Suppression of soliton transverse instabilities in nonlocal nonlinear media

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We analyze transverse instabilities of spatial bright solitons in nonlocal nonlinear media, both analytically and numerically. We demonstrate that the nonlocal nonlinear response leads to a dramatic suppression of the transverse instability of the soliton stripes, and we derive asymptotic expressions for the instability growth rate in both short- and long-wave approximations. © 2008 Optical Society of America

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1. INTRODUCTION

Symmetry-breaking instabilities have been studied in different areas of physics, since they provide a simple means to observe the manifestation of strongly nonlinear effects in nature. One example is the transverse instabilities of spatial optical solitons in nonlinear Kerr media [1] associated with the growth of transverse modulations of quasi-one-dimensional bright and dark soliton stripes for both focusing [2–6] and defocusing [7] nonlinearities. In addition, such instabilities are also studied and observed in materials with quadratic nonlinearities [8]. In particular, this kind of symmetry-breaking instability turns a bright-soliton stripe into an array of two-dimensional filaments [9], and it bends a dark-soliton stripe, creating pairs of optical vortices of the opposite polarities [10]. Consequently, transverse instabilities set severe limits on the observation of quasi-one-dimensional spatial solitons in bulk media [11].

Several different physical mechanisms for suppressing soliton transverse instabilities have been proposed and studied in detail, including the effect of partial incoherence of light [12,13] and anisotropic nonlinear response [13] in photorefractive crystals and the stabilizing action of nonlinear coupling between different modes or polarizations [14]. Recently initiated theoretical and experimental studies of nonlocal nonlinearities revealed many novel features in the propagation of spatial solitons, including the suppression of modulational [15] and azimuthal [16] instabilities.

In this paper we demonstrate that a significant suppression of soliton transverse instabilities can be achieved in nonlocal nonlinear media. We derive analytical results for the instability growth rate in both long- and short-scale asymptotic limits. First, in Section 2 we employ the numerical approach to solve and analyze soliton transverse instability in the framework of the nonlinear Schrödinger equation with diffusion-type nonlocality of the self-focusing nonlinear response. Second, in Section 3 we construct analytical models to describe the transverse instability by means of two asymptotic expansions in long- and short-scale regimes. Finally, in Section 4 we analyze numerically the evolution of the soliton stripe in the nonlinear regime and verify the theoretical predictions. Section 5 concludes the paper.

2. TRANSVERSE INSTABILITY IN NONLOCAL MEDIA

We consider the propagation of an optical beam in a nonlocal nonlinear medium described by the normalized two-dimensional nonlinear Schrödinger equation:

$$i\frac{\partial E}{\partial z} + \frac{1}{2}\Delta_{\perp}E + nE = 0,$$

$$n - d\Delta_{\perp}n = |E|^2, \tag{1}$$

where $\Delta_{\perp}=\partial^2/\partial x^2+\partial^2/\partial y^2$, $E=E(x,y;z)$ is the slowly varying electric field envelope, $n=n(x,y;z)$ is the optical refractive index, and the parameter d stands for the strength of nonlocality. Model (1) describes light propagation in different types of nonlocal nonlinear media, including nematic liquid crystals [17].

We look for stationary solutions of Eqs. (1) in the form of bright-soliton stripes, $E(x,y;z)=u(x)\exp(i\beta z)$, where $u(x)$ is a (numerically found) localized function, $u(\pm\infty)=0$, and β is the (real) propagation constant.

The transverse instability of quasi-one-dimensional solitons in nonlocal nonlinear media is investigated by a standard linear stability analysis [1] by introducing the perturbed solution in the form

$$n = n_0(x) + \epsilon\delta n,$$

$$E=e^{i\beta z}[u_0+\epsilon(v+iw)e^{i\lambda z+ipy}+\epsilon(v^*+iw^*)e^{-i\lambda^* z-ipy}], \quad (2)$$

where (u_0,n_0) is the solution of Eqs. (1), $\epsilon \ll 1$ is a small perturbation, and $v(x)$, $w(x)$, and $\delta n(x)$ are perturbed amplitudes that are modulated in the transverse y direction with the wavenumber p. The instability growth rate is defined as an imaginary part of the eigenvalue λ.

