nonlinear optical response

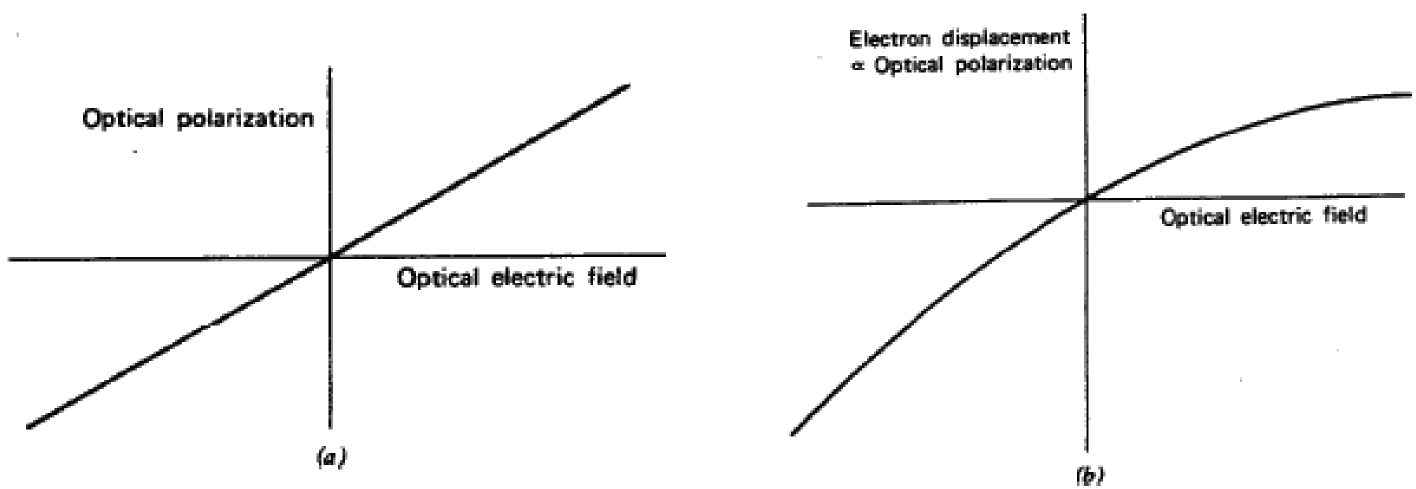
- nonlinear media: the relation between P and E is nonlinear,

$$P = \epsilon_0 \chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots ,$$

- new wave equaiton, with *radiating source* S

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} = -S,$$

$$\mathbf{P} = \chi \mathbf{E} \qquad \mathbf{P} = \chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}\mathbf{E}$$



