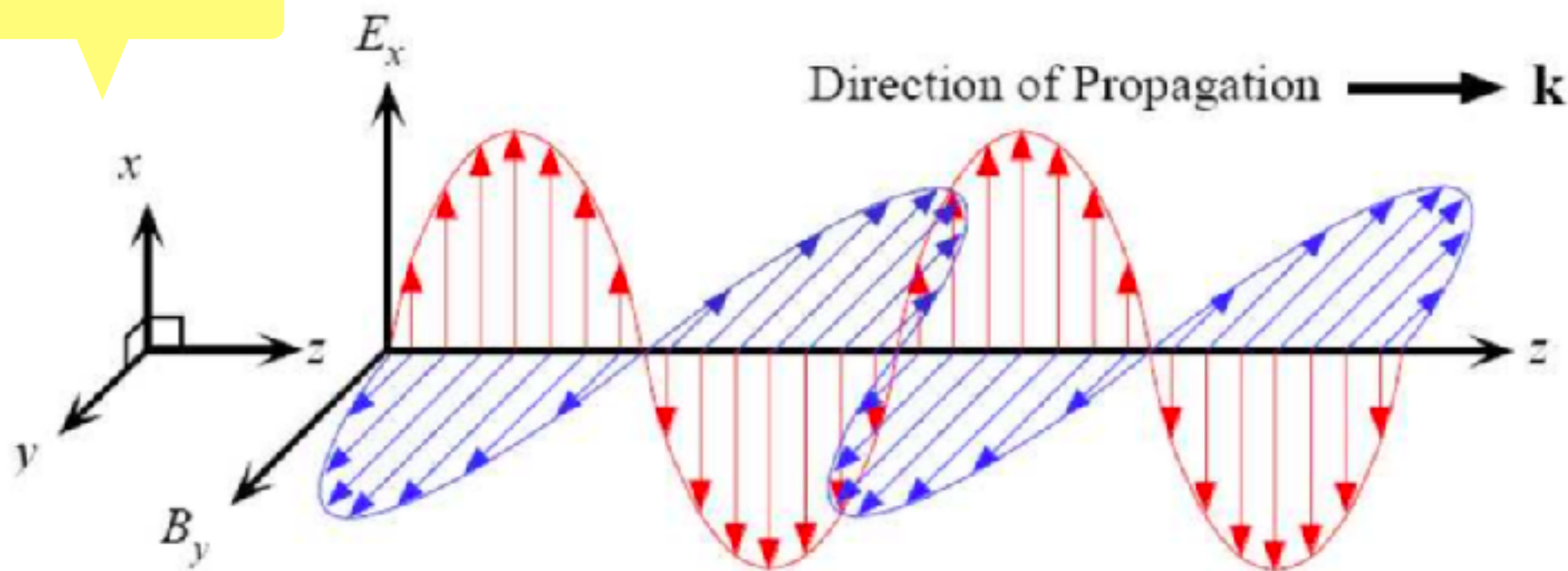
Poynting Energy Theorem

Poynting's theorem:

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = - \int_{\bar{V}} (\vec{E} \cdot \vec{J}) d\bar{V} - \frac{\partial}{\partial t} \int_{\bar{V}} w_e d\bar{V} - \frac{\partial}{\partial t} \int_{\bar{V}} w_m d\bar{V}$$

The quantity $\vec{E} \cdot \vec{J}$ is the **power density** associated with the work done by the field, having to do with the current flow.

Free space



$$w_e = \frac{1}{2} \epsilon_0 |\vec{E}|^2;$$
$$w_m = \frac{1}{2} \mu_0 |\vec{H}|^2;$$

Energy of EM Waves

The *total energy* for the electric and magnetic fields, plus the particles, is

$$\begin{aligned} H &= \sum_{\alpha} \frac{1}{2} m_{\alpha} v_{\alpha}^2(t) + \frac{\epsilon_0}{2} \int d^3 \vec{r} [E^2(\vec{r}, t) + c^2 B^2(\vec{r}, t)], \\ &= \sum_{\alpha} \frac{1}{2} m_{\alpha} v_{\alpha}^2(t) + H_{\perp} + H_{\parallel}, \end{aligned}$$

where the energy contributed from the transverse electric and magnetic field is,

$$H_{\perp} \equiv \frac{\epsilon_0}{2} \int d^3 \vec{r} [E_{\perp}^2(\vec{r}, t) + c^2 B_{\perp}^2(\vec{r}, t)].$$

- Coulomb self energy of the particle and Coulomb interaction energy between pairs of particles,
- **Transverse** Electric and Magnetic fields
- Longitudinal Electric field (the longitudinal Magnetic field is zero)

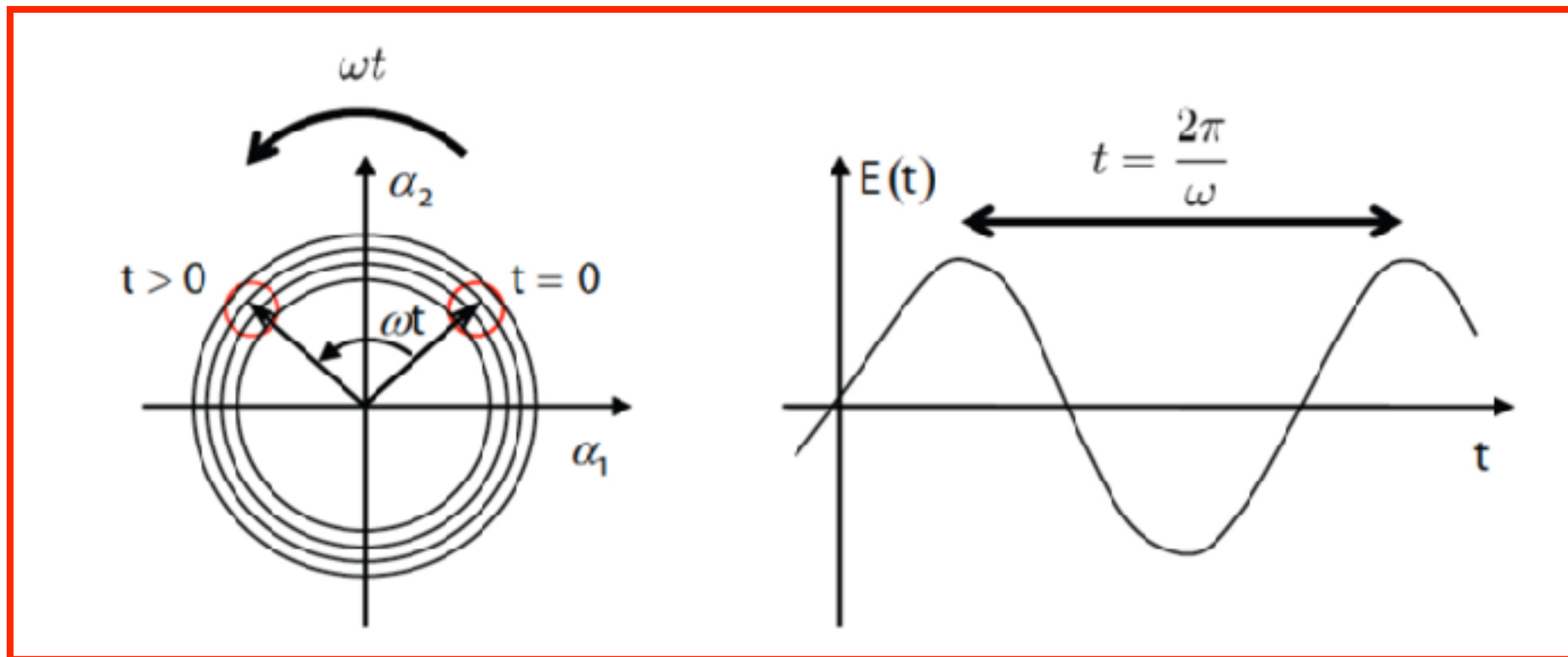


EM Waves

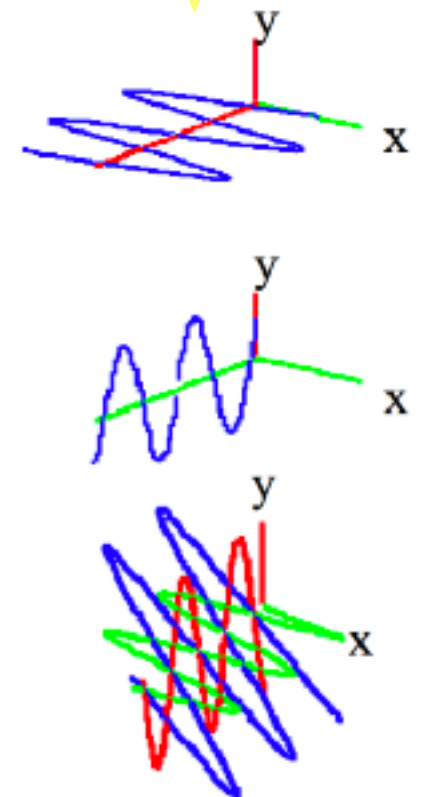
Sinusoidally time-varying plane waves:

$$\vec{E}(x, t) = \hat{x} A \text{Re}\{\text{Exp}[i(\omega t - \beta x)]\} = A \cos(\omega t - \beta x),$$

where $\text{Exp}[i(\omega t - \beta x)]$ is called the **phasor**.



polarization



Degrees of freedom in Classical EM waves:

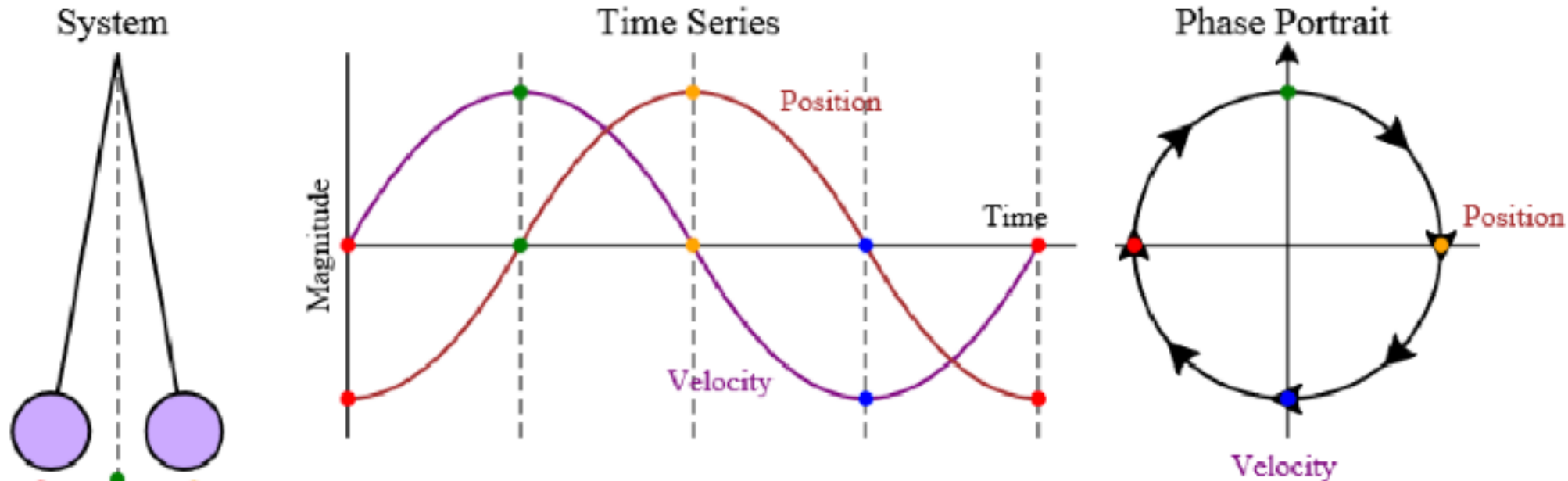
- Amplitude, Phase,
- Polarization (spin), Optical Angular Momentum (OAM),
- Frequency (Wavelength), Frequency (Time) Bandwidth, Intensity (Power)

Simple Harmonic Oscillator, SHO

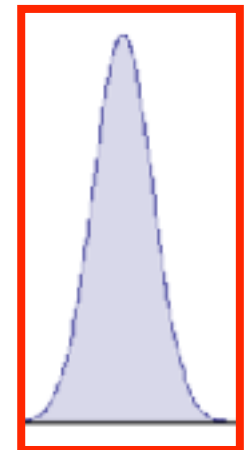
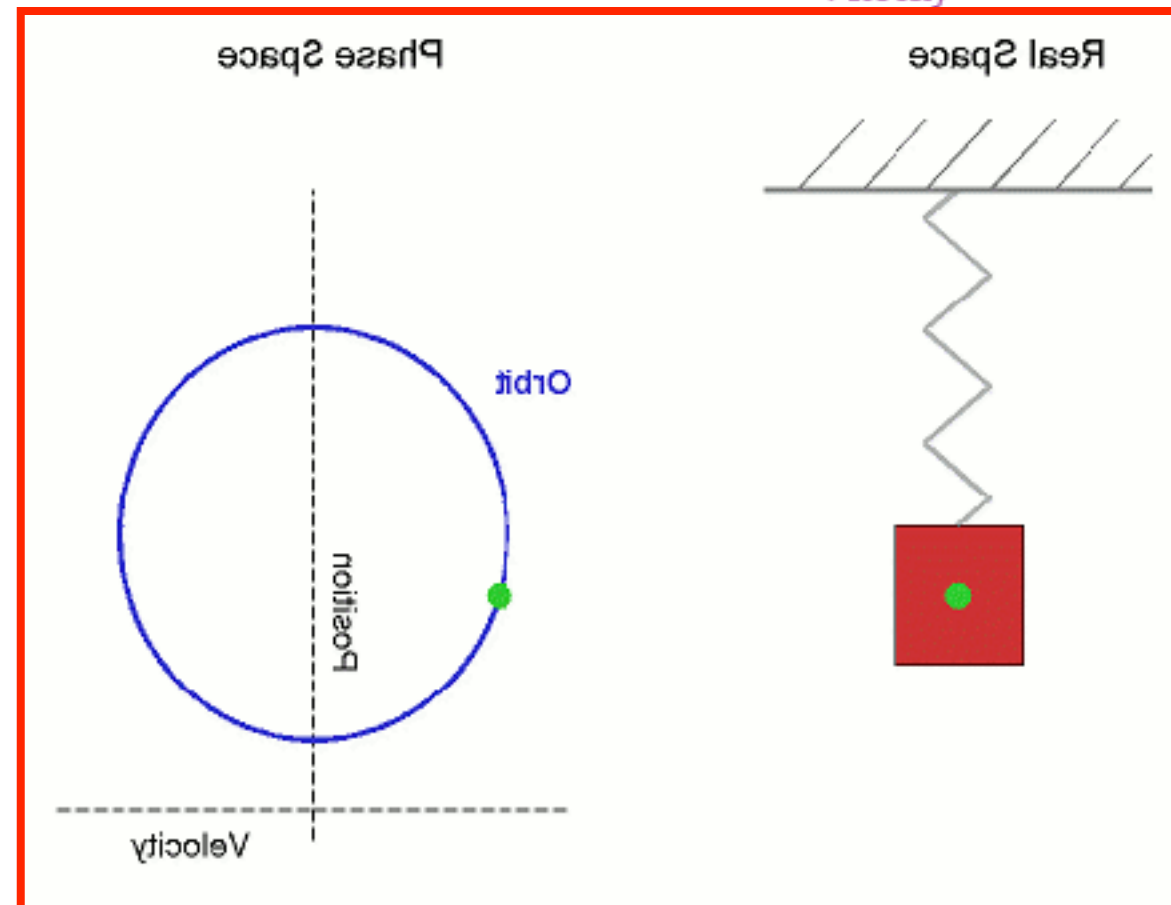
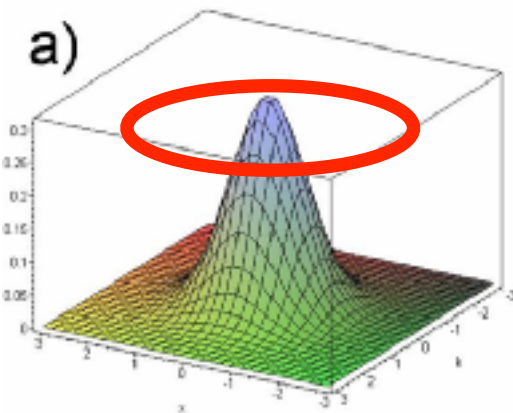
SHO	EM waves	Quantum
particle	(transverse) mode	wave-particle
Newton's law	Maxwell's eqs	Schrodinger/Heisenberg eq. Quantum Liouville eq. Dirac eq.
sinusoidal sol.	plane wave sol.	harmonic waves
KE+PE	Poynting energy	Hamiltonian energy
Kinetic Energy	Diffraction/Dispersion	free-particle expansion
Potential Energy	refractive index change (GRIN lens)	Potential Energy
Trajectory	Phasor	Probability distribution (Husimi function)
x, p canonical coordinates	quadrature	$[\hat{X}, \hat{P}]$

Phase space

Classical Mechanics

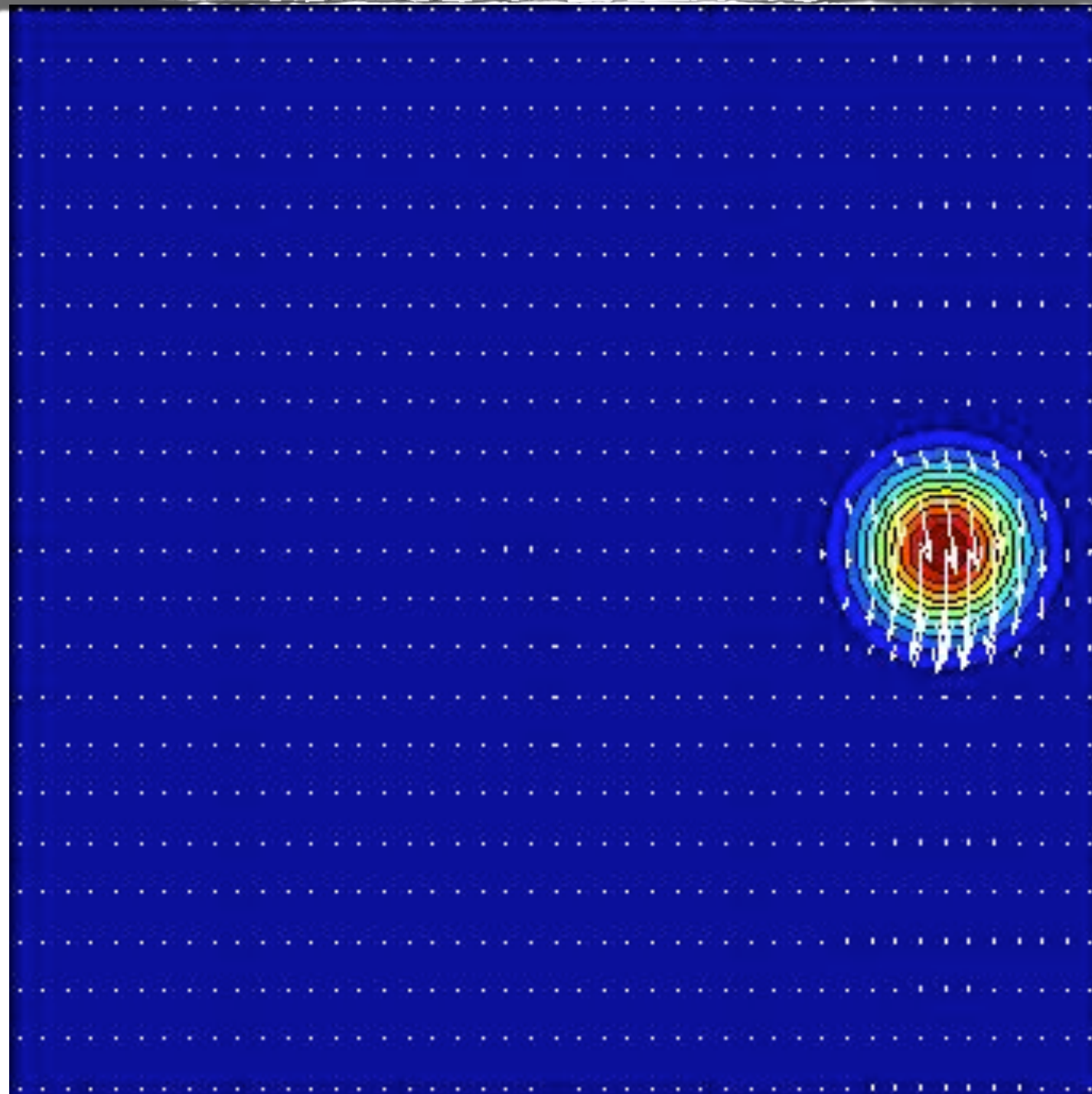


wave-nature



from Wiki

Coherent states



with Popo Yang

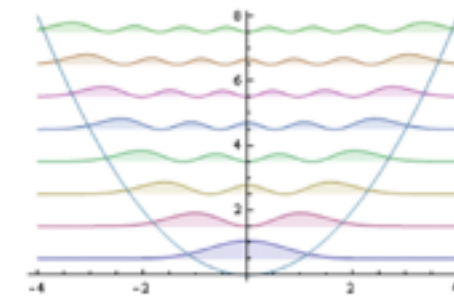
Popo Yang, Ivan F. Valtierra, Andrei B. Klimov, Shin-Tza Wu, RKL, Luis L. Sanchez-Soto, and Gerd Leuchs,
Physica Scripta for the New Focus issue: [Quantum Optics and Beyond - in honour of Wolfgang Schleich](#).

Quantum Simple Harmonic Oscillator (SHO)

coherent states $|\alpha\rangle, |\beta\rangle, |\gamma\rangle$

number states $|n\rangle, |m\rangle$

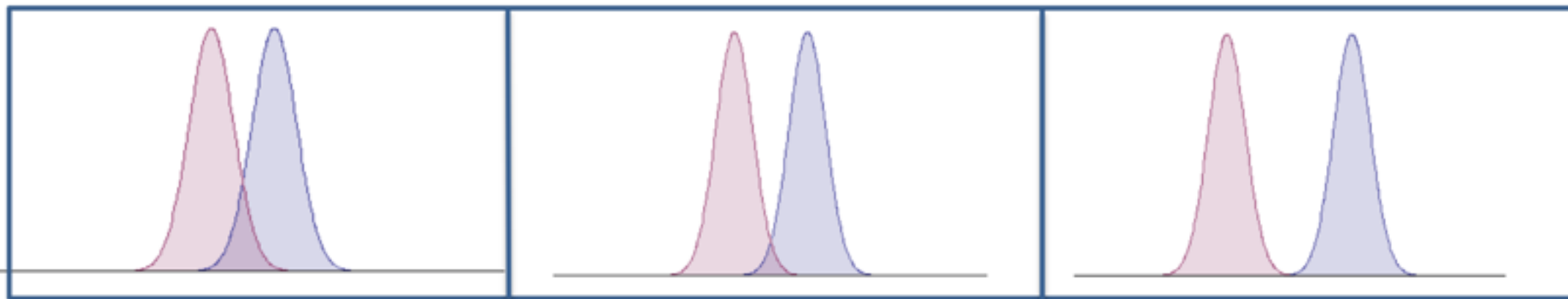
$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$



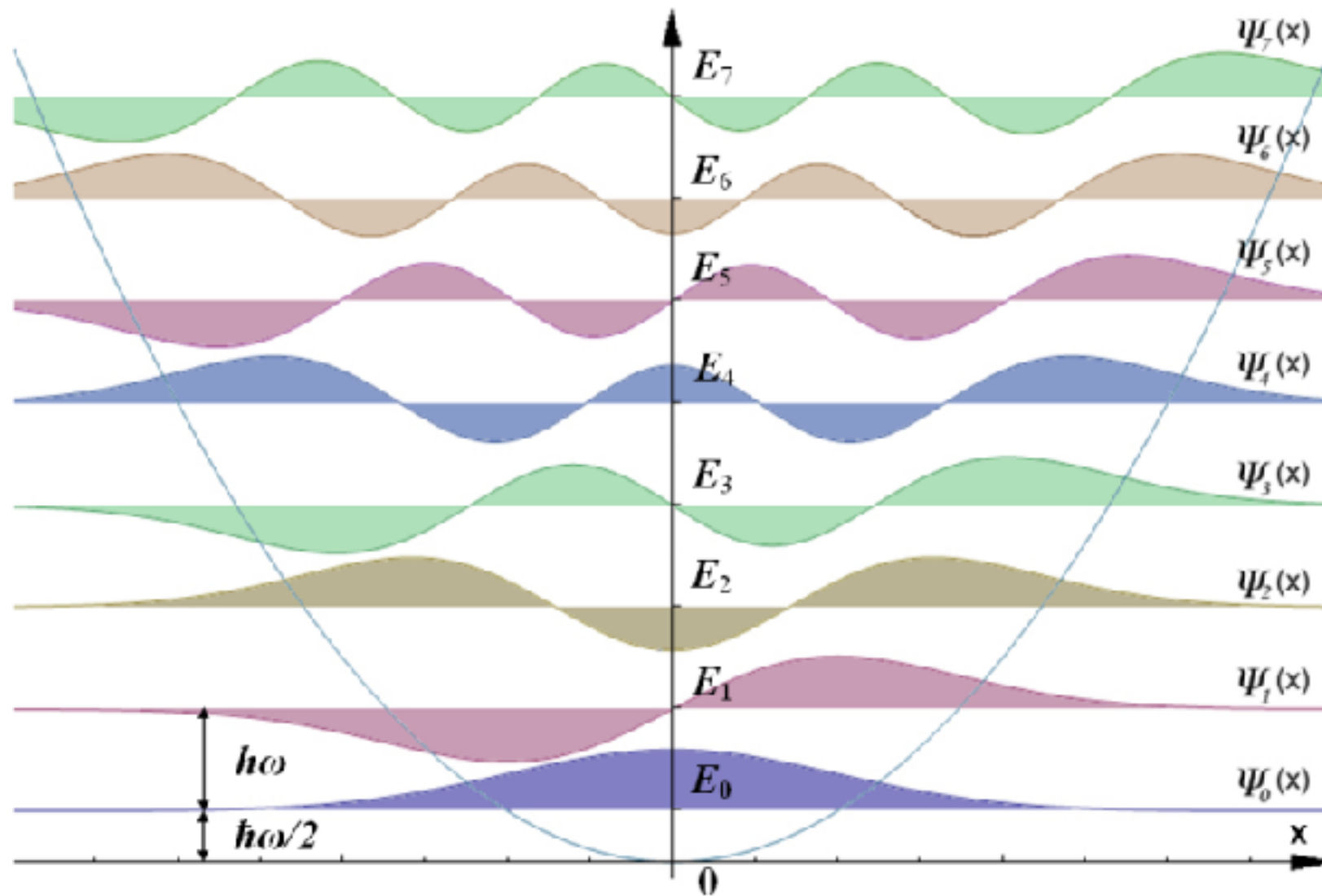
coherent states form a complete set $1 = \int_{\mathbb{R}^2} \frac{d^2\alpha}{2\pi} |\alpha\rangle\langle\alpha|$

but not an orthonormal basis $|\langle\beta|\alpha\rangle|^2 = e^{-|\alpha-\beta|^2}$

two coherent states are never orthogonal to each other
there is always a non-vanishing overlap between them



Quantum Simple Harmonic Oscillator (SHO)



$$\hat{H} = \frac{1}{2} \frac{\hat{p}^2}{m} + \frac{1}{2} k \hat{x}^2, \quad [\hat{x}, \hat{p}] = i\hbar.$$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger \hat{a} + \frac{1}{2}). \quad [\hat{a}, \hat{a}^\dagger] = 1,$$

