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Amplitude-squeezed fiber-Bragg-grating solitons

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Quantum fluctuations of optical fiber-Bragg-grating solitons are investigated numerically by the back-propagation method. It is found that the band-gap effects of the grating act as a nonlinear filter and cause the soliton to be amplitude squeezed. The squeezing ratio saturates after a certain grating length and the optimal squeezing ratio occurs when the pulse energy is slightly above the fundamental soliton energy.

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In the literature, various types of optical soliton phenomena have been studied extensively in the area of nonlinear optical physics. These include the nonlinear Schrödinger solitons in dispersive optical fibers, spatial and vortex solitons in photorefractive materials or waveguides, and cavity solitons in resonators [1]. It has also been well known that fiber Bragg gratings (FBGs) with Kerr nonlinearity can exhibit optical-soliton-like phenomena known as the fiber-Bragg-grating solitons [2,3]. The FBGs are one-dimensional photonic band-gap crystals with weak index modulation. By utilizing the high dispersion of the FBGs near the band edges, one can produce optical solitons in the anomalous dispersion side if the input pulse has suitable pulse width and peak intensity. From the theoretical point of view, solitary waves in one-dimensional periodic structures can travel with different group velocities and have been verified in some experiments [3]. Even for two- or three-dimensional nonlinear photonic band-gap crystals, solitary waves can also exist [4] and have been observed recently [5].

Most of the previous studies on fiber-Bragg-grating solitons have been on the classical effects and there is almost no result on their quantum properties. The quantum theory of traveling-wave optical solitons has been intensively developed during the past 15 years and several approaches have been successfully carried out to calculate the quantum properties of different traveling-wave optical solitons including the family of nonlinear Schrödinger solitons [6,7] as well as the self-induced-transparency solitons [8]. Fiber-Bragg-grating solitons belong to the class of bidirectional pulse propagation problems where the quantum theory is still lack of enough consideration. It is the aim of this study to bridge this gap by developing a general quantum theory for bidirectional pulse propagation problems and particularly applying the theory to the case of fiber-Bragg-grating solitons. It will be shown that the output fiber-Bragg-grating soliton pulses will quantum mechanically get amplitude squeezed and the squeezing ratio can be calculated theoretically. In our modeling, we use the nonlinear coupled mode equations (NCMEs) to describe the two bidirectional waves propagating in a uniform FBG. We use the linearization approach to study the quantum effects of optical solitons in FBGs by extending the back-propagation method we previously developed [9] to the cases of nonlinear bidirectional propagation problems. By following the same spirit of the back propagation method, we will first derive a set of linear adjoint equations from the linearized NCMEs in such a way that any inner product between the solutions of the two equation sets is conserved during the time evolution. Under the linearization approximation, the measurements performed after the time evolution can also be expressed in terms of the inner product between the perturbed quantum field operator and a measurement characteristic function which depends on the measurement to be performed. By back-propagating the measurement characteristic function to $t=0$ through the solution of the adjoint equations, we can express the measured operator in terms of the input field operators which have known quantum characteristics. In this way, the variance of the measured operator as well as its squeezing ratio can be calculated readily for a given measurement characteristic function. To be more explicit, let us consider the wave propagation problem in a one-dimensional fiber grating structure with the nonlinearity coming from the third-order nonlinearity of the optical fiber. With the self-phase modulation and cross-phase modulation effects, we model Bragg solitons by using the following NCMEs that describe the coupling between the forward and the backward propagating waves in a uniform FBG:

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_a(z,t)+\frac{\partial}{\partial z}U_a=i\delta U_a+i\kappa U_b+i\Gamma|U_a|^2U_a+2i\Gamma|U_b|^2U_a, \tag{1}$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_b(z,t)-\frac{\partial}{\partial z}U_b=i\delta U_b+i\kappa U_a+i\Gamma|U_b|^2U_b+2i\Gamma|U_a|^2U_b. \tag{2}$$

Here $U_a(z,t)$ and $U_b(z,t)$ represent the forward and backward propagation pulses, respectively. They are in the units of $GW^{1/2}/cm$. Moreover, v_g is the group velocity of the pulse, κ is the coupling coefficient, λ_B is the Bragg wavelength, δ is the wavelength detuning parameter, and Γ represents the self-phase modulation coefficient. This set of NCMEs has analytical soliton solutions for the case of infinite grating length, as is shown by Aceves and Wabnitz with the introduction of the massive Thirring model [2]. However,

