Summary of Fourier Optics

- Diffraction of the paraxial wave is described by Fresnel diffraction integral,

$$u(x,y,z) \quad = \quad \frac{j}{\lambda z}\int_{-\infty}^{\infty} dx_0 \int_{-\infty}^{\infty} dy_0 u_0(x_0,y_0) e^{-j(k/2z)[(x-x_0)^2+(y-y_0)^2)]},$$

- Fraunhofer diffraction is the limit of Fresnel diffraction for large distances between the input plane and the observation plane via the Fourier transform,
- the radius of the central **Airy disk**, diffraction pattern of a circular disk, define the limit of diffraction, $\theta = 1.22\lambda/D$,
- with a 4f-system, one can modify the image by using a spatial filter in the mask plane.

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Polarization

- polarization of light is determined by the direction of x,y,z components of $E(r,t)$ vary in time with different amplitudes and phases,
- for TEM waves z-component is zero,

$$E(z,t) = \text{Re}\{A\text{exp}(i2\pi\nu(t-\frac{z}{c})\},$$

where

$$A = A_x\hat{e}_x + A_y\hat{e}_y = a_x\text{exp}(i\phi_x)\hat{e}_x + a_y\text{exp}(i\phi_y)\hat{e}_y,$$

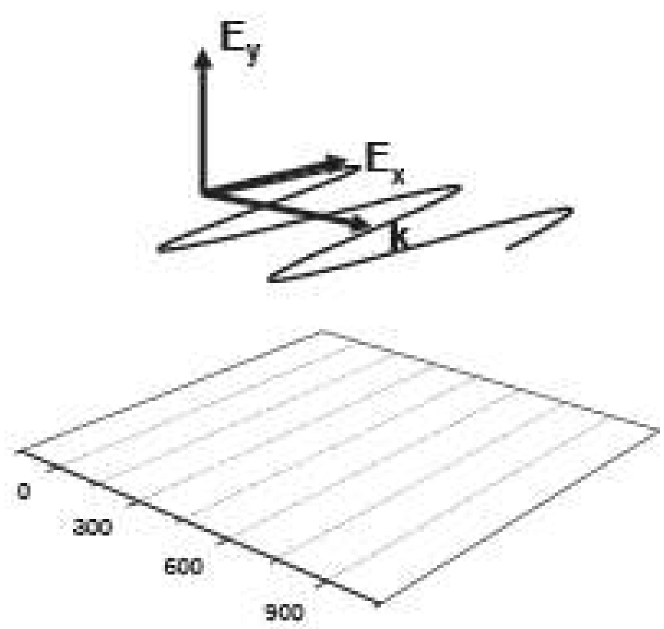
- complex electric field vector,

$$\begin{aligned} E(z,t) &= E_x\hat{e}_x + E_y\hat{e}_y, \\ &= a_x\cos[2\pi\nu(t-\frac{z}{c}) + \phi_x] + a_y\cos[2\pi\nu(t-\frac{z}{c}) + \phi_y], \end{aligned}$$

Linear polarized light

$$E_x = a_x \cos[2\pi\nu(t - \frac{z}{c}) + \phi_x],$$
$$E_y = a_y \cos[2\pi\nu(t - \frac{z}{c}) + \phi_y],$$

- linear polarized light, $a_y = 0$, field points in x direction,
- linear polarized light, $a_x = 0$, field points in y direction,

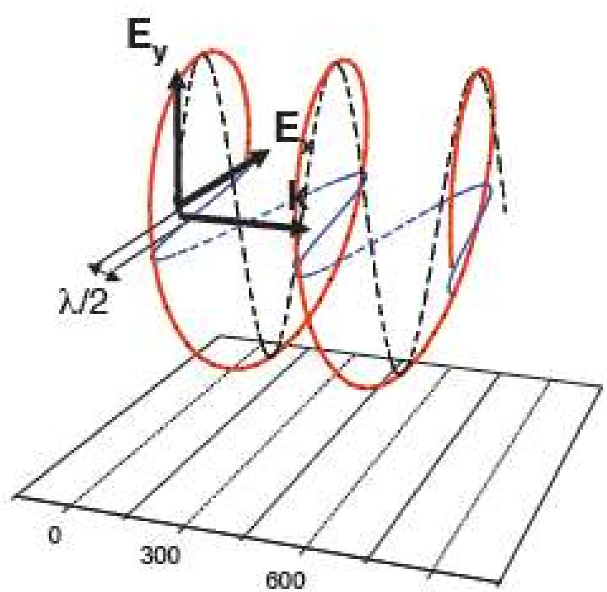


Circular polarized light

- $\phi = \phi_y - \phi_x = \pm\pi/2$ and $a_x = a_y = a_0$,

$$E_x = a_0 \cos[2\pi\nu(t - \frac{z}{c}) + \phi_x],$$
$$E_y = \mp a_0 \sin[2\pi\nu(t - \frac{z}{c}) + \phi_x],$$

- $E_x^2 + E_y^2 = a_0^2$, which is a circle,
- right-circular polarized light, $\phi = +\pi/2$; and left-circular polarized light, $\phi = -\pi/2$,