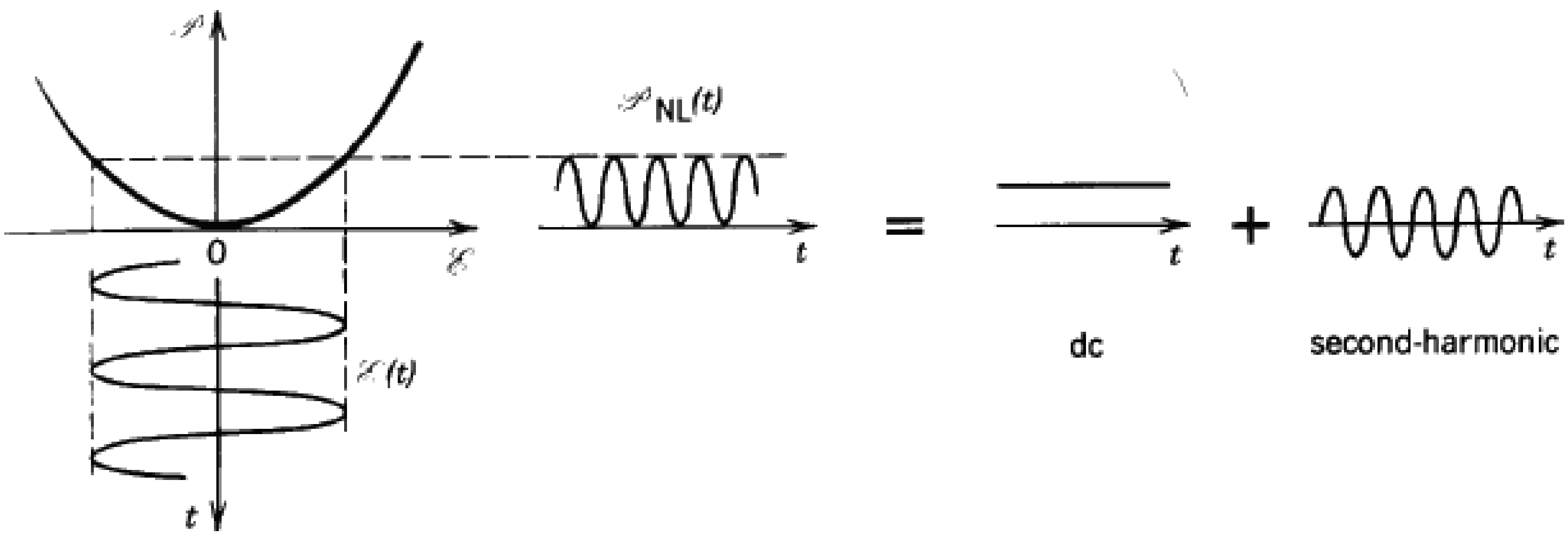
second order nonlinearity

- $P_{NL} = 2dE^2$,
- $E(t) = \text{Re}(E(\omega)\exp(i\omega t))$,

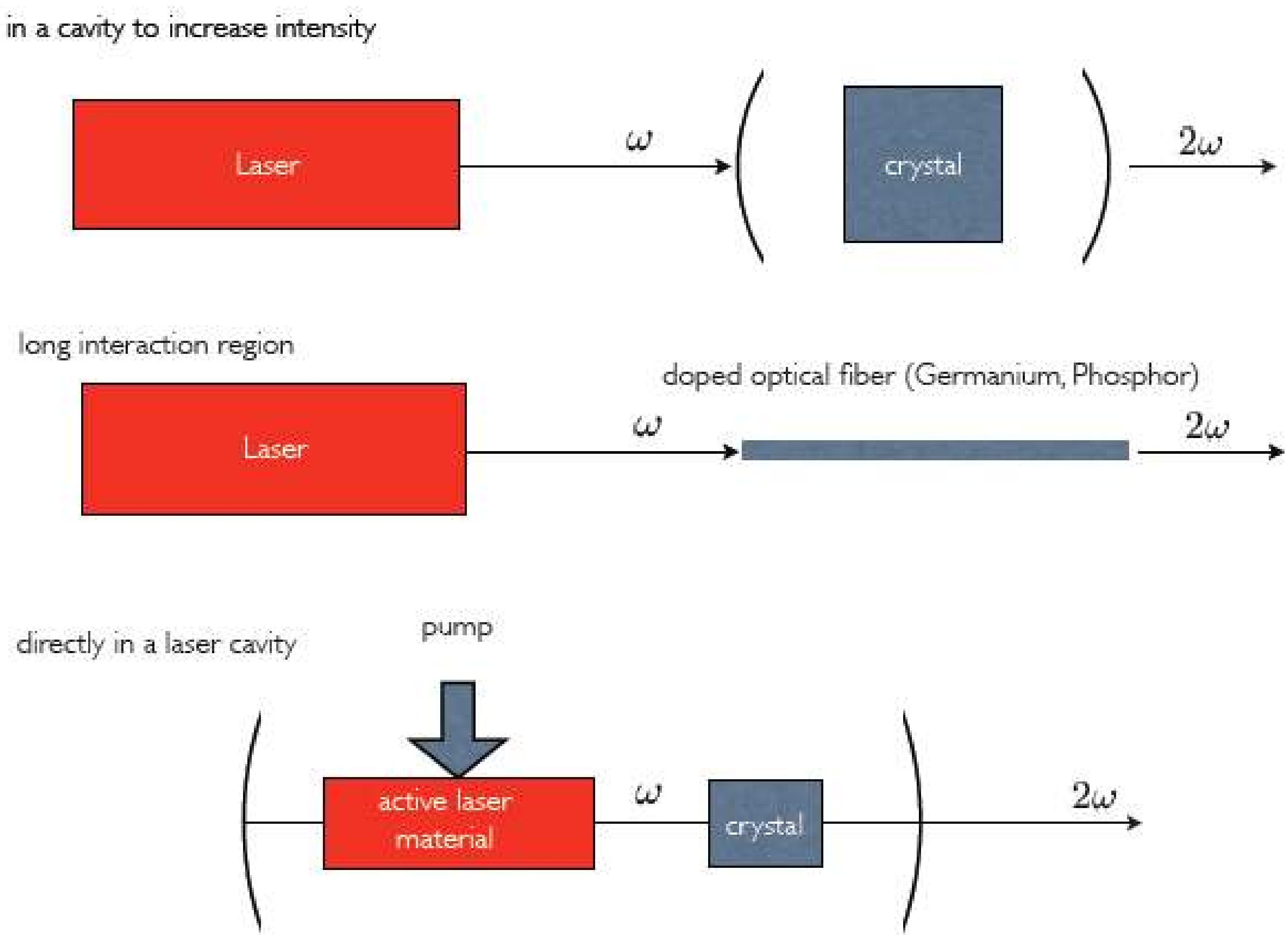
$$P_{NL} = P_{NL}(0) + \text{Re}(P_{NL}(2\omega)\exp(i2\omega t)),$$

where

$$P_{NL}(0) = dE(\omega)E^*(\omega), \qquad P_{NL}(2\omega) = dE(\omega)E(\omega),$$



Second Harmonic Generation



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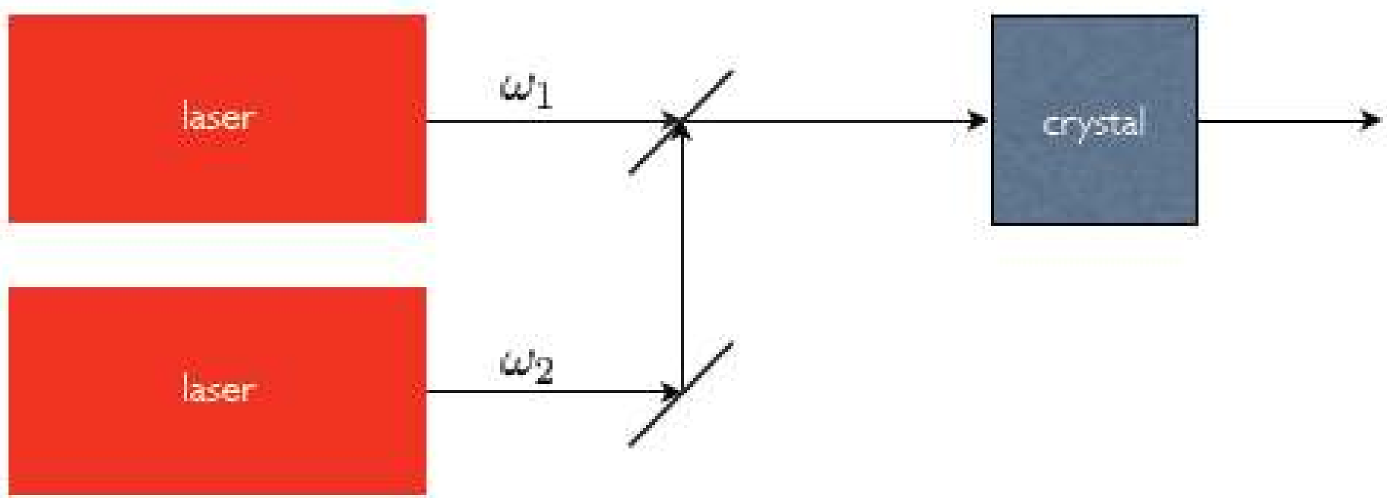
three wave mixing

- $P_{NL} = 2dE^2$,
- $E(t) = \text{Re}(E(\omega_1)\text{exp}(i\omega_1 t) + E(\omega_2)\text{exp}(i\omega_2 t))$,
- gives now components with frequencies:
  $0, 2\omega_1, 2\omega_2, \omega_+ = \omega_1 + \omega_2, \omega_- = \omega_1 - \omega_2$

$$P_{NL}(0) = d(|E(\omega_1)|^2 + |E(\omega_2)|^2),$$
$$P_{NL}(2\omega_1) = dE(\omega_1)E(\omega_1), \qquad P_{NL}(2\omega_2) = dE(\omega_2)E(\omega_2),$$
$$P_{NL}(\omega_+) = dE(\omega_1)E(\omega_2), \qquad P_{NL}(\omega_-) = dE(\omega_1)E^*(\omega_2),$$