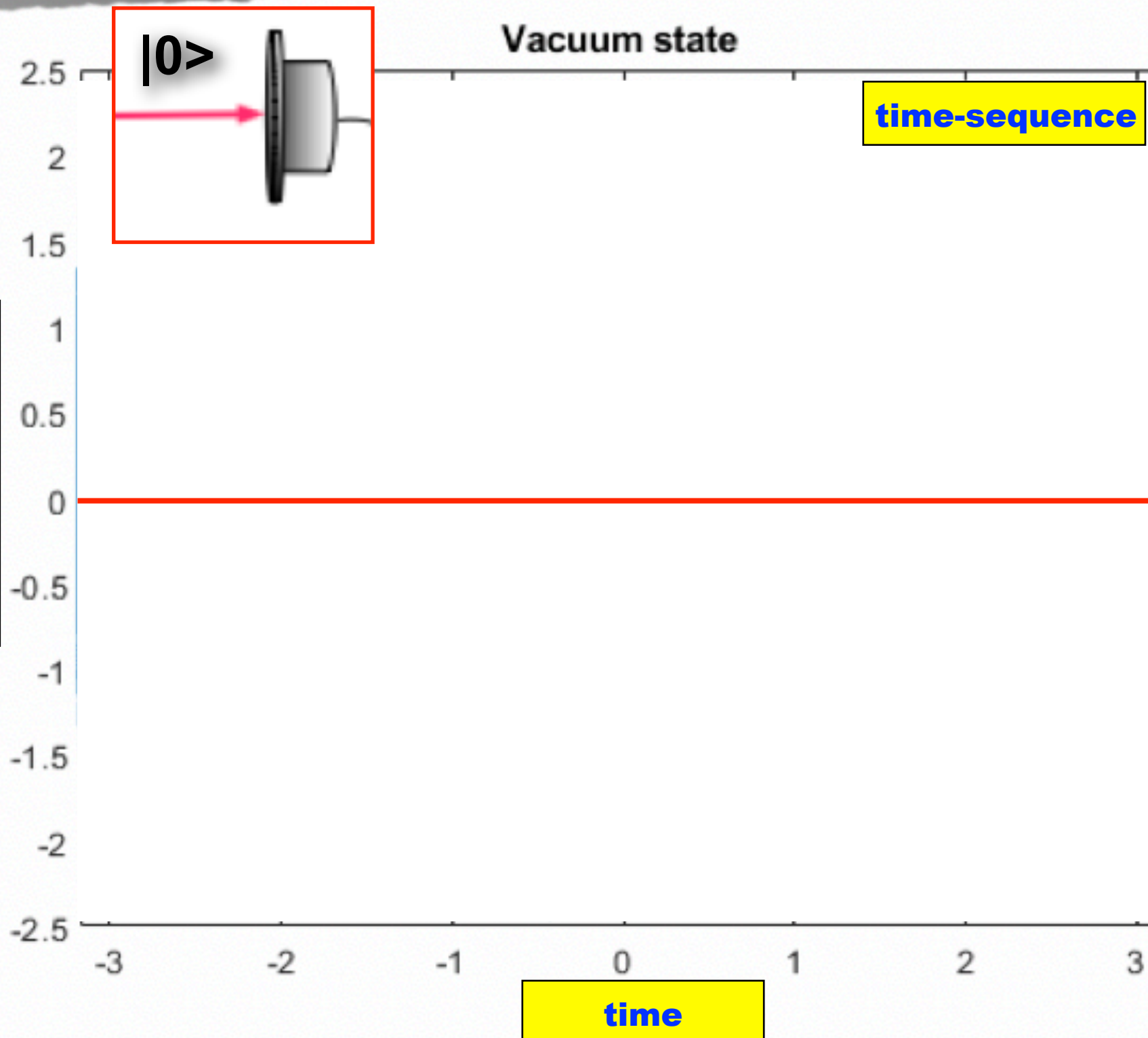
$$\begin{aligned} \hat{N}|n\rangle &= n|n\rangle, \\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle, \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle, \\ E_n &= \hbar\omega(n + \frac{1}{2}). \end{aligned}$$

$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \quad \epsilon = 2n + 1, \quad n = 0, 1, 2, 3 \dots$$

$$E = \frac{\hbar\omega}{2} \epsilon = \hbar\omega(n + \frac{1}{2}), \quad n = 0, 1, 2, 3, \dots$$



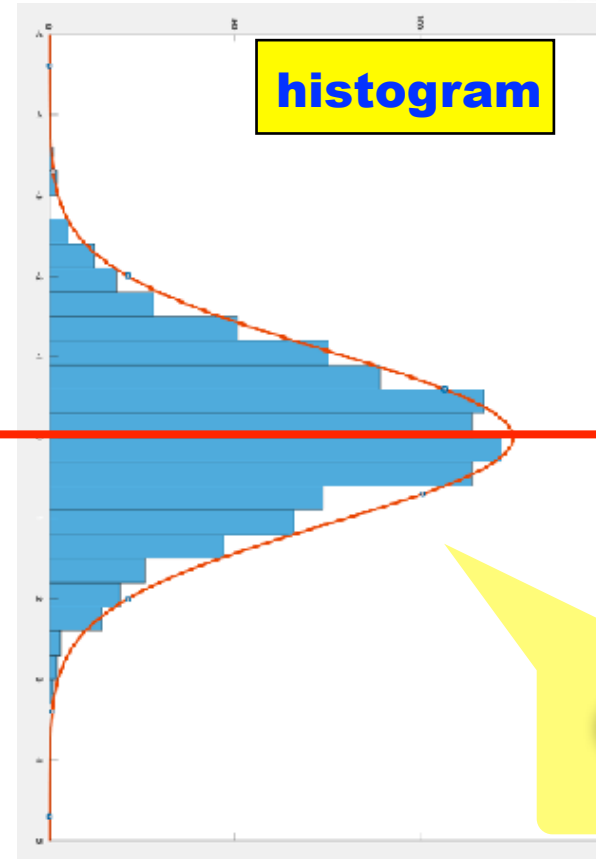
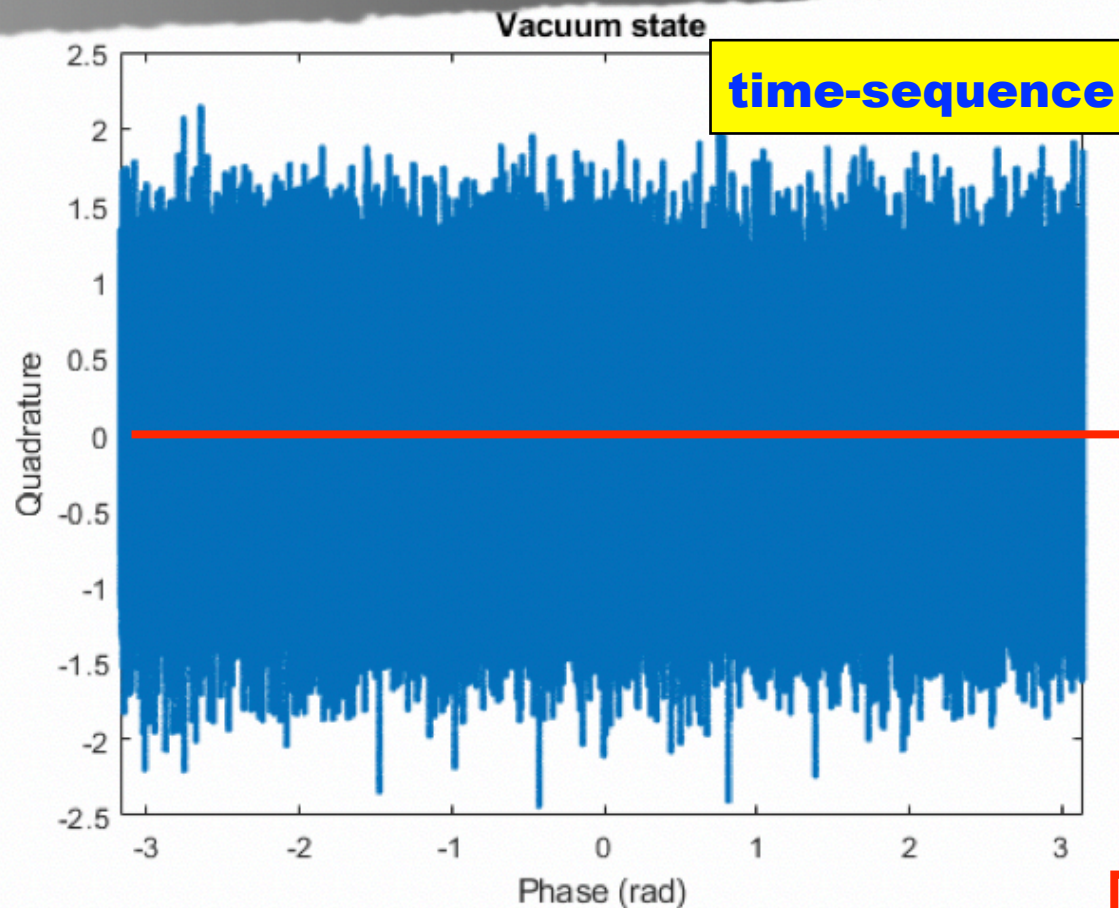
Vacuum State: $|0\rangle$



Gaussian wave-package

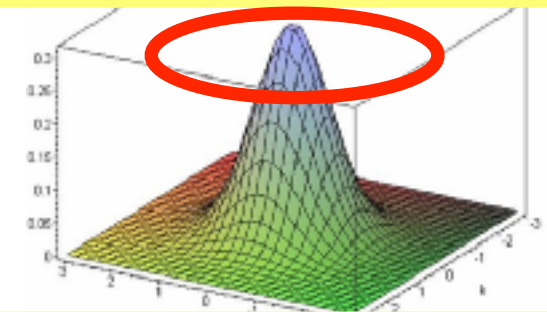
with Zero Mean

Vacuum State: $|0\rangle$



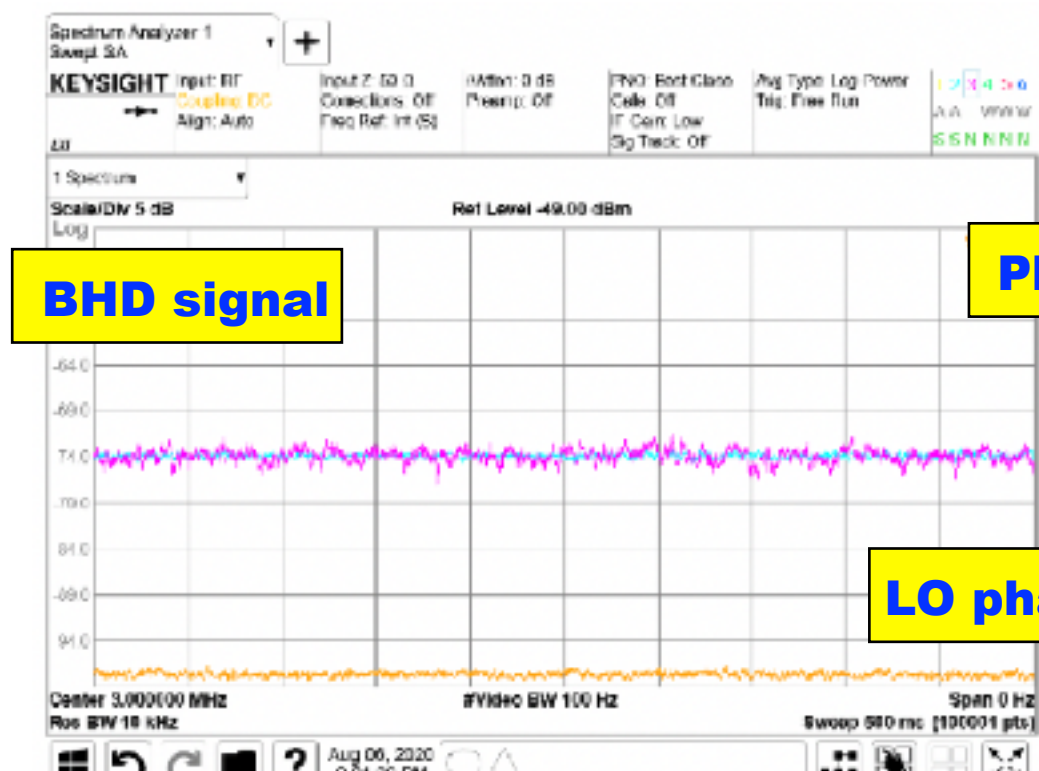
with Zero Mean

Gaussian wave-package

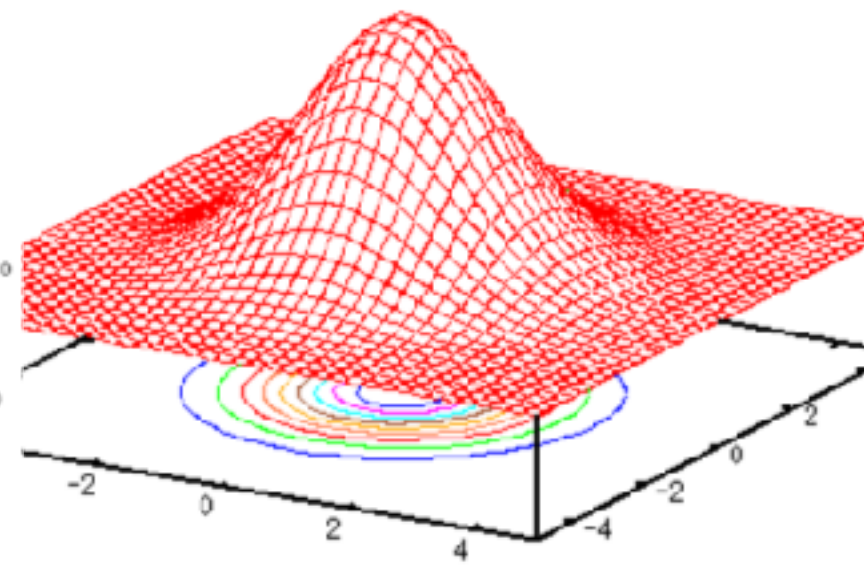
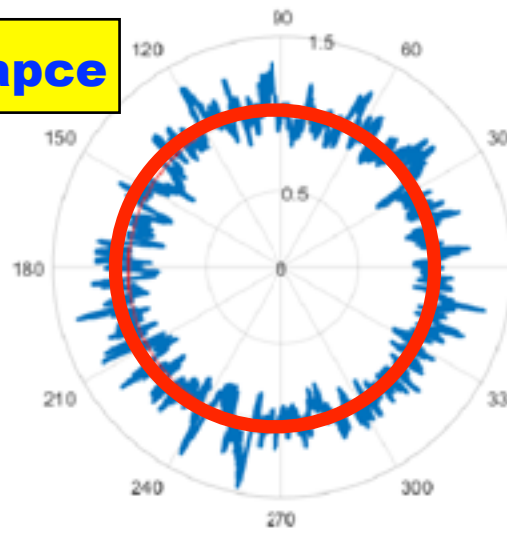


$$\Psi(x) = \langle x|0 \rangle = C \exp[-x^2 / \Delta x^2]$$

$$\tilde{\Psi}(p) = \langle p|0 \rangle = C \exp[-\Delta x^2 p^2]$$



Phase Sapce

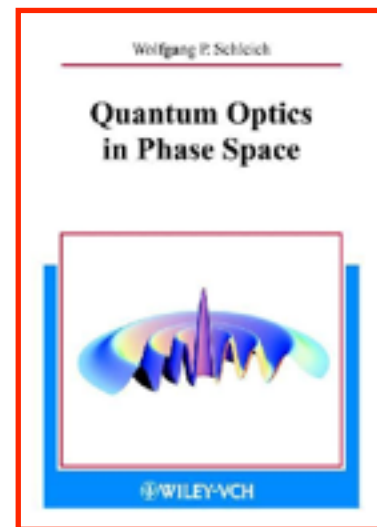


Number (Fock) states

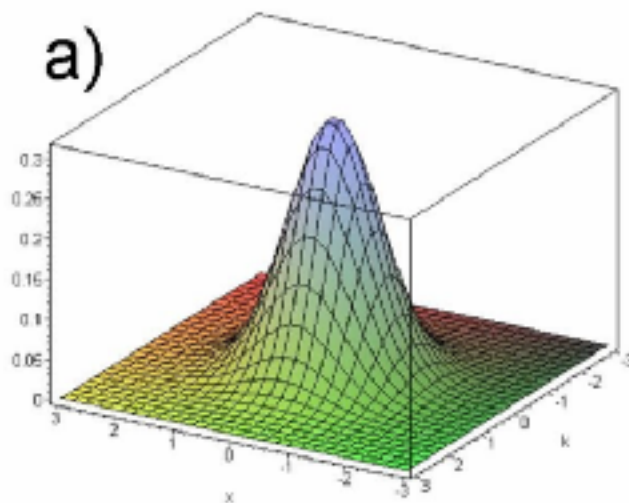
**Non-classical
states**

- Wigner quasiprobability distribution

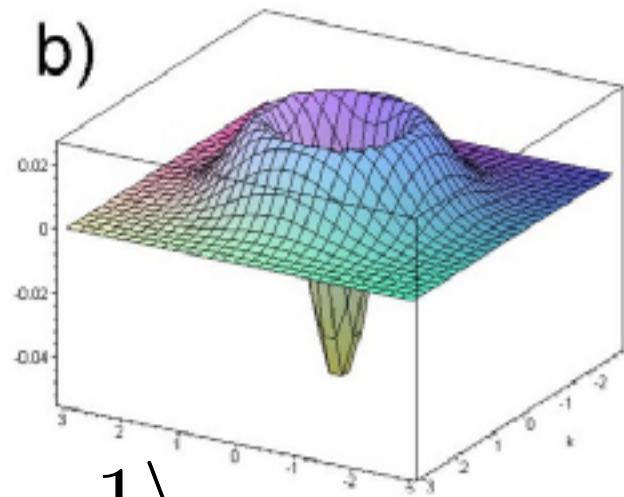
$$W(x, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\xi \psi^* \left(x - \frac{\xi}{2}\right) \psi \left(x + \frac{\xi}{2}\right) e^{-ik\xi}$$



$|0\rangle$

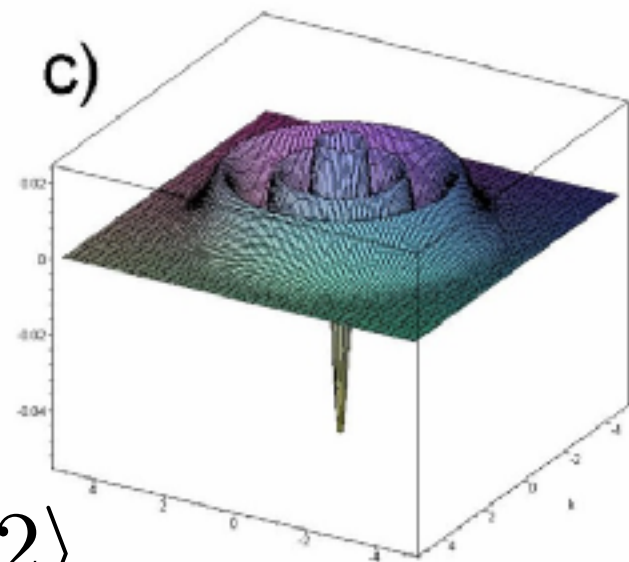


$|n = 1\rangle$



negative probability

$|n = 2\rangle$



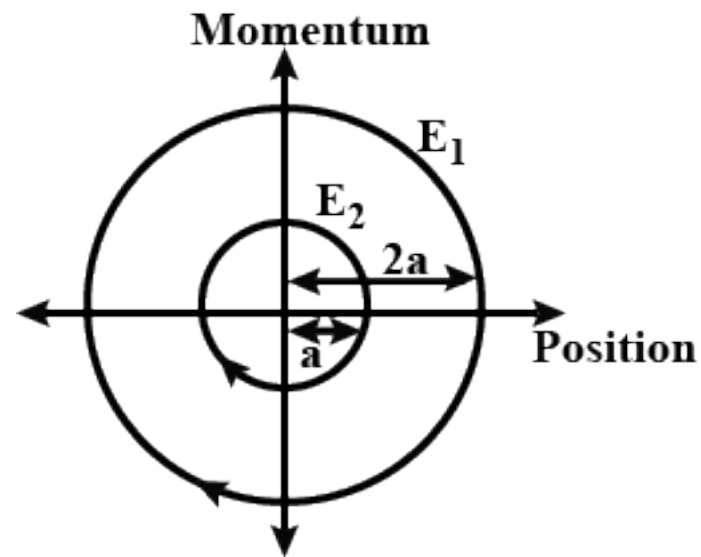
with Ludmila Praxmeyer

More on SHO



Damped SHO

J. B. Marion and S. T. Thornton, "Classical dynamics of particles and systems," (1995).



- Energy $\propto$ radius
- damped SHO
 - critically damped
 - over-damped
 - under-damped

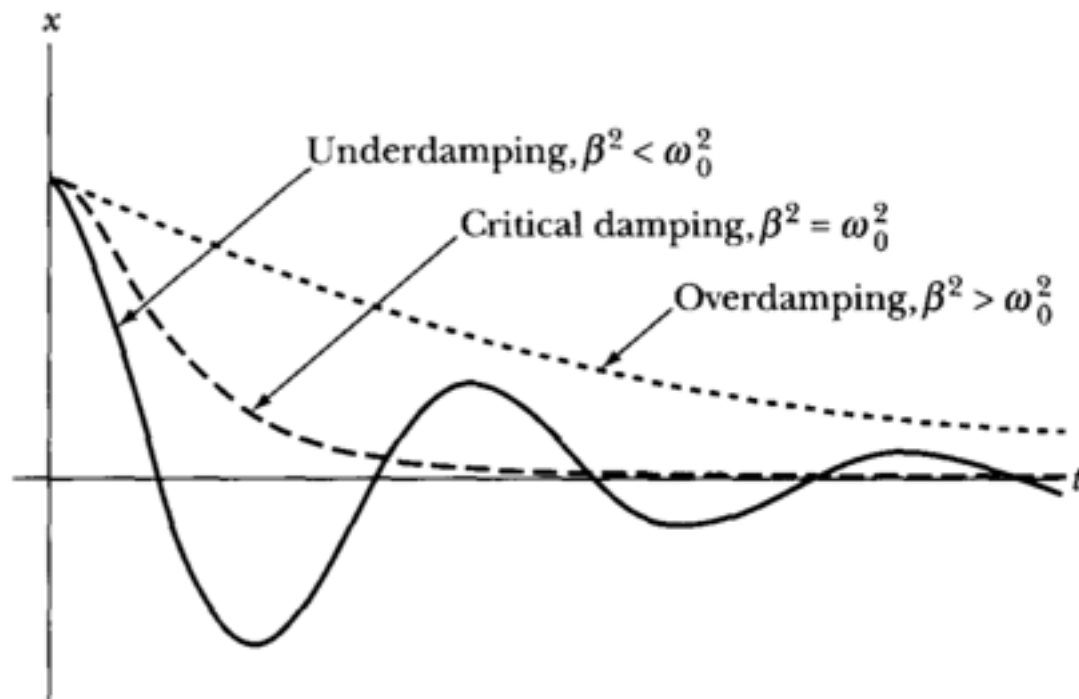
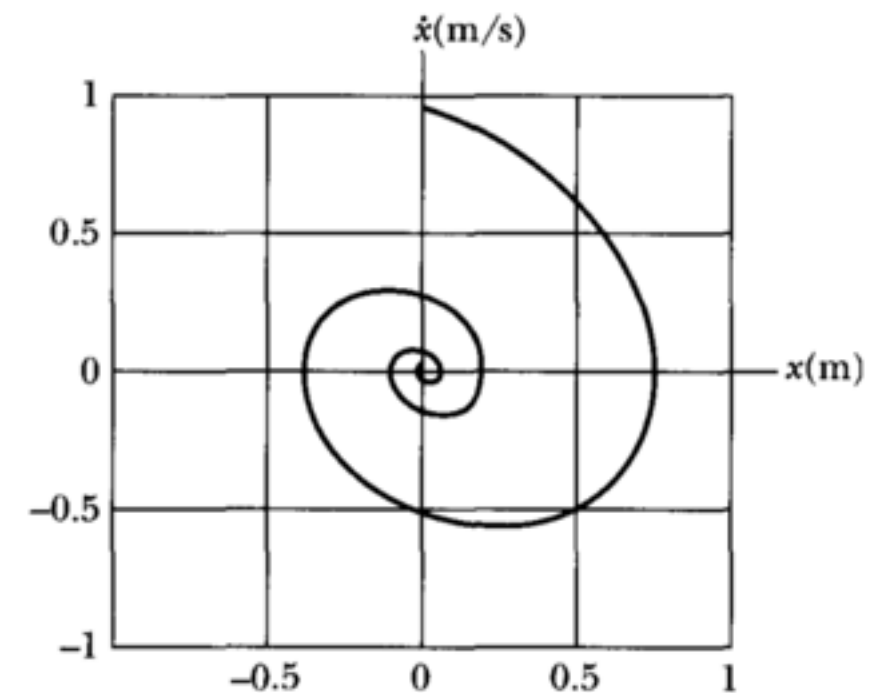


FIGURE 3-6 Damped oscillator motion for three cases of damping.

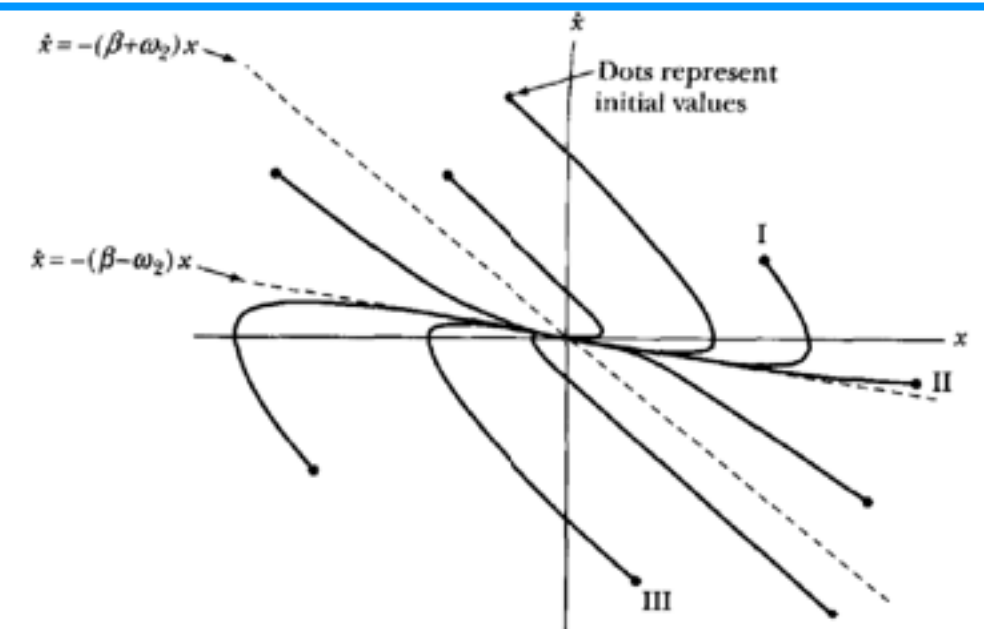


FIGURE 3-11 The phase paths for overdamped motion are shown for several initial values of $(x, \dot{x})$. We examine more closely the paths labeled I, II, and III.