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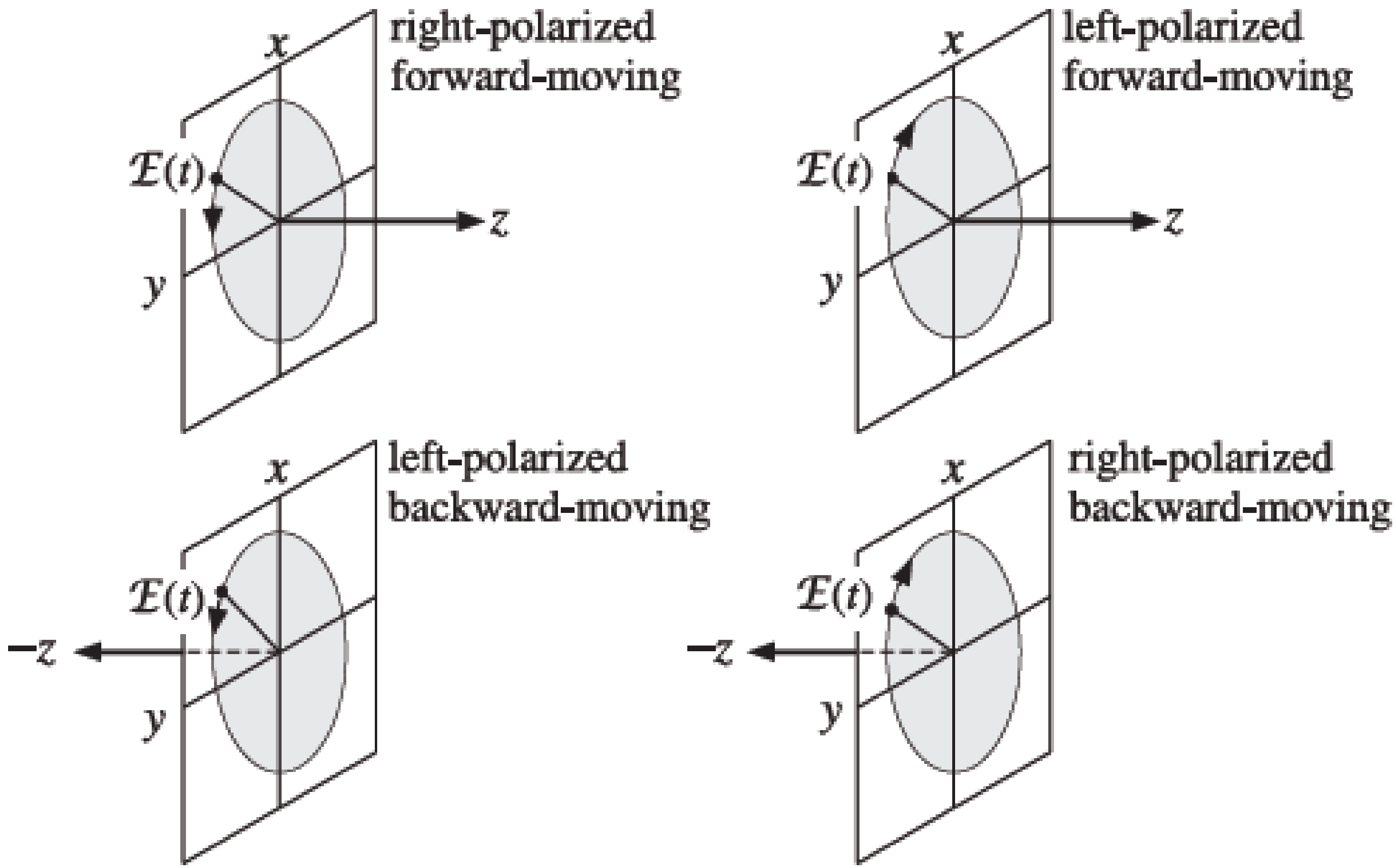
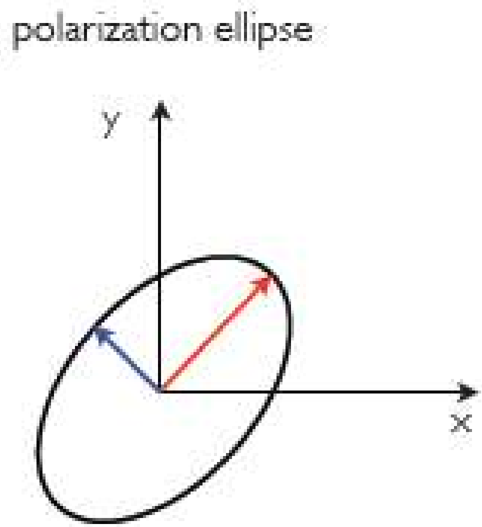
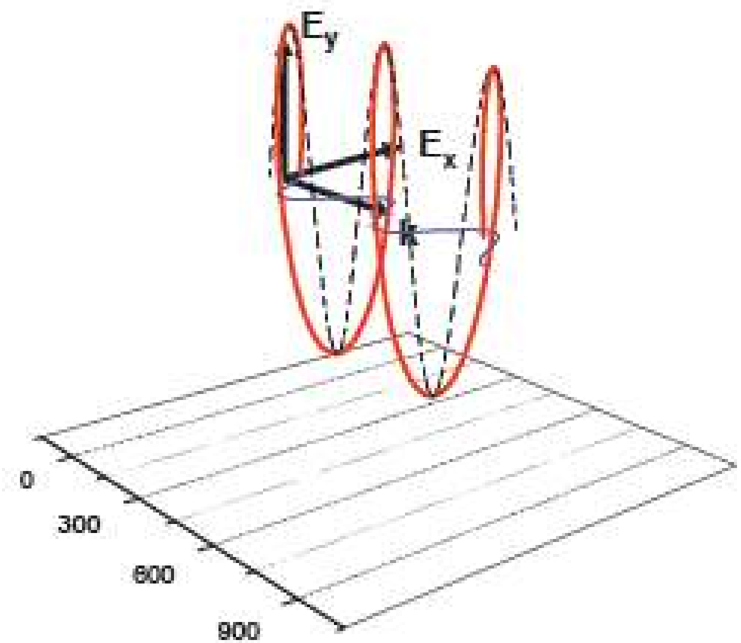
Polarization circulation



Polarization ellipse

$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\phi \frac{E_x E_y}{a_x a_y} = \sin^2\phi,$$