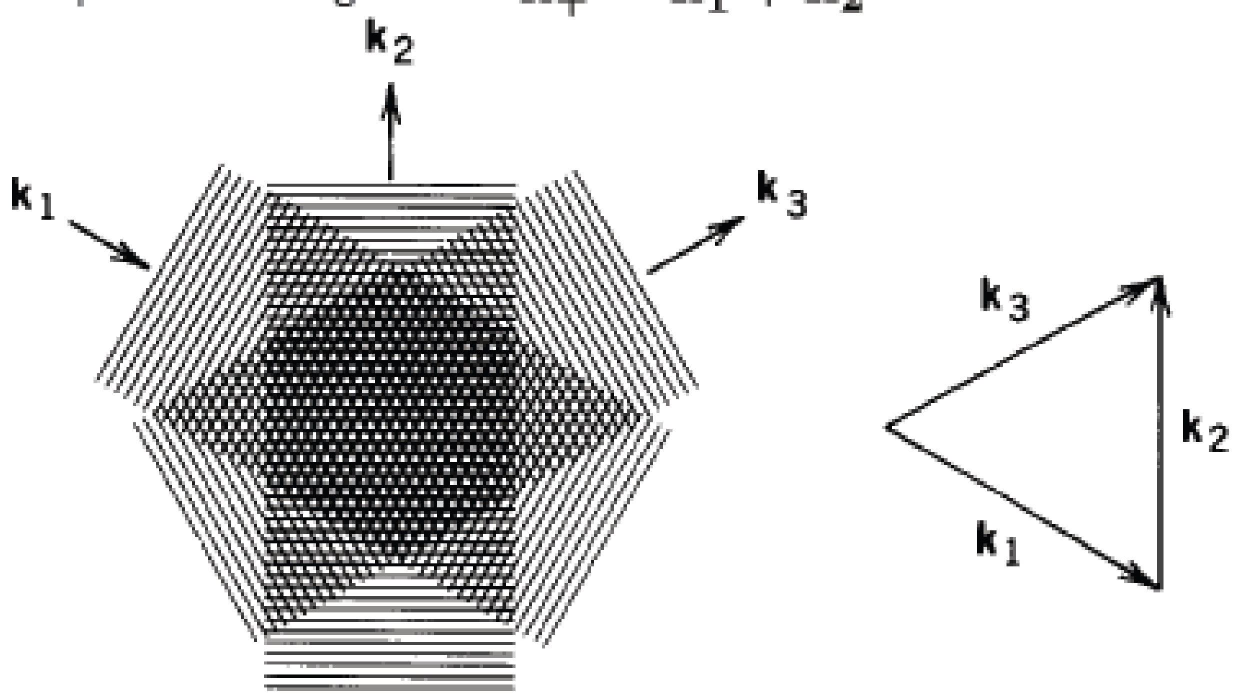
three wave mixing-phase matching

- two plane waves,

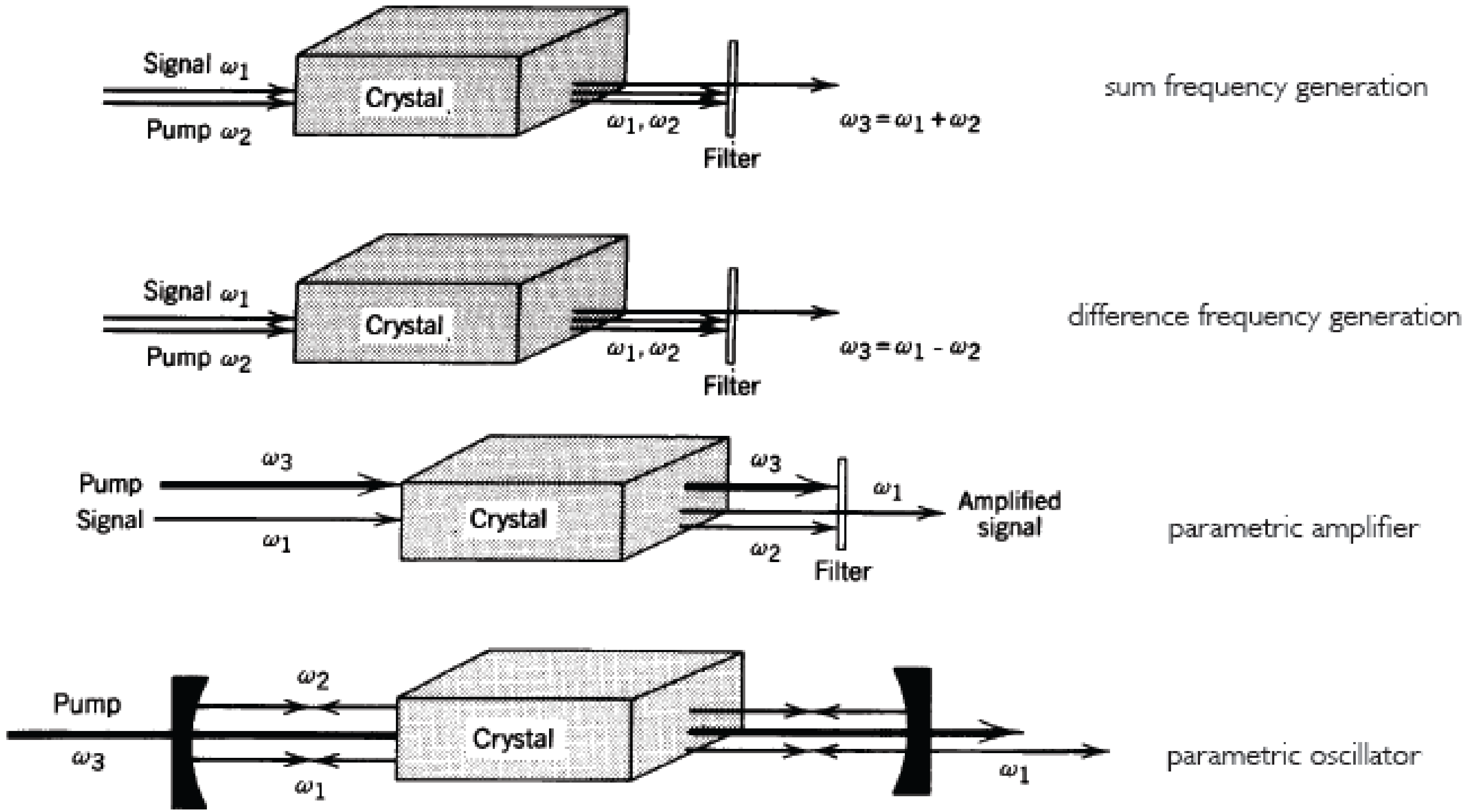
$$E(\omega_1) = A_1 \text{exp}(-ik_1 \cdot r), \quad E(\omega_2) = A_2 \text{exp}(-ik_2 \cdot r),$$
$$P_{NL}(\omega_+) = 2dA_1 A_2 \text{exp}(-ik_+ \cdot r),$$