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for gratings of finite length, no analytic solution can be found. So in our studies we directly use the finite difference numerical simulation method with the parameters based on the first experiment reported in the literature [3]. We consider a 60 ps full width at half maximum (FWHM) sech-shaped pulse incidents into a uniform grating with 15.0 cm^{-1} wavenumber detuning from the center of the band gap. The coupling strength of the fiber grating is 10 cm^{-1}, the nonlinear coefficient Γ is 0.018 cm/GW, and the group velocity v_g is chosen to be c/n with $n=1.5$ and c being the speed of light in free space. When the input peak intensity is below the required value for forming a solitary pulse in the FBGs (about 4.5 $\mathrm{GW/cm^2}$ in this case), the peak intensity of the pulse will decrease along the propagation. On the other hand, as shown in Fig. 1, when the input peak intensity is above 4.5 $\mathrm{GW/cm^2}$, the peak intensity of the pulse oscillates during the propagation within the grating. Only when the nonlinearity can exactly compensate the dispersion induced by the FBGs, one can have a stable solitary pulse inside the grating.

After obtaining these classical solutions, we now turn to the calculation of their quantum properties. In quantum theory the NCMEs become the quantum nonlinear coupled mode equations (QNCMEs):

$$\frac{1}{v_g}\frac{\partial}{\partial t}\hat{U}_a(z,t)+\frac{\partial}{\partial z}\hat{U}_a=i\delta\hat{U}_a+i\kappa\hat{U}_b+i\Gamma\hat{U}_a^{\dagger}\hat{U}_a\hat{U}_a+2i\Gamma\hat{U}_b^{\dagger}\hat{U}_b\hat{U}_a, \tag{3}$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}\hat{U}_b(z,t)-\frac{\partial}{\partial z}\hat{U}_b=i\delta\hat{U}_b+i\kappa\hat{U}_a+i\Gamma\hat{U}_b^{\dagger}\hat{U}_b\hat{U}_b+2i\Gamma\hat{U}_a^{\dagger}\hat{U}_a\hat{U}_b, \tag{4}$$

where $\hat{U}_a$ and $\hat{U}_b$ represent the forward and backward normalized fields which satisfy the usual equal time bosonic commutation relations:

$$[\hat{U}_a(z_1,t),\hat{U}_a^{\dagger}(z_2,t)]=\delta(z_1-z_2),$$

$$[\hat{U}_b(z_1,t),\hat{U}_b^{\dagger}(z_2,t)]=\delta(z_1-z_2),$$

$$[\hat{U}_a(z_1,t),\hat{U}_a(z_2,t)]=[\hat{U}_a^{\dagger}(z_1,t),\hat{U}_a^{\dagger}(z_2,t)]=0,$$

$$[\hat{U}_b(z_1,t),\hat{U}_b(z_2,t)]=[\hat{U}_b^{\dagger}(z_1,t),\hat{U}_b^{\dagger}(z_2,t)]=0,$$

$$[\hat{U}_a(z_1,t),\hat{U}_b(z_2,t)]=[\hat{U}_a(z_1,t),\hat{U}_b^{\dagger}(z_2,t)]=0.$$

This is a set of coupled operator equations in the Heisenberg picture and can be derived from the following Hamiltonian under the effective-mass approximation [10]:

$$\begin{aligned}\mathcal{H}=-v_g\Bigg\{&\delta\int dz(\hat{U}_a^{\dagger}\hat{U}_a+\hat{U}_b^{\dagger}\hat{U}_b)+i\int dz\left(\hat{U}_a^{\dagger}\frac{\partial}{\partial z}\hat{U}_a-\hat{U}_b^{\dagger}\frac{\partial}{\partial z}\hat{U}_b\right)+\kappa\int dz(\hat{U}_a^{\dagger}\hat{U}_b+\hat{U}_b^{\dagger}\hat{U}_a)\\&+\frac{\Gamma}{2}\int dz(\hat{U}_a^{\dagger}\hat{U}_a^{\dagger}\hat{U}_a\hat{U}_a+\hat{U}_b^{\dagger}\hat{U}_b^{\dagger}\hat{U}_b\hat{U}_b)\\&+\Gamma\int dz\,(\hat{U}_a^{\dagger}\hat{U}_b^{\dagger}\hat{U}_b\hat{U}_a+\hat{U}_b^{\dagger}\hat{U}_a^{\dagger}\hat{U}_a\hat{U}_b)\Bigg\}.\end{aligned} \tag{5}$$

This derivation automatically proves that the QNCMEs preserve the commutation brackets.