Nonlinear, Inverted SHO

J. B. Marion and S. T. Thornton, "Classical dynamics of particles and systems," (1995).

- Anharmonic Potential
- Nonlinear Force

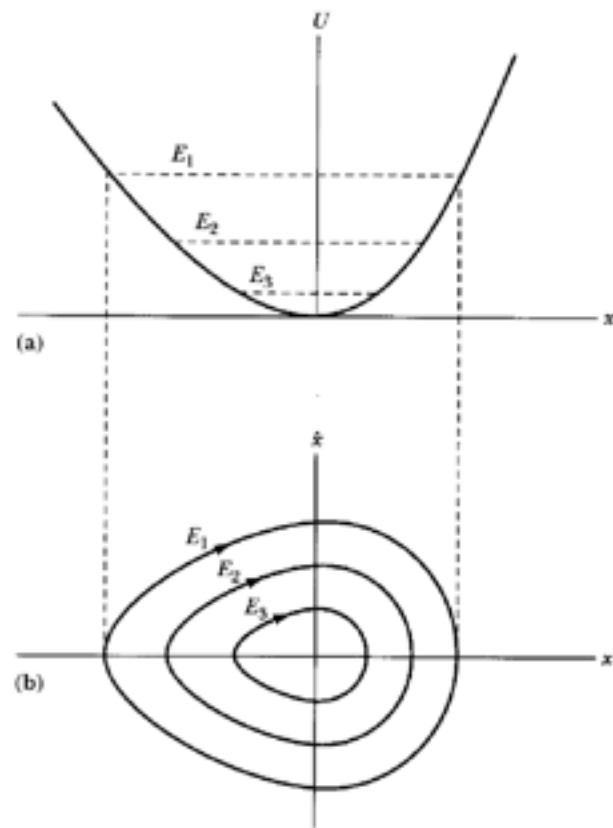


FIGURE 4-5 (a) Asymmetric potential and (b) phase diagram for bounded motion.

- Inverted SHO

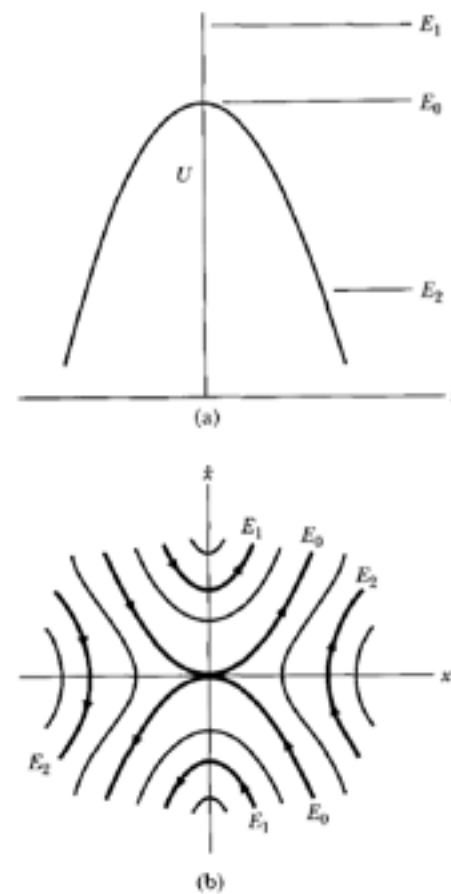


FIGURE 4-6 (a) Inverted asymmetric potential and (b) phase diagram for unbounded motion.

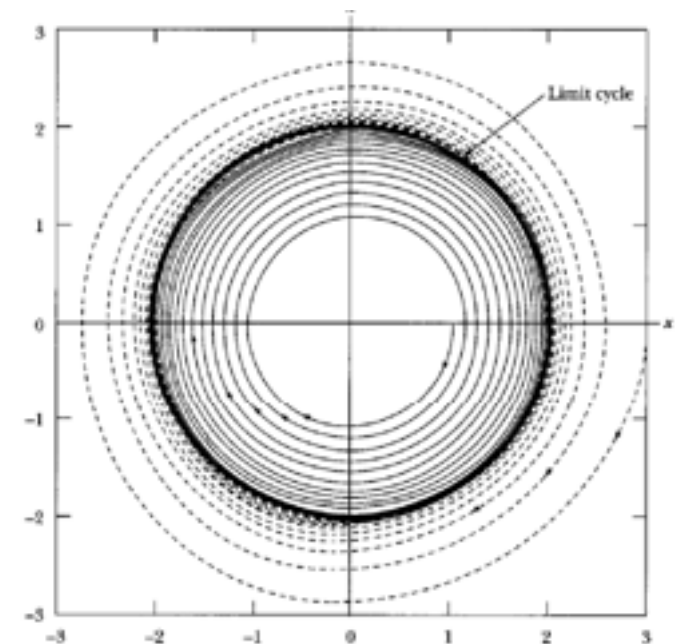
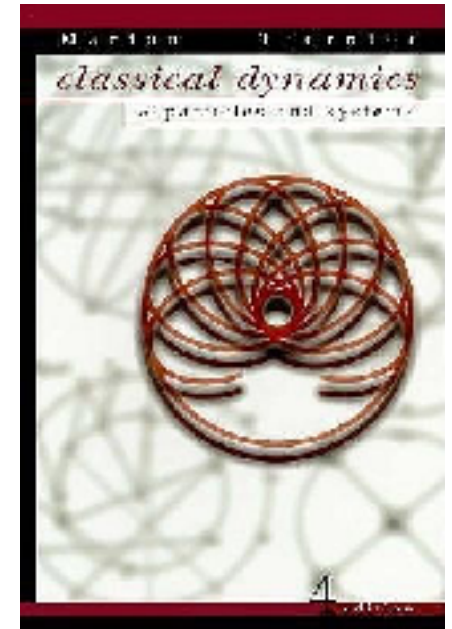
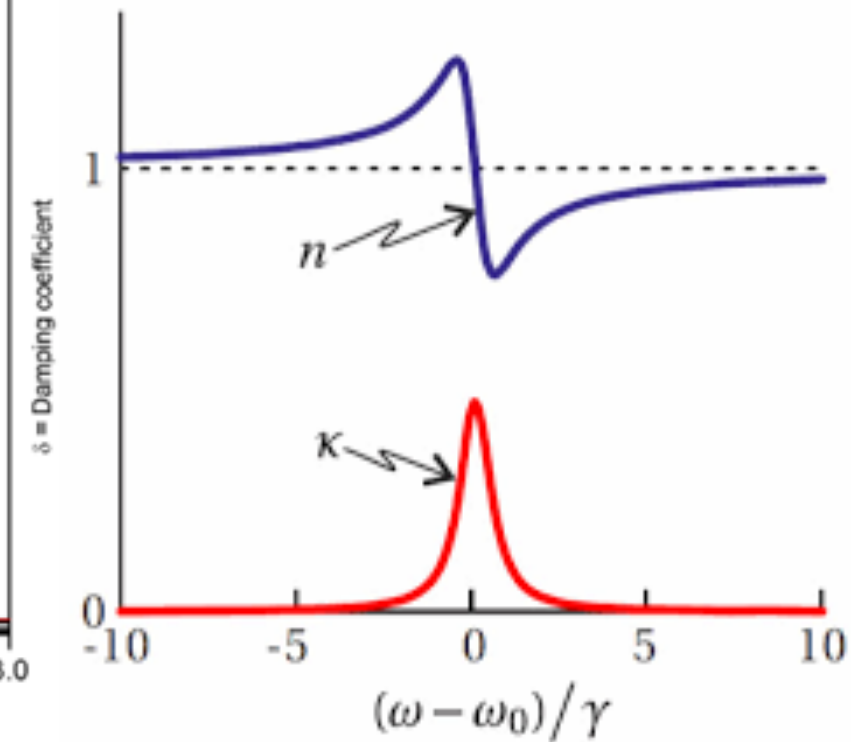
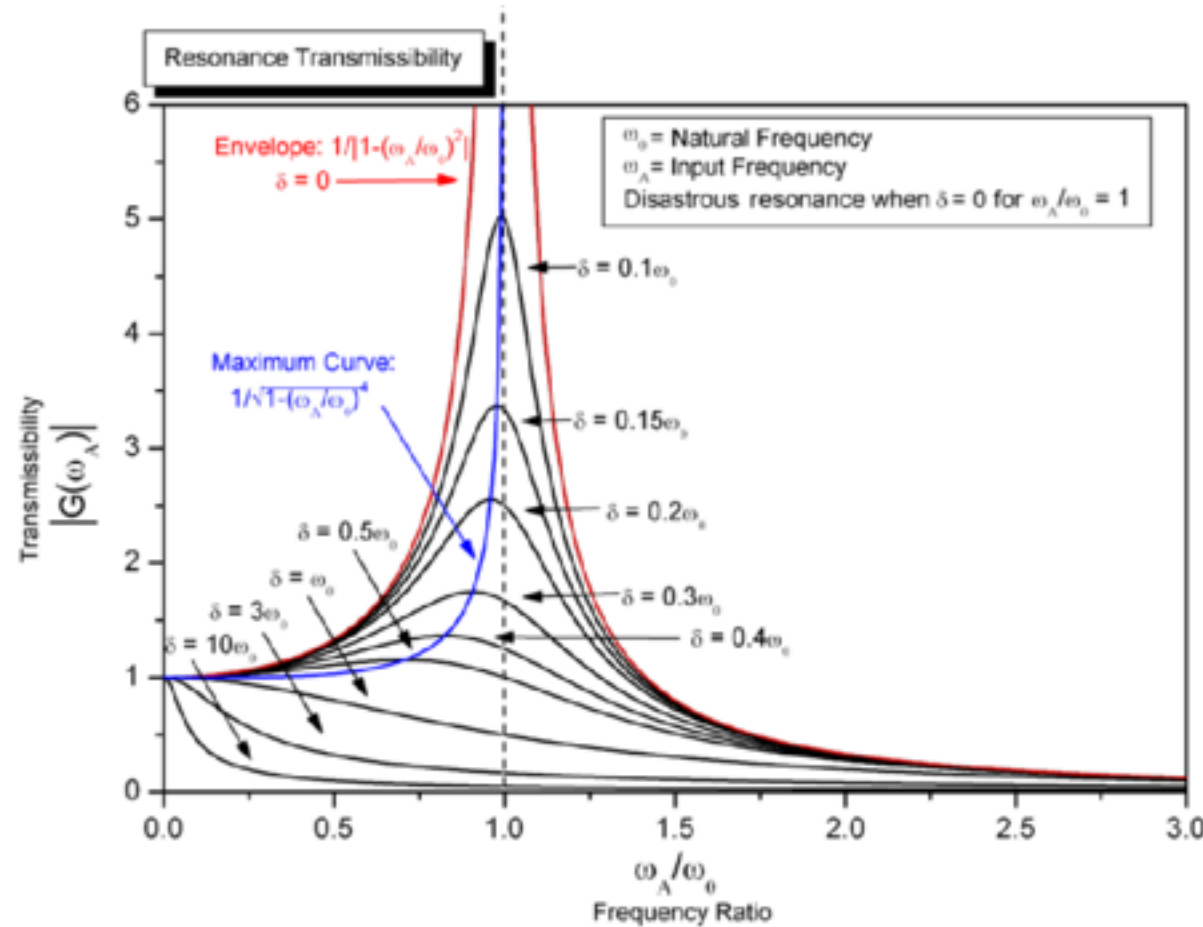
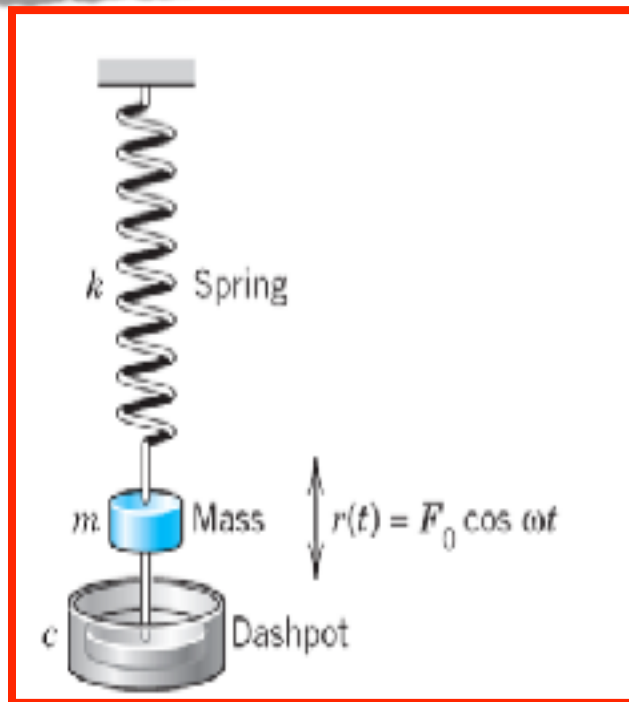


FIGURE 4-7 Phase diagram for the solution of the van der Pol Equation 4.20. The damping term is $\mu = 0.05$, and the solution very slowly approaches the limit cycle at 2. Positive and negative damping occur, respectively, for $|\dot{x}|$ values outside and inside the limit cycle at 2. The solid and dashed lines have initial $(x, \dot{x})$ values of $(1.0, 0)$ and $(3.0, 0)$, respectively.

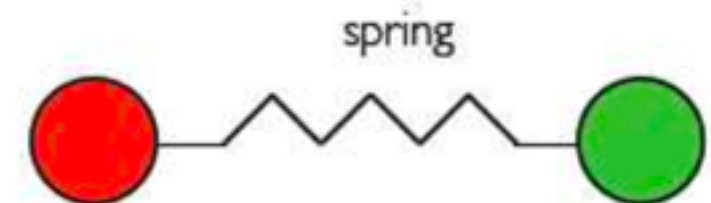
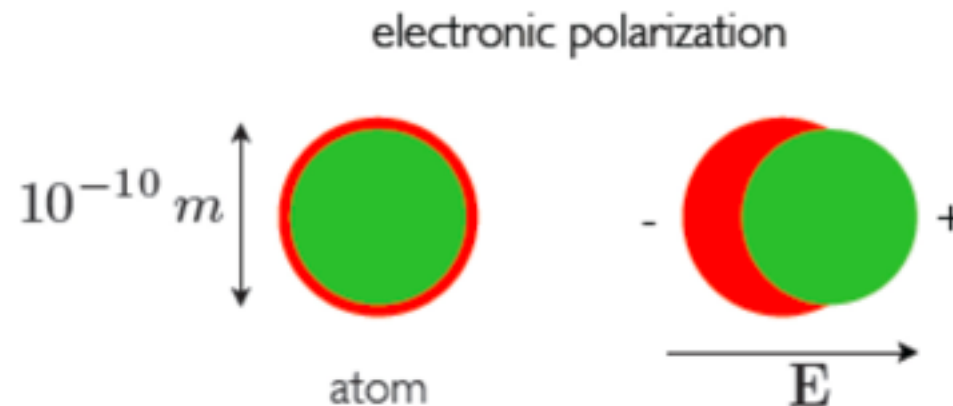
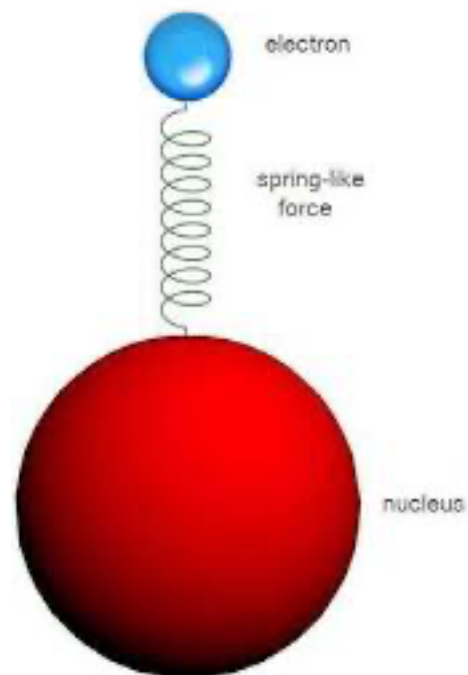
- limit cycle

Driven SHO and Lorentz model



Increase of amplitude as damping decreases and frequency approaches resonant frequency of a driven damped simple harmonic oscillator.^{[1][2]}

Dispersion/Absorption spectrum



Chaos

J. B. Marion and S. T. Thornton, "Classical dynamics of particles and systems," (1995).

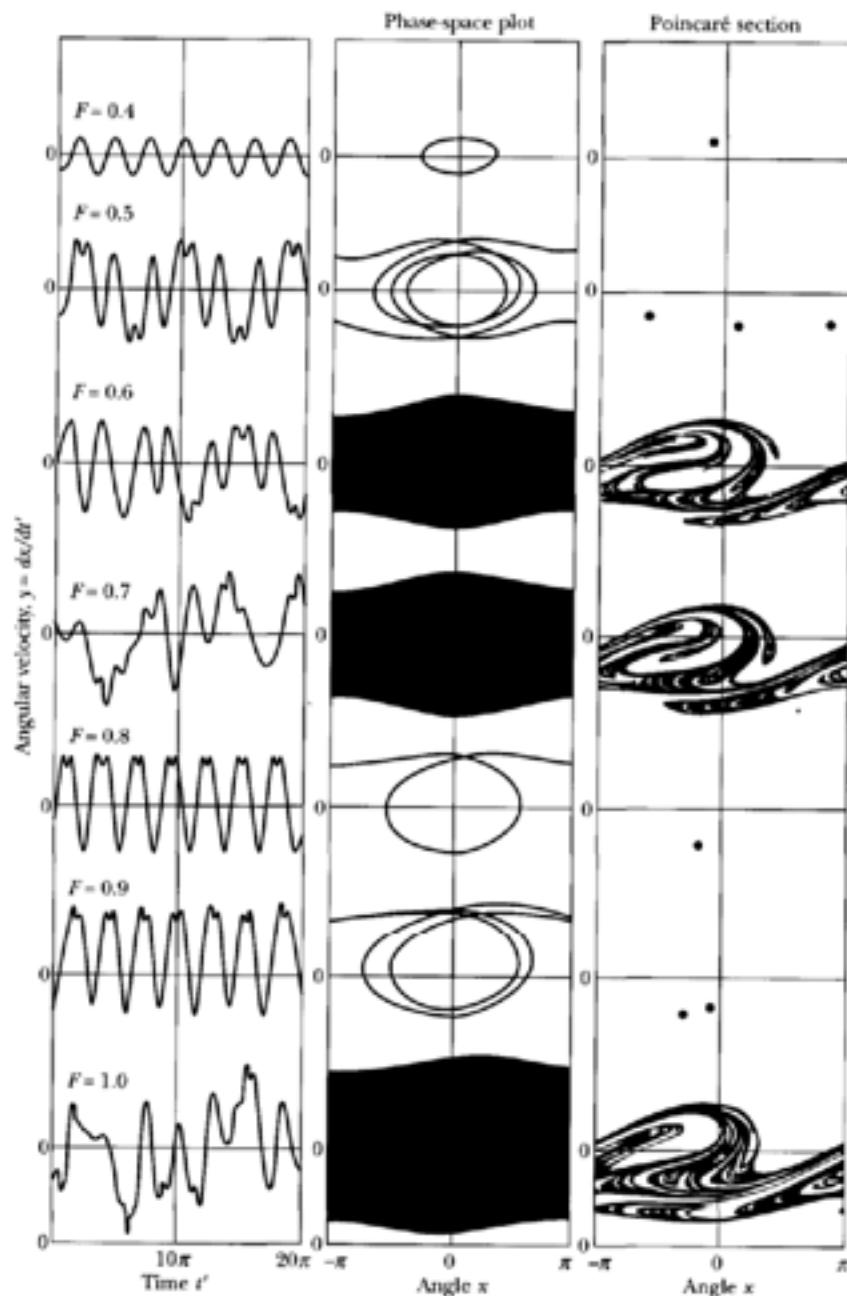
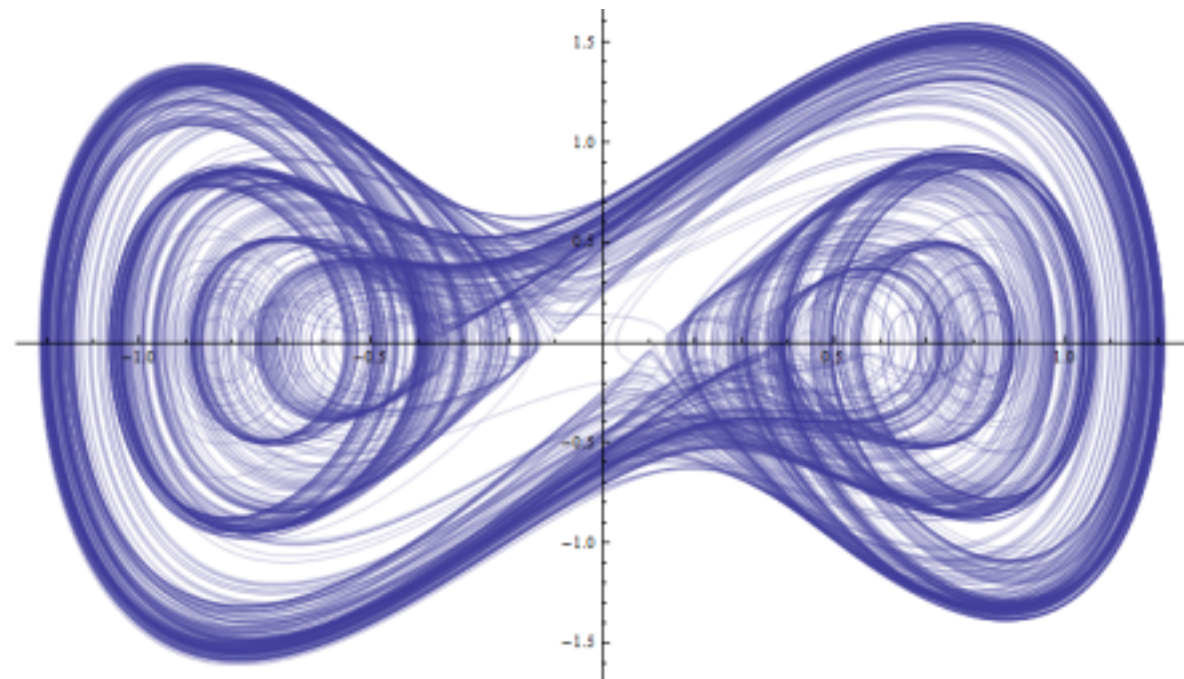


FIGURE 4-19 The damped and driven pendulum for various values of the driving force strength. The angular velocity versus time is shown on the left, and phase diagrams are in the center. Poincaré sections are shown on the right. Note that motion is chaotic for the driving force F values of 0.6, 0.7, and 1.0.

Chaos in Lorenz Model



SHO in 2D: Lissajous curves

$$\mathbf{F} = -k\mathbf{r}$$

$$\left. \begin{aligned} \ddot{x} + \omega_0^2 x &= 0 \\ \ddot{y} + \omega_0^2 y &= 0 \end{aligned} \right\}$$

which can be resolved in polar coordinates into the components

$$\left. \begin{aligned} F_x &= -kr \cos \theta = -kx \\ F_y &= -kr \sin \theta = -ky \end{aligned} \right\}$$

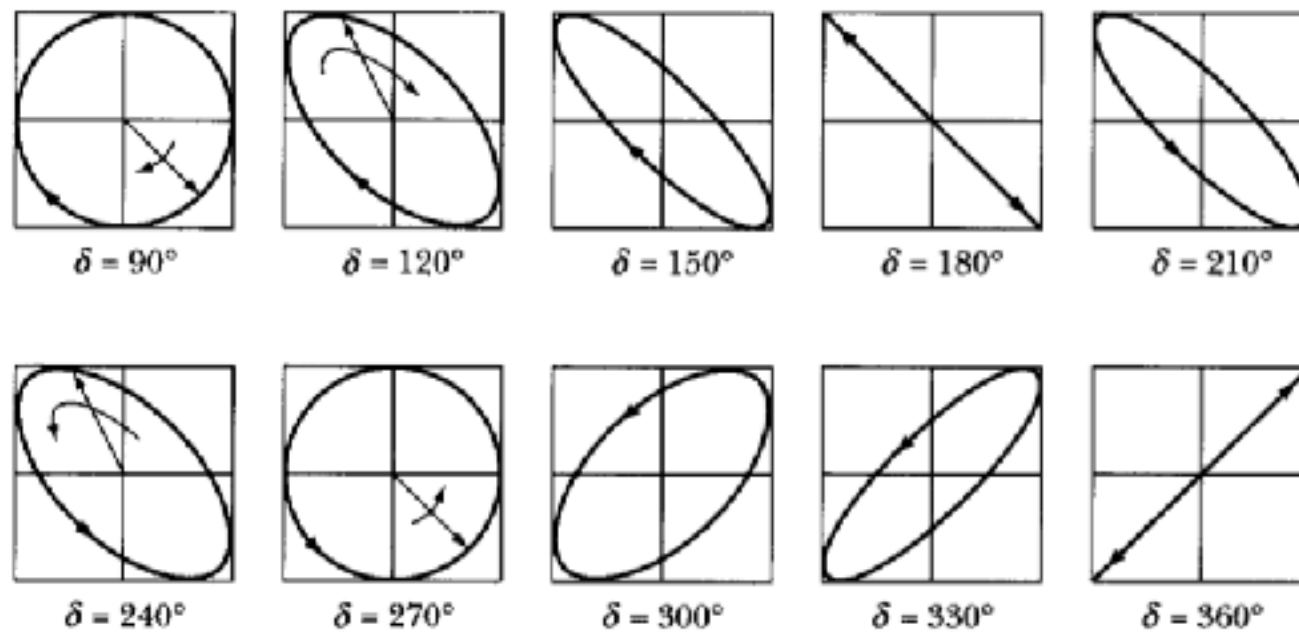
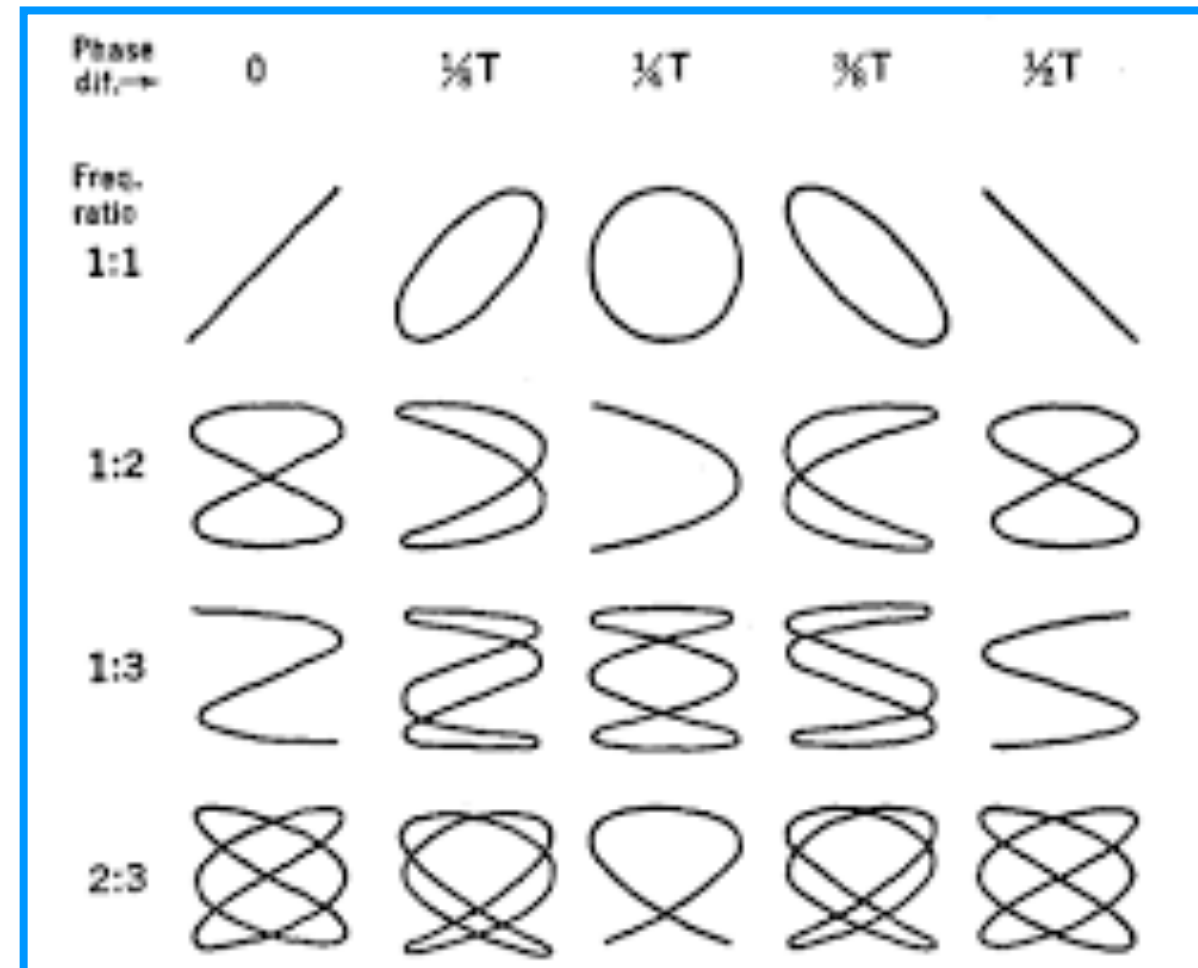


FIGURE 3-2 Two-dimensional harmonic oscillation motion for various phase angles $\delta = \alpha - \beta$.



