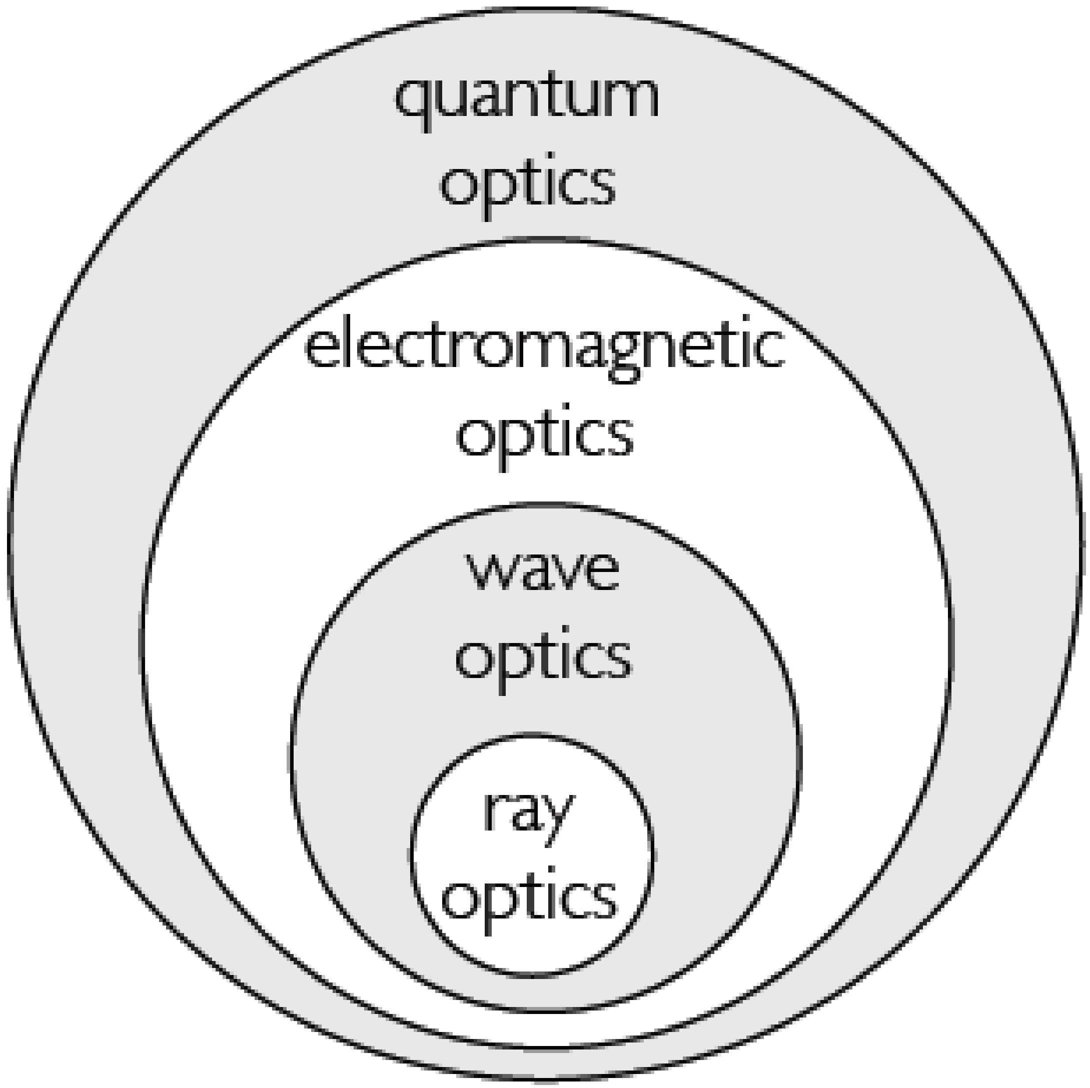
Summary of Beam Optics

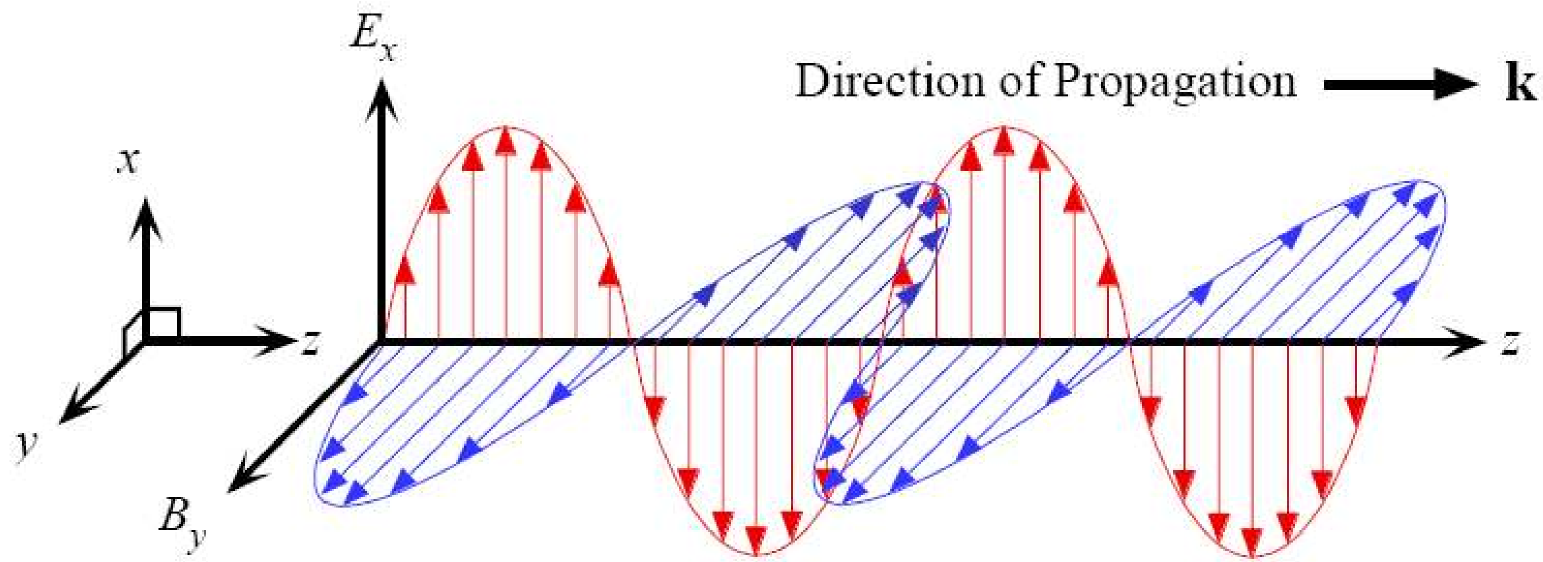
- Gaussian beams,
- waves with limited spatial extension perpendicular to propagation direction,
- Gaussian beam is solution of paraxial Helmholtz equation,
- Gaussian beam has parabolic wavefronts, (as seen in lab experiment),
- Gaussian beams characterized by focus waist and focus depth,

General Optics



ElectroMagnetic waves

- light is a wave of electric and magnetic fields,
- electric and magnetic fields are vectors → polarization,
- microscopic nature of the refractive index, from atomic dipoles,



Syllabus

1. Introduction to modern photonics (Feb. 26),
2. Ray optics (lens, mirrors, prisms, et al.) (Mar. 7, 12, 14, 19),
3. Wave optics (plane waves and interference) (Mar. 26, 28),
4. Beam optics (Gaussian beam and resonators) (Apr. 9, 11, 16, 18),
5. Electromagnetic optics (reflection and refraction) (Apr. 23, 25, 30),
   Midterm (May 7-th),
6. Fourier optics (diffraction and holography) (May 2, 9),
7. Crystal optics (birefringence and LCDs) (May 14, 16),
8. Waveguide optics (waveguides and optical fibers) (May 21, 23),
9. Photon optics (light quanta and atoms) (May 28, 30),
10. Laser optics (spontaneous and stimulated emissions) (June 4),
11. Semiconductor optics (LEDs and LDs) (June 6),
12. Nonlinear optics (June 18),
13. Quantum optics (June 20),
    Final exam (June 27),
14. Semester oral report (July 4),

Maxwell's equations

- Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B},$$

- Ampére's law:

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} + \mathbf{J},$$

- Gauss's law for the electric field:

$$\nabla \cdot \mathbf{D} = \rho,$$

- Gauss's law for the magnetic field:

$$\nabla \cdot \mathbf{B} = 0,$$

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Plane electromagnetic waves

- Maxwell's equations in free space, there is vacuum, no free charges, no currents, $\mathbf{J} = \rho = 0$,
- both **E** and **B** satisfy wave equation,

$$\nabla^2 \mathbf{E} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2},$$

- we can use the solutions of *wave optics*,

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= E_0 \text{exp}(i\omega t) \text{exp}(-i\mathbf{k} \cdot \mathbf{r}), \\ \mathbf{B}(\mathbf{r}, t) &= B_0 \text{exp}(i\omega t) \text{exp}(-i\mathbf{k} \cdot \mathbf{r}), \end{aligned}$$

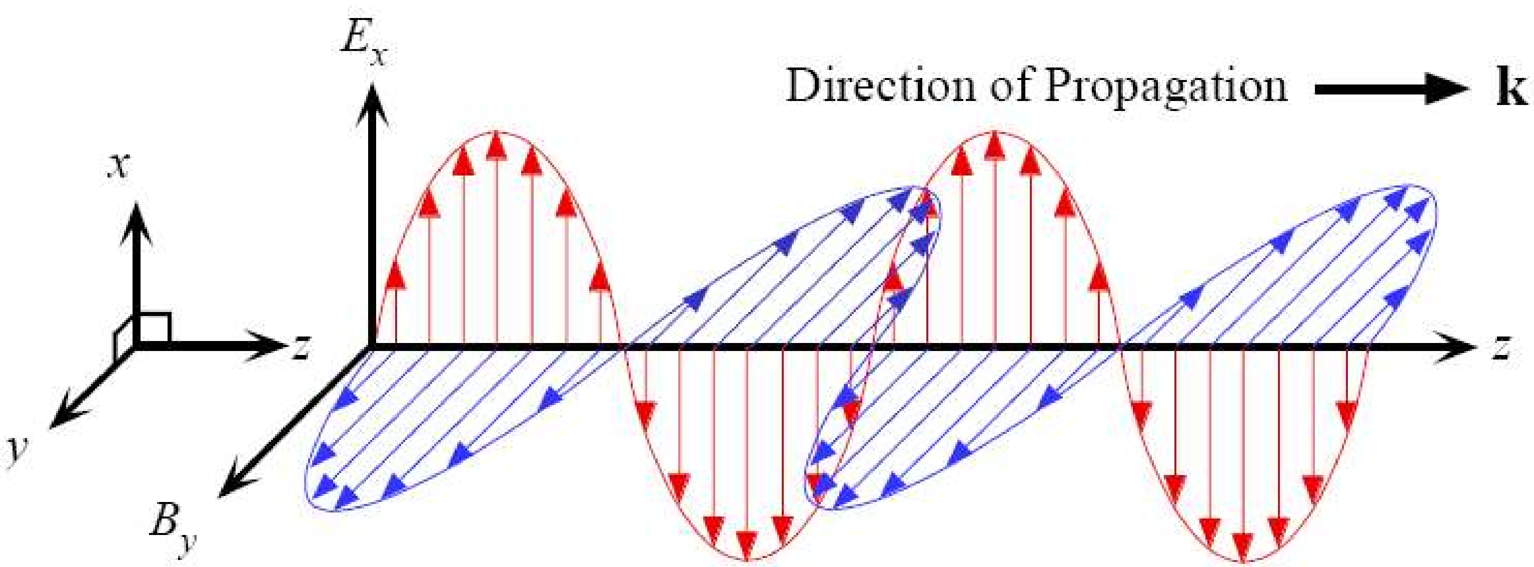
Elementary electromagnetic waves

- The **k**, $\mathbf{B}_0$, and $\mathbf{E}_0$ are standing perpendicular on each other,

$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2}\mathbf{E}, \qquad \mathbf{k} \times \mathbf{E} = \omega \mathbf{B},$$

$$|\mathbf{B}_0| = |\mathbf{E}_0|/c,$$

- light is a TEM wave,



Poynting's theorem

- Poynting's theorem is the law of power conservation for electromagnetic fields,

$$\nabla \cdot (E \times H) + \frac{\partial}{\partial t}(\frac{1}{2}\epsilon_0 E^2) + \frac{\partial}{\partial t}(\frac{1}{2}\mu_0 H^2) + E \cdot \frac{\partial P}{\partial t} + H \cdot \frac{\partial}{\partial t}(\mu_0 M) + E \cdot J = 0.$$

- for the linear constitutive law, $E \cdot \frac{\partial P}{\partial t} = \frac{\partial}{\partial t}(\frac{1}{2}\epsilon_0 \chi_e E^2)$,
- then the Poynting's theorem for the *linear*, *isotropic* medium becomes,

$$\nabla \cdot (E \times H) + \frac{\partial}{\partial t}(w_e + w_m) + E \cdot J = 0,$$

where $w_e = \frac{1}{2}\epsilon E^2$ and $w_m = \frac{1}{2}\mu H^2$.

- Or write the Poynting's theorem in integral form,

$$\oint_S E \times H \cdot dA + \frac{\partial}{\partial t}\int_V (w_e + w_m)\, dV + \int_V E \cdot J\, dV = 0,$$

Energy density and intensity of a plane wave

- energy density: $u = u_E + u_B$,

$$u_E = \frac{\epsilon_0}{2}|\mathbf{E}|^2, \qquad u_B = \frac{1}{2\mu_0}|\mathbf{B}|^2,$$

- for $|\mathbf{B}_0| = |\mathbf{E}_0|/c$,

$$u = \frac{\epsilon_0}{2}|\mathbf{E}|^2 + \frac{1}{2\mu_0 c^2}|\mathbf{E}|^2 = \epsilon_0|\mathbf{E}|^2,$$

- energy is carried in equal parts by magnetic and electric field,
- energy flow:

  Poynting vector : $$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B},$$

  in the direction of **k**,
- time average: $I = \langle|\mathbf{S}|\rangle$, (times longer than optical cycle)
- intensity for a plane wave, $I = \frac{|\mathbf{E}|^2}{2\eta_0}$, where $\eta_0 = \sqrt{\epsilon_0/\mu_0} \approx 377\Omega$, is the impedance of vacuum,

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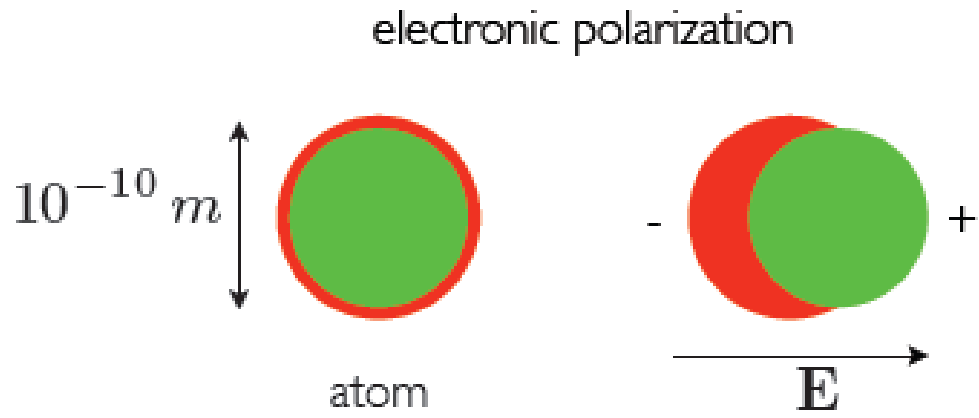
Maxwell's equations in a medium



- electron cloud displaced by Δr,
- atomic dipole: $p = q\Delta r$
- many atomic dipoles in a medium will sum up to a larger dipole,
- this sum of dipoles is measured by **P**, dipole moment per volume,

$$\mathbf{P} = Np,$$

where N is the atomic number density.