Since for optical solitons the average photon number is usually very large, we can safely use the linearization approximation to study their quantum effects. By setting $\hat{U}_a(z,t)=U_{a0}(z,t)+\hat{u}_a(z,t)$, $\hat{U}_b(z,t)=U_{b0}(z,t)+\hat{u}_b(z,t)$, and substituting them into Eqs. (3) and (4) for linearization, we obtain the linear quantum operator equations in Eq. (6) that describe the evolution of the quantum fluctuations associated with the fiber-Bragg-grating solitons. The quantum perturbation fields $\hat{u}_a(z,t)$ and $\hat{u}_b(z,t)$ in Eq. (6) also have to satisfy the same equal time commutation relations as the original field operators $\hat{U}_a(z,t)$ and $\hat{U}_b(z,t)$:

$$\begin{aligned}\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix}\hat{u}_a\\ \hat{u}_b\end{pmatrix}=&\begin{pmatrix}-\frac{\partial}{\partial z}+i\delta+2i\Gamma|U_{a0}|^2+2i\Gamma|U_{b0}|^2 & i\kappa+2i\Gamma U_{a0}U_{b0}^{*}\\ i\kappa+2i\Gamma U_{a0}^{*}U_{b0} & \frac{\partial}{\partial z}+i\delta+2i\Gamma|U_{a0}|^2+2i\Gamma|U_{b0}|^2\end{pmatrix}\begin{pmatrix}\hat{u}_a\\ \hat{u}_b\end{pmatrix}\\&+\begin{pmatrix}i\Gamma U_{a0}^2 & 2i\Gamma U_{a0}U_{b0}\\ 2i\Gamma U_{a0}U_{b0} & i\Gamma U_{b0}^2\end{pmatrix}\begin{pmatrix}\hat{u}_a^{\dagger}\\ \hat{u}_b^{\dagger}\end{pmatrix},\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix}u_a^A\\ u_b^A\end{pmatrix}=&\begin{pmatrix}-\frac{\partial}{\partial z}+i\delta+2i\Gamma|U_{a0}|^2+2i\Gamma|U_{b0}|^2 & i\kappa+2i\Gamma U_{a0}U_{b0}^{*}\\ i\kappa+2i\Gamma U_{a0}^{*}U_{b0} & \frac{\partial}{\partial z}+i\delta+2i\Gamma|U_{a0}|^2+2i\Gamma|U_{b0}|^2\end{pmatrix}\begin{pmatrix}u_a^A\\ u_b^A\end{pmatrix}\\&+\begin{pmatrix}-i\Gamma U_{a0}^2 & -2i\Gamma U_{a0}U_{b0}\\ -2i\Gamma U_{a0}U_{b0} & -i\Gamma U_{b0}^2\end{pmatrix}\begin{pmatrix}u_a^{A*}\\ u_b^{A*}\end{pmatrix}.\end{aligned} \tag{7}$$

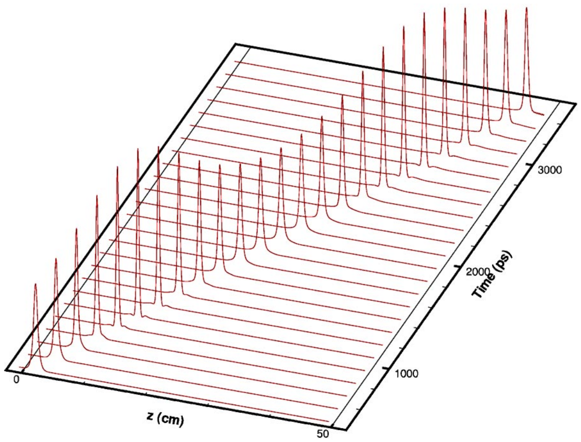

FIG. 1. Evolution of the fiber-Bragg-grating soliton with the input peak intensity $I = 9.0\ \mathrm{GW/cm^2}$.