J. B. Marion and S. T. Thornton, "Classical dynamics of particles and systems," (1995).

Pendulum

$$\ddot{\theta} + \omega_0^2 \sin\theta = 0$$

$$\ddot{\theta} + \omega_0^2 \theta = 0$$

$$\omega_0^2 \equiv \frac{g}{l}$$

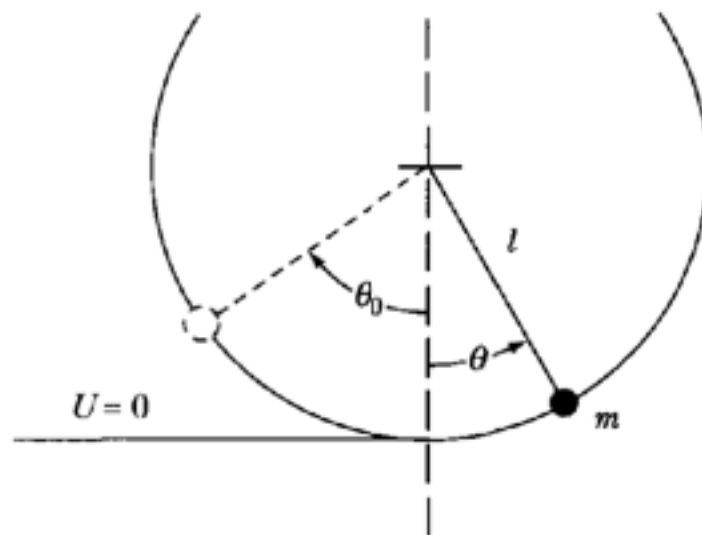


FIGURE 4-9 The plane pendulum where the mass m is not required to oscillate in small angles. The angle $\theta > 0$ is in the counterclockwise direction so that $\theta_0 < 0$.

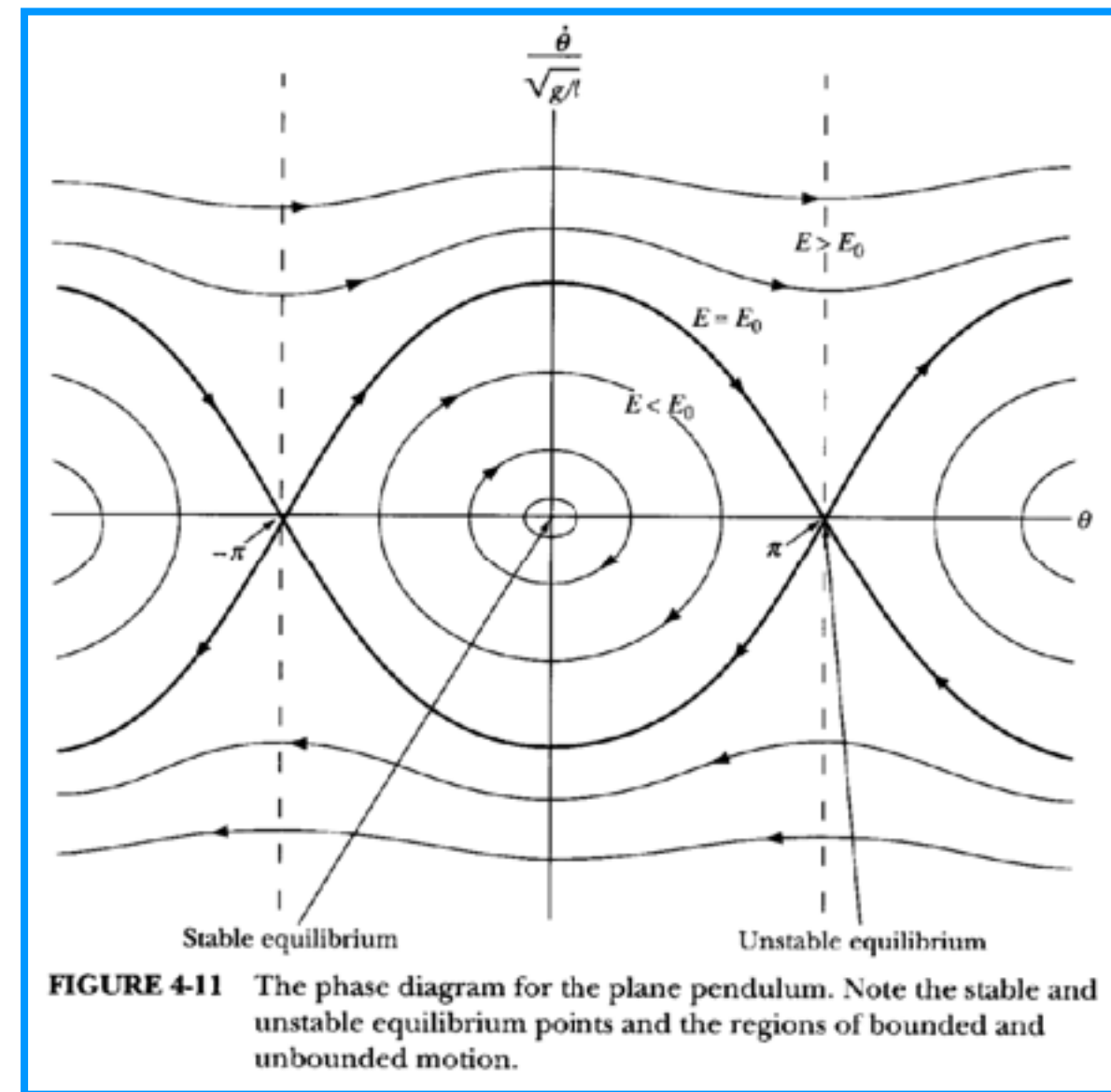
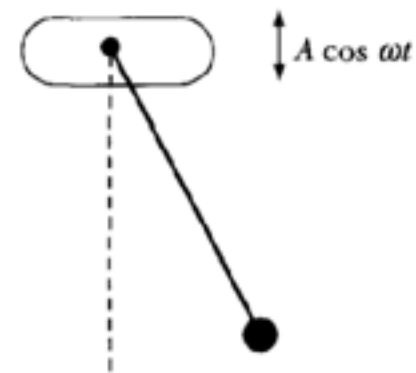


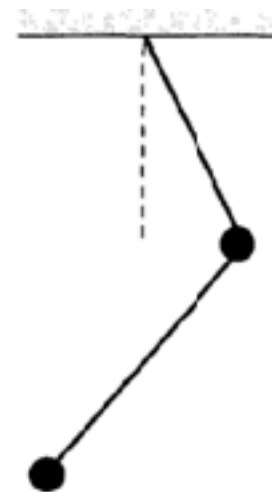
FIGURE 4-11 The phase diagram for the plane pendulum. Note the stable and unstable equilibrium points and the regions of bounded and unbounded motion.

More Pendulums

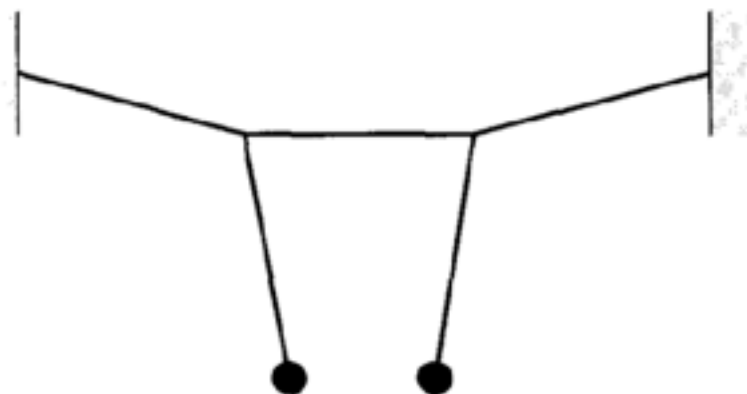
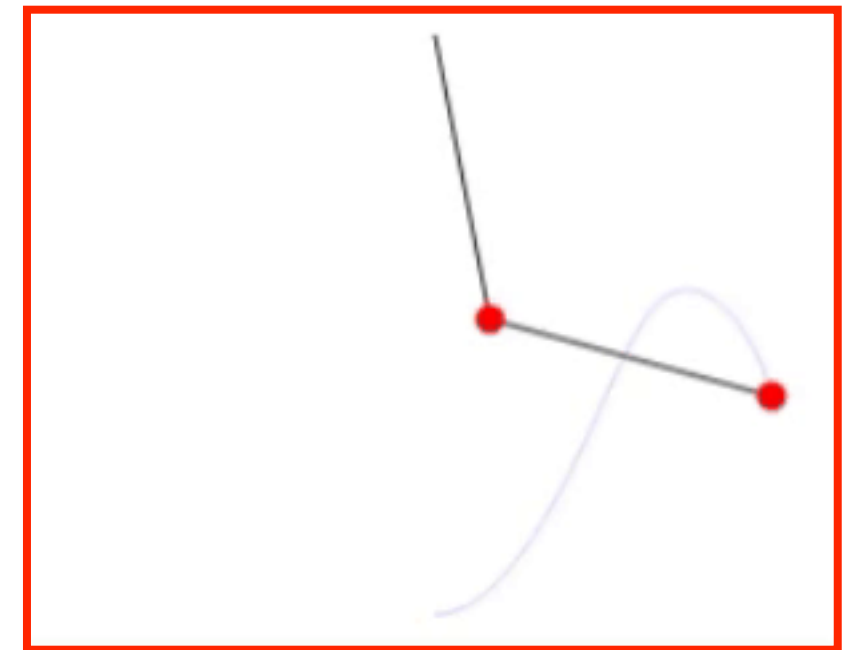
<https://math.stackexchange.com/questions/2971394/periodic-solutions-of-the-double-pendulum>



(a) Forced pivot



(b) Double pendulum



(c) Coupled pendulums



(d) Magnetic pendulum

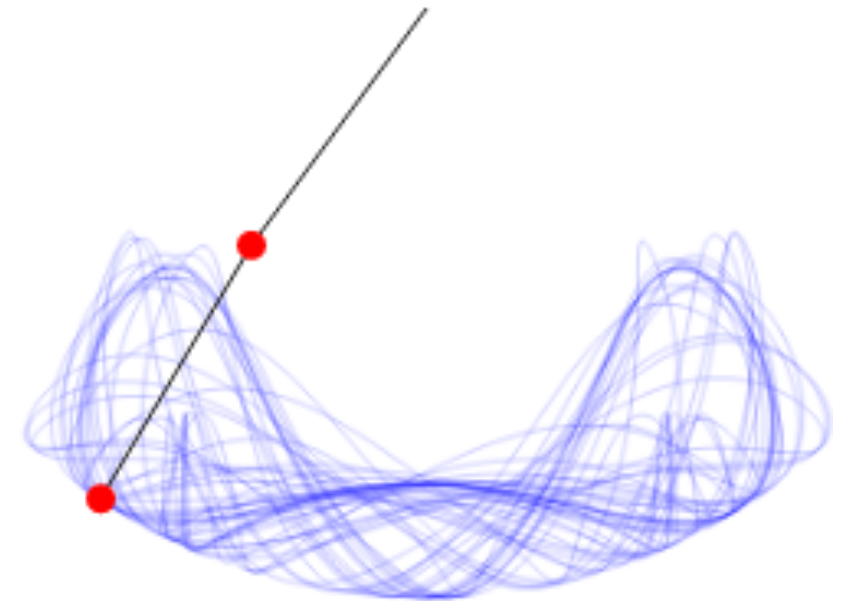


FIGURE 4-17 Examples of pendulums that have chaotic motion.

SHO for EIT/Fano resonance

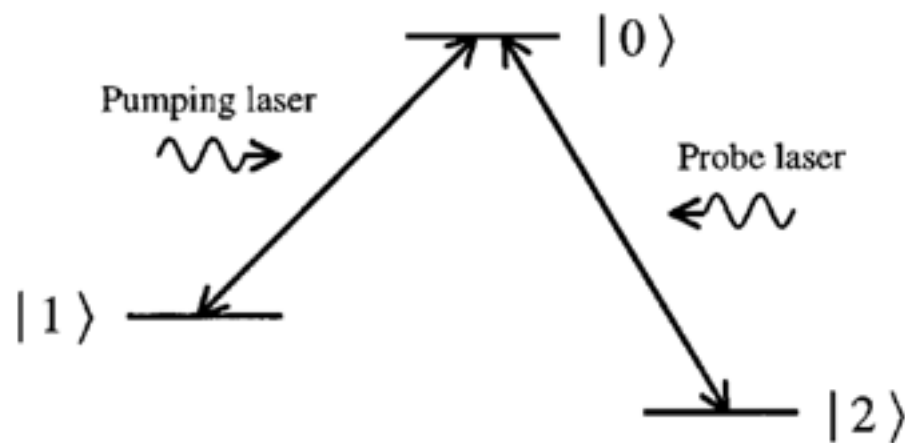


Fig. 1. The energy diagram of a three-level Λ -type atom interacting with two laser beams, coupling the two ground states to a common excited atomic state. Note that, if we connect each ground state to the excited state by a straight line, we draw the Greek letter Λ , hence the name.

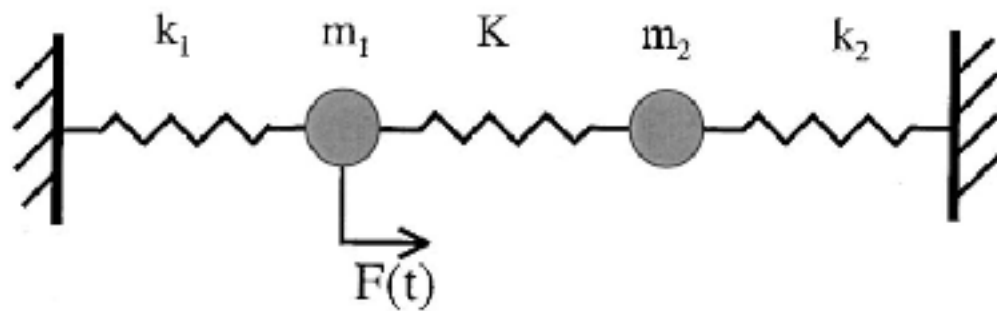
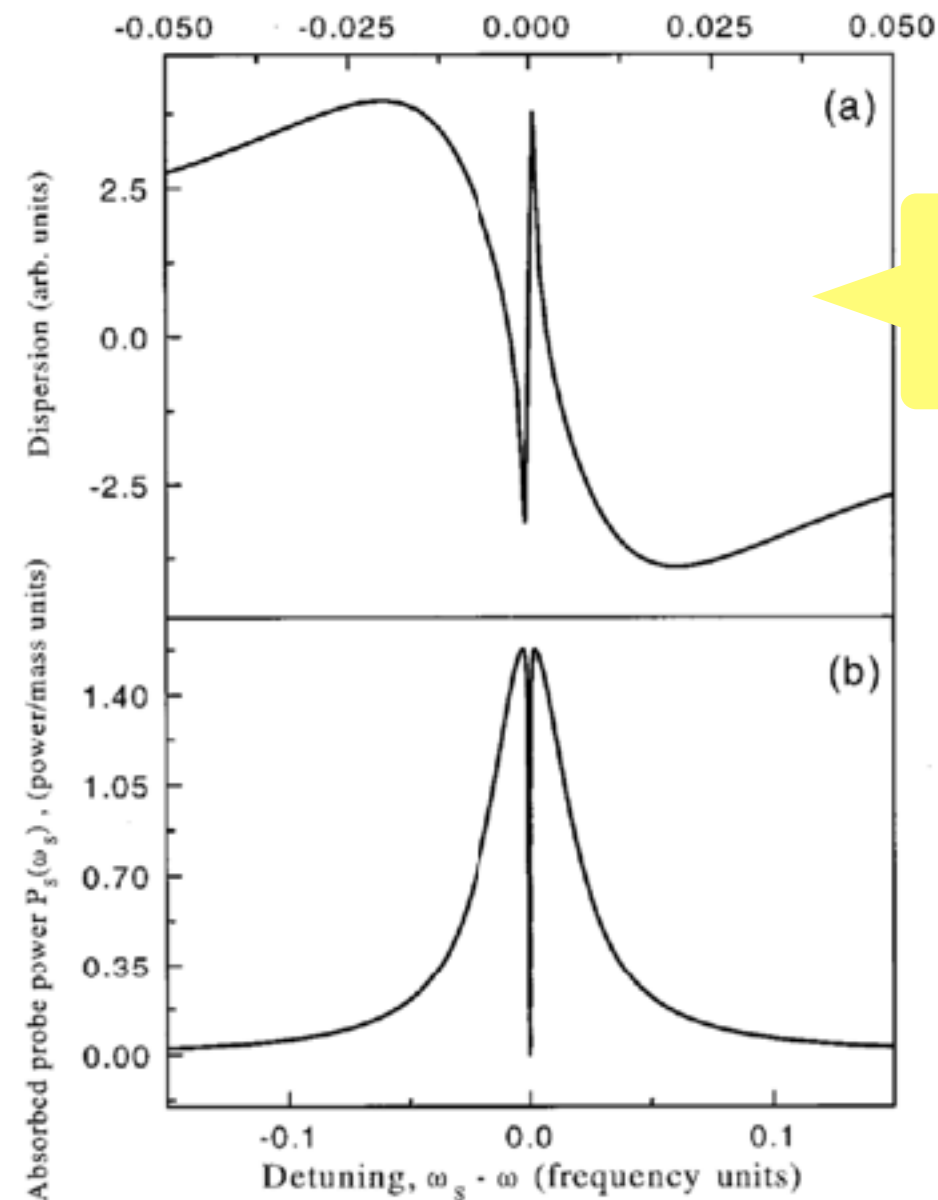


Fig. 2. The mechanical model used to simulate EIT.



Assignment

Fig. 4. The dispersive (a) and absorptive (b) responses of the probe power transferred to particle 1 for $\Omega_r = 0.1$ in frequency units.

C. L. Garrido Alzar et al., "Classical analog of electromagnetically induced transparency," Am. J. Phys. 70, 37 (2002).

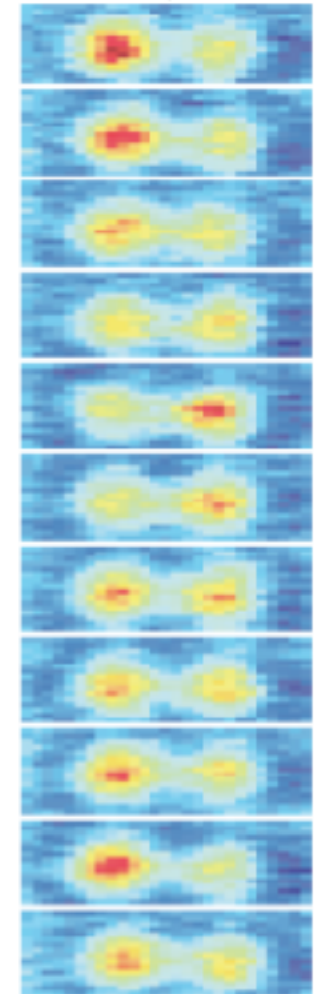
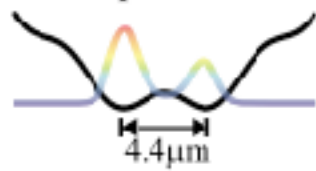
More on SHO and Phase Space

- ☐ **Simple Harmonic Oscillator, SHO**
- ☐ **Pendulum**
 - ☐ Damped SHO (over-damped, critically-damped, under-damped)
 - ☐ Inverted SHO
 - ☐ Nonlinear SHO
 - ☐ Periodic (Orbit) and Chaotic Trajectories
 - ☐ RKC Circuit
- ☐ **Two-Level (State) systems**
 - ☐ **Left-Right, Ground-Excited, Down-Up (spin 1/2), Dimer states,**
 - ☐ Bose-Einstein Condensates in Double Wells
 - ☐ Double pendulum/Coupled SHOs
 - ☐ Parity-Time symmetric SHO
- ☐ **EIT and Fano resonance**
- ☐ Pseudo-Potential

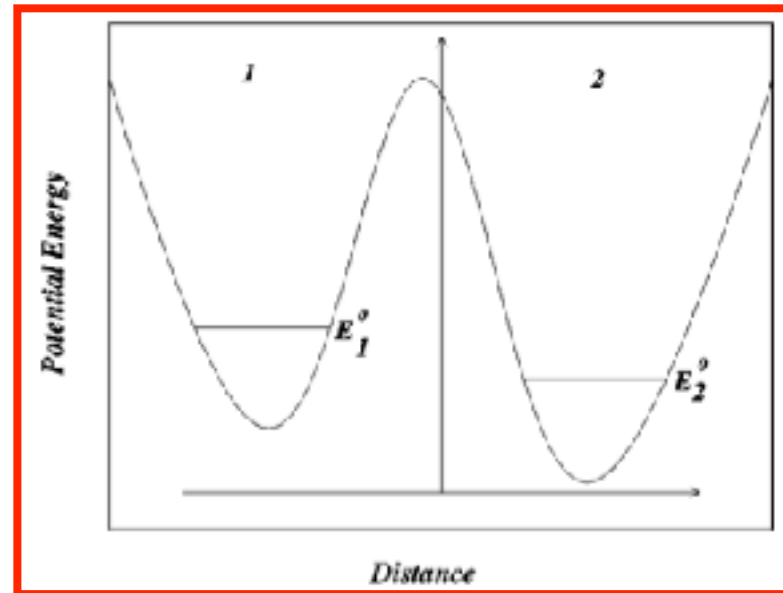
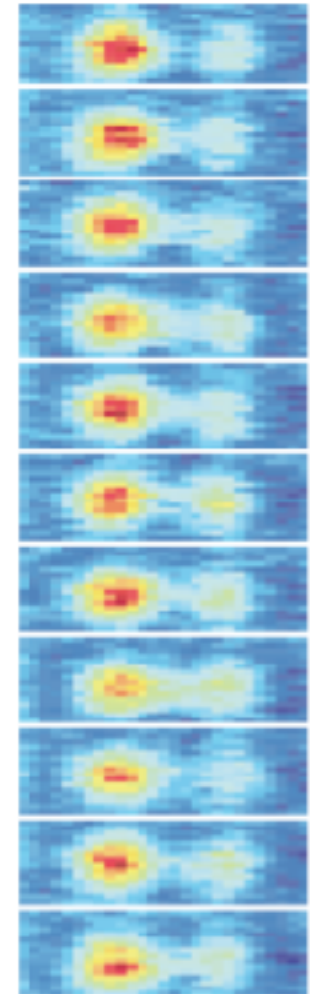
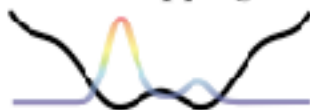
*<http://mx.nthu.edu.tw/~rkleee>

Josephson tunneling for BEC in double well

a Josephson oscillations



b Self-trapping



S. Raghavan, et al.,
“Coherent oscillations between two weakly coupled Bose-Einstein condensates: Josephson effects, π oscillations, and macroscopic quantum self-trapping,”
Phys. Rev. A **59**, 620 (1999).