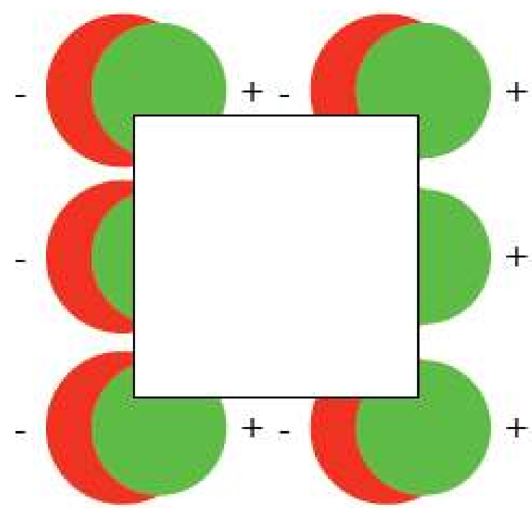
Maxwell's equations in a medium

- Total electron charges Q,

$$
\begin{aligned}
Q &= -\oint \mathbf{P}\,\mathrm{d}\mathbf{A}, \\
&= -\iiint \nabla\cdot\mathbf{P}\,\mathrm{d}\mathbf{V}, \quad \text{Gauss theorem}, \\
&= \iiint \rho\,\mathrm{d}\mathbf{V},
\end{aligned}
$$

- $\rho = -\nabla\cdot\mathbf{P}$,
- time dependent polarization creates also current,

$$
\begin{aligned}
\mathbf{J} &= Nqv, \\
&= Nq\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}, \\
&= N\frac{\mathrm{d}p}{\mathrm{d}t}, \\
&= \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t},
\end{aligned}
$$



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Maxwell's equations in a medium

$$\nabla(\epsilon_0 \cdot \mathbf{E}) = -\nabla \cdot \mathbf{P},$$
$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial \mathbf{P}}{\partial t} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t},$$

- electric flux density, **D**,

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P},$$

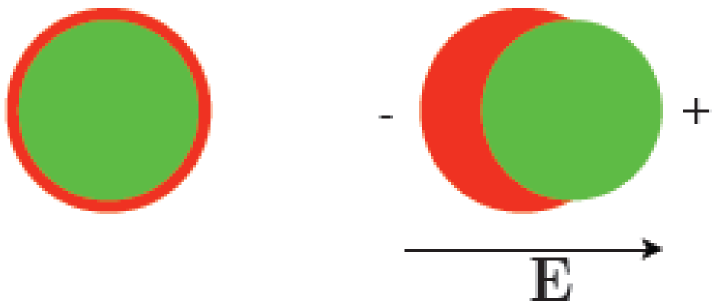
- polarization density, **P**,
- magnetic flux density, **B**,

$$\mathbf{D} = \mu_0 \mathbf{H} + \mathbf{M},$$

- magnetization, **M**,
- both **P** and **M** are vector fields,
- we will deal mainly with nonmagnetic media, $\mathbf{M} = 0$,

Types of polarization

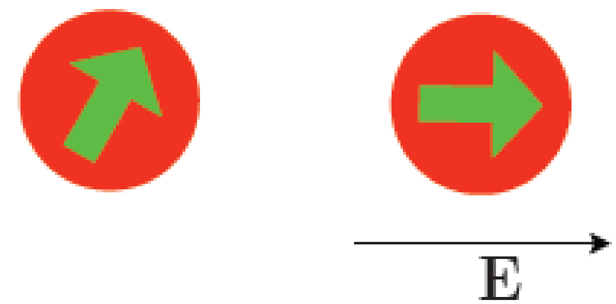
electronic polarization



important systems with extended electronic wave functions

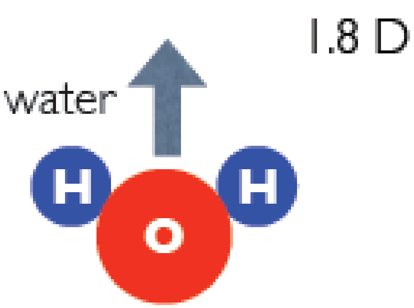
molecules with π orbitals

orientational polarization



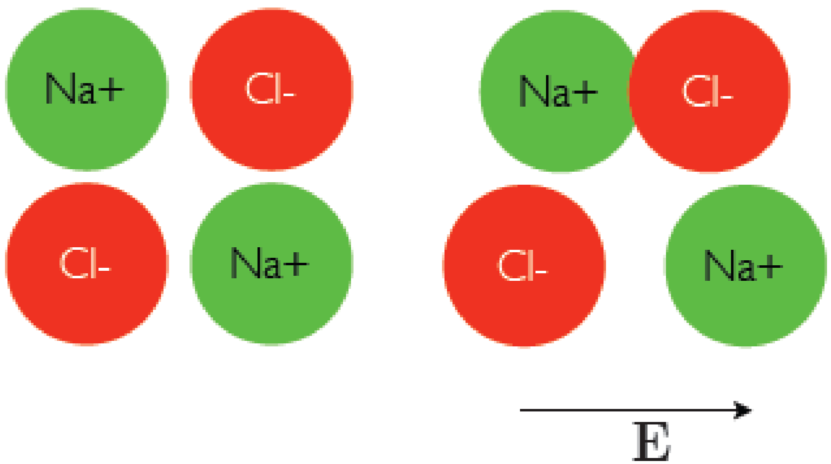
important for polar systems, molecules with dipole moment

example: water



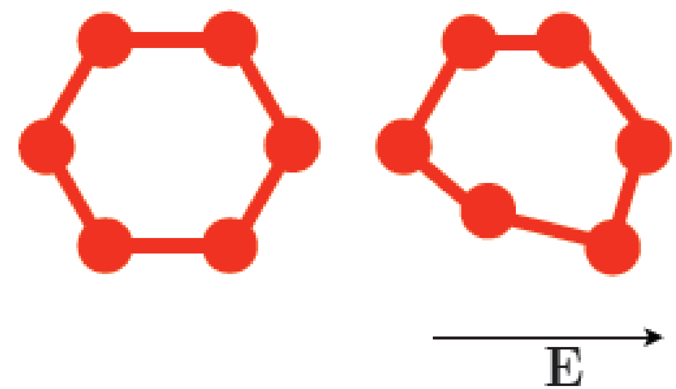
Types of polarization

ionic polarization



important for ionic crystals

molecular deformation



important for flexible systems
polymers ...

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Dielectric media

- linear: a medium is said to be linear if **P**(r, t) is linearly related to **E**(r, t), this is important for superposition (no superposition possible in a nonlinear medium),
- nondispersive a medium is said to be nondispersive if

$$\mathbf{P}(t) = \mathbf{E}(t),$$

  medium responds instantaneously (idealization, since polarization is never really instantaneous),
- homogeneous: a medium is said to be homogeneous if the relation between **P** and **E** are not a function of **r**,
- isotropic: a medium is said to be isotropic if the relation between **P** and **E** are not a function of direction **P**$||$**E**, example for anisotropy: birefringence,

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Simple media

Constitutive relation: **B** $= \mu$**H** and **D** $= \epsilon$E.

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E},$$

where D is the electric flux density (C/m^2), E is the electric field strength (V/m), and P is the *dipole moment density* (C/m^2).

- linear: $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$, where ϵ is the permittivity (F/m), χ is the electric susceptibility,
- isotropic: $\chi(x) = \chi(y) = \chi(z)$,
- homogeneous: $\chi(r)$ is independent of r,
- dispersion-free media: $\chi(\omega)$ is independent of ω

Material equations: **D** $= \epsilon$E, where

$$\mu\epsilon = \mu_0 \epsilon_0 (1 + \chi) = \frac{n^2}{c^2},$$

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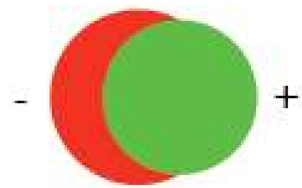
Model for the polarization response



- damped harmonic oscillator

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\sigma\frac{\mathrm{d}x}{\mathrm{d}t}+\omega_0^2x=\frac{q}{m}E(t),$$

- assume $E(t)=E_0\exp(-i\omega t)$, and $x(t)=x_0\exp(-i\omega t)$, then

$$x(t)=\frac{1}{\omega_0^2-i\omega\sigma-\omega^2}\frac{q}{m}E(t),$$

- electronic dipole, $p(t)=qx(t)$, and the polarization, $\mathbf{P}(t)=Nq\Delta x(t)=\epsilon_0\chi\mathbf{E}(t)$, where χ is a complex number,