Optics of anisotropic media

- anisotropy - optical properties depend on the orientation of the medium,

$$P_i = \sum_j \epsilon_0 \chi_{ij} E_j \, ,$$

- each component of P is a linear combination of the components of E,
- χ is now a tensor (susceptibility tensor),
- P are not necessarily parallel to E anymore,

$$D = \epsilon E + P = \epsilon(1 + \chi)E = \epsilon E$$

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the permeability tensor

- choosing coordinate system that way that ϵ is diagonal,

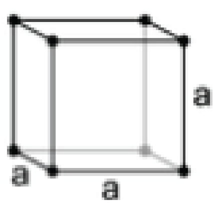
$$\epsilon = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

- the components of ϵ are then the principal axes,
- principle refractive indices, $n_x = \sqrt{\frac{\epsilon_{11}}{\epsilon_0}}$, $n_y = \sqrt{\frac{\epsilon_{22}}{\epsilon_0}}$, and $n_z = \sqrt{\frac{\epsilon_{33}}{\epsilon_0}}$,
- optically isotropic, $n_x = n_y = n_z$,
- uniaxial, $n_x = n_y = n_o$ (ordinary index),
  and $n_z = n_e$ (extraordinary index, optical axis),
- biaxial, $n_x \neq n_y \neq n_z$,

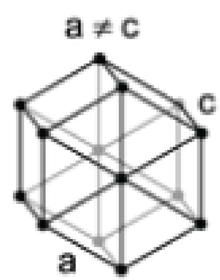
optical anisotropy

optical anisotropy is related crystal structure

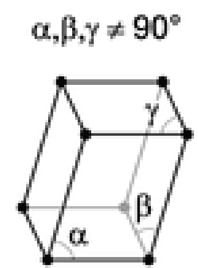
Symmetry	Lattice	$\overline{\overline{\chi}}$	Indices of Refraction	
Isotropic	Cubic	$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$	$n = \sqrt{1+a}$	diamond
Uniaxial	Triagonel Tetragonel Hexagonal	$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}$	$n_O = \sqrt{1+a}$ $n_E = \sqrt{1+6}$	Calcit
Biaxial	Triclinic Monoclinic Orthorhombic	$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$	$n_1 = \sqrt{1+a}$ $n_2 = \sqrt{1+b}$ $n_3 = \sqrt{1+c}$	MICA



cubic



hexagonal



triclinic

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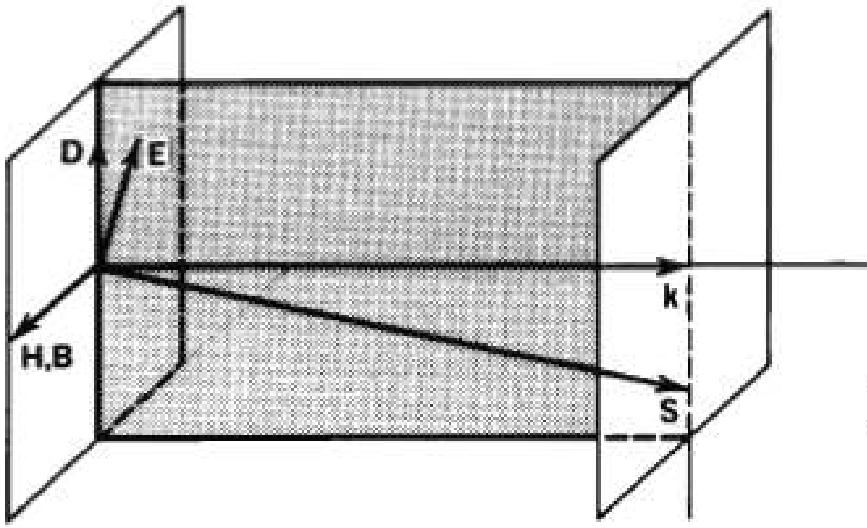
vectors of electromagnetic waves

general considerations for biaxial crystals



plane wave

$$\mathbf{k} \times \mathbf{H} = -\omega\epsilon\mathbf{E} = -\omega\mathbf{D}$$

$$\mathbf{k} \times \mathbf{E} = \omega\mu_0\mathbf{H}$$

$\mathbf{k}$ $\mathbf{H}$ and $\mathbf{D}$ are orthogonal now

$$\mathbf{D} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$\mathbf{D}$ not always parallel to $\mathbf{E}$

$$S = \frac{1}{\mu_0} E \times B,$$

energy flow is not in the direction of k,

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Propagation along a principle axis

- a linear polarized light propagates along principle axis,

$$k = \begin{bmatrix} 0 \\ 0 \\ k_z \end{bmatrix}, \qquad D = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ 0 \\ 0 \end{bmatrix},$$

- phase velocity $\frac{c_0}{n_x}$, and the wave vector $k = n_x k_0$,
- $D = \epsilon_{11} E$, D parallel to E,
- equivalent for polarization in y-direction,
- these special cases are called "normal modes" of the crystal for propagation along a principle axis