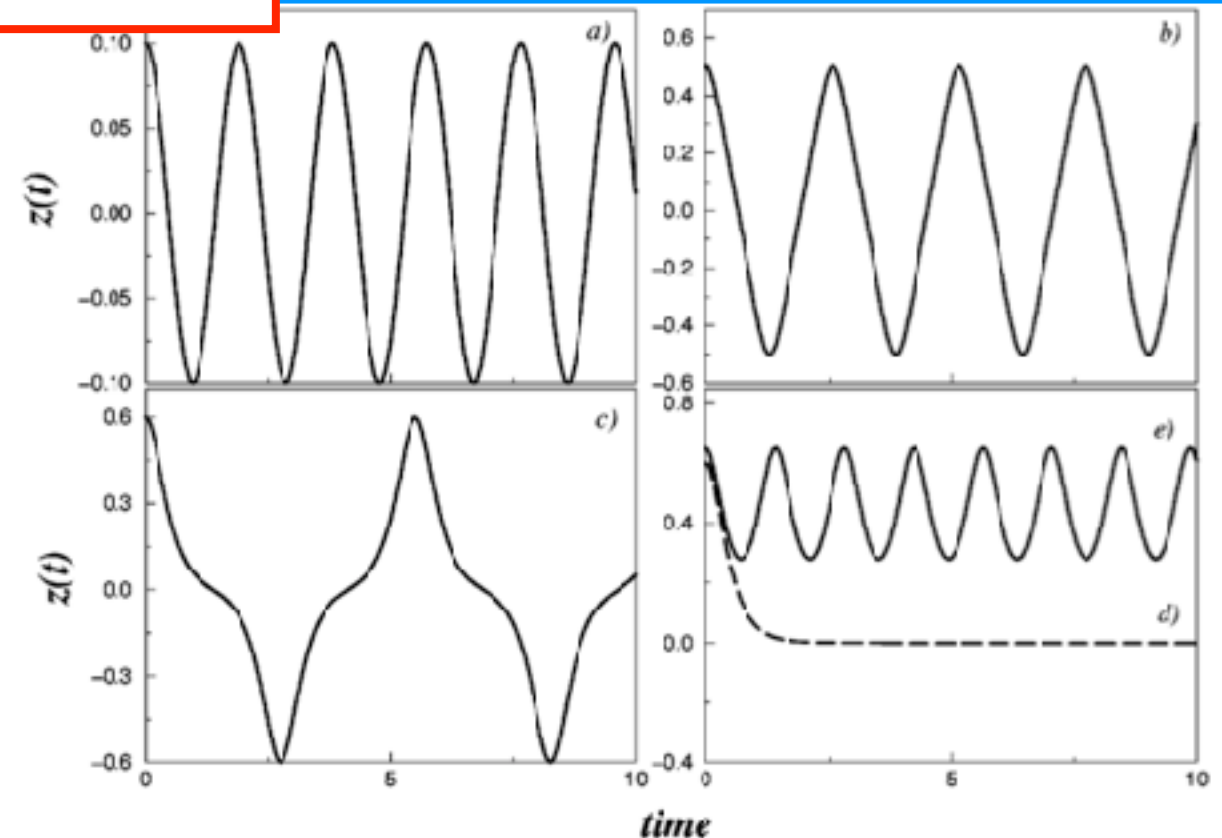


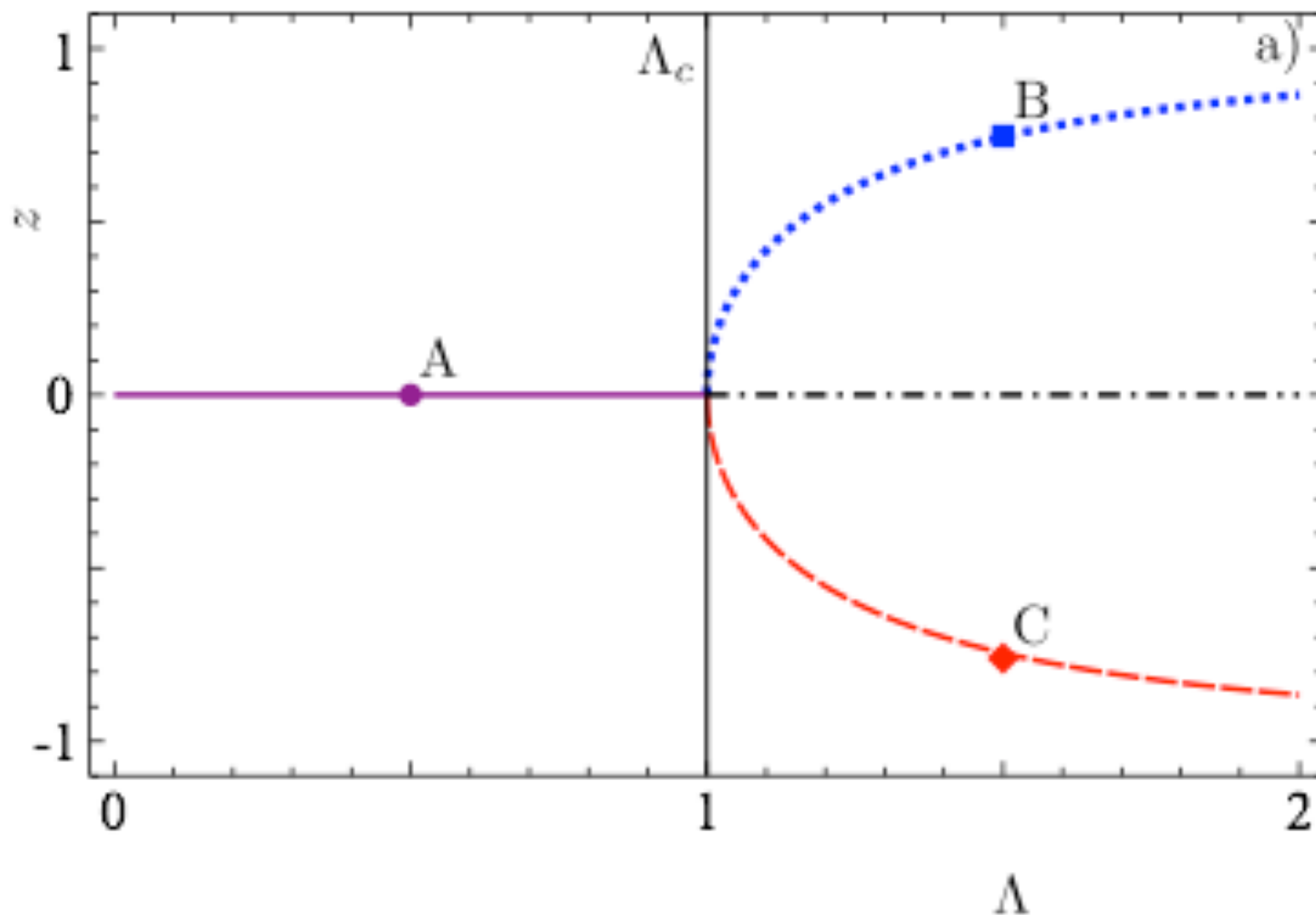
FIG. 2. Population imbalance $z(t)$ as a function of dimensionless time $2Kt$ (in units of $\hbar$), with conditions $\Lambda=10$ and $\phi(0)=0$ in a symmetric trap. The initial population imbalance $z(0)$ takes the values (a) 0.1, (b) 0.5, (c) 0.59, (d) 0.6, and (e) 0.65.

M. Albiez, et al.,
Phys. Rev. Lett. **95**, 010402 (2005).

Josephson tunneling for BEC in double well

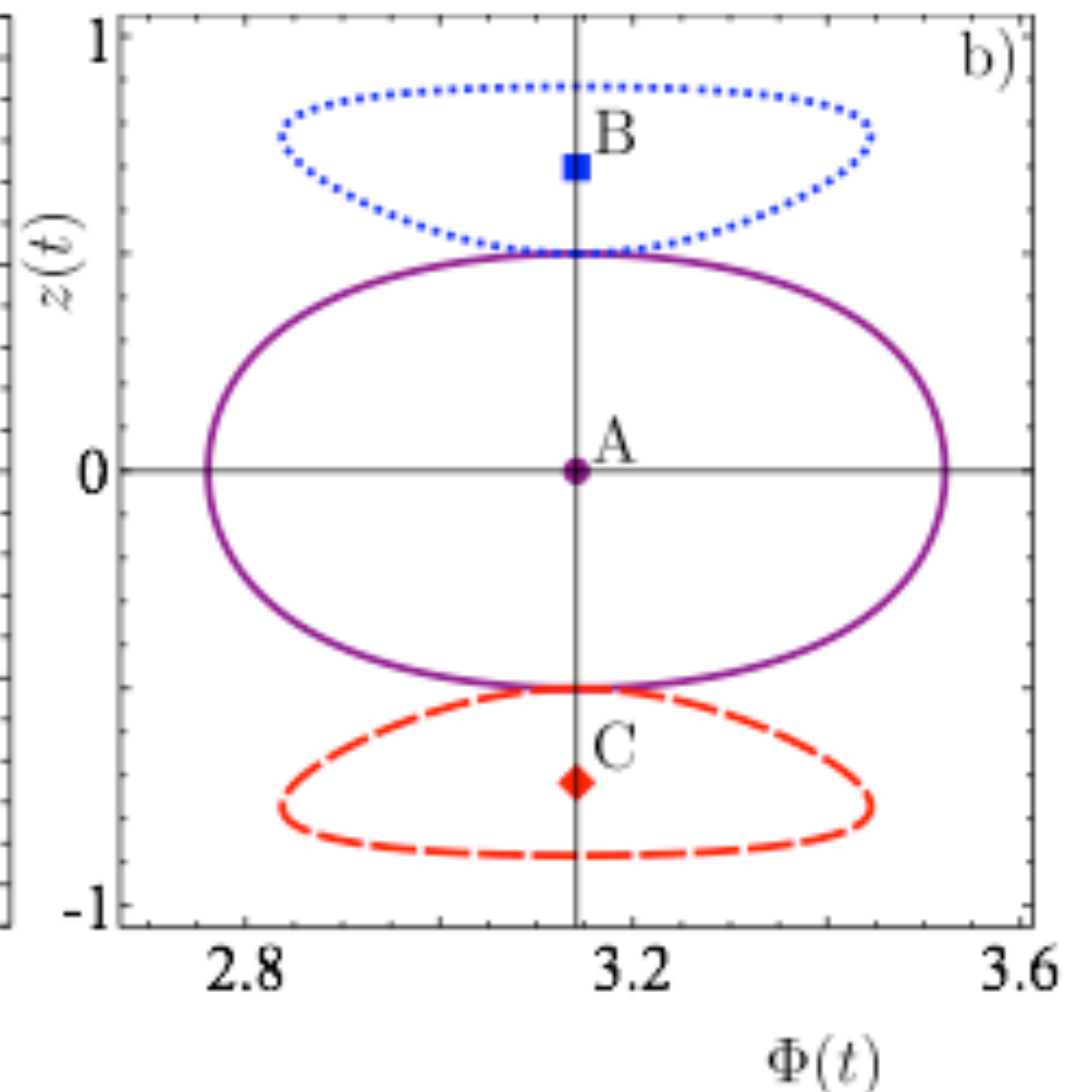
Rigid Pendulum

$$H = \frac{p_\theta^2}{2ml^2} - mgl \cos \theta.$$



Non-Rigid Pendulum

$$H_1 = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2} \cos \Phi,$$



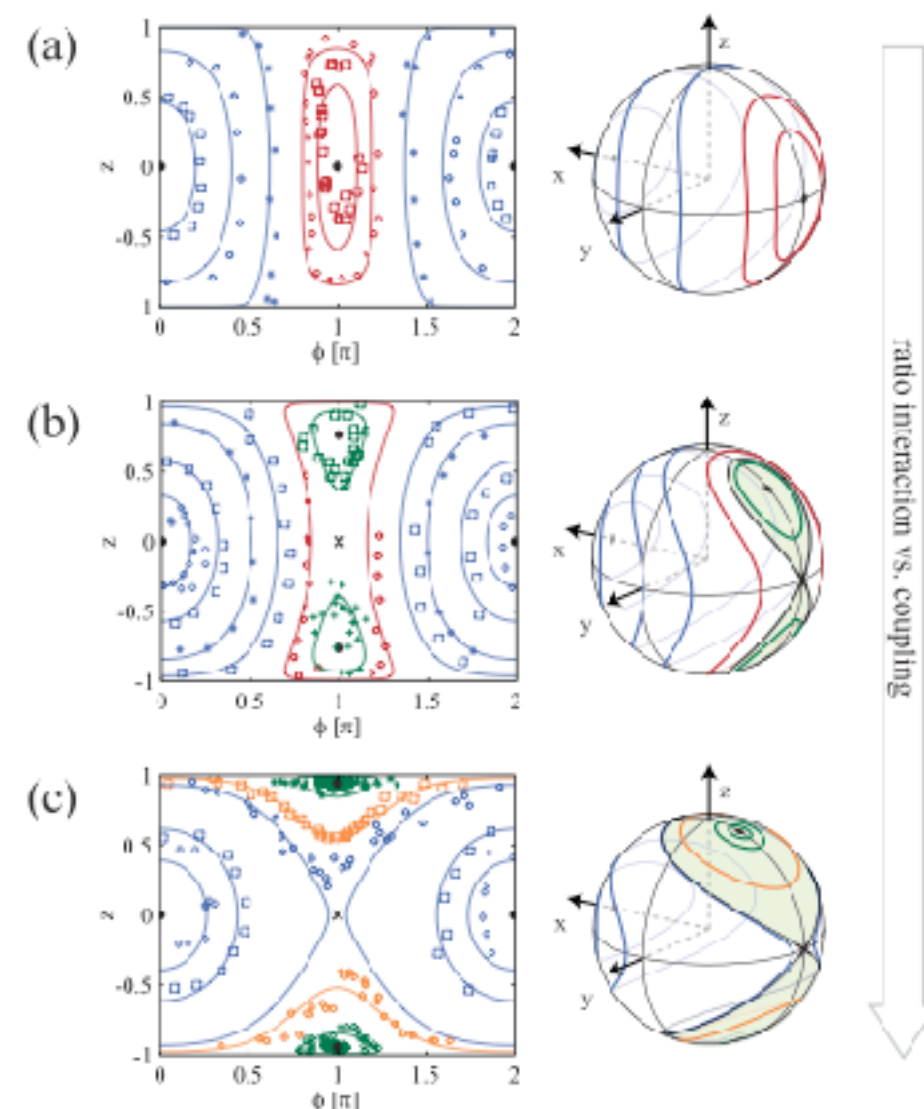
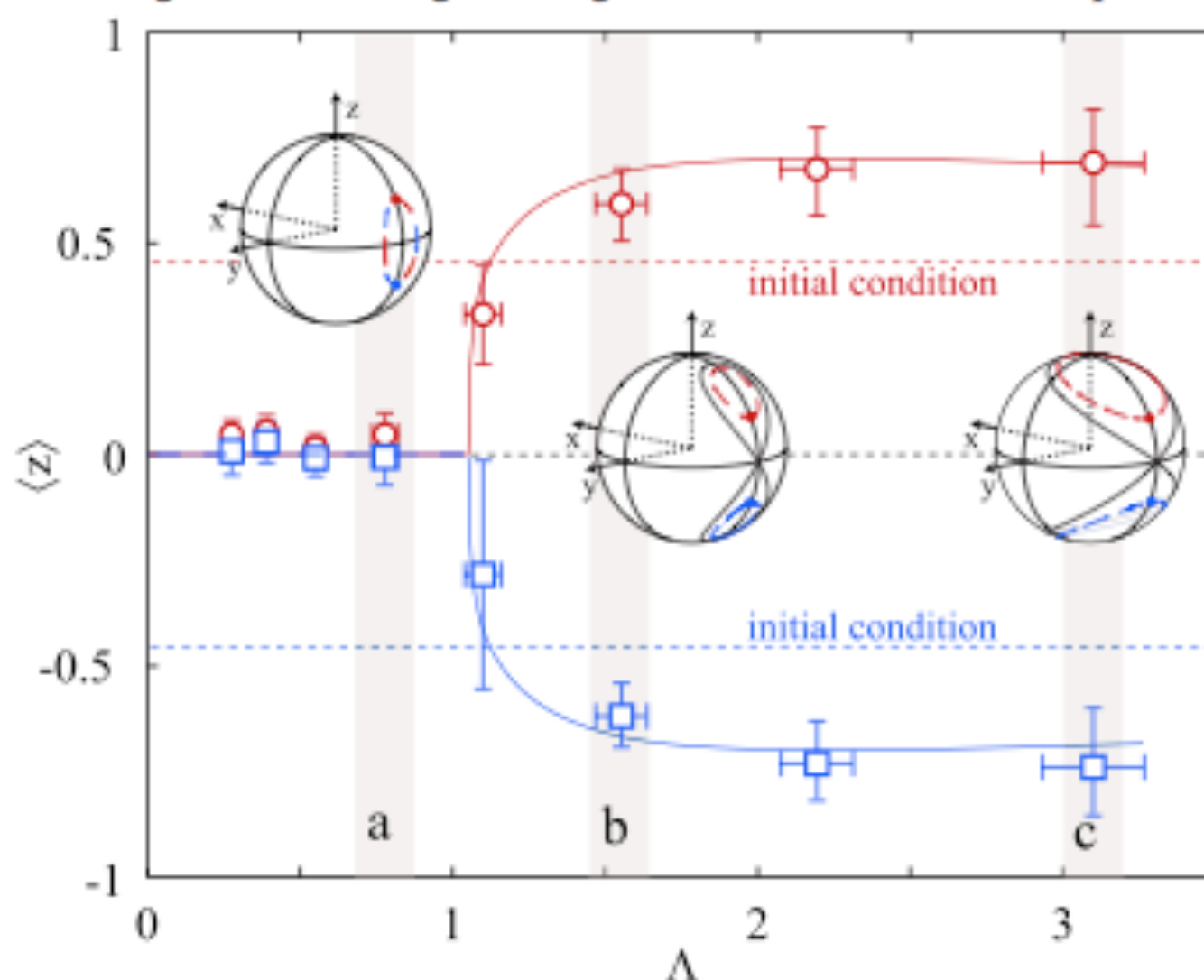
Classical Bifurcation at the Transition from Rabi to Josephson Dynamics

Tilman Zibold,^{*} Eike Nicklas, Christian Gross, and Markus K. Oberthaler

Kirchhoff Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, D-69120, Germany

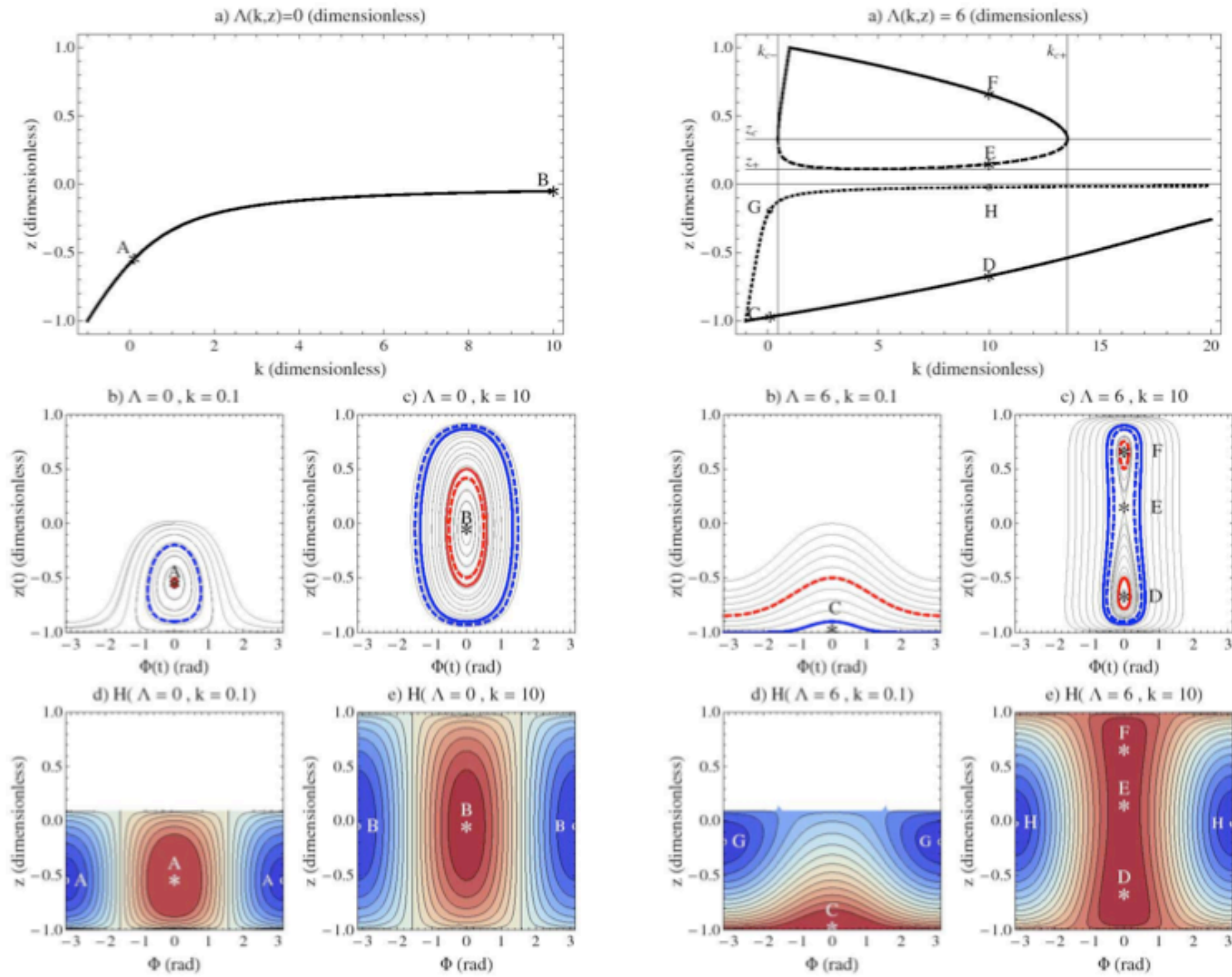
(Received 12 July 2010; published 12 November 2010)

We report on the experimental demonstration of the internal bosonic Josephson effect in a rubidium spinor Bose-Einstein condensate. The measurement of the full time dynamics in phase space allows the characterization of the theoretically predicted π -phase modes and quantitatively confirms analytical predictions, revealing a classical bifurcation. Our results suggest that this system is a model system which can be tuned from classical to the quantum regime and thus is an important step towards the experimental investigation of entanglement generation close to critical points.