Substituting these asymptotic expansions into Eqs. (1), to the first order of ϵ we obtain a set of linear equations,

$$\lambda w=\left(\beta+\frac{1}{2}p^2\right)v-\frac{1}{2}\frac{\partial^2 v}{\partial x^2}-n_0 v-u_0\delta n, \quad (3)$$

$$\lambda v=\left(\beta+\frac{1}{2}p^2\right)w-\frac{1}{2}\frac{\partial^2 w}{\partial x^2}-n_0 w, \quad (4)$$

$$\delta n=d\left(\frac{\partial^2}{\partial x^2}-p^2\right)\delta n+2u_0 v, \quad (5)$$

which we then investigate both numerically and analytically.

Typical localized solutions of model (1) for bright solitons are shown in Fig. 1(a) for local (solid curve) and nonlocal (dashed and dashed–dotted curves) media. Figure 1(b) shows the growth rate of the soliton transverse instabilities versus the modulation wavenumber p for local ($d=0$) and nonlocal ($d=0.5$ and $d=1$) nonlinearities. It can be seen that nonlocality reduces the growth rate of the transverse instability of the soliton stripe. Moreover, the cutoff transverse wavenumber p_c of the gain spectrum becomes smaller as the value of the nonlocality parameter d grows.

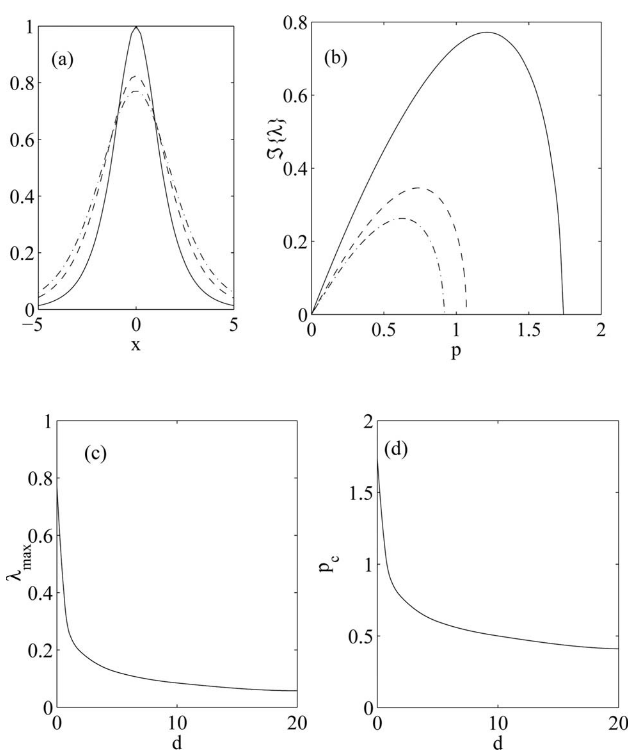

Fig. 1. (a) Intensity profiles of the bright solitons in local ($d=0$, solid) and nonlocal ($d=0.5$, dashed; $d=1$, dashed–dotted) nonlinear media. (b) Instability growth rate for bright solitons versus the transverse wavenumber p for local (solid) and nonlocal ($d=0.5$, dashed; $d=1$, dashed–dotted) nonlinear media. (c), (d) Cutoff value of the transverse wavenumber and maximum growth rate versus the nonlocality parameter d.

To describe the suppression of the soliton transverse instabilities quantitatively, we calculate the dependence of the cutoff transverse wavenumber p_c and the maximum growth rate on the strength of nonlocality, d, shown in Figs. 1(c) and 1(d). We observe that the maximum growth rate decreases significantly at large values of the nonlocality parameter, and eventually it approaches zero when $d\to\infty$. The cutoff wavenumber p_c of the transverse instability domain becomes smaller as the value of nonlocality grows. In the limit of very large values of d, the cutoff wavenumber vanishes as well. Consequently, the soliton stripe becomes more stable when the degree of nonlocality increases.