- real media have usually multiple resonances,

More general

$$\chi(\omega) = \frac{Nq^2}{\epsilon_0 m} \frac{1}{\omega_0^2 - i\omega\sigma - \omega^2}$$

- Quantum mechanics,

$$\chi(\omega) = \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{\omega_j^2 - i\omega\sigma_j - \omega^2}$$

where f_j is the oscillator strength,

- redefine,

$$\chi(\nu) = \chi_0 \frac{\nu_0^2}{\nu_0^2 - \nu^2 + i\nu\Delta\nu}$$

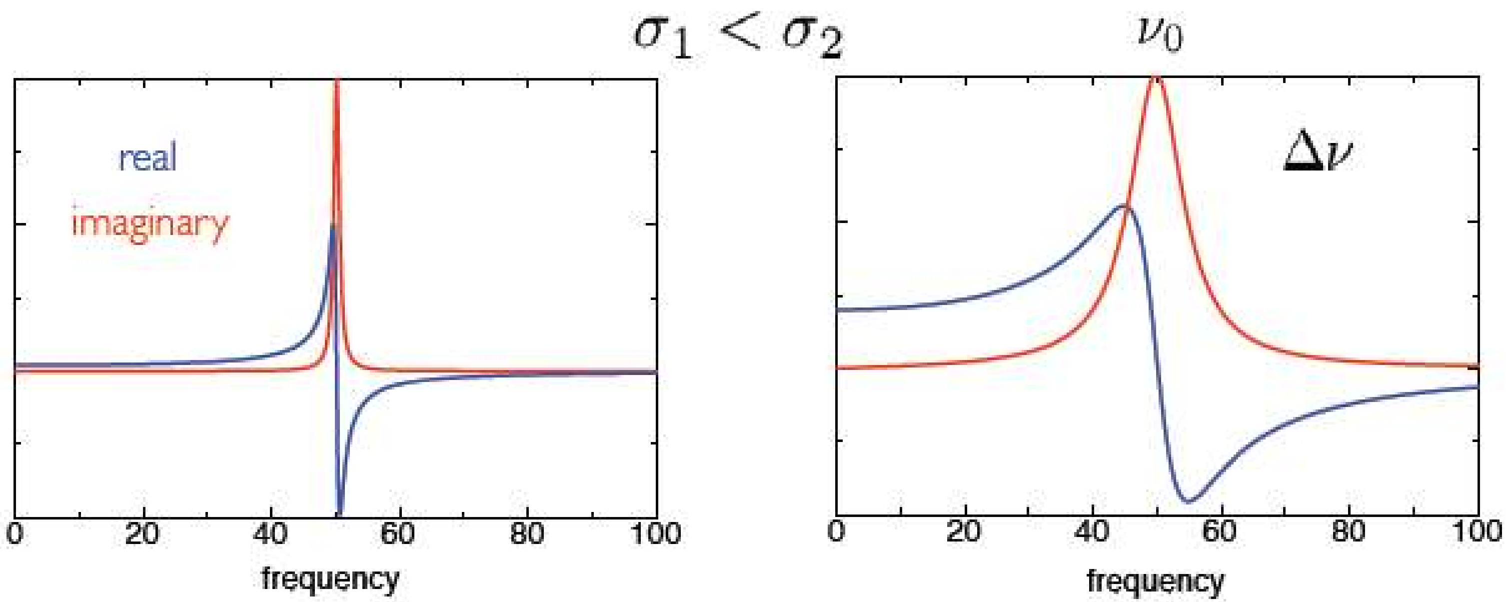
- $\chi = \chi' + i\chi"$, where

$$\chi'(\nu) = \chi_0 \frac{\nu_0^2(\nu_0^2 - \nu^2)}{(\nu_0^2 - \nu^2)^2 + (\nu\Delta\nu)^2},$$

$$\chi"(\nu) = \chi_0 \frac{\nu_0^2 \nu\Delta\nu}{(\nu_0^2 - \nu^2)^2 + (\nu\Delta\nu)^2},$$

Lorentzian function,

Consequences of the simple model



- complex susceptibility, $\chi = \chi' + i\chi"$,

$$\epsilon = \epsilon_0(1+\chi),$$

- Helmholtz equation still valid but

$$k = \omega\sqrt{\epsilon\mu_0}^{1/2} = k_0(1+\chi)^{1/2} = k_0(1+\chi' + i\chi")^{1/2},$$

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Relation to refractive index

- plane waves: exp$(-ikz)$,

$$k = \omega\sqrt{\epsilon\mu_0}^{1/2} = k_0(1+\chi)^{1/2} = k_0(1+\chi' + i\chi")^{1/2},$$

- simplify

$$k = \beta - i\frac{\alpha}{2} = n\frac{\omega}{c},$$

- refractive index is now also a complex number,
- plane waves,

$$\text{exp}(-ikz) = \text{exp}(-i\beta z)\text{exp}(-\frac{\alpha}{2}),$$

- intensity,

$$I \propto |\text{exp}(-ikz)|^2 = \text{exp}(-\alpha z),$$

where α is absorption coefficient,

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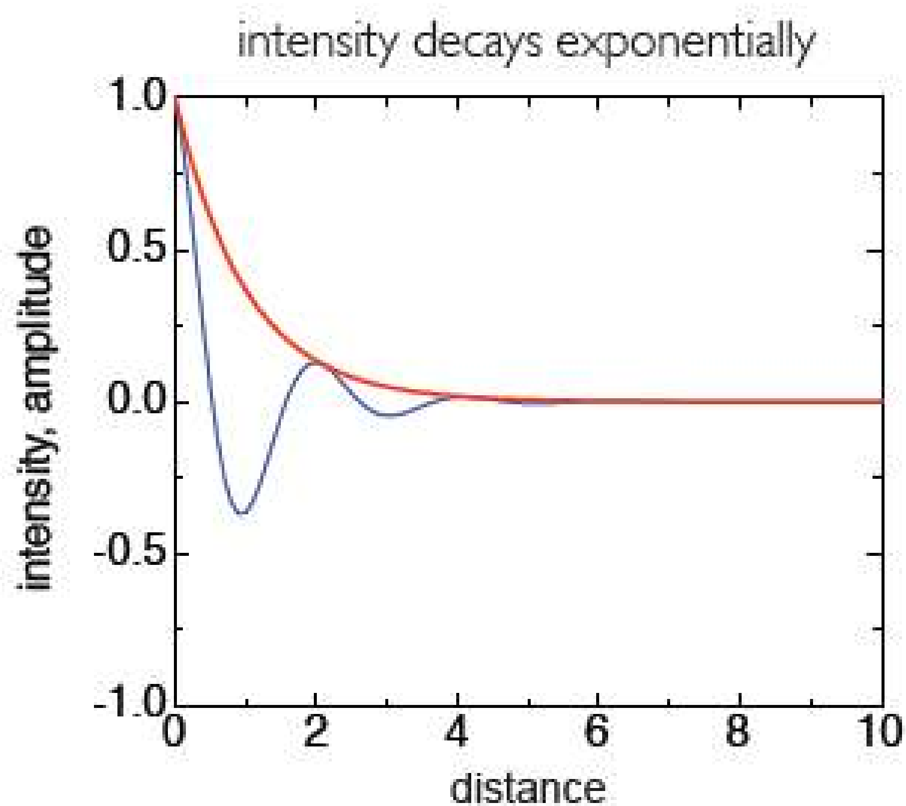
Absorption coefficient

- plane waves, $\exp(-ikz) = \exp(-i\beta z)\exp(-\frac{\alpha}{2})$,
- intensity, $I \propto |\exp(-ikz)|^2 = \exp(-\alpha z)$, where α is absorption coefficient,



Weakly absorbing media

- weakly absorbing media: $\chi' \ll 1, \qquad \chi" \ll 1,$
- dispersion:

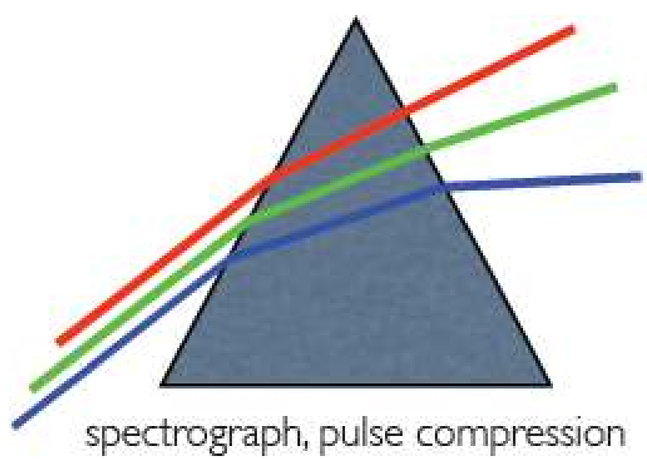
$$n(\nu) \approx 1 + \frac{\chi'}{2},$$

phase velocity is a function of frequency, typically n decreases with increasing frequency

- absorption:

$$\alpha \approx -k_0 \chi",$$

absorption is a function of frequency, characteristic for the material,



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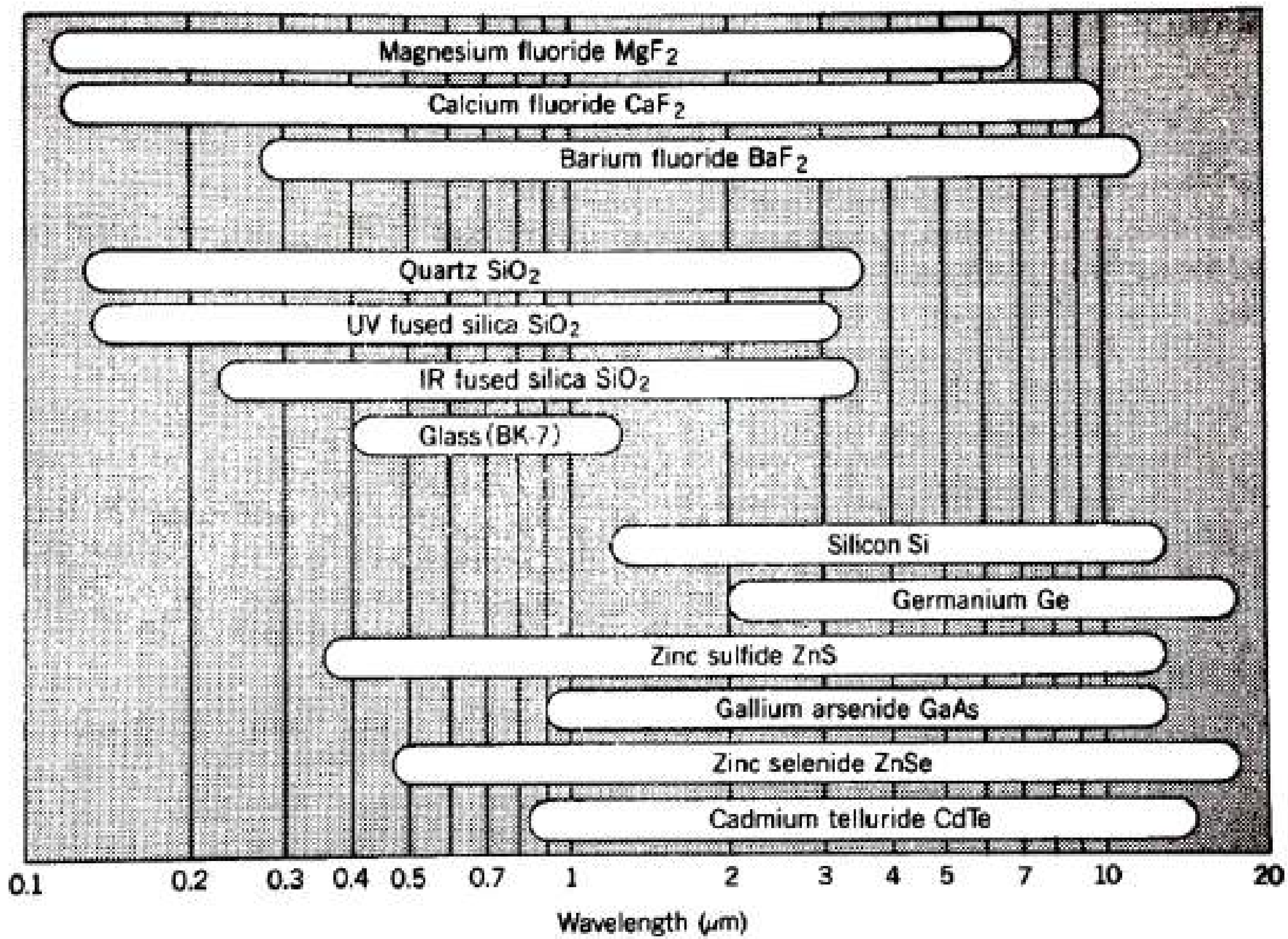
Absorption and Dispersion

materials are not transparent to all optical wavelength



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Kramers Kronig Relation

- absorption and dispersion are always related

$$\chi'(\nu) = \frac{2}{\pi}\int_0^{\infty} \frac{s\chi''(s)}{s^2-\nu^2}\mathrm{d}s,$$
$$\chi''(\nu) = \frac{2}{\pi}\int_0^{\infty} \frac{s\chi'(s)}{\nu^2-s^2}\mathrm{d}s,$$

- group velocity,

$$v_G = \frac{c}{n+\omega\frac{\partial n}{\partial \omega}},$$

- large $\frac{\partial n}{\partial \omega}$ always with large α,

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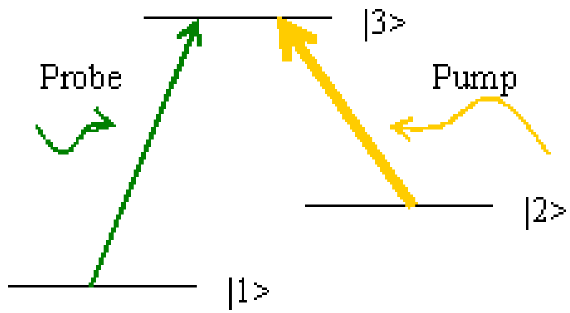
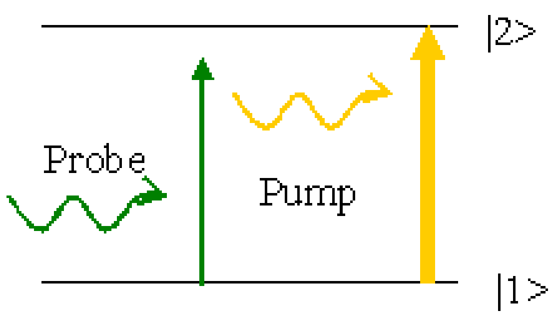
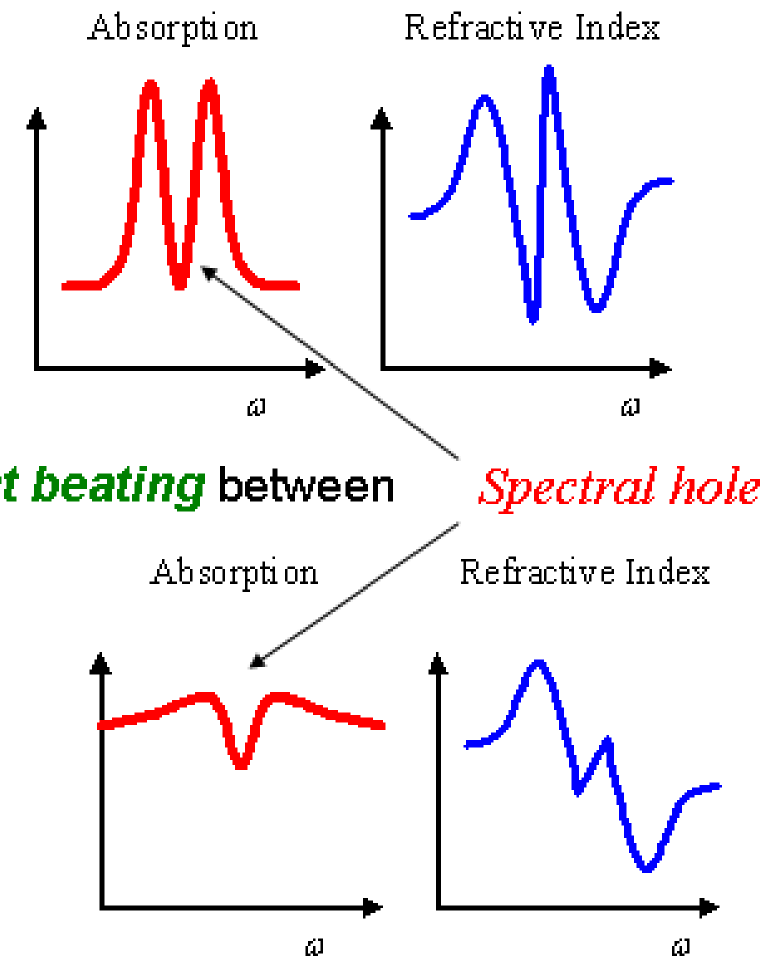
Slow-light