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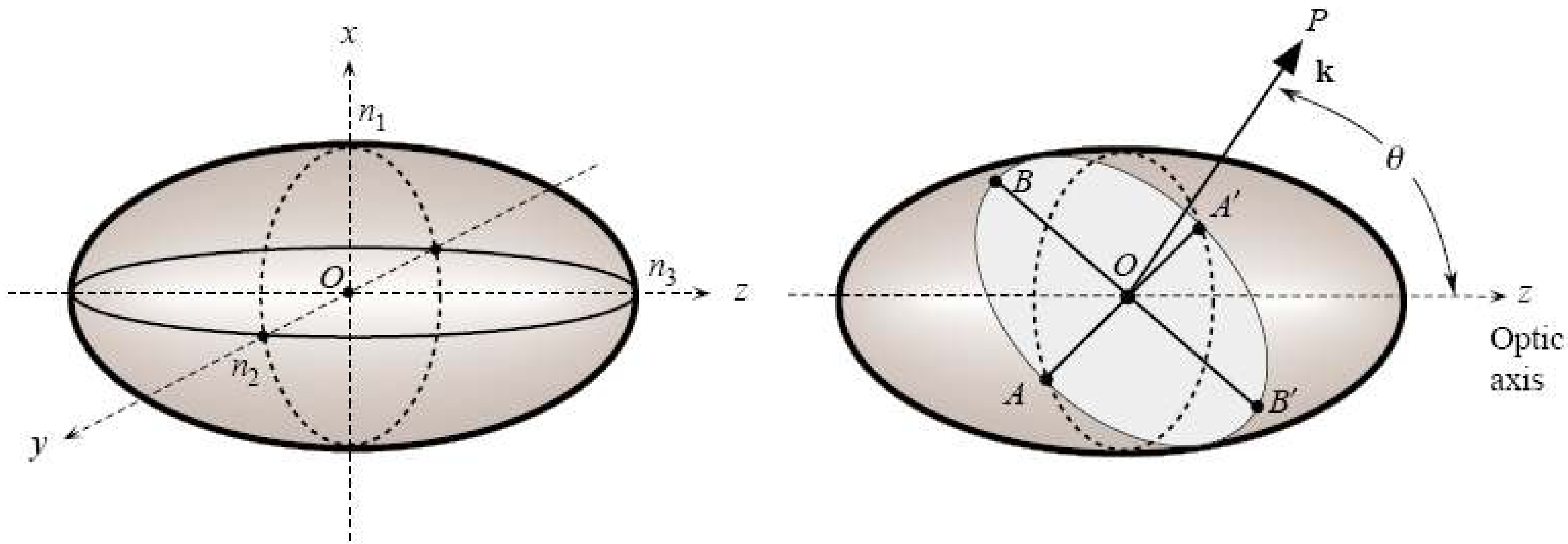
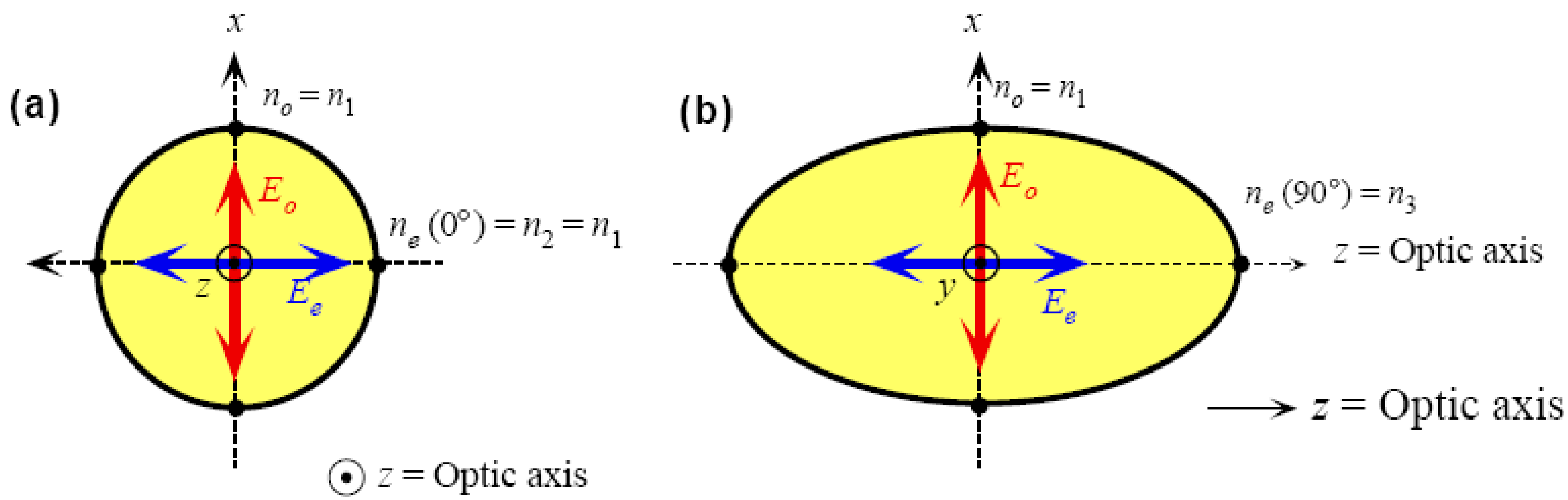
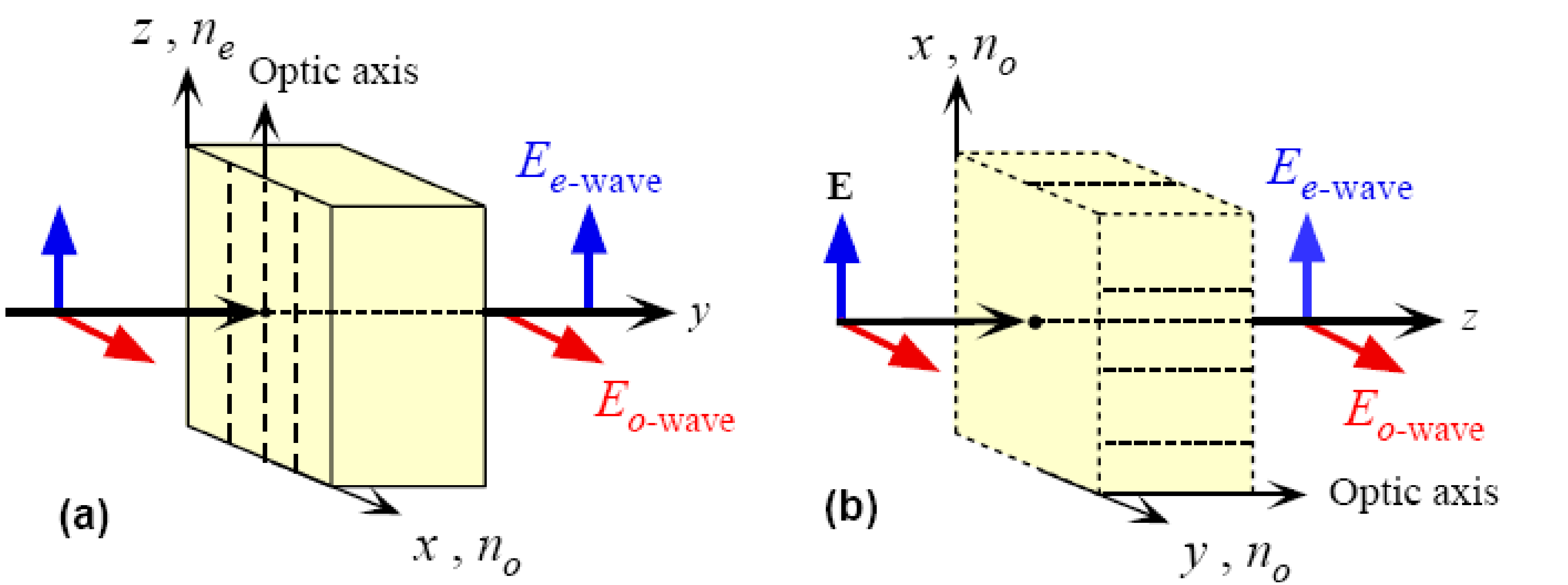
Index ellipsoid