frequency matching $\omega_+ = \omega_1 + \omega_2$

phase matching $\mathbf{k}_+ = \mathbf{k}_1 + \mathbf{k}_2$



three wave mixing-phase matching



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coupled wave equations - three wave mixing

- new wave equaiton, with *radiating source* S

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} = -S,$$

where $P_{NL} = 2dE^2$,

- three wave mixing involves three different electric fields superposed,

$$E(t) = \sum_{q=1,2,3} \text{Re}[E_q \text{exp}(i\omega_q t)] = \sum_{q=1,2,3} \frac{1}{2}[E_q \text{exp}(i\omega_q t) + E_q^* \text{exp}(-i\omega_q t)],$$

- nonlinear polarization density,

$$P_{NL} = \frac{1}{2} d \sum_{q,r=\pm1,\pm2,\pm3} [E_q E_r \text{exp}(i(\omega_q + \omega_r)t)],$$

where $\omega_{-q} = -\omega_q$ and $E_{-q} = E_q^*$,

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coupled Helmholtz equations

- radiating source,

$$S = \frac{1}{2} d\mu_0 \sum_{q,r=\pm1,\pm2,\pm3} [(\omega_q + \omega_r)^2 E_q E_r \text{exp}(i(\omega_q + \omega_r)t)],$$

- three wave mixing coupled equations for $\omega_3 = \omega_1 + \omega_2$,

$$(\nabla^2 + k_1^2)E_1 = -2d\mu_0\omega_1^2 E_3 E_2^*, \quad \text{waves 2 and 3 grow wave 1,}$$

$$(\nabla^2 + k_2^2)E_2 = -2d\mu_0\omega_2^2 E_3 E_1^*, \quad \text{waves 1 and 3 grow wave 2,}$$

$$(\nabla^2 + k_3^2)E_3 = -2d\mu_0\omega_3^2 E_1 E_2, \quad \text{waves 1 and 2 grow wave 3,}$$