3. ANALYTICAL APPROACH

Next, we analyze the transverse instability of bright solitons in nonlocal nonlinear media by applying a variation method. We proceed in accord with the following steps. First, we expand the nonlocal refractive index function into series in the nonlocality parameter [18] in the terms involving the refractive index in Eq. (5). Second, we apply the method of [4] to construct the asymptotic expansions of the elliptical problem defined by Eqs. (3)–(5). We employ the corresponding ansatz defined as $v=v_0+\Gamma v_1$ and $w=w_0+\Gamma w_1$, where v_0 and w_0 are the neutral mode, and Γ is an imaginary part of the eigenvalue of the linear stability problem. To begin with, we derive an approximate analytical solution for the soliton in a nonlocal medium by expanding the nonlinear refractive index to the first order in the nonlocality parameter d:

$$n(x)=\sum_{m=0}^{\infty}\frac{1}{m!}h_m\frac{\partial^m|E|^2}{\partial x^m}\approx|E|^2+d\left[\frac{\partial^2}{\partial x^2}(|E|^2)\right]+\mathbf{O}(d^2), \quad (6)$$

where

$$h_m=i^m\frac{\mathrm{d}^m}{\mathrm{d}\omega^m}H(\omega)|_{\omega=0}$$

is the expansion of the Fourier transform, $H(\omega)$, of the nonlocal kernel function. The approximate stationary solution E can then be calculated by minimizing the Lagrangian:

$$L=\int \mathrm{d}x\left\{\frac{i}{2}(E_zE^*-E_z^*E)-\frac{1}{2}|E_x|^2+\frac{1}{2}|E|^4+|E|^2 +\frac{d}{2}\left(-\ |E|^2|E|_x^2-\frac{1}{2}E^2E_x^{*2}-\frac{1}{2}E^{*2}E_x^2\right)+\mathbf{O}(d^2)\right\}, \quad (7)$$

where the subscripts z and x stand for the derivatives with respect to the longitudinal and transverse coordinates, respectively. Given the ansatz of the stationary state $E=u_0(x)\exp(i\beta z)$, the wavenumber β and solution $u_0=A\,\mathrm{sech}(ax)$ are obtained by minimizing the Lagrangian of the nonlocal nonlinear Schrödinger equation (7).

For a given power P and nonlocality parameter d, we obtain

$$\beta = (1/6)Pa + (1/6)a^2 + (2/15)dPa^3,$$

$$A = (Pa/2)^{1/2},$$

$$a = [-5 + (25 + 60dP^2)^{1/2}](12dP)^{-1}, \quad (8)$$

which are solved to the first order in d. The results of the variation method in the first and second orders of nonlocality parameter d are compared with numerical calculations, and they are shown in Fig. 2. We note that, with a sufficient order of expansion, the variation approach provides a good approximation to the numerical results.

A. Long-Scale Expansion

In the long-scale expansion, we expand the solution E around the stationary solution u_0 and use perturbations for its neutral mode: v_0 and w_0 at $p=0$ such that

$$0 = \beta v_0 - \frac{1}{2}\frac{\partial^2 v_0}{\partial x^2} - n_0 v_0 - u_0 \delta n_0, \quad (9)$$

$$0 = \beta w_0 - \frac{1}{2}\frac{\partial^2 w_0}{\partial x^2} - n_0 w_0, \quad (10)$$

$$\delta n_0 = d\frac{\partial^2 \delta n_0}{\partial x^2} + 2u_0 v_0, \quad (11)$$

which can be solved as $v_0=0$ and $w_0=u_0$. By substituting $v=v_0+\Gamma v_1$ and $w=w_0+\Gamma w_1$ into Eqs. (3)–(5) and equating the first-order terms in Γ, we obtain