Uniaxial crystals





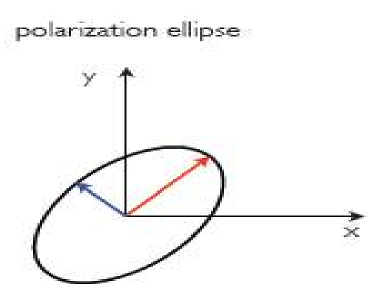
Arbitrary polarization in the x-y plane

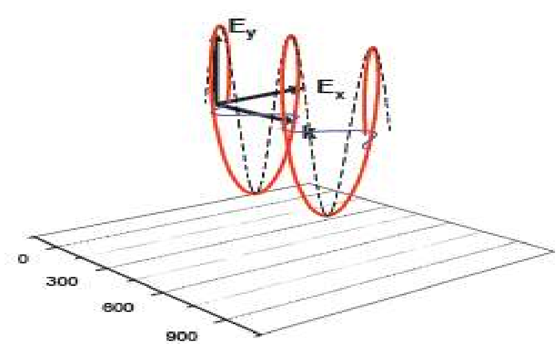
$$D = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix},$$

- both field components travel with different phase velocity,
- for a travelled distance d, the phase differnce between x- and y-components,

$$\begin{aligned} E_x &= a_x \cos[2\pi\nu(t - \frac{z}{c}) + \phi_x], \\ E_y &= a_y \cos[2\pi\nu(t - \frac{z}{c}) + \phi_y], \end{aligned}$$

$$\phi = \phi_y - \phi_x = (n_y - n_x)k_0 d,$$





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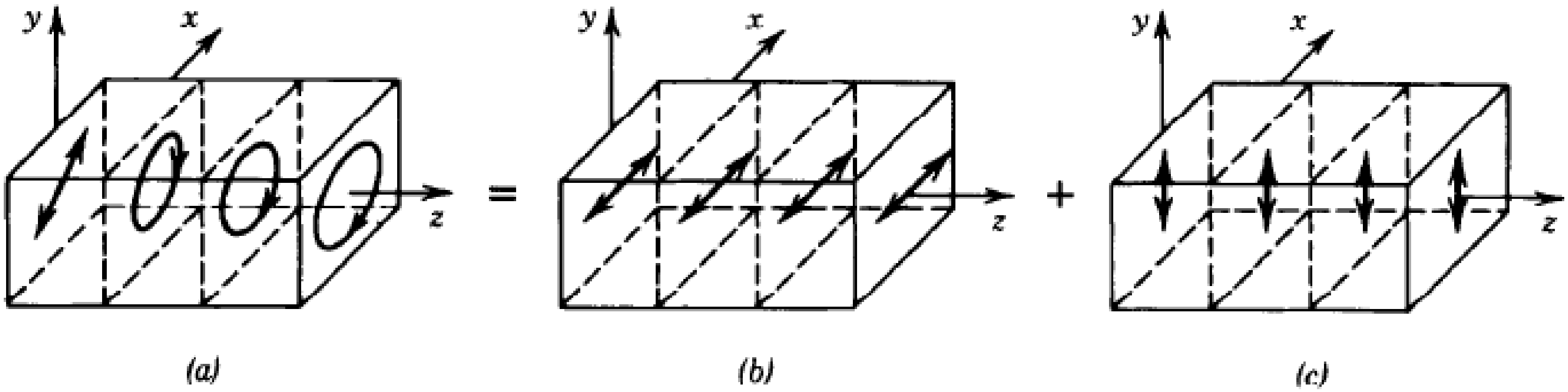
Normal modes