Classical dynamics of a two-species condensate driven by a quantum field

B. M. Rodríguez-Lara^{1,2} and Ray-Kuang Lee^{1,3}

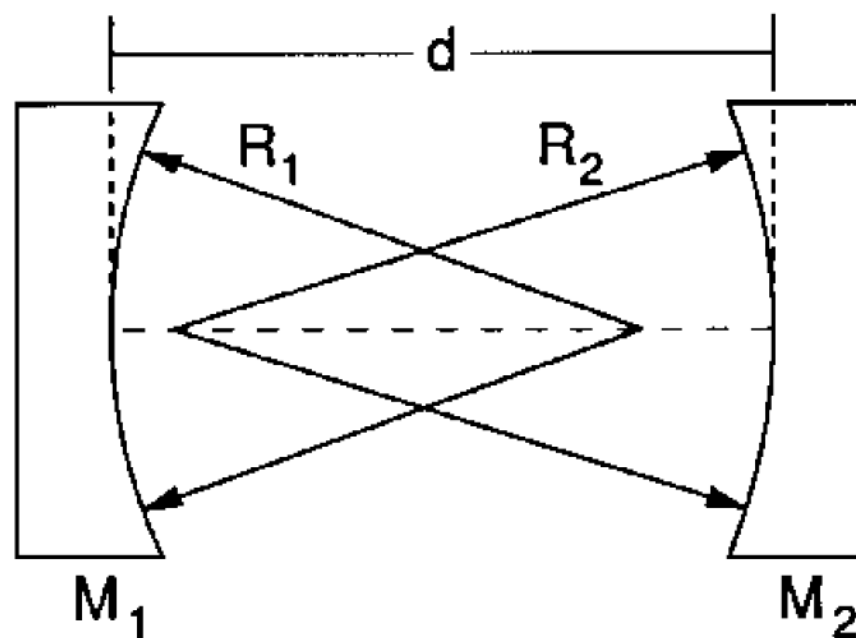


Ray-Wave Analogy

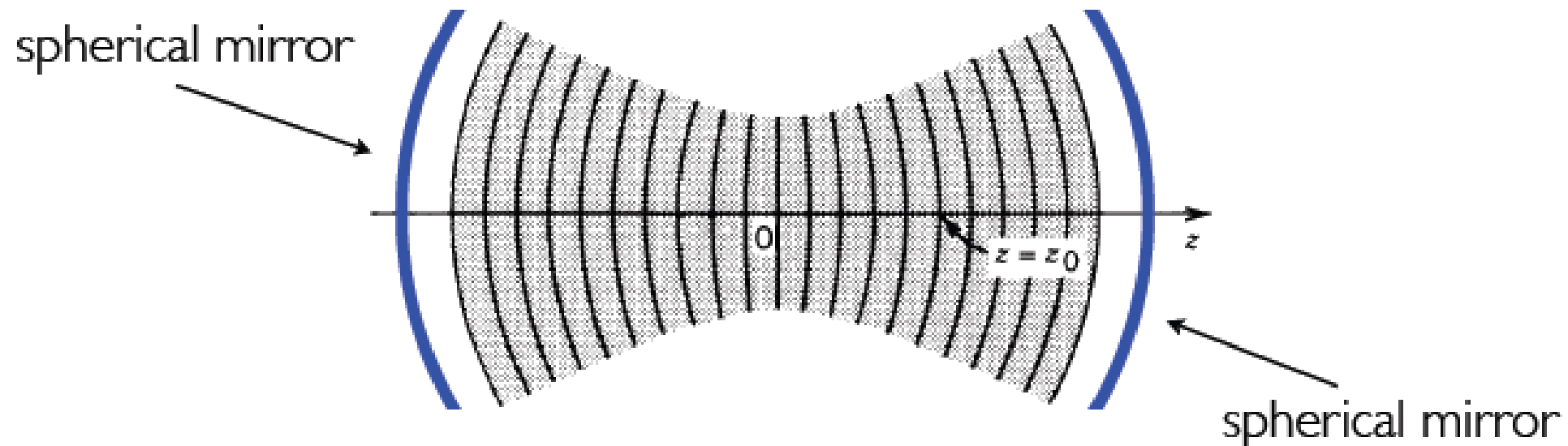


Optical Cavity

Ray Optics

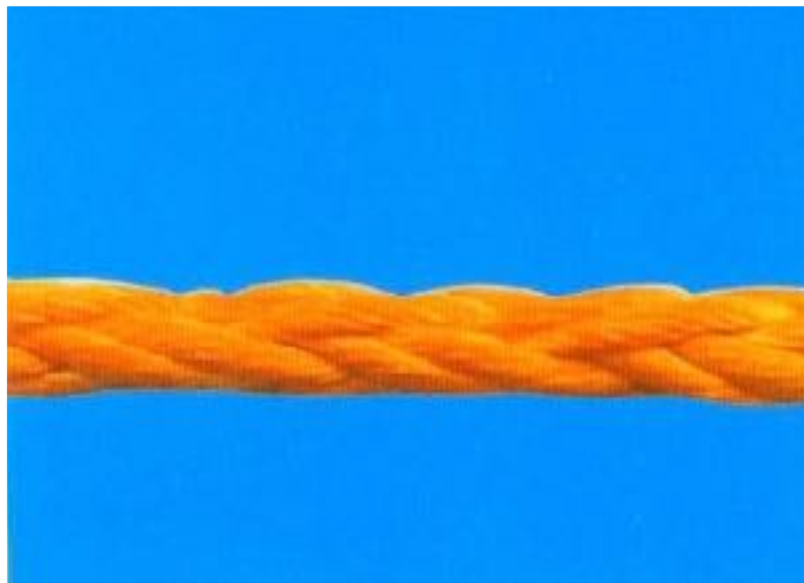


Wave Optics

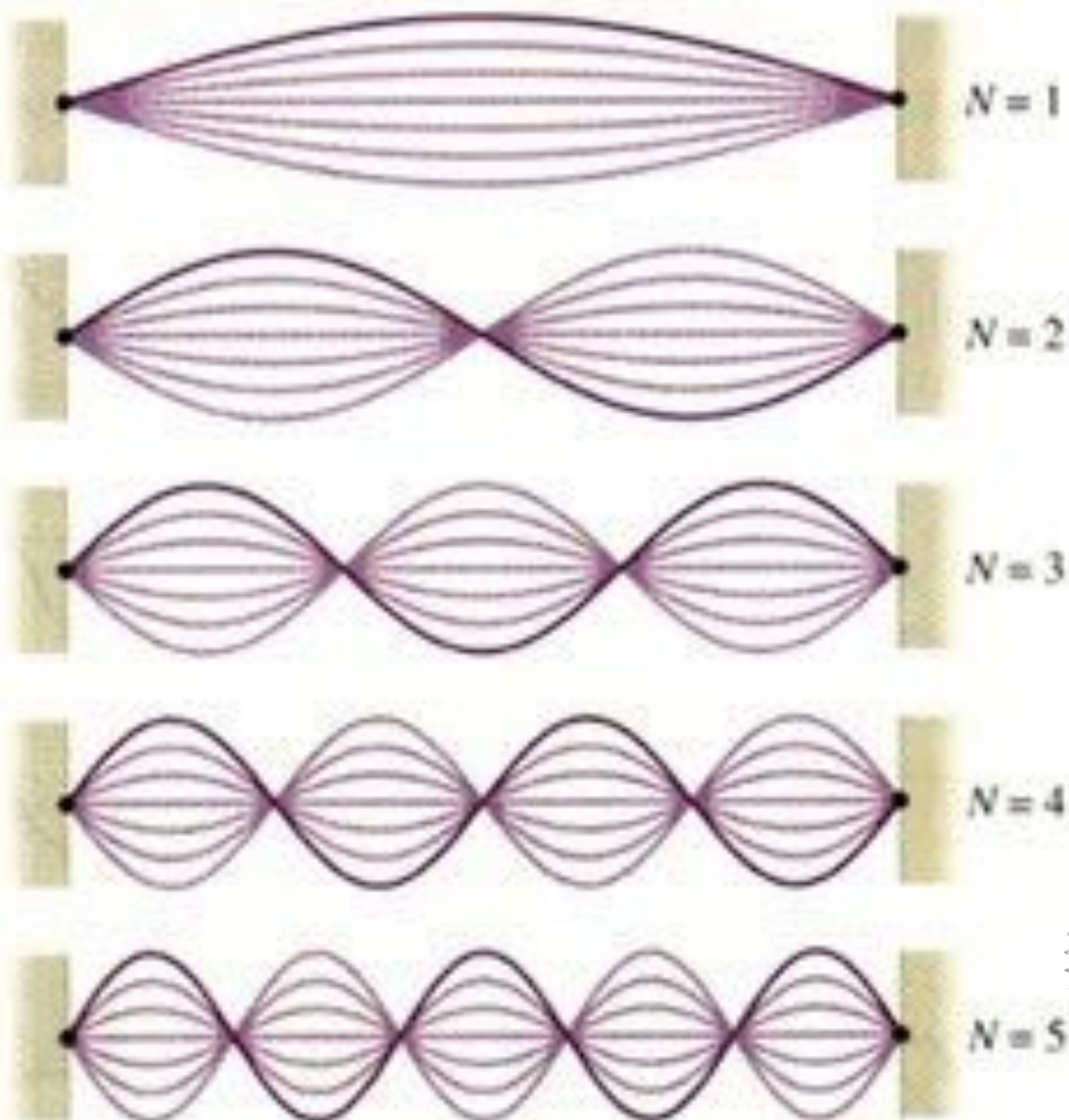


Harmonics: String

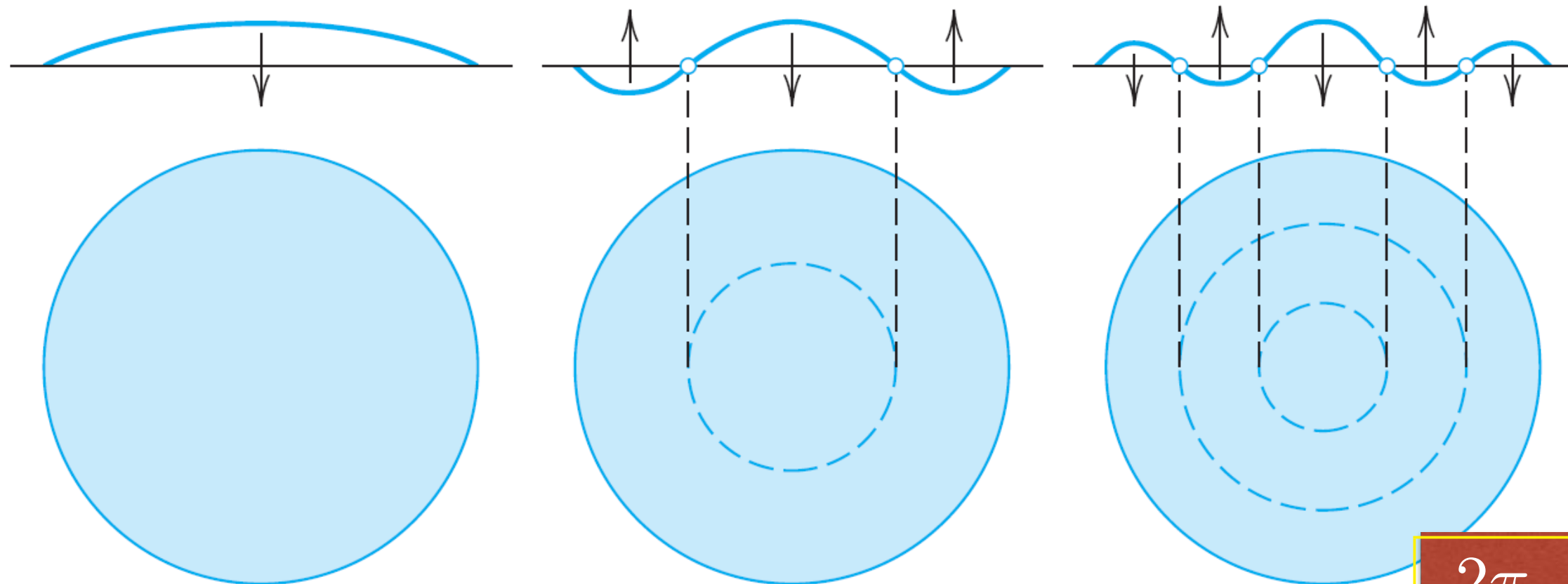
1st thru 5th harmonics of a vibrating string



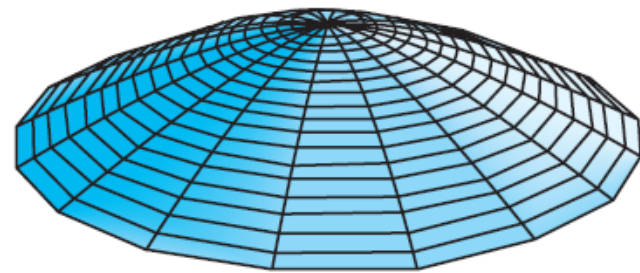
$$\frac{2\pi}{\lambda} L = N \pi.$$



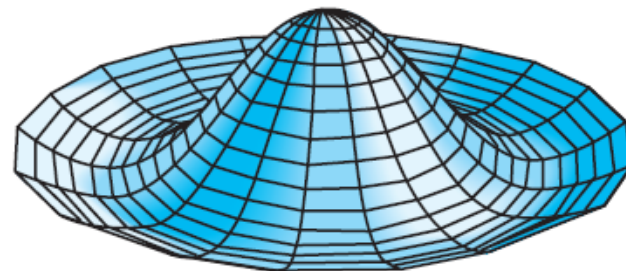
Disk:



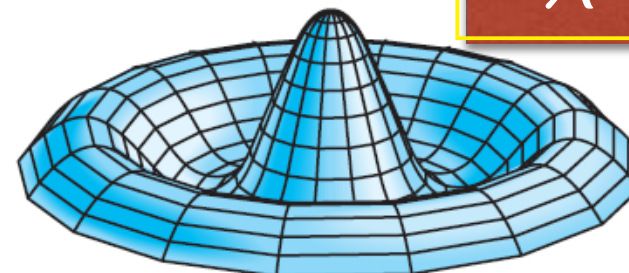
$$\frac{2\pi}{\lambda} D = N \pi.$$



$m = 1$



$m = 2$



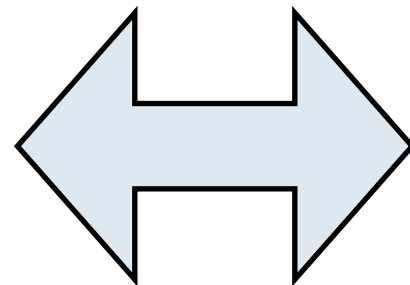
$m = 3$



Classical-Quantum correspondence

Ray Optics
Fermat's principle

Particle



Trajectory

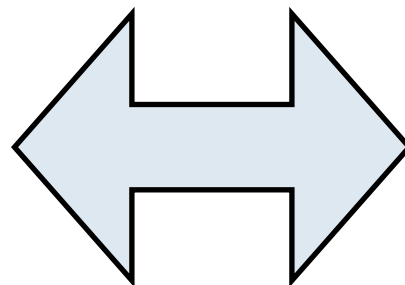
Husimi function

Mode

Gutzwilla trace formula

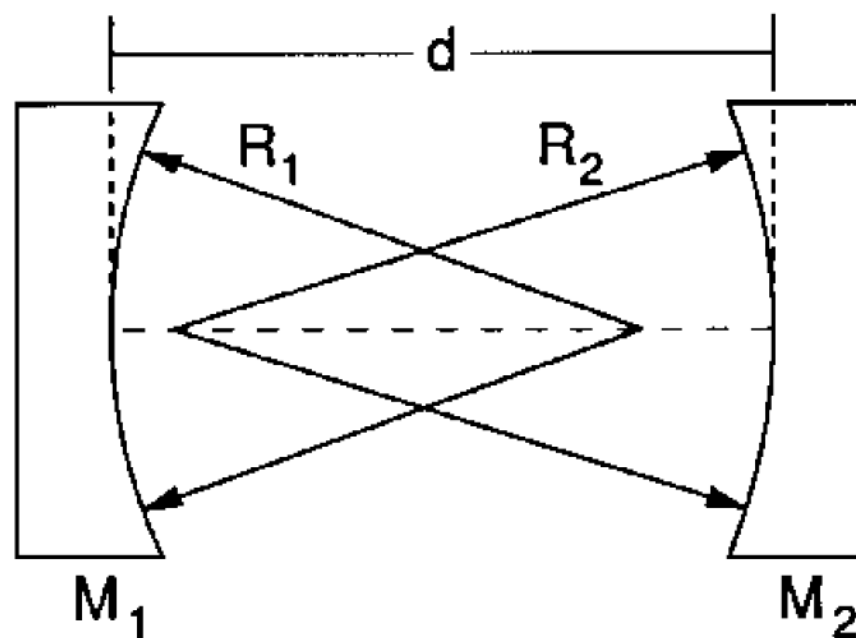
EM Optics
Maxwell's Eq.

Wave

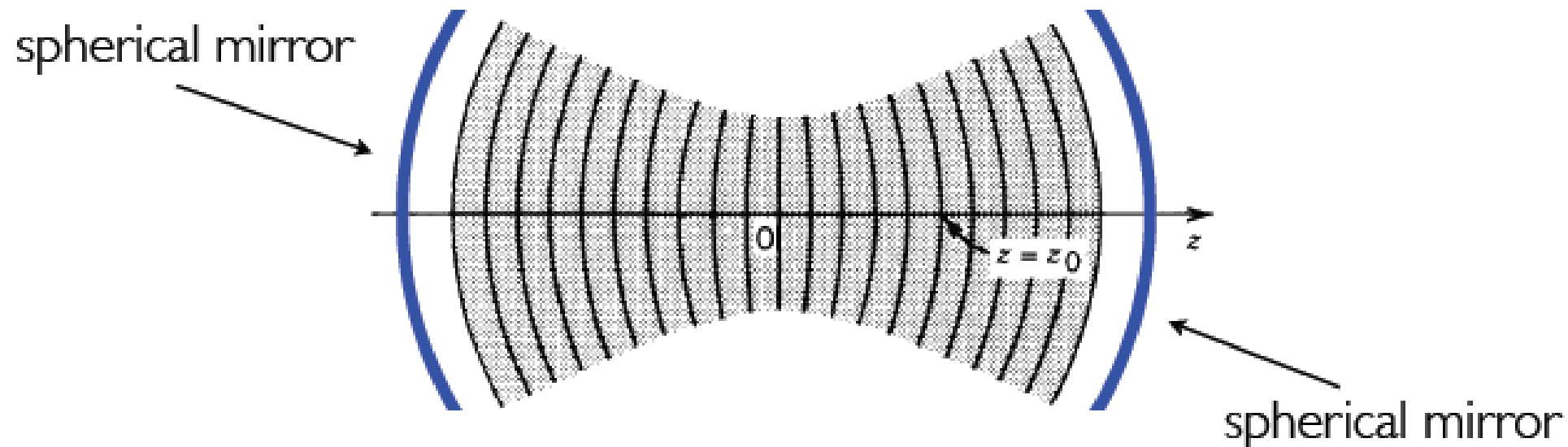


Optical Cavity

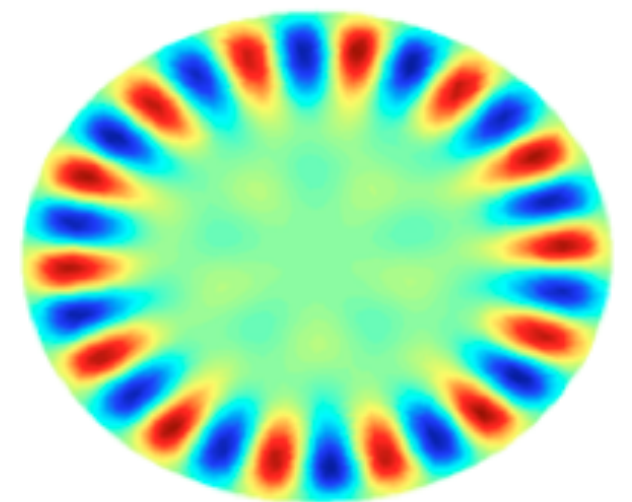
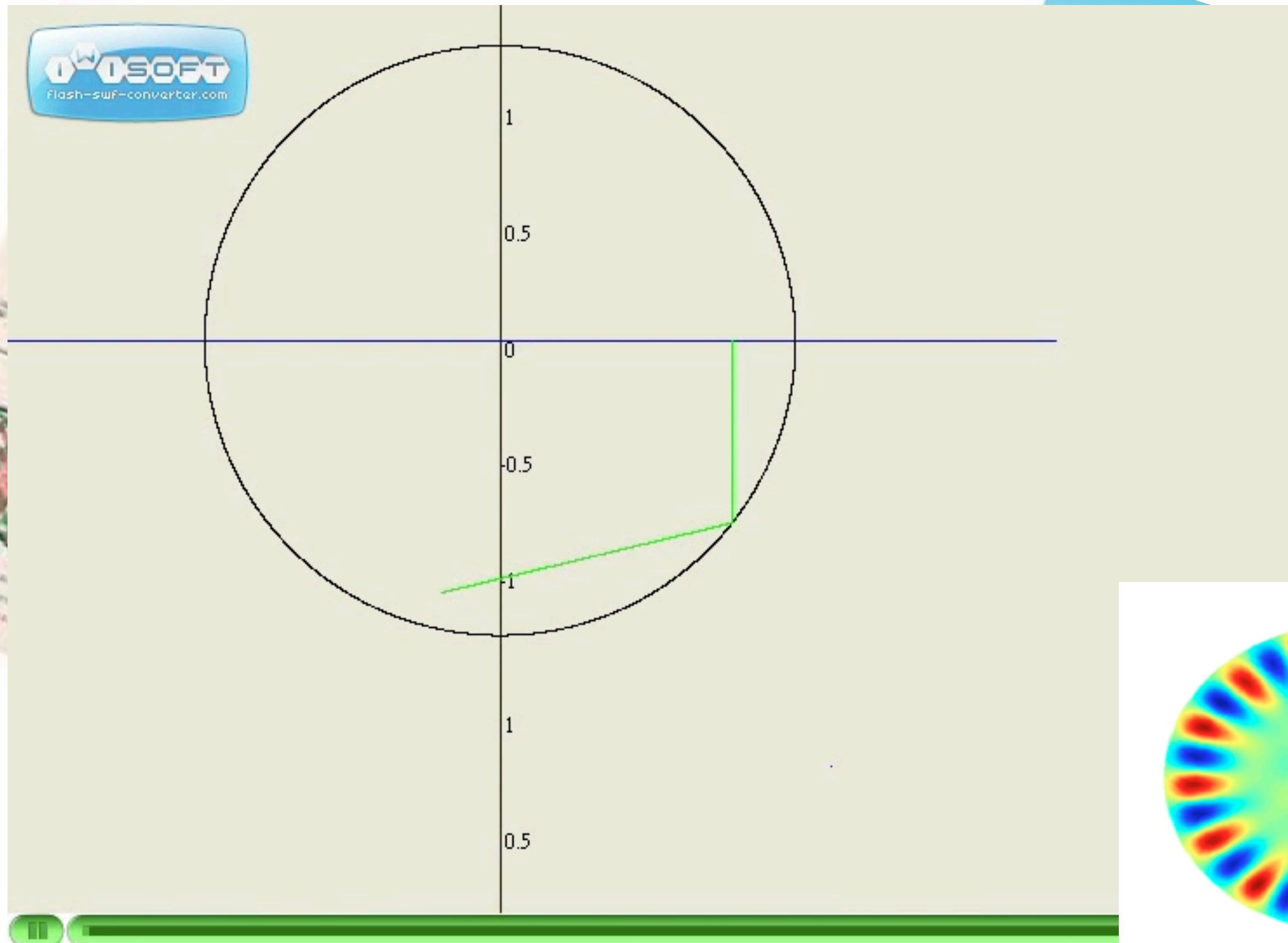
Ray Optics



Wave Optics

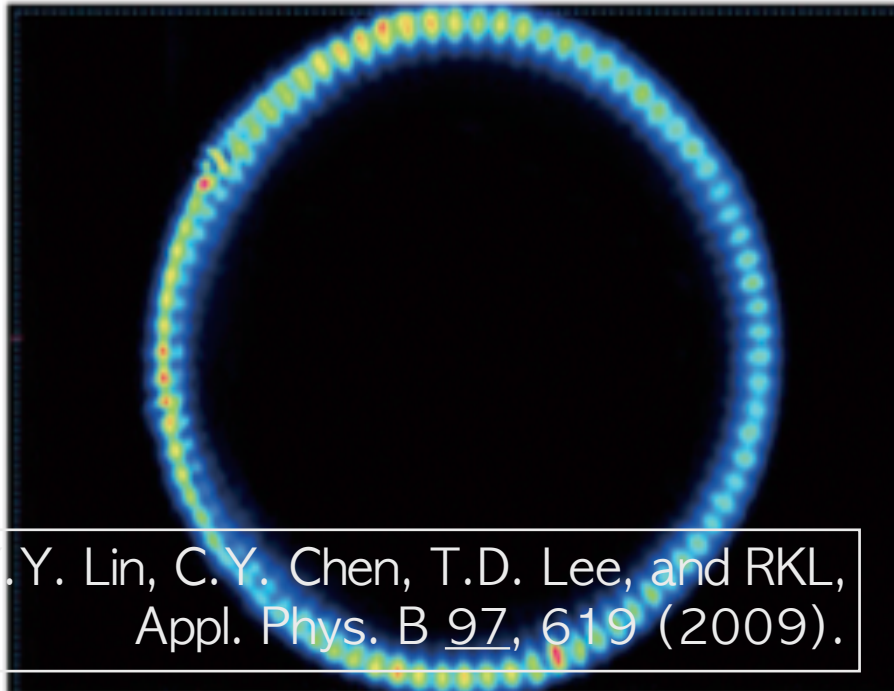
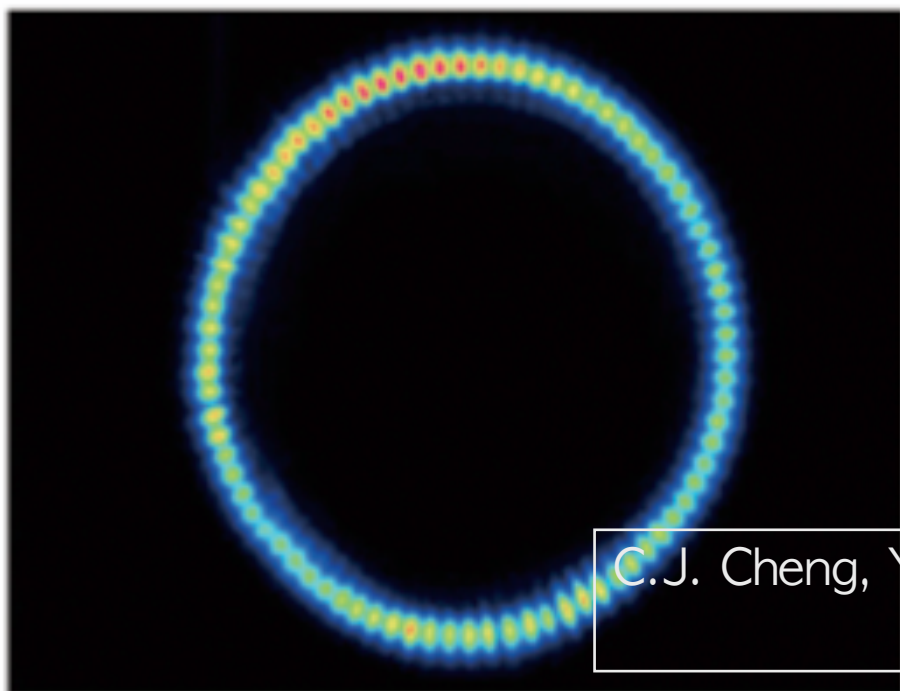
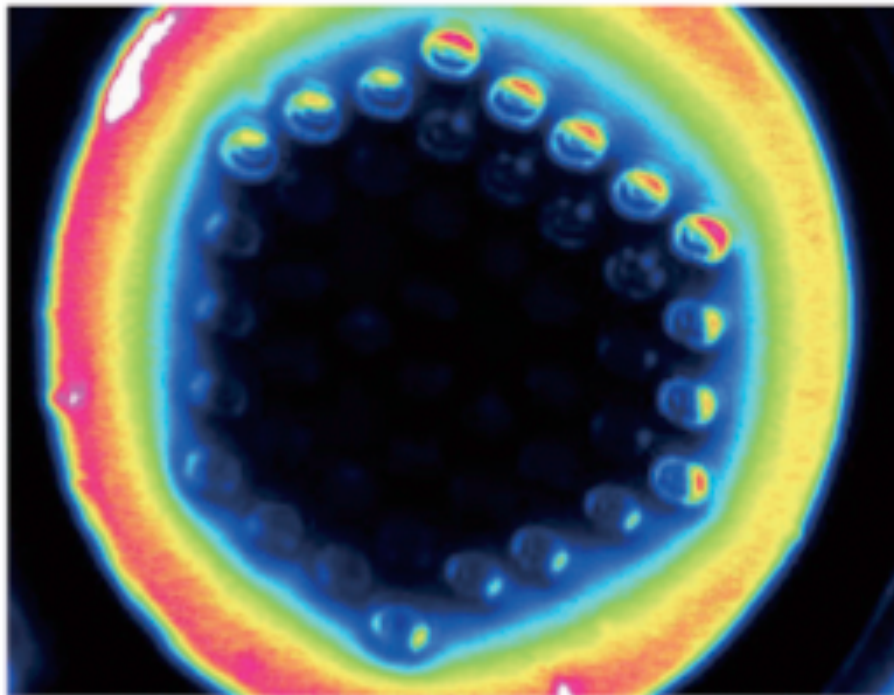
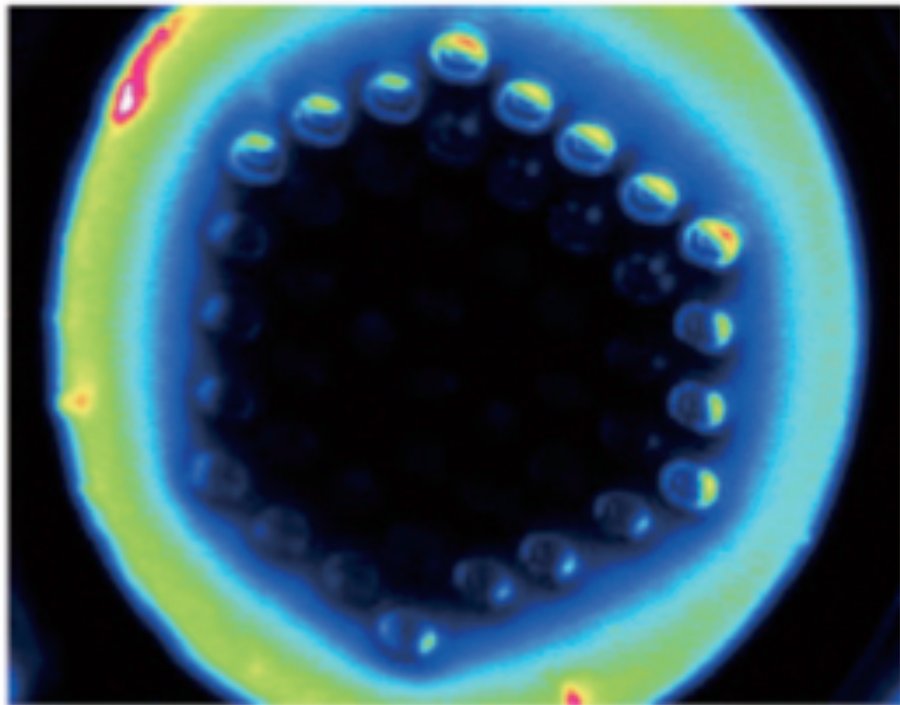


Ray Optics: Poincare Section of Surface

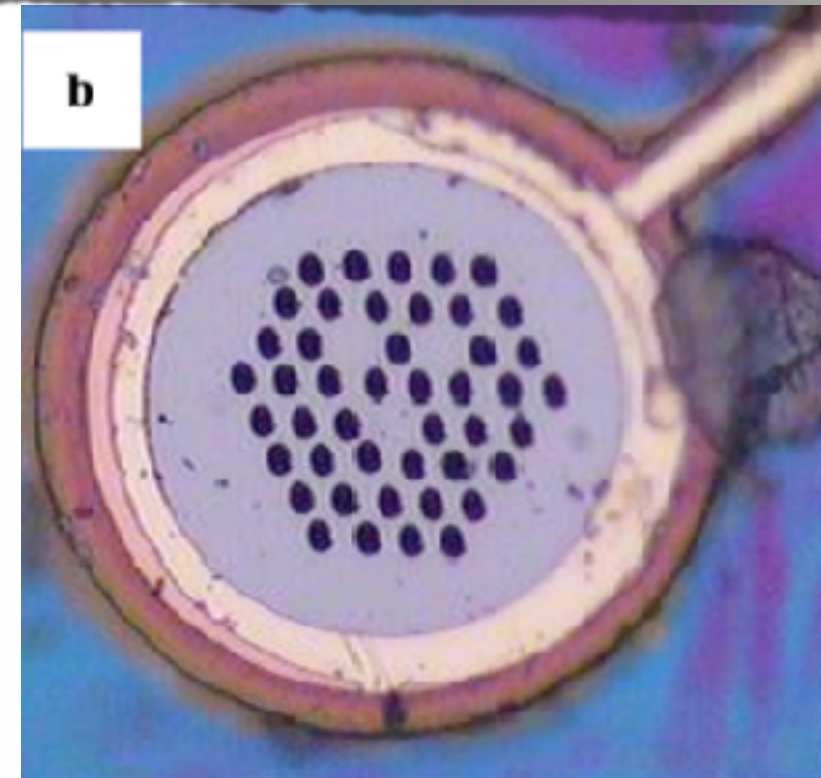


Recorded **Highest**-order **Whispering-Gallery Mode**

experiment in VCSELs:



C.J. Cheng, Y.Y. Lin, C.Y. Chen, T.D. Lee, and RKL,
Appl. Phys. B 97, 619 (2009).