- **Electromagnetically Induced Transparency (EIT)**
  — Formation of ***dark state*** by intense pump in a three-level system



- **Population Oscillation**
  — Absorption dip generated by ***coherent beating*** between pump and probe in a two-level system





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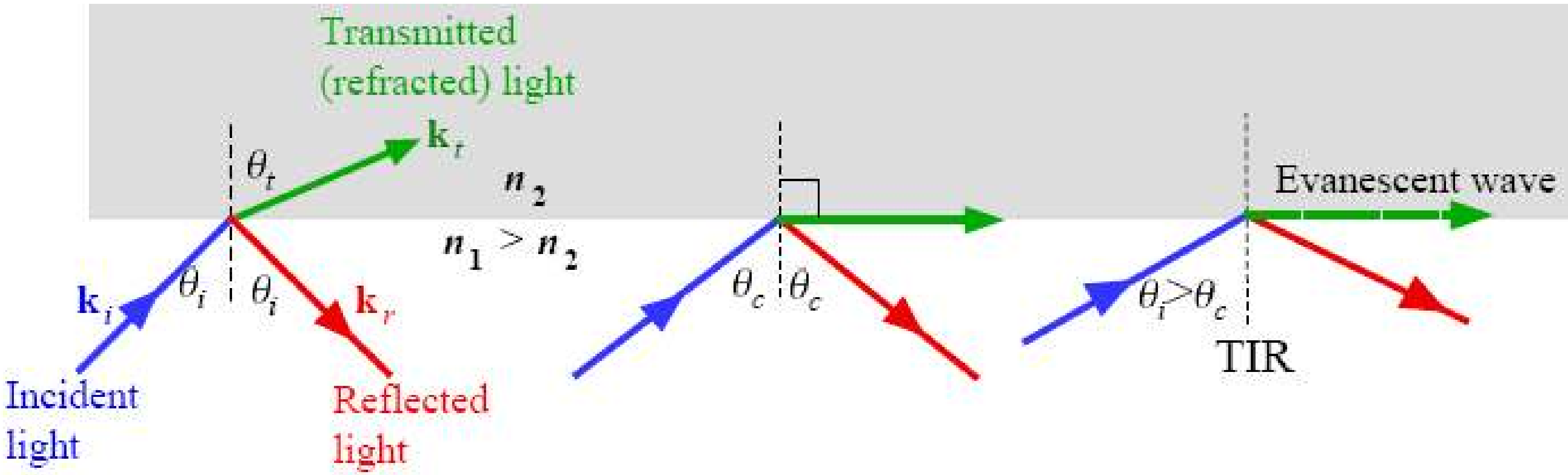
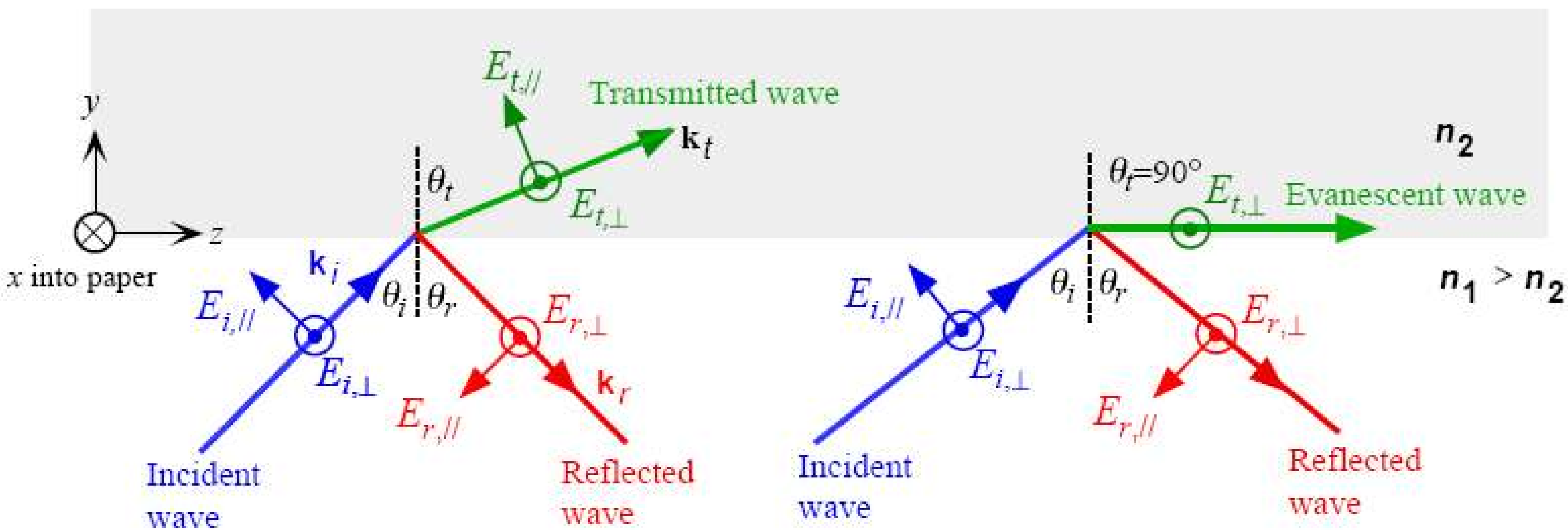
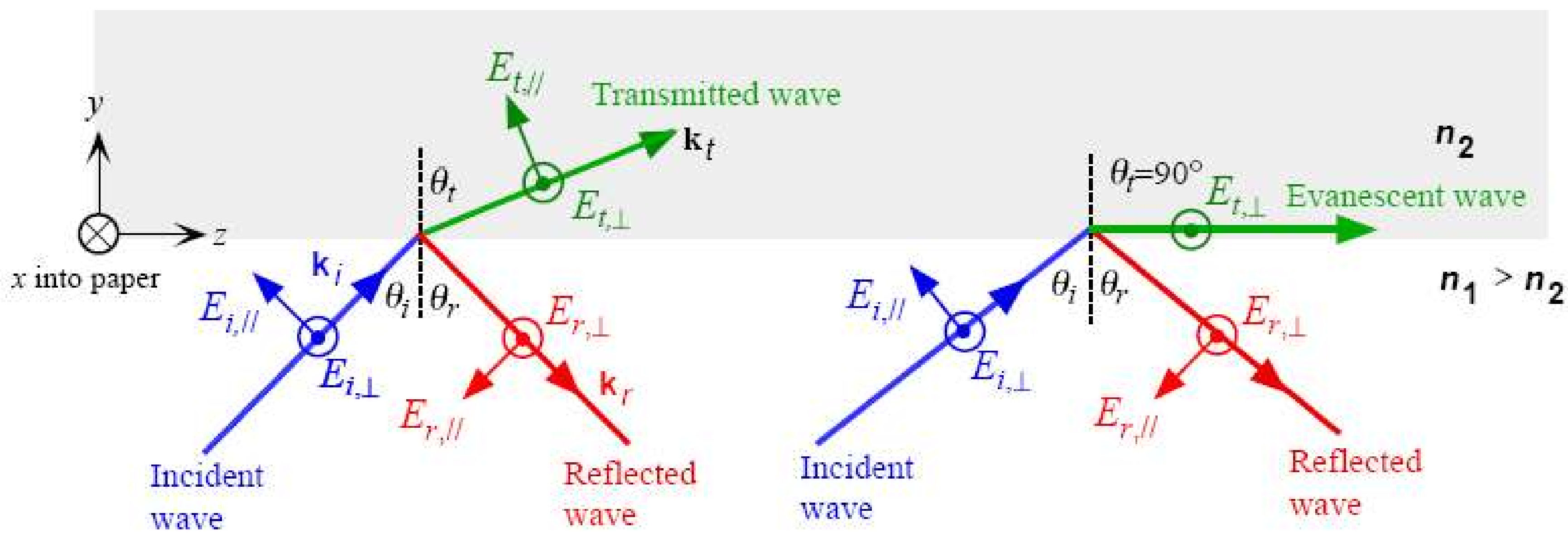
Reflection, Refraction, and TIR