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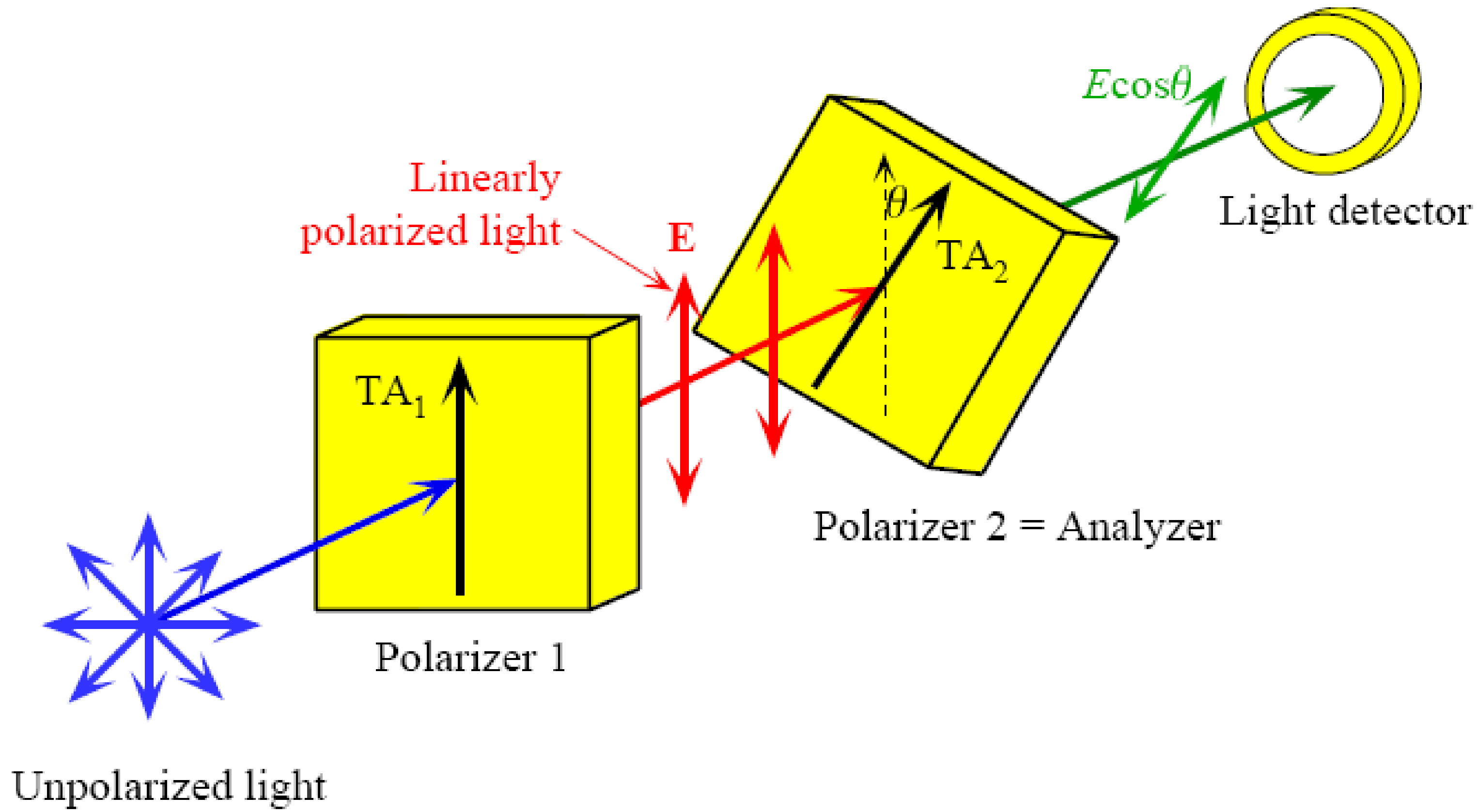
Polarizers



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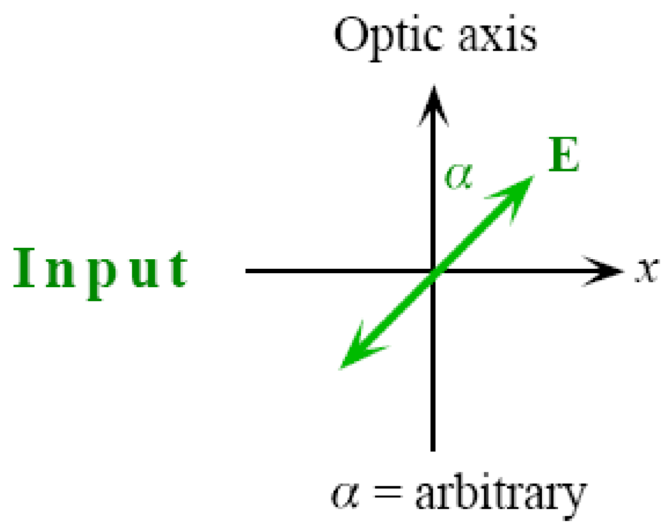
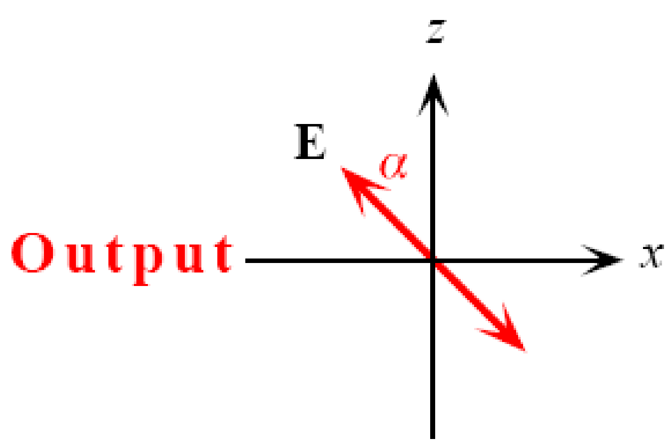
Half-wavelength-plate