If we define the inner product operation according to

$$\langle \vec{f} | \vec{\hat{g}} \rangle = \frac{1}{2} \int dz [f_a^* \hat{g}_a + f_a \hat{g}_a^\dagger + f_b^* \hat{g}_b + f_b \hat{g}_b^\dagger] \tag{8}$$

then Eq. (7) is the corresponding set of adjoint equations for the perturbed QNCMEs, which have the following desired property:

$$(d/d\,t) \langle \vec{u}^A | \vec{\hat{u}} \rangle = 0, \tag{9}$$

where $\vec{u}^A = (u_a^A, u_b^A)^T$ is the solution of the adjoint equation defined in Eq. (7). The important thing is that the inner product between the solutions of the two equation sets is preserved along the time axis.

By taking advantage of the preservation of the inner product, we can express the inner product of the output quantum perturbation operator with a projection function in terms of the input quantum field operators by the *back-propagation method*. This will allow us to calculate the quantum uncertainty for the inner product of the output quantum operator with any given projection function. Under the linearization approximation, any measurement of a physical quantity can be expressed as an inner product between a measurement characteristic function and the perturbed quantum field operator [9]. The squeezing ratio of the measured quantity thus can be calculated according to

$$R(T) = \frac{\mathrm{var}[\langle \vec{f} | \vec{\hat{u}}(t=T) \rangle]}{\mathrm{var}[\langle \vec{f} | \vec{\hat{u}}(t=0) \rangle]} = \frac{\mathrm{var}[\langle \vec{F}_T | \vec{\hat{u}}(t=0) \rangle]}{\mathrm{var}[\langle \vec{f} | \vec{\hat{u}}(t=0) \rangle]}. \tag{10}$$

Here $\mathrm{var}[\cdot]$ means the variance, $\vec{f}$ is the original projection function, and $\vec{F}_T$ is the back-propagated projection function. The choice of the characteristic function $\vec{f}$ will depend on the measurement to be performed. For the photon number measurement, $\vec{f}$ is simply the normalized output classical pulse from the grating [11]. For the homodyne detection, it will be the local oscillator pulse. In the following we consider a solitary pulse incident into a uniform FBG, and calculate the quantum fluctuation of its first transmitted pulse based on the formulation given above.

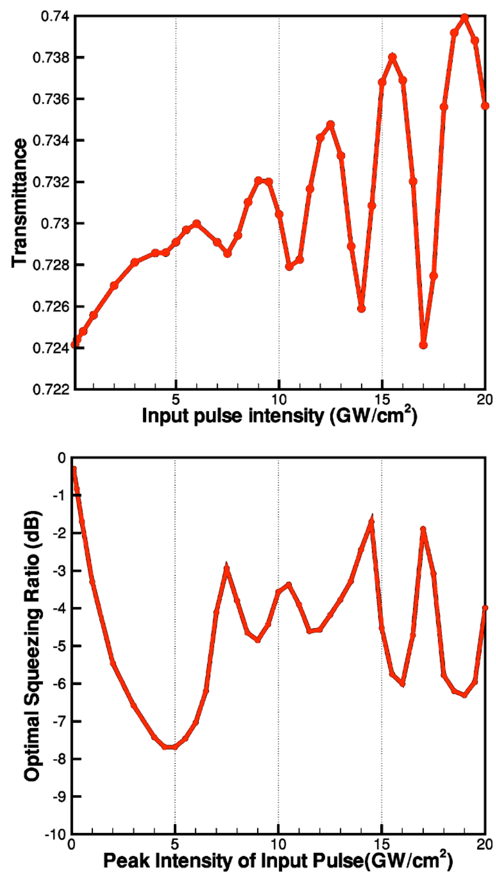

FIG. 2. Transmittance (top) and photon number squeezing ratio (bottom) for fiber-Bragg-grating solitons with different input intensities.

The transmittance of the FBGs with different input intensities of solitons for a constant FBG length (50 cm) is shown in the top curve of Fig. 2. The calculated photon number squeezing ratio is shown in the bottom for the same parameters. When the input peak intensity is smaller than that of the fundamental soliton, the output squeezing ratio monotonically decreases when the input peak intensity is increased. The output squeezing ratio will begin to oscillate strongly with respect to the changing input intensity when the input intensity is much larger than that of the fundamental soliton. The oscillation behaviors of the FBG transmittance and the squeezing ratio match very well. That is, the squeezing ratio has a local minimum when the transmission has a local maximum. Intuitively the periodic grating structure acts like a spectral filter which can filter out the noisier high-frequency components in the soliton spectrum and produce a net amplitude squeezing effect just as in the previous soliton amplitude squeezing experiments where a spectral filter is cascaded after a nonlinear fiber [12,13]. And the minimum squeezing ratio occurs when the pulse energy of soliton is slightly larger than that of the fundamental soliton.