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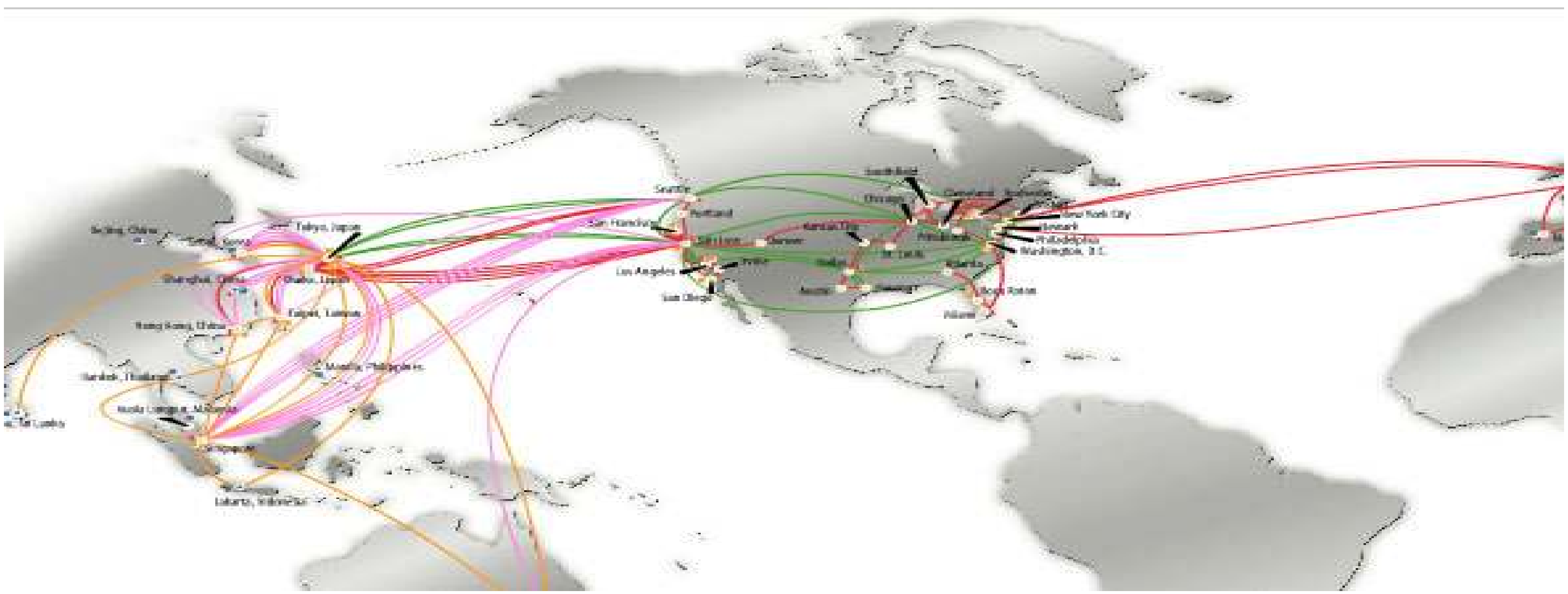
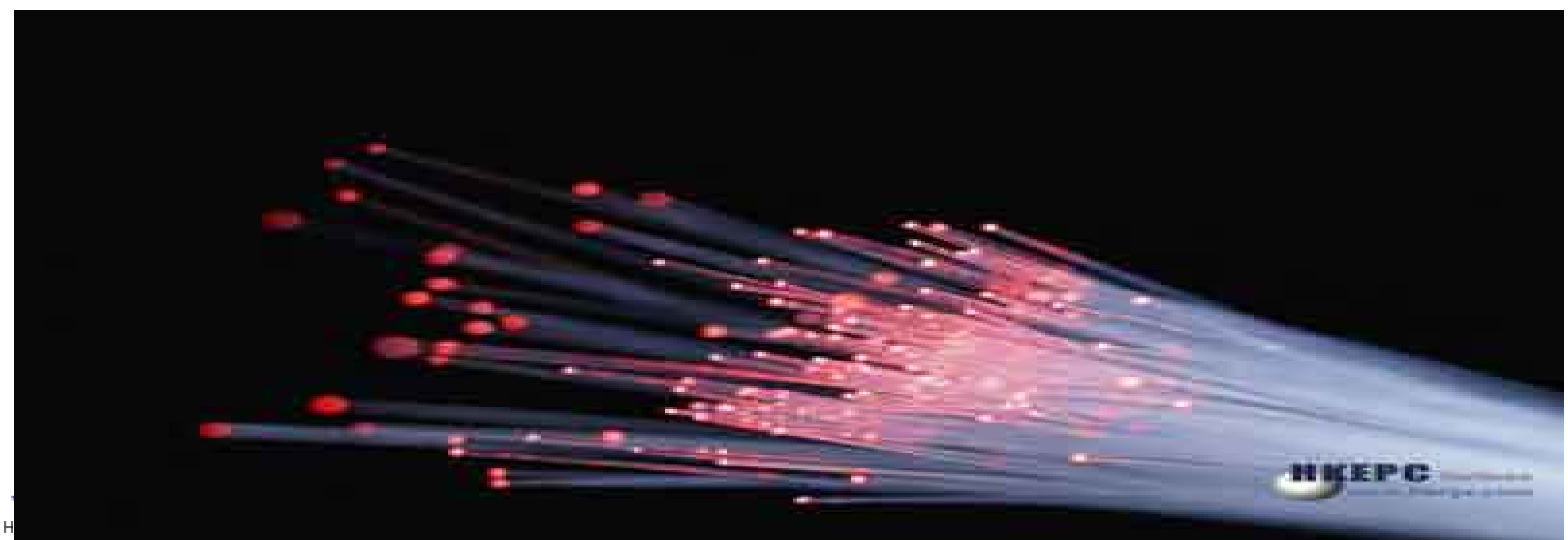
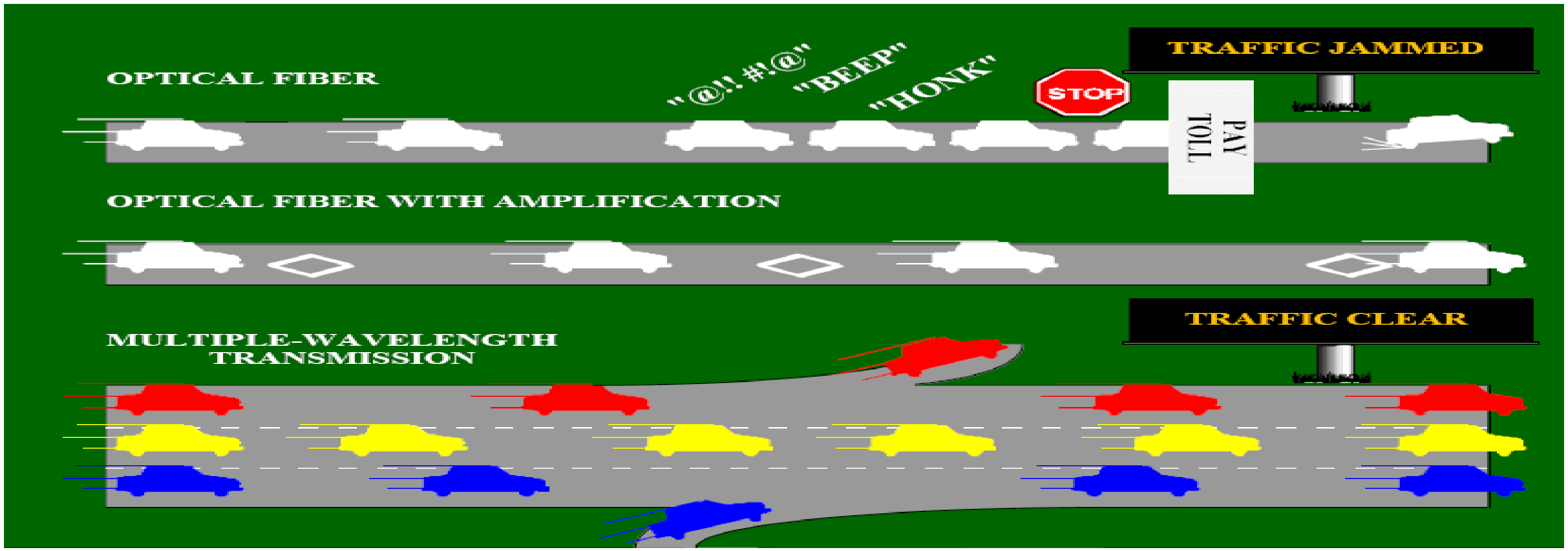
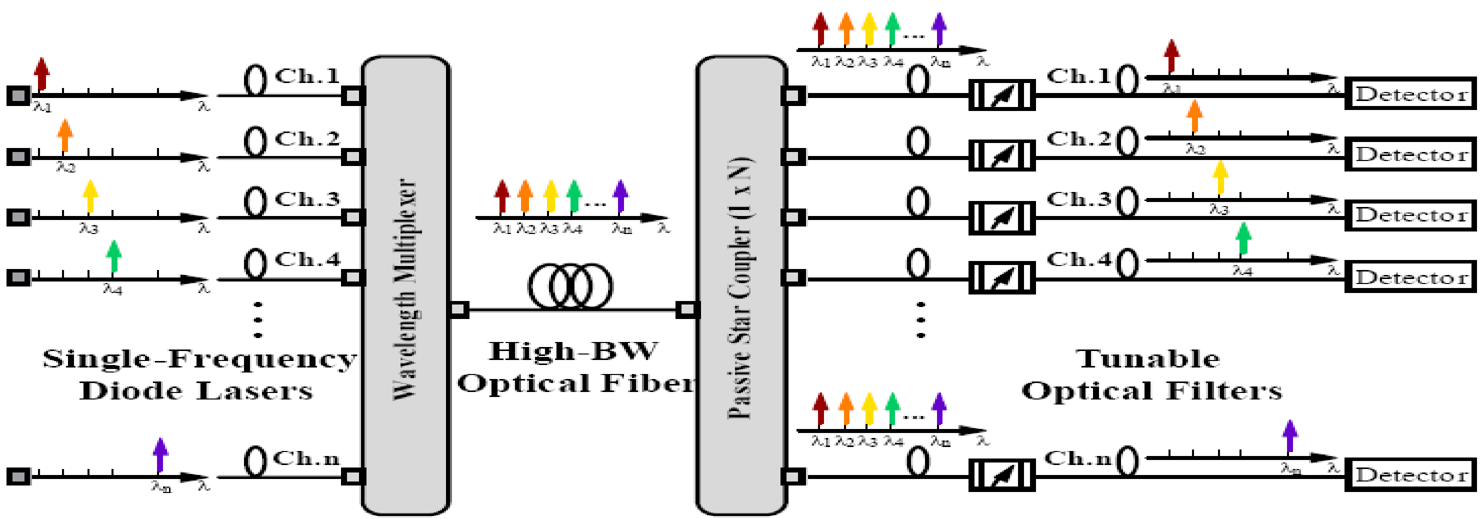
Global overseas fiber network