Half wavelength plate: $\phi = \pi$





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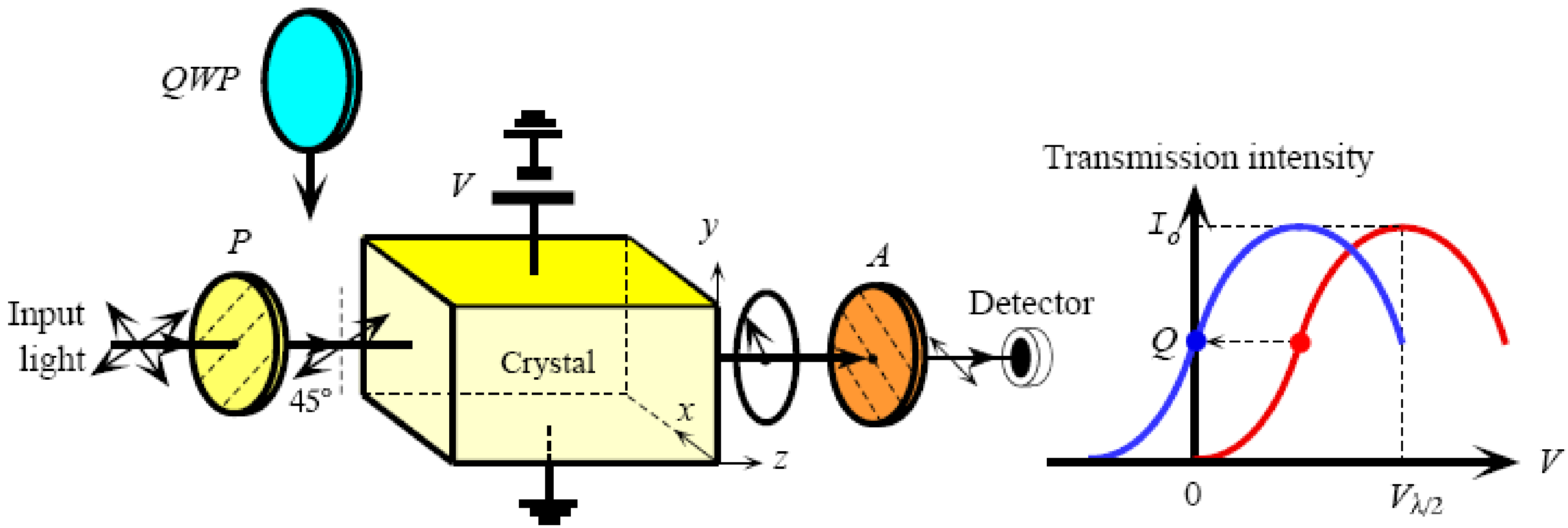
Pockels cell



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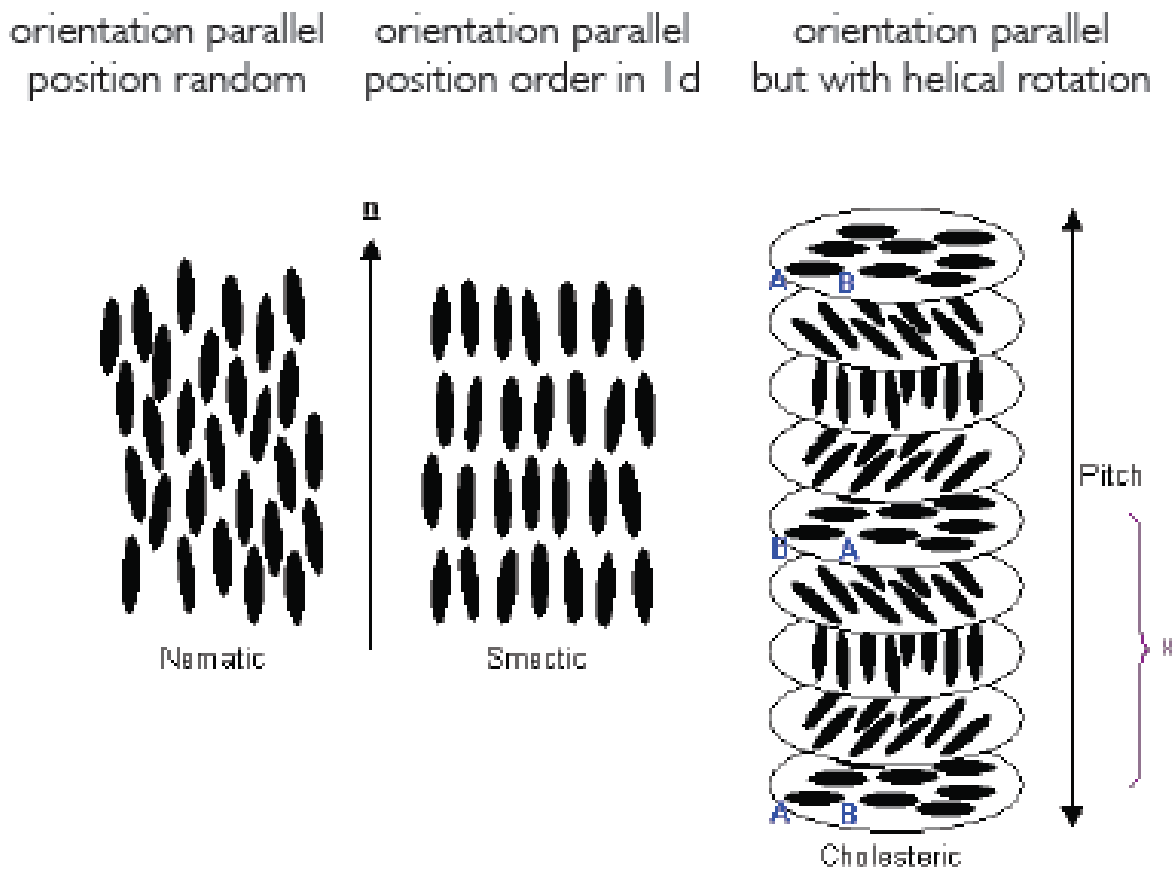
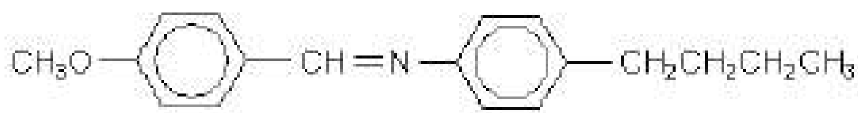
Liquid Crystals