EM wave approach



- Consider a plane wave with its electric field polarized parallel to the surface of an interface between two media, (transverse electric, or TE, wave),

$$E_{in} = \hat{x} E_+ e^{-j\mathbf{k}^{(1)}\cdot\mathbf{r}}$$

- the tangential E and E must be continuous at $z = 0$. This implies ,

$$k_z^{(1)} = k_z^{(2)} = k_z, \qquad \text{phase matching,}$$

EM wave approach

- The consequence is Snell's law,

$$\sqrt{\mu_1\epsilon_1}\sin\theta_1 = \sqrt{\mu_2\epsilon_2}\sin\theta_2.$$

- At $y < 0$, the superposition of the incident and reflected waves is,

$$E_x = [E_+^{(1)}e^{-jk_y^{(1)}y} + E_-^{(1)}e^{+jk_y^{(1)}y}]e^{-jk_z z},$$

- from Faraday's law,

$$H_z = -\frac{k_y^{(1)}}{\omega\mu_1}[E_+^{(1)}e^{-jk_y^{(1)}y} - E_-^{(1)}e^{+jk_y^{(1)}y}]e^{-jk_z z},$$

where

$$\frac{k_y^{(1)}}{\omega\mu_1} = \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_1 \equiv Y_0^{(1)},$$

is the *characteristic admittance* by medium 1 to a TE wave at inclination θ_1 with respect at the y direction. The inverse of $Y_0^{(1)}$ is the *characteristic impedance* $Z_0^{(1)}$.

EM wave approach

- At $y > 0$, the transmitted waves is,

$$E_x = E_+^{(2)} e^{-jk_y^{(2)} y} e^{-jk_z z},$$

with the z component of the H field,

$$H_z = -\frac{k_y^{(2)}}{\omega \mu_2} E_+^{(2)} e^{-jk_y^{(2)} y} e^{-jk_z z},$$

with the characteristic admittance in medium 2,

$$\frac{k_y^{(2)}}{\omega \mu_2} = \sqrt{\frac{\epsilon_2}{\mu_2}} \cos\theta_2 \equiv Y_0^{(2)}.$$

- Continuity of the tangential components of E and H requires the ratio

$$Z \equiv -\frac{E_x}{H_z}$$

to be continuous. Z is the *wave impedance* at the interface.

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EM wave approach

- At $y = 0$,

$$Z_0^{(1)} \frac{E_+^{(1)} + E_-^{(1)}}{E_+^{(1)} - E_-^{(1)}} = Z_0^{(2)}.$$

- The quantity,

$$\Gamma \equiv \frac{E_-}{E_+},$$

is the *reflection coefficient*.

EM wave approach

- For $E_-^{(1)}/E_+^{(1)}$,

$$\Gamma^{(1)} = \frac{Z_0^{(2)} - Z_0^{(1)}}{Z_0^{(2)} + Z_0^{(1)}},$$

using Snell's law,

$$\Gamma^{(1)} = \frac{\sqrt{1-\sin^2\theta_1} - \sqrt{1-\sin^2\theta_1 \frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_2\mu_1}{\epsilon_1\mu_2}}}{\sqrt{1-\sin^2\theta_1} + \sqrt{1-\sin^2\theta_1 \frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_2\mu_1}{\epsilon_1\mu_2}}}.$$

- The density of power flow in the y direction is

$$\frac{1}{2}\text{Re}[E \times H^*] \cdot \hat{y} = -\frac{1}{2}\text{Re}[E_x H_z] = \frac{1}{2} Y_0^{(1)} |E_+^{(1)}|^2 (1 - |\Gamma^{(1)}|^2).$$

Thus $|\Gamma|^2$ is the ratio of reflected to incident power flow.

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Fresnel's equations: TE

- for TE waves, $E_\perp$, the reflection coefficient,

$$r_\perp = \frac{E_-^{(1)}(0)}{E_+^{(1)}(0)} = \frac{\sqrt{1-\sin^2\theta_1} - \sqrt{1-\sin^2\theta_1\frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_2\mu_1}{\epsilon_1\mu_2}}}{\sqrt{1-\sin^2\theta_1} + \sqrt{1-\sin^2\theta_1\frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_2\mu_1}{\epsilon_1\mu_2}}},$$

$$= \frac{\cos\theta_1 - [n^2-\sin^2\theta_1]^{1/2}}{\cos\theta_1 + [n^2-\sin^2\theta_1]^{1/2}},$$

where $n \equiv \frac{n_2}{n_1} = (\frac{\epsilon_2}{\epsilon_1})^{1/2}$, and the transmission coefficients,

$$t_\perp = \frac{2\cos\theta_1}{\cos\theta_1 + [n^2-\sin^2\theta_1]^{1/2}},$$

- relations between reflection and transmission coefficients,

$$r_\perp + 1 = t_\perp,$$

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Fresnel's equations: TM

- for TM waves, $E_{\parallel}$, the reflection coefficient,

$$
\begin{aligned}
r_{\parallel} &= -\frac{\sqrt{1-\sin^2\theta_1} - \sqrt{1-\sin^2\theta_1\,\frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_1\mu_2}{\epsilon_2\mu_1}}}{\sqrt{1-\sin^2\theta_1} + \sqrt{1-\sin^2\theta_1\,\frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_1\mu_2}{\epsilon_2\mu_1}}}, \\
&= \frac{[n^2-\sin^2\theta_1]^{1/2} - n^2\cos\theta_1}{[n^2-\sin^2\theta_1]^{1/2} + n^2\cos\theta_1}, \\
t_{\parallel} &= \frac{2n\cos\theta_1}{n^2\cos\theta_1 + [n^2-\sin^2\theta_1]^{1/2}},
\end{aligned}
$$