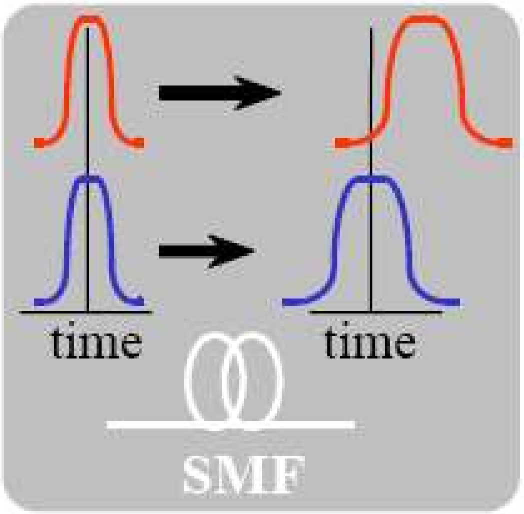


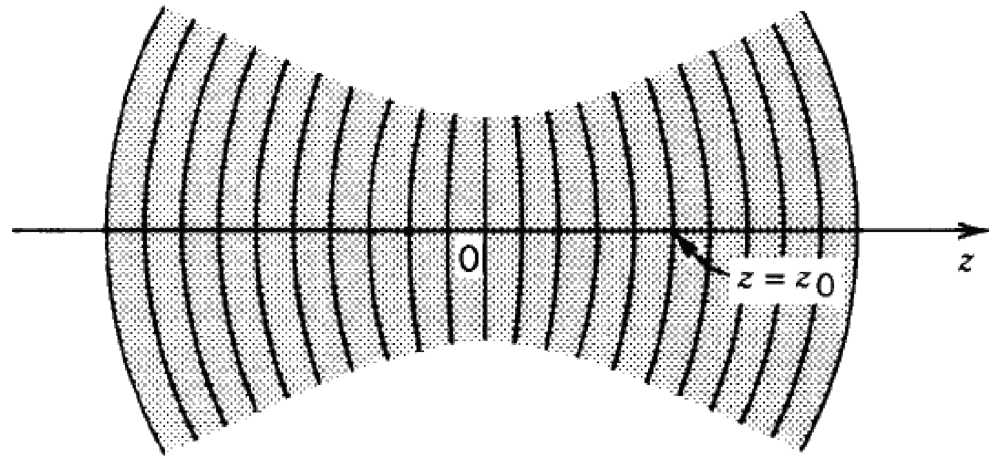
Wavelength-Division-Multiplex

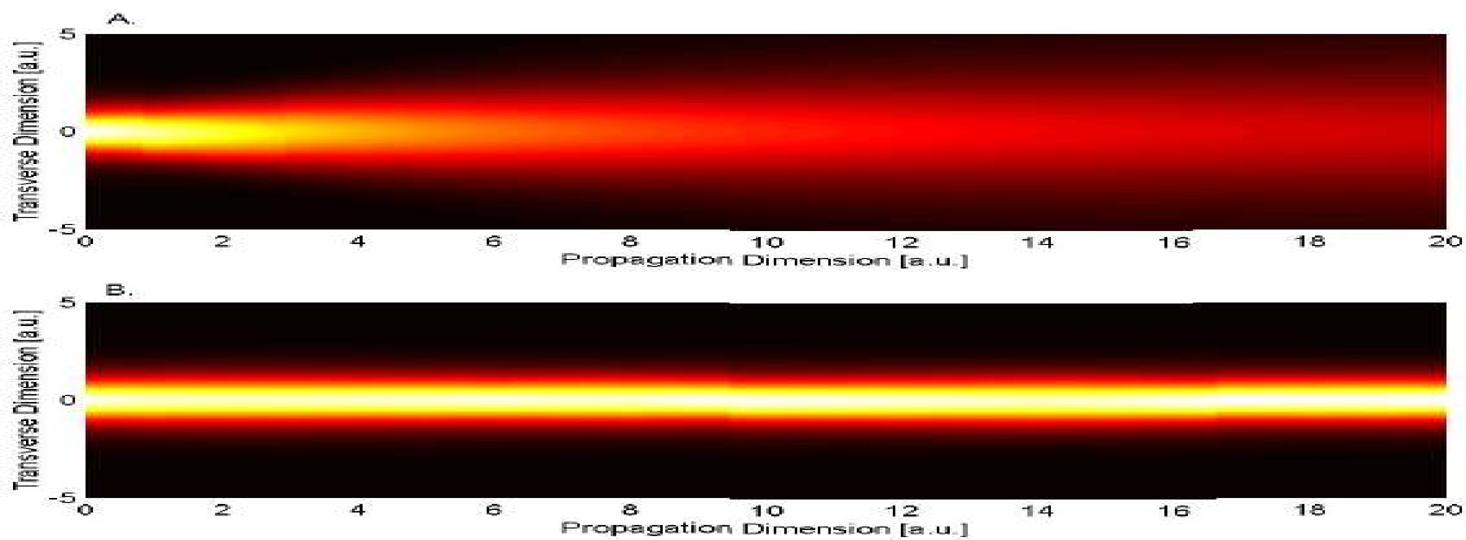




Dispersion/Diffraction effect