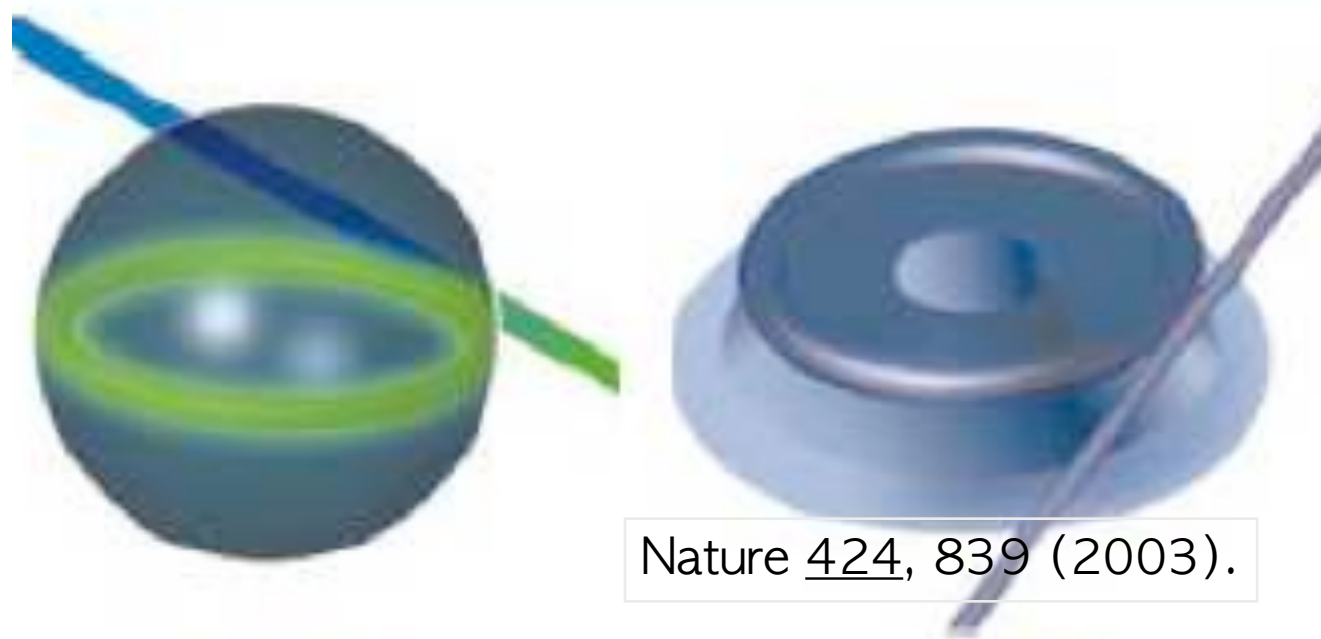
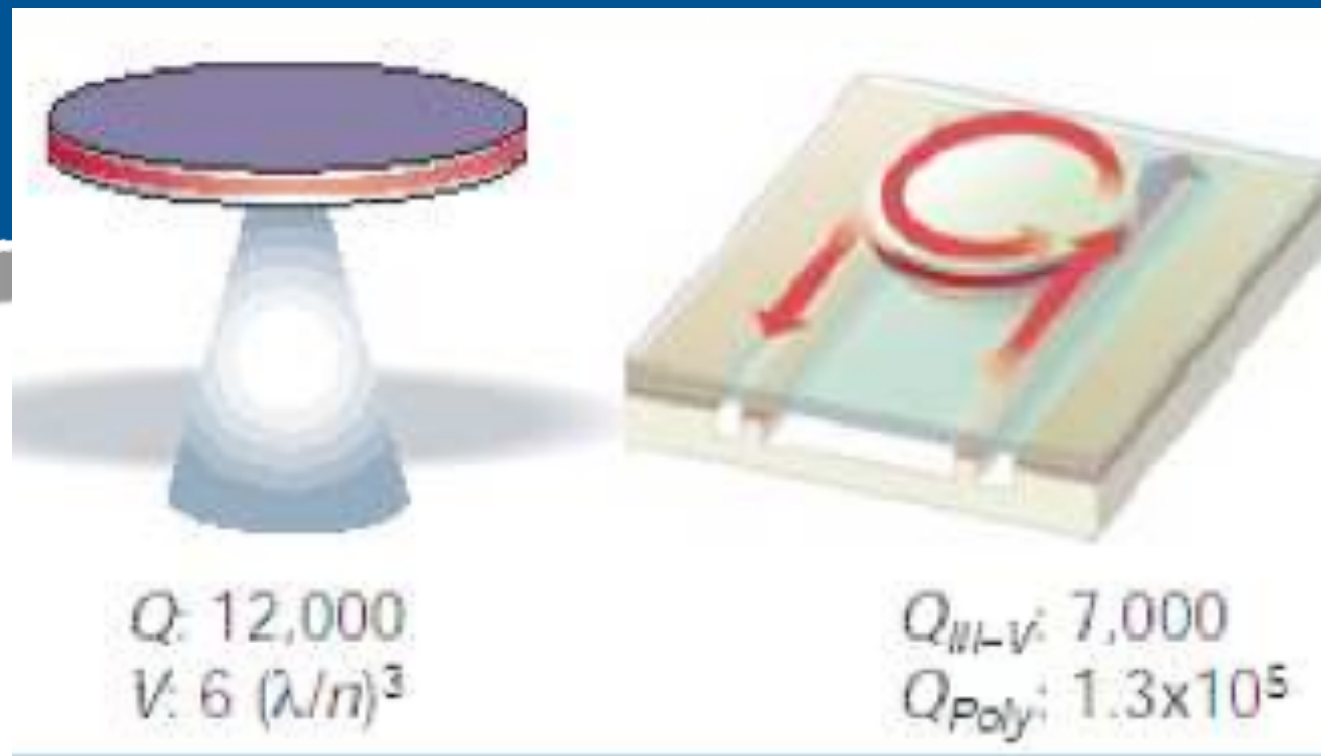


✓ 41st-order

✓ Room Temperature



es: patterns



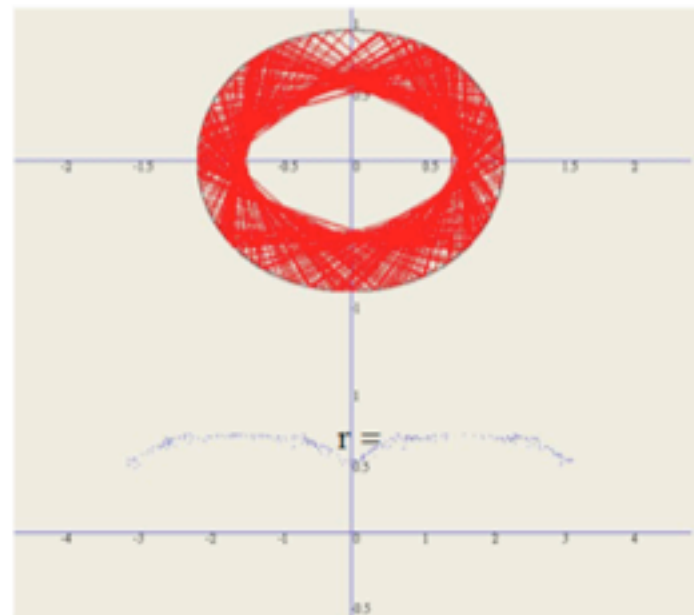
- 1, **Decrease** the size of the cavity => micro-disk and μ -sphere.
- 2, **Suppress** the lower-modes and keep the cavity size large.

Periodic Orbits

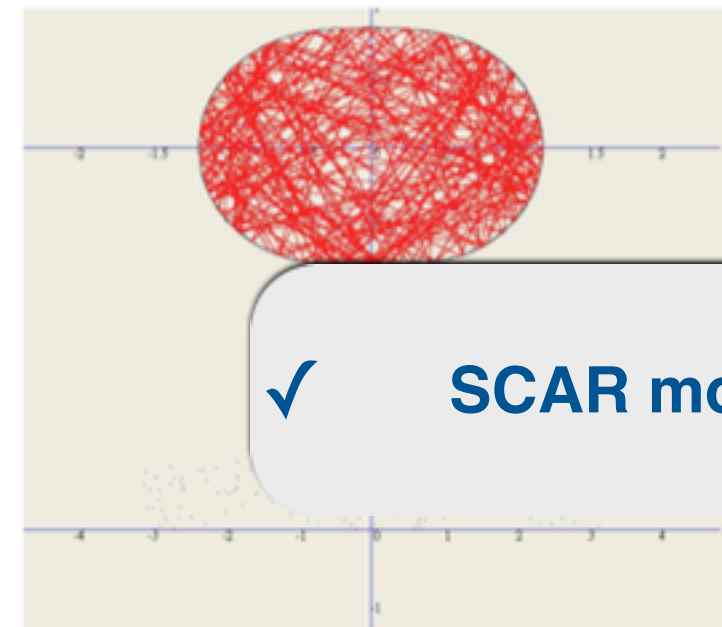
Function: $r = 1 + \varepsilon \cdot \cos 2\phi$



$\varepsilon=0$



$\varepsilon=0.03$



SCAR modes

$\varepsilon=0.08$

Integrated



chaotic

- **Integral system: Stable Periodic Orbit**
- **Non-integral system: Unstable Periodic Orbit**

Surface-Structure-Assisted Chaotic Mode Lasing in Vertical Cavity Surface Emitting Lasers

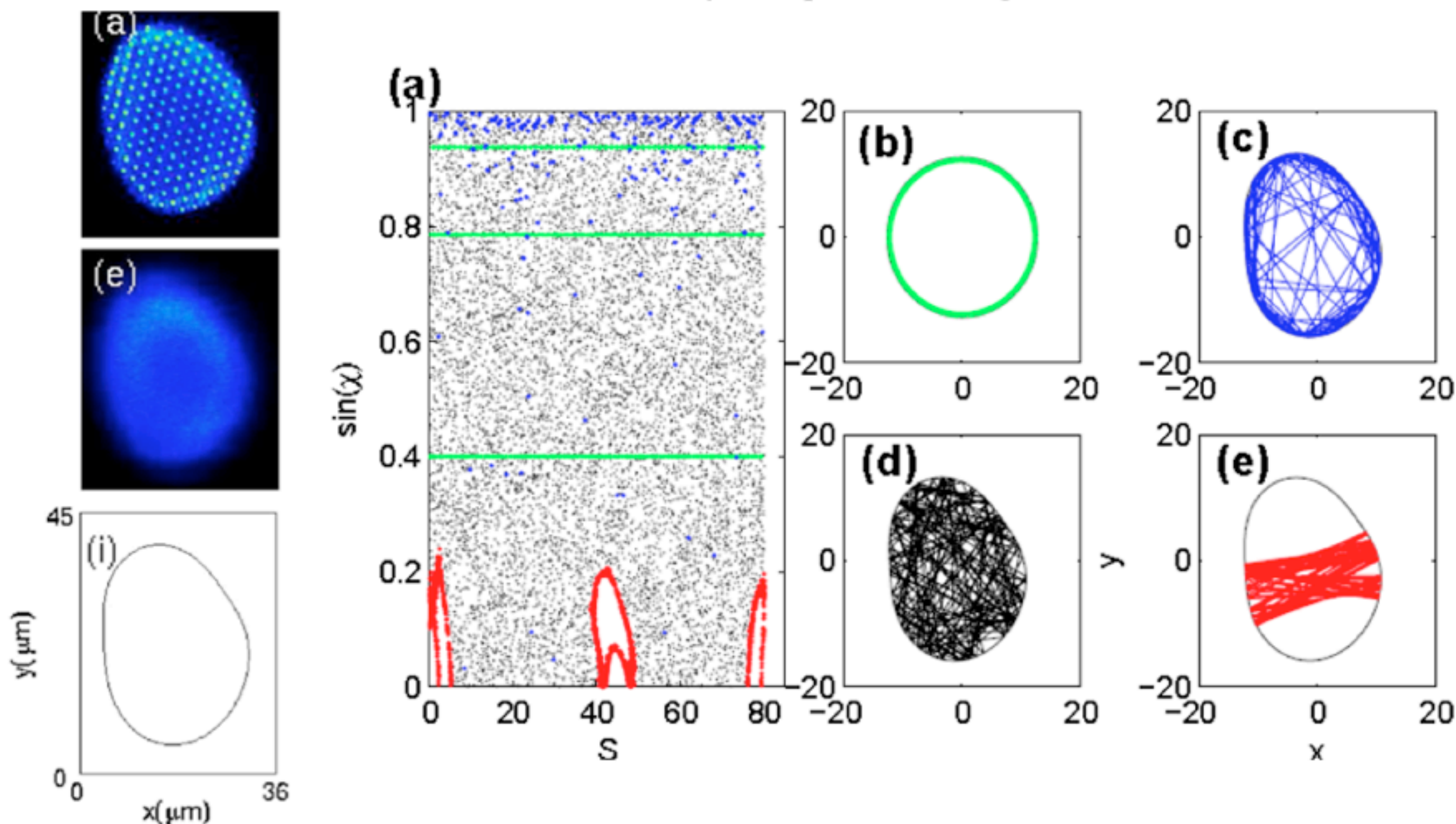
Tsin-Dong Lee,¹ Chih-Yao Chen,² YuanYao Lin,² Ming-Chih Chou,¹ Te-ho Wu,³ and Ray-Kuang Lee²

¹Graduate School of Optoelectronics, National Yunlin University of Science and Technology, Yunlin, 640 Taiwan

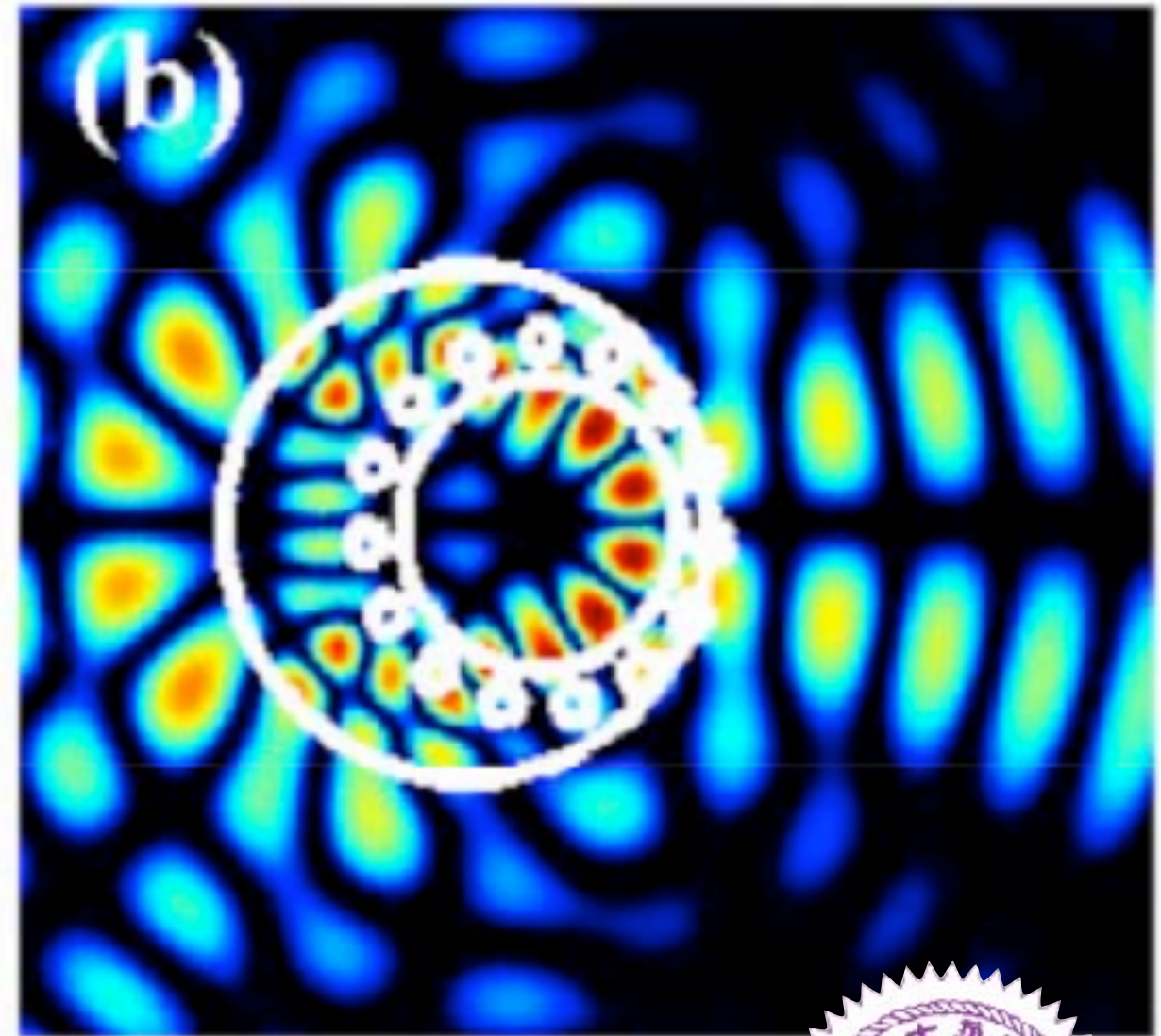
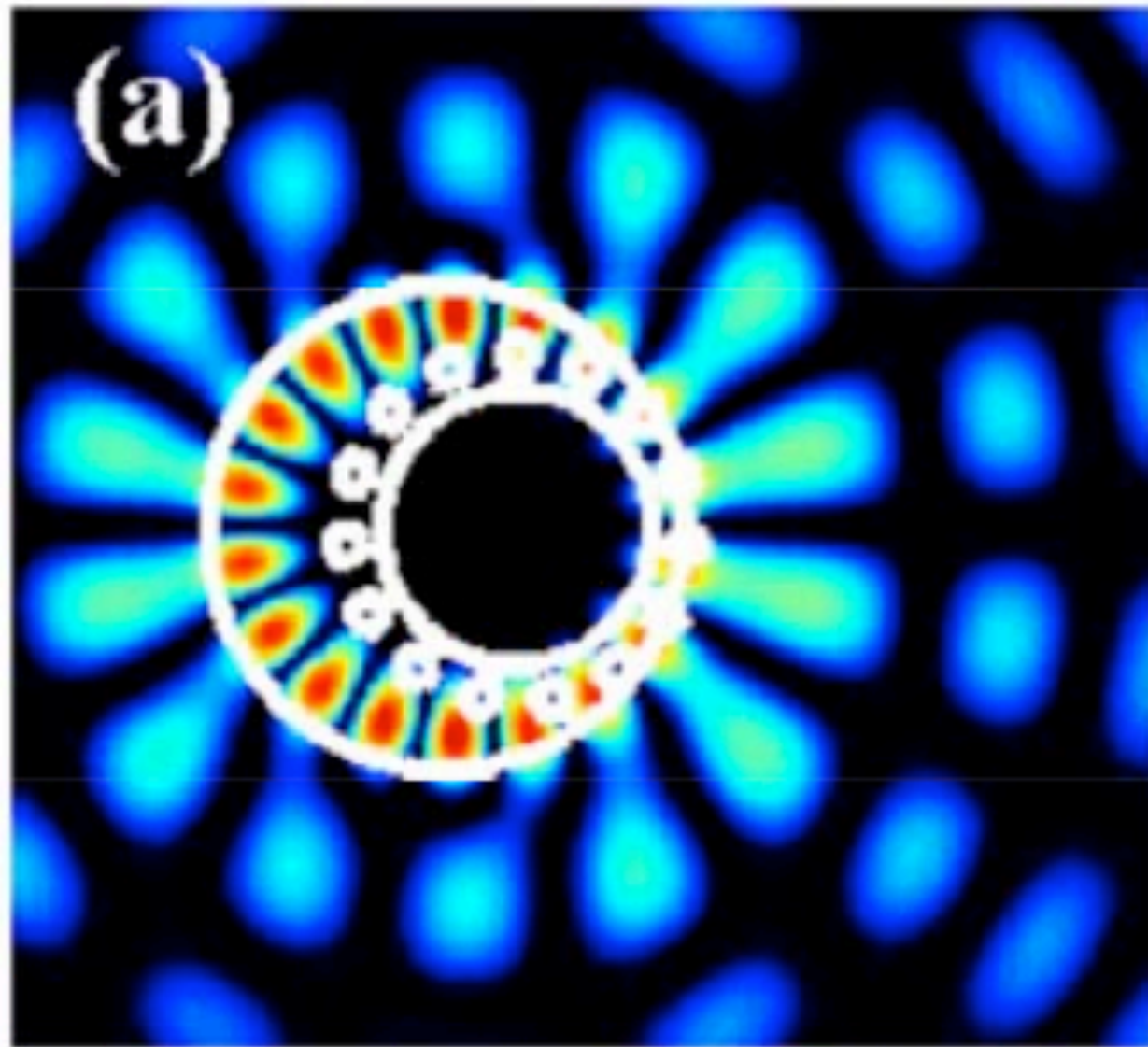
²Institute of Photonics Technologies, National Tsing-Hua University, Hsinchu, 300 Taiwan

³Graduate School of Material Sciences, National Yunlin University of Science and Technology, Yunlin, 640 Taiwan

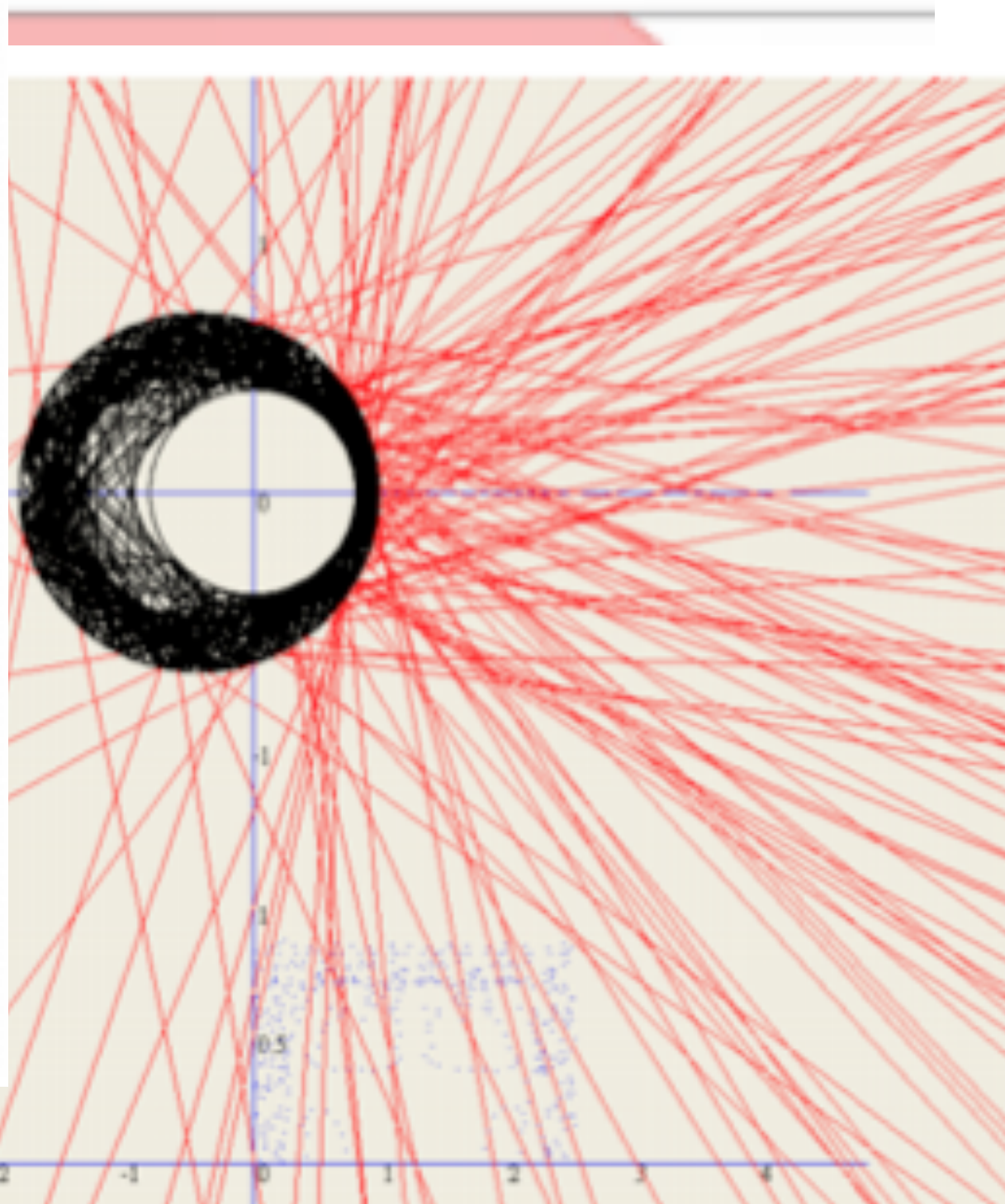
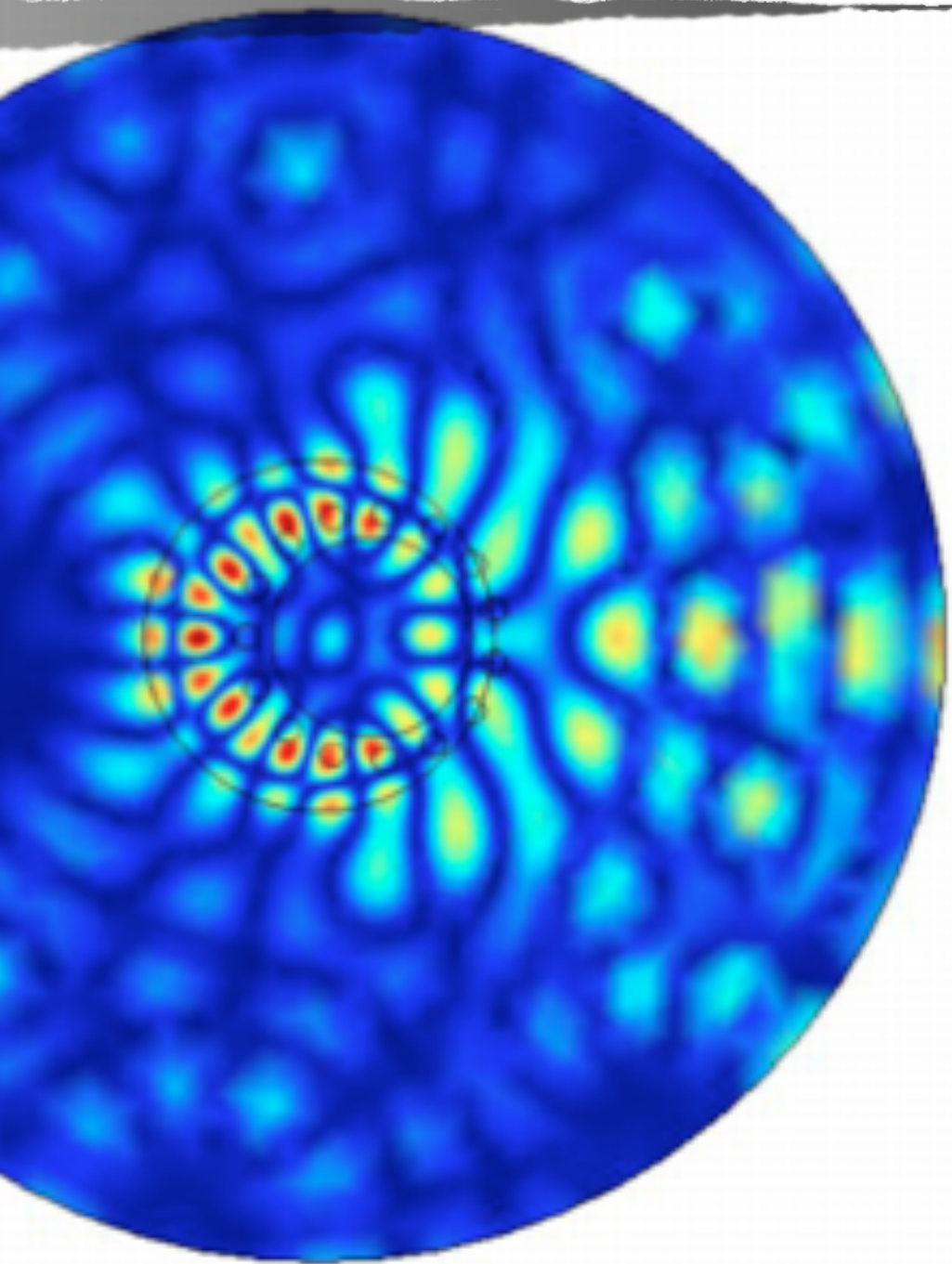
(Received 2 May 2008; published 22 August 2008)