- fluid materials, which have orientational order but typically positional disorder,
- phases of liquid crystal,

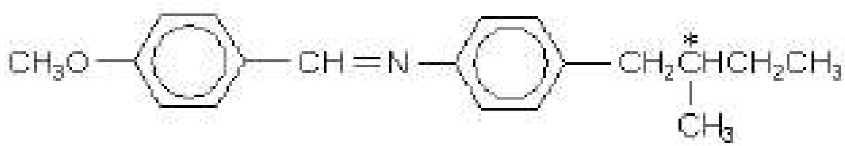


Molecules for liquid crystals

4-methoxylbenzylidene-4'-butylaniline (MBBA)

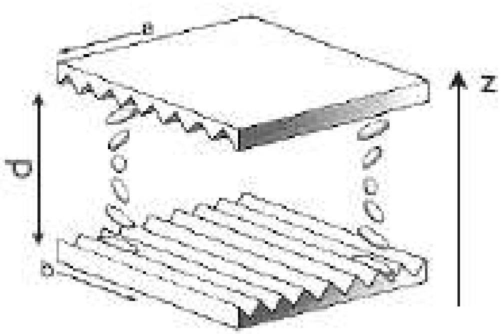


nematic at 20 °C, isotropic at 74 °C



similar molecule but cholesteric phase

twisted liquid crystals
(induced cholesteric phase)



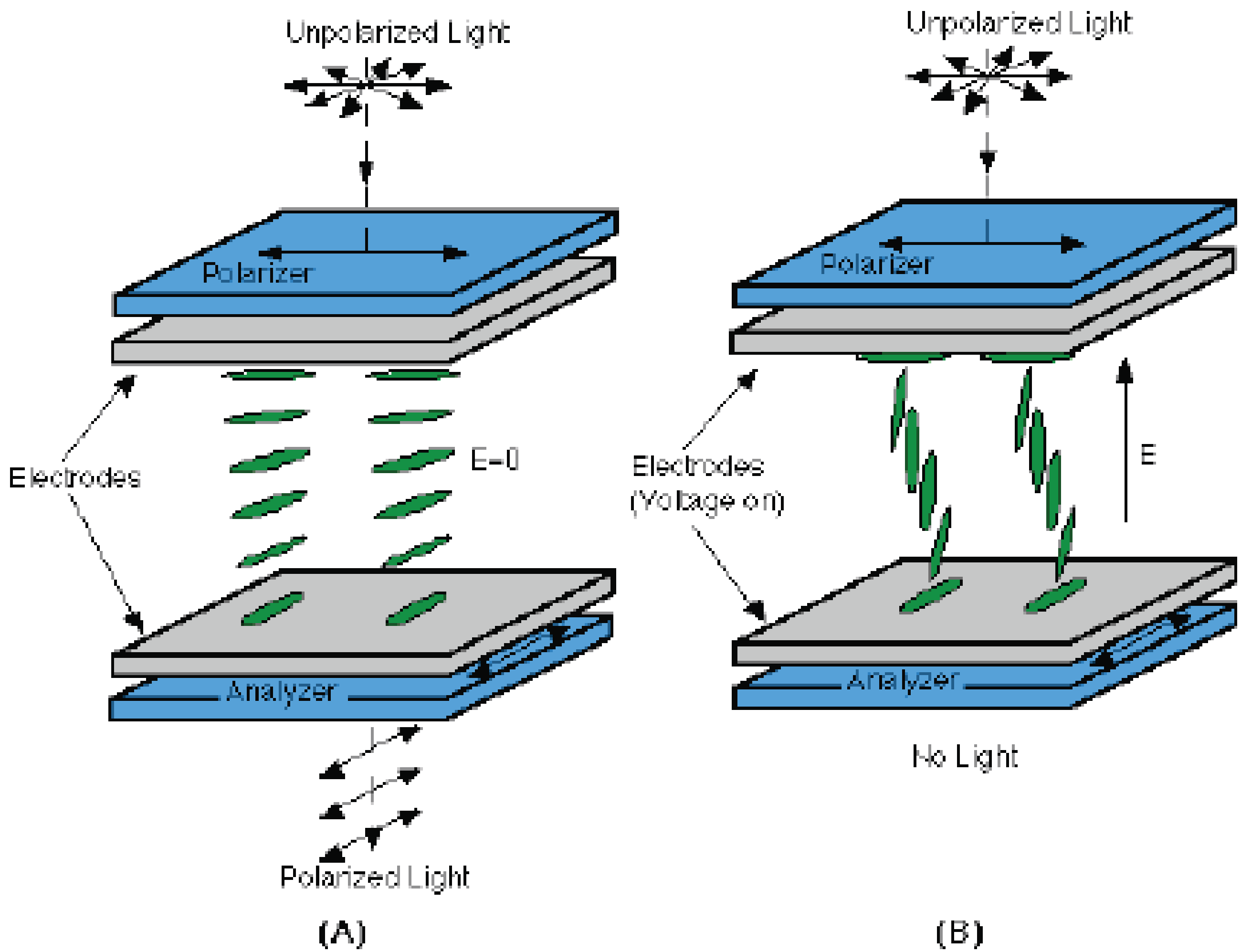
twist angle $\theta = \alpha z$

phase retardation $\beta = (n_e - n_o) k_0$

typically $\beta >> \alpha$

thus linearly polarized light at z=0 stays linear polarized but turns its polarization direction by αd

Liquid Crystal Displays - LCD



depend on molecular collective reorientation - slow