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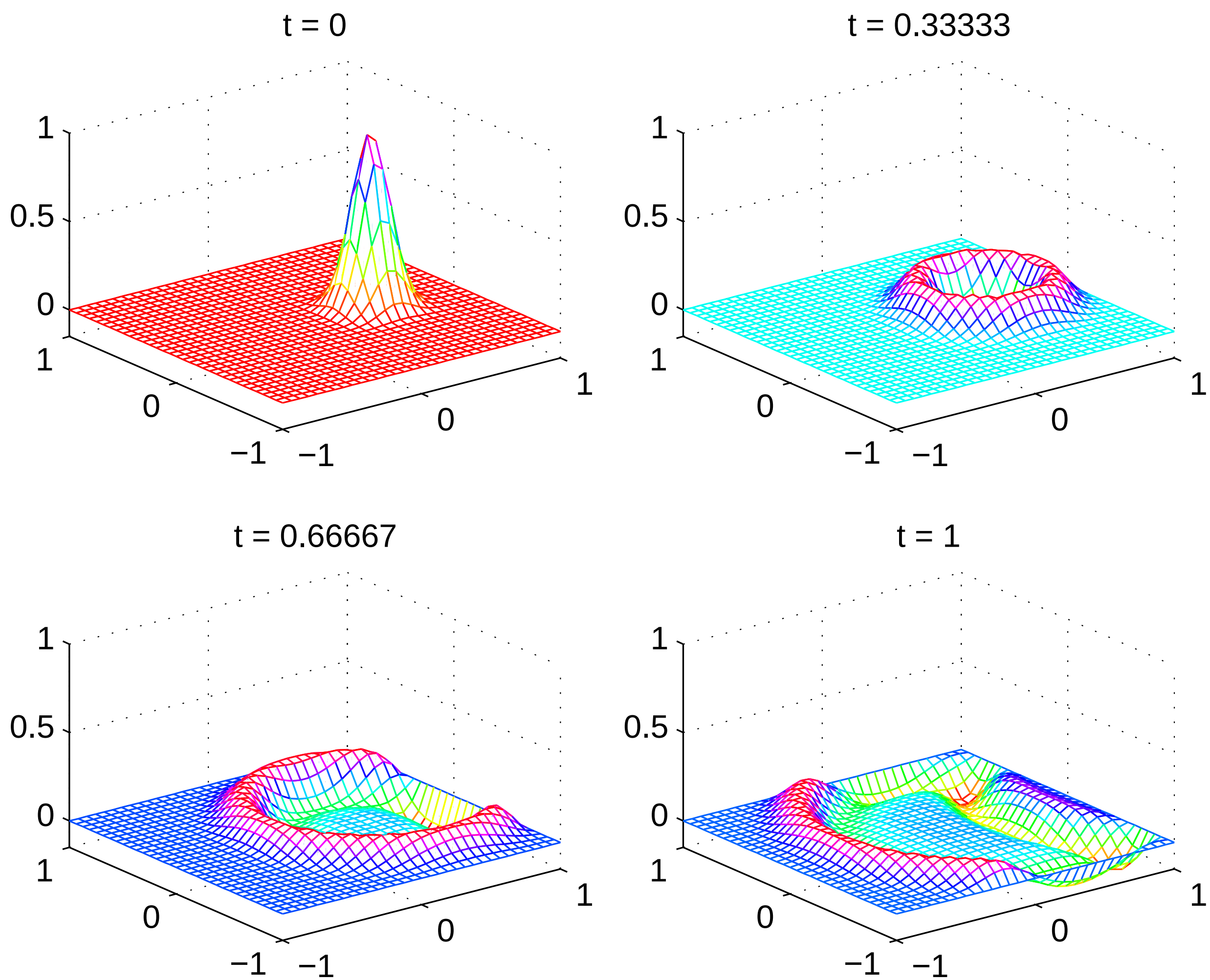
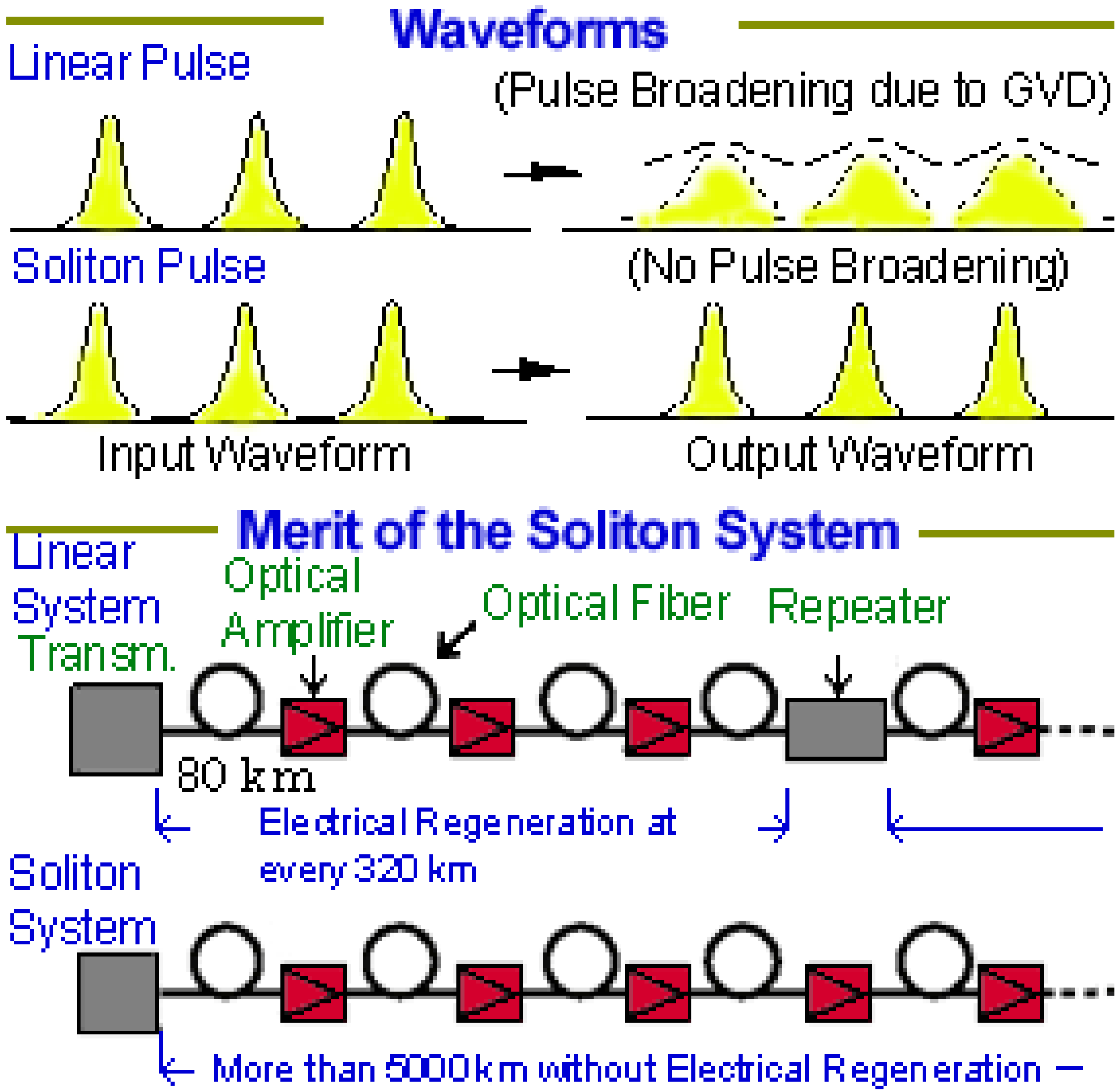
Dispersive/Diffractive/Diffused Wave