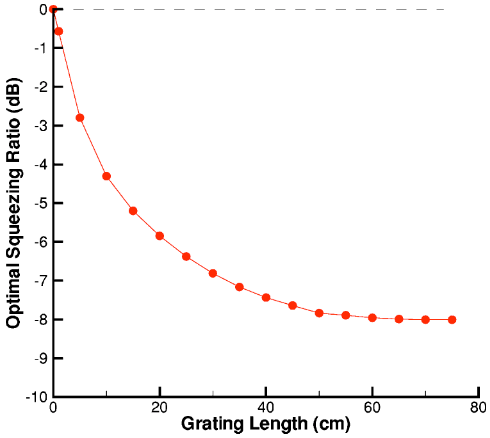

FIG. 3. Optimal squeezing ratio for Bragg solitons propagating through different length of FBGs.

It is also intuitively clear that larger amplitude squeezing should occur when the transmittance curve is saturated. Figure 3 shows the dependence of the optimal squeezing ratios for different FBG lengths with a constant input intensity ($I = 4.5$ GW/cm^2). If we only consider the gratings with the length longer than 1 cm, we find that the squeezing ratio monotonically decreases with the FBG length and saturates at the length around 60 cm. Intuitively this is because the filtering effect of the grating will unavoidably introduce additional noises on the light fields and eventually cause the squeezing ratio to become saturated.

So far we have shown that the FBG solitons will get amplitude squeezed during propagation. Under the linearization approximation, the amplitude squeezing corresponds to the squeezing of the in-phase quadrature field component. To further determine the maximum squeezing phase angle of the quadrature field components of the FBG soltions, we perform another calculation to simulate the squeezing ratio when the homodyne detection scheme is used and when the local oscillator pulse is exactly the classical output pulses. With the homodyne detection scheme, one has the additional degree of freedom to adjust the relative phase between the local oscillator and the signal for detecting different quadrature field components. Figure 4 plots the squeezing ratio for different FBG lengths and for different local oscillator phases with a constant input intensity ($I = 4.5$ GW/cm^2).

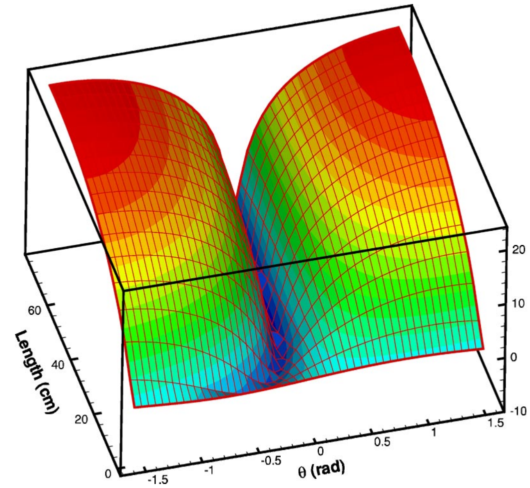

FIG. 4. Squeezing ratio for different FBG lengths and different local oscillator phases.

One can see that for short FBG lengths the quadrature squeezing direction is close to but not exactly in the in-phase (or amplitude) quadrature, $\theta = 0$. However, when the FBG length is long enough, the squeezing direction will approach the in-phase quadrature. This proves that the FBG solitons will indeed be squeezed in the amplitude direction when the FBG length is long enough.

To summarize, we have developed a general quantum theory for bidirectional nonlinear optical pulse propagation problems and have used it to calculate the squeezing ratio of fiber-Bragg-grating solitons in one-dimensional photonic band-gap crystals. We find that the output pulses can get amplitude squeezed and the squeezing ratio exhibits interesting relations with the fiber grating length as well as with the intensity of the input pulse. To measure the quantum fluctuations of fiber-Bragg-grating soliton experimentally, one needs to apply the direct measurement for the first transmitted pulse from the grating by gating out other smaller multiple reflected pulses. It will be very interesting to see if one can actually observe these effects experimentally in the future.

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