Whispering-gallery modes: Radiation pattern

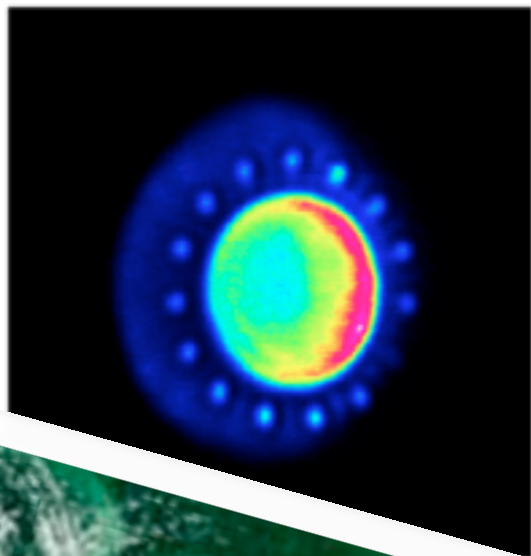


Un-directional lasing with Surface-microstructure

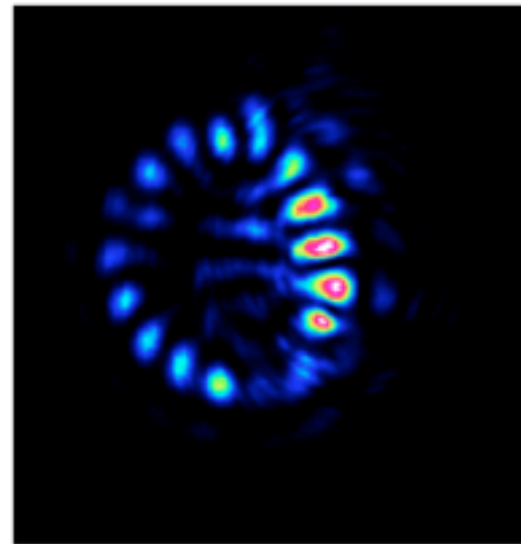


Un-directional lasing: Experimental data

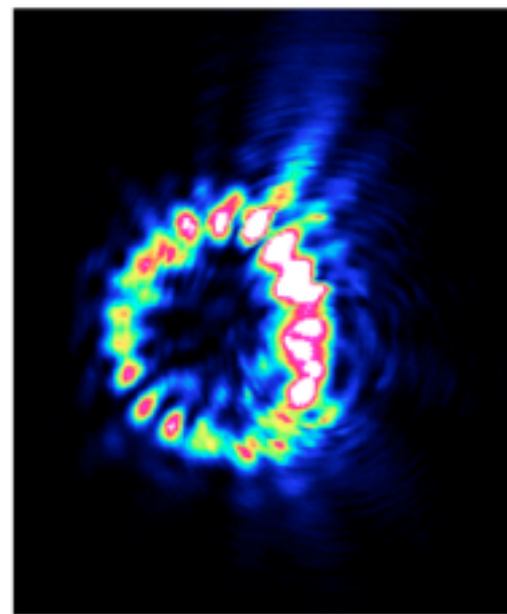
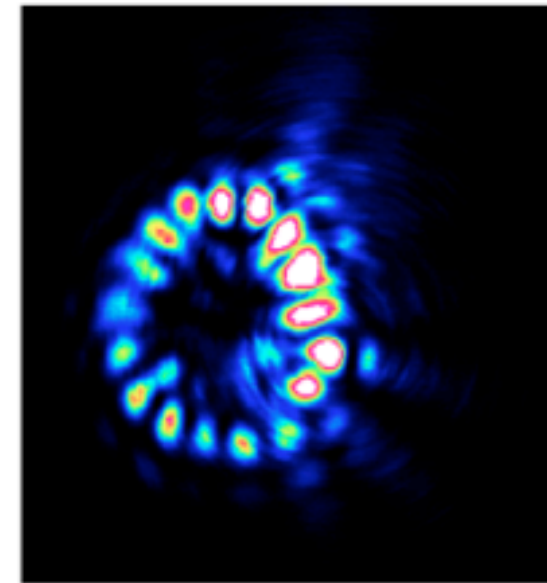
3.5mA



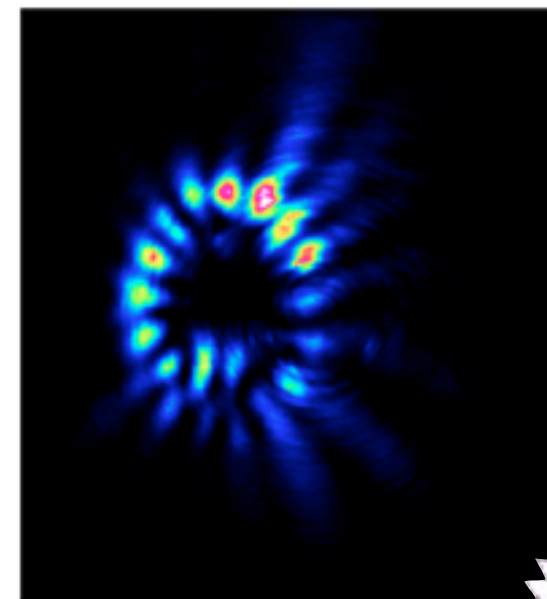
17mA



19mA



30mA



19mA--defocus

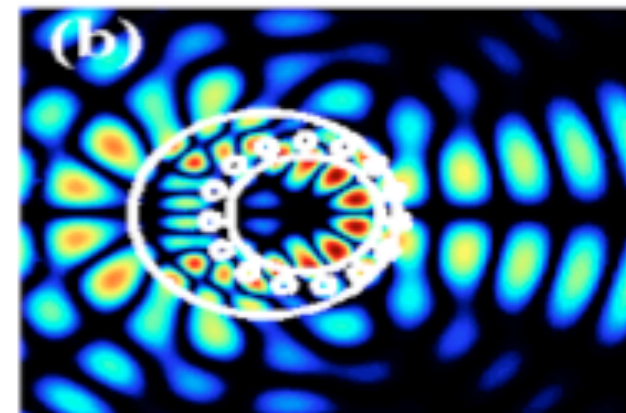
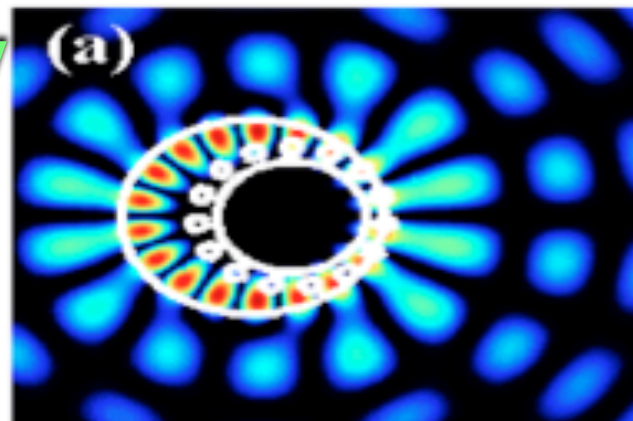


Y.Y. Lin, C.Y. Chen, W. Chien, J.S. Pan, T.D. Lee, and RKL,
Appl. Phys. Lett. 94, 221112 (2009).

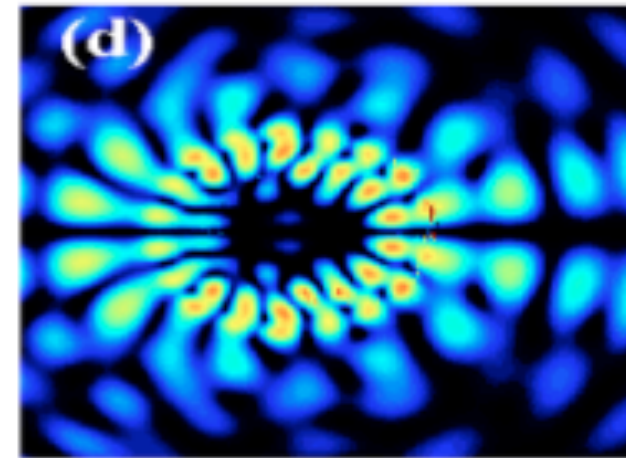
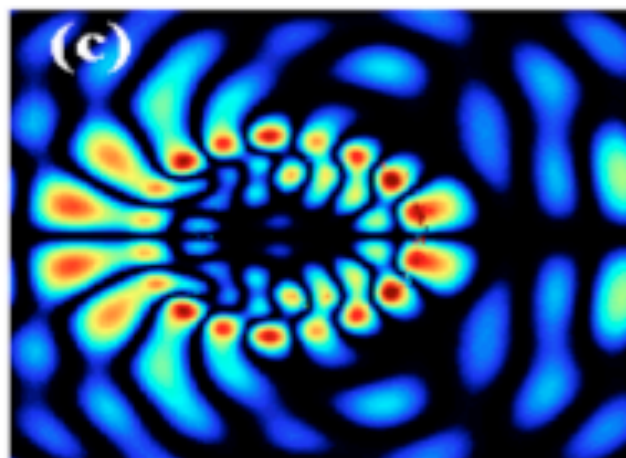


Superposition between Stable and Unstable POs

Whispering-Gallery Mode



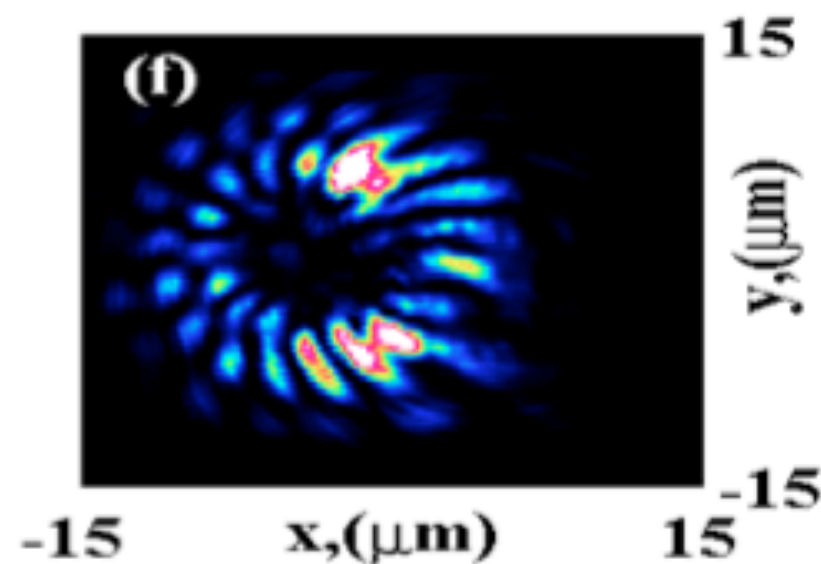
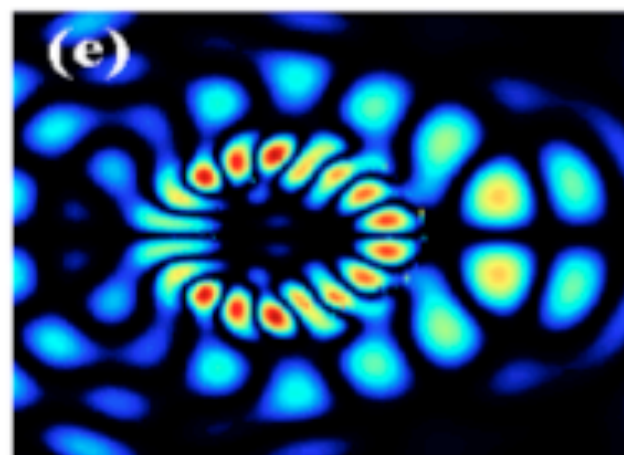
Dynamical Localized Mode



$$\Delta\phi = \pi/2$$

$$\Delta\phi = \pi$$

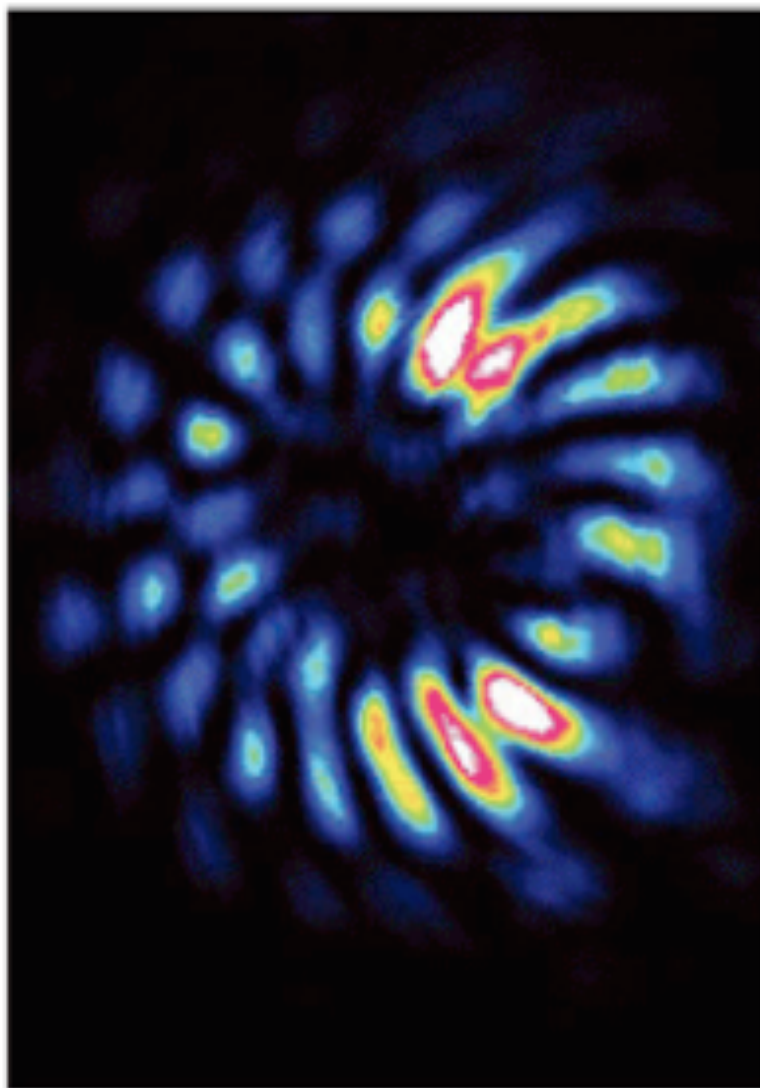
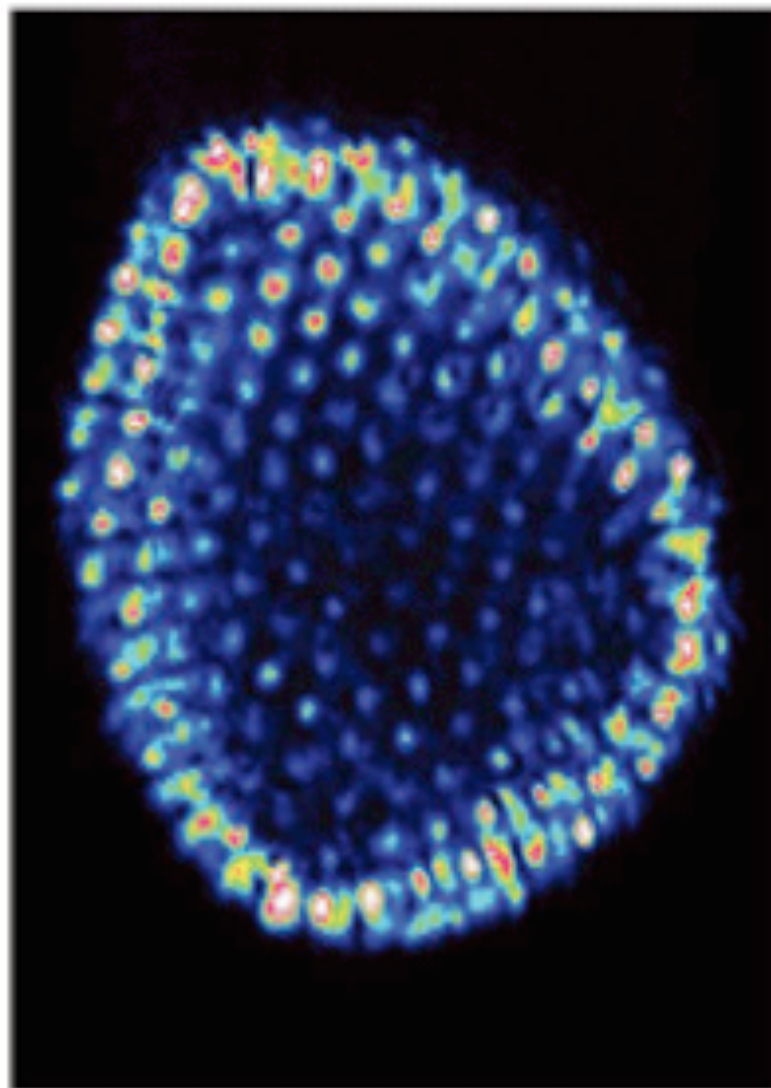
$$\Delta\phi = 0$$



Experimental measurement



OSA: After Image Photon Contest, 2008



Second Place

Ray-Kuang Lee

National Tsing-Hua University

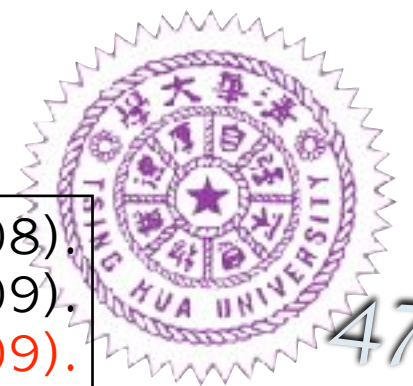
Hsinchu, Taiwan

Our judges were blown away by these two near-field images, which we treated as a single submission for the purposes of this contest. The image on the right shows chaotic mode lasing in a vertical cavity surface emitting laser (VCSEL); the one on the left is of rotating traveling-wave modes in a VCSEL.

T.D. Lee, C.Y. Chen, Y.Y. Lin, M.C. Chou, T.h. Wu, and RKL, Phys. Rev. Lett. 101, 084101 (2008).

Y.Y. Lin, C.Y. Chen, W. Chien, J.S. Pan, T.D. Lee, and RKL, Appl. Phys. Lett. 94, 221112 (2009).

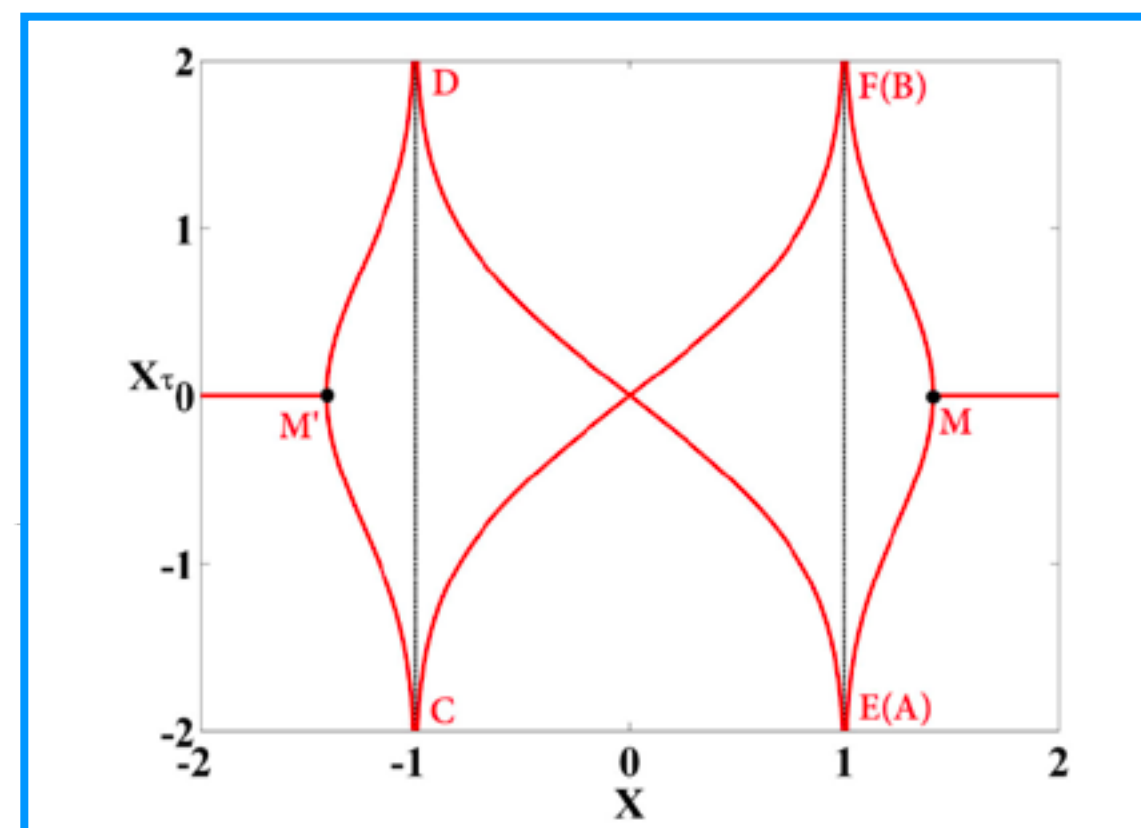
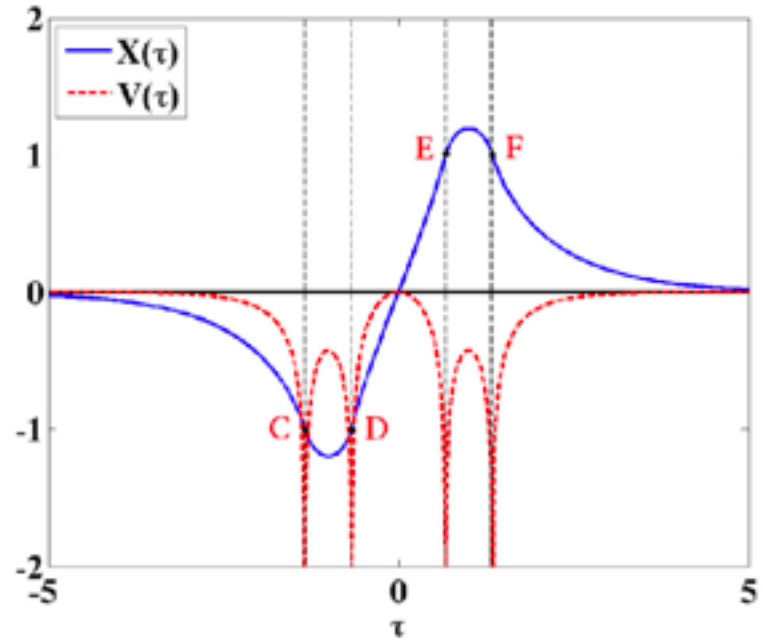
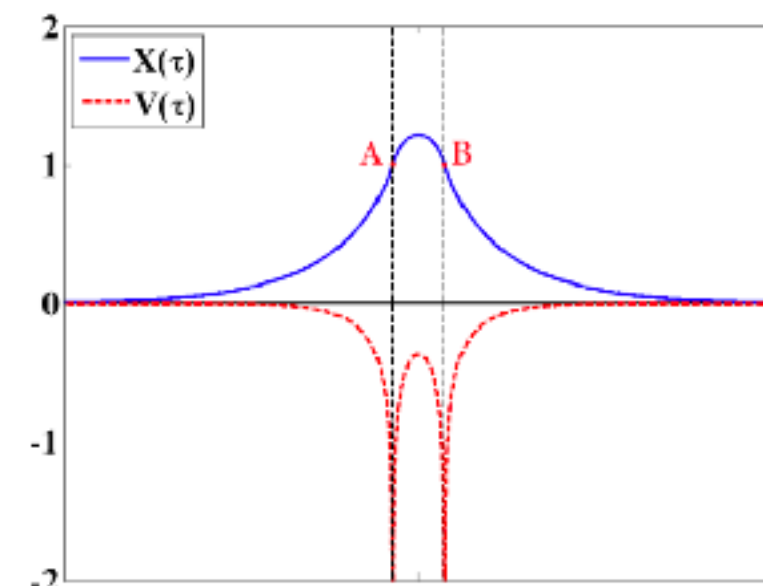
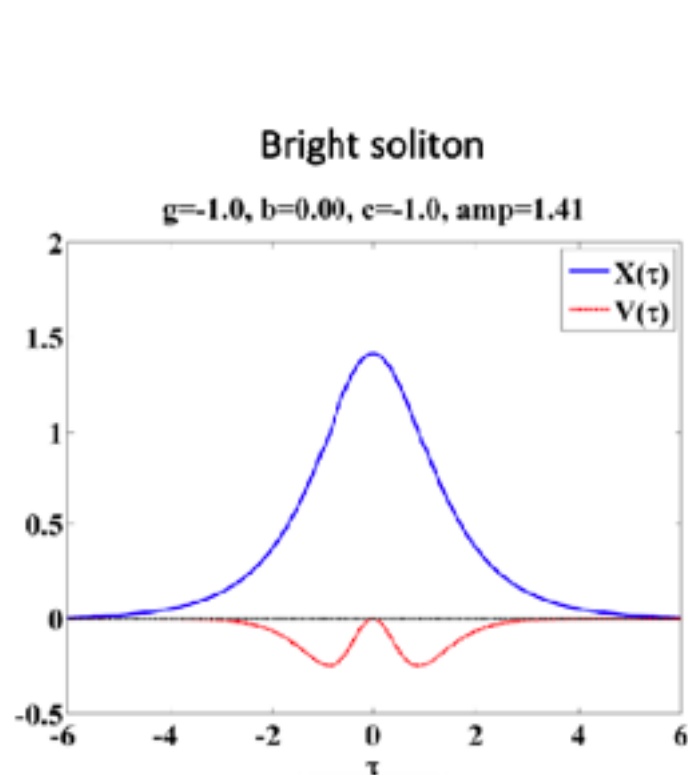
OSA, Optics & Photonics News Nov., 20 (2008) and OPN calendar (2009).



$$cX = (-1 + b|X|^2) \frac{\partial^2}{\partial \tau^2} X + g|X|^2 X$$

Solitons supported by intensity-dependent dispersion

CHUN-YAN LIN,¹ JEN-HSU CHANG,² GERSHON KURIZKI,³ AND RAY-KUANG LEE^{1,4,*} 



Next and Questions

- ☒ **Simple Harmonic Oscillator, SHO** → **Quantum SHO**
- ☒ **Hamiltonian energy** → **Quantization**
- ☒ **Phase Space** → **Quantum Phase Space**
- ☒ **EM wave as a SHO** → **Quantum Field**
- ☐ **Sinusoidal waves** → **Beyond Plane waves**
- ☐ **Poynting density** → **Lagrangian density**
- **More on SHO**
 - ☐ **Damped and Driven SHO** → **dissipation/amplification, Lorentz model**
 - ☐ **Inverted SHO** → **Quantum inverted SHO**
 - ☐ **Nonlinear SHO**
 - ☐ **Periodic (Orbit)** → **WKB (Wentzel–Kramers–Brillouin) approximation**
 - ☐ **Chaotic Trajectories** → **Bohmian, Quantum Chaos**
 - ☐ **Bose-Einstein Condensates in Double Wells** → **BEC**
 - ☐ **Parity-Time symmetric SHO** → **Quantum PT-symmetry**
 - ☐ **Double pendulum/Coupled SHOs**
 - ☐ **EIT and Fano resonance** → **EIT**
 - ☐ **Pseudo-Potential**

download

*<http://mx.nthu.edu.tw/~rkleee>

If you want to know more

- Hamiltonian mechanics
- Poisson bracket
- Symplectic Approach
- Canonical Quantization
- Classical-Quantum Correspondence
- Bohmian Mechanics
- Wigner Flow
- More ...