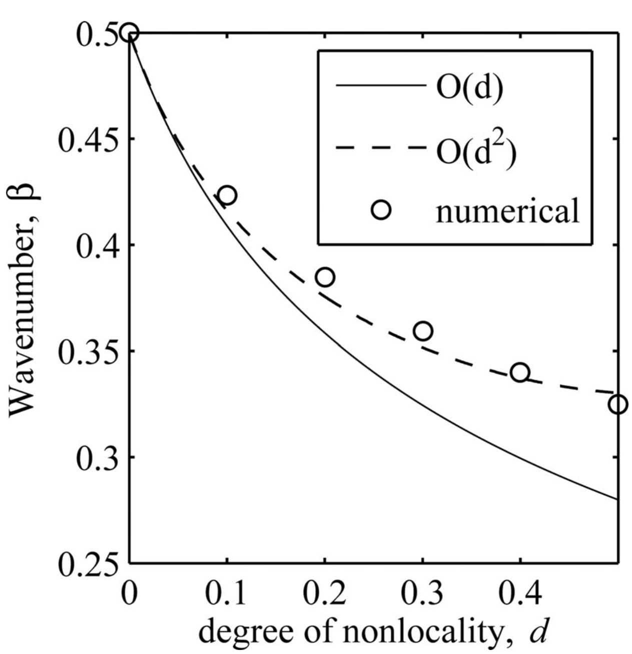

Fig. 2. Wavenumber versus nonlocality parameter d at fixed power ($P=2$). Solid and dashed curves show the solution obtained by the variational method to the first and second order in d, respectively. Circles mark the numerical results.

$$w_0 = \left(\beta + \frac{1}{2}p^2\right)v_1 - \frac{1}{2}\frac{\partial^2 v_1}{\partial x^2} - n_0 v_1 - u_0 \delta n_1, \quad (12)$$

$$v_0 = \left(\beta + \frac{1}{2}p^2\right)w_1 - \frac{1}{2}\frac{\partial^2 w_1}{\partial x^2} - n_0 w_1, \quad (13)$$

$$\delta n_1 = d\left(\frac{\partial^2}{\partial x^2} - p^2\right)\delta n_1 + 2u_0 v_1. \quad (14)$$

Then we multiply Eq. (4) times w_0 and integrate over x to obtain

$$\Gamma(w_0,v) = \left(w_0, \left[\left(\beta + \frac{1}{2}p^2\right) - \frac{1}{2}\frac{\partial^2}{\partial x^2} - n_0\right]w\right), \quad (15)$$

where we define

$$(w,v) = \int_{-\infty}^{\infty} dx w^* v. \quad (16)$$

Subsequently, considering the relation given in Eq. (10), we obtain the expression

$$\frac{1}{2}p^2(w_0,w) = -\Gamma(w_0,v). \quad (17)$$

Repeating the same operations for Eqs. (3)–(5) yields

$$\Gamma(w_0,w) = \frac{1}{2}p^2(w_0,v) - (w_0,u_0\delta n_1), \quad (18)$$

$$\delta n_1 = \frac{2u_0 v_1}{1+dp^2} + d\frac{\frac{\partial^2}{\partial x^2}(2u_0 v_1)}{(1+dp^2)^2}. \quad (19)$$

When d and p are small, δn_1 can be approximated by

$$\delta n_1 \approx 2u_0 v_1 + d\left[\frac{\partial^2}{\partial x^2}(2u_0 v_1)\right] \equiv \Delta n(v_1;p=0). \quad (20)$$

Using Eq. (17) to replace (w_0,w) by (w_0,v), recalling $v=v_0+\Gamma v_1=\Gamma v_1$ to represent its neutral mode, we obtain the instability growth rate Γ and transverse wavenumber p in the form

$$\Gamma^2 = \frac{(w_0,u_0\Delta n(v_1;p=0))}{(w_0,v_1)}p^2 - \frac{1}{4}p^4. \quad (21)$$

The task now is to find the solution v_1 according to Eq. (12). Herein we again apply the variation method by using the ansatz $v_1 = d^2[\mathrm{sech}(b_1 x)]/dx^2$, where the parameter b_1 is obtained by minimizing the system Lagrangian corresponding to Eq. (12):

$$L_{v1}=\int_{-\infty}^{\infty}\mathrm{d}x\Bigg\{-w_0|v_1|^2+\frac{1}{2}\left|\frac{\partial v_1}{\partial x}\right|^2+\left(\beta+\frac{p^2}{2}\right)|v_1|^2 -\left(|u_0|^2+d\frac{\partial^2|u_0|^2}{\partial x^2}\right)|v_1|^2-\frac{2u_0^2|v_1|^2}{1+dp^2}-\frac{2d}{(1+dp^2)^2} \times\left(u_0\frac{\partial^2 u_0}{\partial x^2}|v_1|^2-u_0^2\left|\frac{\partial v_1}{\partial x}\right|^2\right)\Bigg\}. \quad (22)$$