- relations between reflection and transmission coefficients,

$$r_{\parallel} + n t_{\parallel} = 1,$$

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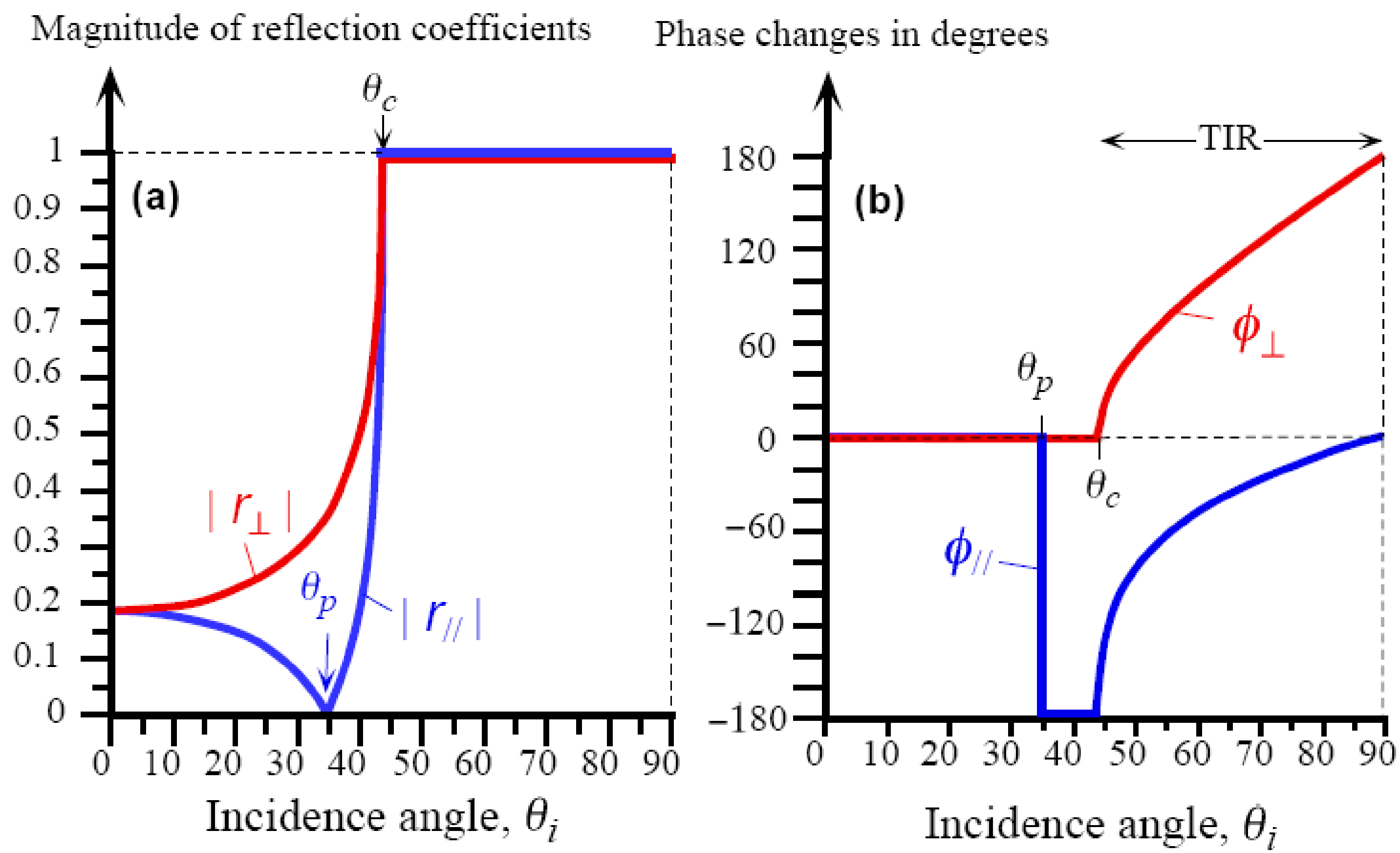
Reflection and transmission, $n_1 > n_2$,



- θ_c is the critical angle for *total internal reflection*,
- there is a *phase change* for TIR,
- for $\theta_i = 90$, there is a $phaseshift$ for the reflection TE wave even $r = 1$,
- for TM waves, the reflection can be zero at the Brewster's angle, θ_p,

EM wave approach: TM

- Consider a plane wave with its magnetic field polarized parallel to the surface of an interface between two media, (transverse magnetic, or TM, wave),
- at $y < 0$, the superposition of the incident and reflected waves is,

$$H_x = [H_+^{(1)} e^{-jk_y^{(1)} y} + H_-^{(1)} e^{+jk_y^{(1)} y}] e^{-jk_z z},$$

- from Ampére's law,

$$E_z = \frac{k_y^{(1)}}{\omega \epsilon_1} [H_+^{(1)} e^{-jk_y^{(1)} z} - H_-^{(1)} e^{+jk_y^{(1)} z}] e^{-jk_z z}.$$

- At $y > 0$, the transmitted waves is,

$$H_x = H_+^{(2)} e^{-jk_y^{(2)} y} e^{-jk_z z},$$

with the z component of the E field,

$$E_z = \frac{k_y^{(2)}}{\omega \epsilon_2} H_+^{(2)} e^{-jk_y^{(2)} y} e^{-jk_z z},$$

EM wave approach: TM

- with the characteristic admittance of the traveling TM wave,

$$Y_0 = \frac{\omega\epsilon}{k_y} = \sqrt{\frac{\epsilon}{\mu}}\frac{1}{\cos\theta}.$$

- continuity of the tangential components of E and H requires the ratio

$$Z \equiv \frac{E_z}{H_x}$$

to be continuous. Z is the *wave impedance* at the interface.

- at $y = 0$,

$$Z_0^{(1)}\frac{H_+^{(1)} - H_-^{(1)}}{H_+^{(1)} + H_-^{(1)}} = Z_0^{(2)}.$$

- The quantity, $\Gamma \equiv -\frac{H_-}{H_+}$, is the *reflection coefficient*.

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EM wave approach: TM

- For $E_-^{(1)}/E_+^{(1)}$,

$$\Gamma^{(1)} = \frac{Z_0^{(2)} - Z_0^{(1)}}{Z_0^{(2)} + Z_0^{(1)}},$$

using Snell's law,

$$\begin{aligned}
\Gamma^{(1)} &= -\frac{\sqrt{1-\sin^2\theta_1} - \sqrt{1-\sin^2\theta_1 \frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_1\mu_2}{\epsilon_2\mu_1}}}{\sqrt{1-\sin^2\theta_1} + \sqrt{1-\sin^2\theta_1 \frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}}\sqrt{\frac{\epsilon_1\mu_2}{\epsilon_2\mu_1}}}, \\
&= \frac{[n^2 - \sin^2\theta_1]^{1/2} - n^2\cos\theta_1}{[n^2 - \sin^2\theta_1]^{1/2} + n^2\cos\theta_1}, \\
t_{\|} &= \frac{2n\cos\theta_1}{n^2\cos\theta_1 + [n^2 - \sin^2\theta_1]^{1/2}},
\end{aligned}$$

- TM waves can be transmitted reflection-free at a dielectric interface, when $\mu_1 = \mu_2 = \mu_0$, for the angle $\theta_1 = \theta_B$, the so-called *Brewster angle*,

$$\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}.$$

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Total internal reflection

- If medium 1 has a larger value of $\sqrt{\mu\epsilon}$, *optical denser*, than medium 2, Snell's law fails to yield a real angle θ_2 for a certain range of angle of incidence.
- for $\mu_1 = \mu_2 = \mu_0$,

$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}.$$

- When no real solution of θ_2 are found, the propagation constant must be allowed to become negative imaginary,

$$k_z^{(2)} = k_z^{(1)}, \qquad k_y^{(2)} = -j\alpha_y^{(2)}.$$

In this case,

$$[k_z^{(2)}]^2 + [k_y^{(2)}]^2 = [k_z^{(2)}]^2 - [\alpha_y^{(2)}]^2 = \omega^2\mu_0\epsilon_2,$$

and

$$k_z^{(2)} = \sqrt{\omega^2\mu_0\epsilon_2 + [\alpha_z^{(2)}]^2}.$$

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Total internal reflection

- In the case of a TE wave, the transmitted fields become,

$$
\begin{aligned}
E_x &= E_+^{(2)} e^{-\alpha_y^{(2)} y} e^{-jk_z z}, \\
H_z &= \frac{j\alpha_y^{(2)}}{\omega\mu_0} E_+^{(2)} e^{-\alpha_y^{(2)} y} e^{-jk_z z}.
\end{aligned}
$$

- The wave impedance, $-E_y/H_x$,

$$
\frac{\omega\mu_0}{k_y^{(1)}} \frac{E_+^{(1)} + E_-^{(1)}}{E_+^{(1)} - E_-^{(1)}} = \frac{j\omega\mu_0}{\alpha_y^{(2)}} = Z_0^{(2)}.
$$

- The characteristic impedance of medium 2 is now *imaginary*, $Z_0^{(2))} = jX_0^{(2)}$, with $X_0^{(2)}$ real.

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Total internal reflection

- Then the reflection coefficient, $\Gamma = E_-^{(1)}/E_+^{(1)}$,

$$\Gamma^{(1)} = \frac{E_-^{(1)}}{E_+^{(1)}} = \frac{jX_0^{(2)} - Z_0^{(1)}}{jX_0^{(2)} + Z_0^{(1)}},$$

- $|\Gamma^{(1)}| = 1$, and the magnitude of the reflected wave, $E_-^{(1)}$, equals to the magnitude of the incident wave, $E_+^{(1)}$.
- At $y < 0$,

$$\begin{aligned} E_x &= E_+^{(1)}[e^{-jk_y^{(1)}y} + \Gamma^{(1)}e^{+jk_y y}]e^{-jk_z z}, \\ &= 2e^{-j\phi}E_+^{(1)}\cos(k_y^{(1)}y - \phi)e^{-jk_z z}, \end{aligned}$$

where $\phi = -\frac{1}{2}\arg(\Gamma^{(1)})$, is the Goos-Hänchen shift.

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Goos-Hänchen shift