While evaluating the system Lagrangian, we use the functional expansion, $\mathrm{sech}(ax)\approx\mathrm{sech}(b_1x)-\mathrm{sech}(b_1x)\tanh(b_1x)(a-b_1)x$, to deduce a closed form for the Lagrangian in Eq. (22). The parameter b_1 can therefore be obtained as a function of both d and P by solving

$$0=\left(\frac{5134}{315}daP+\frac{155}{21}\right)b_1^4-\frac{1048}{225}dPa^2b_1^3 +\left(\frac{14}{5}\beta+\frac{494}{105}dPa^3-\frac{139}{35}Pa-\frac{31}{35}\pi\right)b_1^2 +\left(\frac{19}{210}\pi a-\frac{92}{105}da^4P+\frac{10}{21}Pa^2\right)b_1. \quad (23)$$

Next, substituting the variational parameters subjected to the solution ansatz into Eq. (21), we express the long-scale approximation of nonlocal transverse instabilities in terms of d and P as

$$\Gamma^2=\frac{\left[-\frac{2}{5}ab_1+d\left(-\frac{139}{630}a^4+\frac{53}{210}ab_1^3-\frac{61}{630}a^2b_1^2+\frac{473}{630}a^3b_1\right)\right]P}{-\frac{a+b_1}{3}}p^2-\frac{1}{4}p^4,$$

where a, β are obtained from Eqs. (8) and b_1 is calculated by Eq. (23).

B. Short-Scale Expansion

For the asymptotic expansion near the cutoff wavenumber p_c, we employ the same procedures as those described above for the long-scale expansion, except that we use another ansatz, $v_0=\mathrm{sech}^2(b_0x)$, $w_0=0$, and $w_1=\mathrm{sech}(c_1x)$, which yields

$$0=\left(\beta+\frac{1}{2}p_c^2\right)v_0-\frac{1}{2}\frac{\partial^2 v_0}{\partial x^2}-n_0v_0-u_0\delta n, \quad (24)$$

$$0=\left(\beta+\frac{1}{2}p_c^2\right)w_0-\frac{1}{2}\frac{\partial^2 w_0}{\partial x^2}-n_0w_0, \quad (25)$$

$$\delta n_0=d\left(\frac{\partial^2}{\partial x^2}-p_c^2\right)\delta n_0+2u_0v_0. \quad (26)$$

Again, to obtain b_0 we minimize the Lagrangian L_{v_0} corresponding to Eq. (24) at $p=p_c$:

$$L_{v0}=\int_{-\infty}^{\infty}\mathrm{d}x\Bigg\{\frac{1}{2}\left|\frac{\partial v_0}{\partial x}\right|^2+\left(\beta+\frac{p_c^2}{2}\right)|v_0|^2-\left(|u_0|^2+d\frac{\partial^2|u_0|^2}{\partial x^2}\right) \times|v_1|^2-\frac{2u_0^2|v_0|^2}{1+dp_c^2}-\frac{2d}{(1+dp_c^2)^2} \times\left(u_0\frac{\partial^2 u_0}{\partial x^2}|v_0|^2-u_0^2\left|\frac{\partial v_0}{\partial x}\right|^2\right)\Bigg\}. \quad (27)$$

Then the cutoff wavenumber p_c is found from the condition $L_{v_0}=0$ with an optimized value of b_0, which is obtained by simultaneously solving the equations

$$0=\frac{\partial L_{v_1}}{\partial b_0}=\left(\frac{8}{15}+\frac{352}{315}\frac{d}{(1+dp_c^2)^2}Pa\right)b_0-\frac{32}{63}\frac{d}{(1+dp_c^2)^2}Pa^2 +\left(\frac{4}{3}\beta+\frac{2}{3}p_c^2-\frac{32}{45}Pa+\frac{80}{63}dPa^3-\frac{64}{45}\frac{1}{1+dp_c^2}aP +\frac{416}{315}\frac{d}{(1+dp_c^2)^2}a^5P\right)b_0^{-1}+\left(-\frac{208}{315}da^4P+\frac{8}{45}Pa^2 -\frac{176}{315}\frac{d}{(1+dp_c^2)^2}a^6P+\frac{16}{45}\frac{1}{1+dp_c^2}a^2P\right)b_0^{-2}, \quad (28)$$

$$0=L_{v_1}=\frac{8}{15}+\frac{352}{315}\frac{d}{(1+dp_c^2)^2}Pa+\left(\frac{32}{45}Pa+\frac{64}{45}\frac{1}{1+dp_c^2}aP -\frac{4}{3}\beta-\frac{2}{3}p_c^2-\frac{416}{315}\frac{d}{(1+dp_c^2)^2}a^5P-\frac{80}{63}dPa^3\right)b_0^{-2} +\left(-\frac{32}{45}\frac{1}{1+dp_c^2}a^2P+\frac{416}{315}da^4P-\frac{16}{45}Pa^2 +\frac{352}{315}\frac{d}{(1+dp_c^2)^2}a^6P\right)b_0^3. \quad (29)$$

Introducing a small perturbation near the cutoff wavenumber, $\bar{p}=p-p_c$, the resulting parameters b_0 and p_c are then deemed as constant in the Lagrangian for w_1,

$$L_{w1} = \int_{-\infty}^{\infty} \mathrm{d}x \left\{ -v_0|w_1|^2 + \frac{1}{2}\left|\frac{\partial w_1}{\partial x}\right|^2 + \left(\beta + \frac{\bar{p}^2}{2}\right)|w_1|^2 - \left(|u_0|^2 + d\frac{\partial^2|u_0|^2}{\partial x^2}\right)|w_1|^2 \right\}, \quad (30)$$

to deduce the dependence of c_1 on P and d. Similarly, in the short-scale expansion the amplitude growth rate can be calculated as

$$\Gamma^2 = \frac{(v_0, u_0\Delta n(w_1;\bar{p}=0))}{(v_0,w_1)}\bar{p}^2 - \frac{1}{4}\bar{p}^4, \quad (31)$$

where

$$(v_0,w_1) = \frac{5}{6}\pi b_0^{-1} - \frac{1}{3}\pi c_1 b_0^{-2},$$

$$(v_0,u_0\Delta n(w_1;\bar{p}=0)) = P\Bigg[\left(-\frac{3}{80}\pi + \frac{1}{2}d\left(\frac{27}{140}\pi + \frac{1}{70}a^2\pi\right)\right)c_1 + \left(-\frac{3}{40}\pi + \frac{1}{2}d\left(\frac{1}{35}\pi - \frac{13}{140}a^2\pi\right)\right) + \frac{3}{16}\pi + \frac{9}{80}a\pi + \frac{1}{2}d\left(-\frac{1}{4}\pi - \frac{31}{140}a\pi - \frac{4}{5}a^2 + \frac{11}{140}a^3\pi + \frac{1}{8}a^2\pi\right)\Bigg]. \quad (32)$$

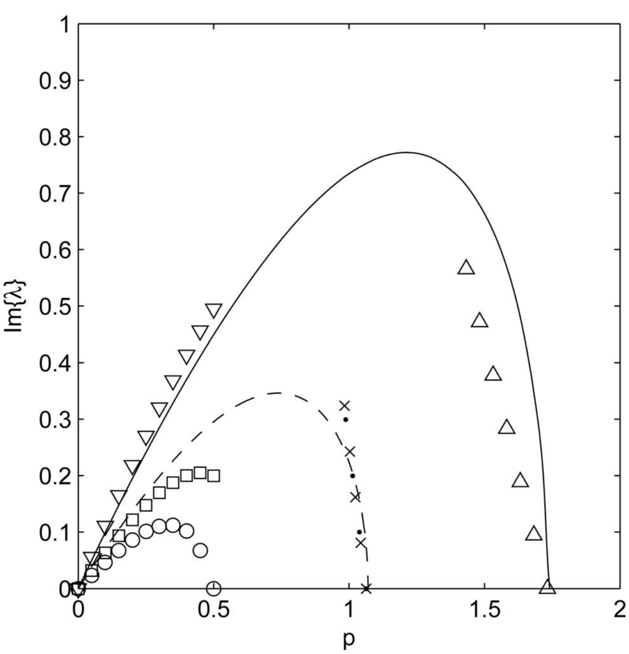

Fig. 3. Growth rate of the soliton transverse instabilities defined from numerical simulations and the variational approach. Solid and dashed curves are numerical data for local ($d=0$) and nonlocal ($d=0.5$) nonlinearities. Triangles above and below the solid curve are the long-scale and short-scale asymptotic expansions for local nonlinearity. Squares and circles are the long-scale asymptotic expansion based on the linearized nonlocal eigenvalue system expanded up to the first and second orders in d. Dots and crosses are the short-scale asymptotic expansion up to the first and second orders in d.

Figure 3 compares the analytical results with the results of our direct numerical calculations. For nonlocal media, the long-scale expansion to the first order in d (circles) is insufficient, but the second-order expansion (squares) gives a good result. In the short-scale expansion, both the first- (dots) and second-order (crosses) expansions are in good agreement with the numerics. It is worth mentioning that in the calculation of the second-order expansion, we employ the generalized variation equation from the averaged Lagrangian containing higher-order derivatives, as discussed in [19].

4. NONLINEAR WAVE EVOLUTION

To observe the consequence of the predicted instability suppression, we study numerically the evolution of the soliton stripe described by Eqs. (1). Figure 4 compares the transverse instability of a bright-soliton stripe with the fixed power 2 in local and nonlocal media, with the initial field modulated transversely with the maximum growth rate. In Figs. 4(a)–4(c) we show snapshots of the soliton stripe evolution in a local nonlinear medium at $z=1.0$, 6.0, and 10.0, respectively. We observe that at $z=10.0$ [Fig. 4(c)] the bright-soliton stripe decays into a sequence of filaments due to the modulation in the y direction. In sharp contrast, there is no visible decay of the soliton stripe in a nonlocal medium ($d=1$) [Figs. 4(d)–4(f)], therefore confirming our major conclusion that the soliton transverse instabilities are substantially suppressed in nonlocal nonlinear media.

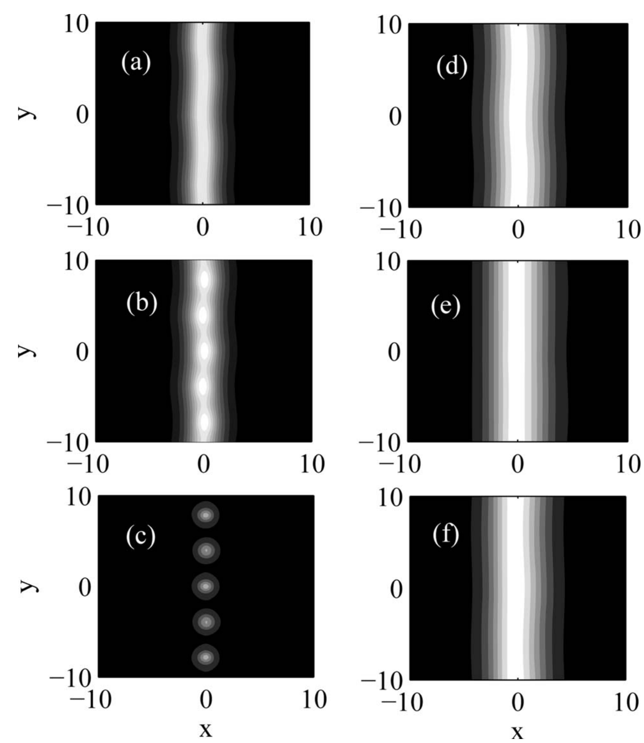

Fig. 4. Evolution of a modulated bright-soliton stripe in (a)–(c) local and (d)–(f) nonlocal nonlinear media at the distances $z=1.0$, 6.0, and 10.0, in that order.

5. CONCLUSIONS

We have analyzed the transverse instabilities of spatial optical solitons in nonlocal nonlinear media by employing both linear stability analysis and numerical simulations. We have demonstrated that the nonlocal nonlinear response can significantly suppress soliton transverse instabilities, allowing experimental observations of the stable propagation of soliton stripes in nonlocal media.

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