$$u_{tt} = u_{xx} + u_{yy}, \qquad -1 < x, y < 1, \quad t > 0, \quad u = 0 \quad \text{on the boundary}$$



Soliton communication system



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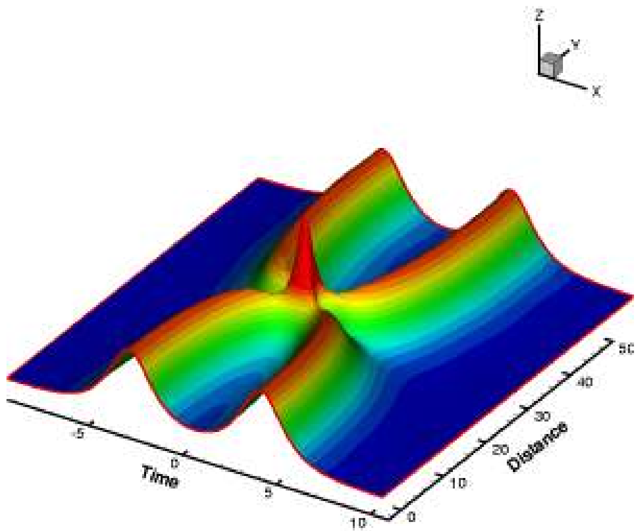
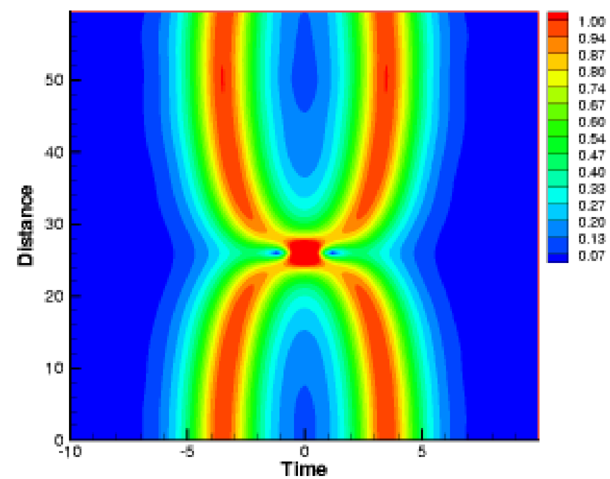
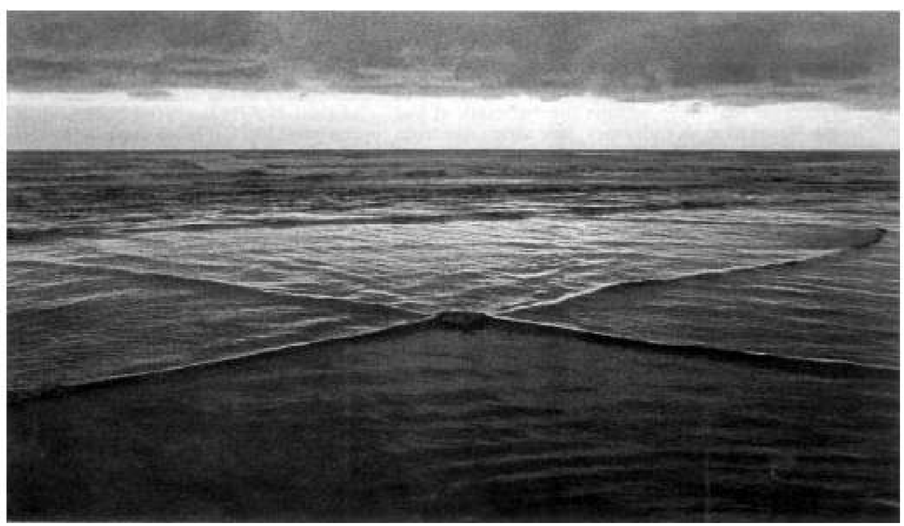
Wave-particle characteristics of solitons

Collision between solitons







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Courtesy of T. Toedterneier

Nonlinear waves in optics

- For an nonlinear medium, with an index that depends on the optical intensity in the medium,

$$n = n_0 + n_2 I,$$

- the wave equation,

$$\nabla^2 A + \omega^2 \mu_0 \epsilon A = 0,$$

with the dielectric constant, $\epsilon = \epsilon_0 n^2 = \epsilon_0 (n_0 + n_2 I)^2$, becomes

$$\nabla_T^2 u - 2jk_0 \frac{\partial u}{\partial z} = -\omega^2 \mu_0 \epsilon_0 (n^2 - n_0^2) u.$$

where u is a slow-varying envelope function,

$$A \propto \hat{y}\, u(x, y, z) e^{-jk_0 z},$$

- In the normalized units, we have *nonlinear Schrödinger equation*,

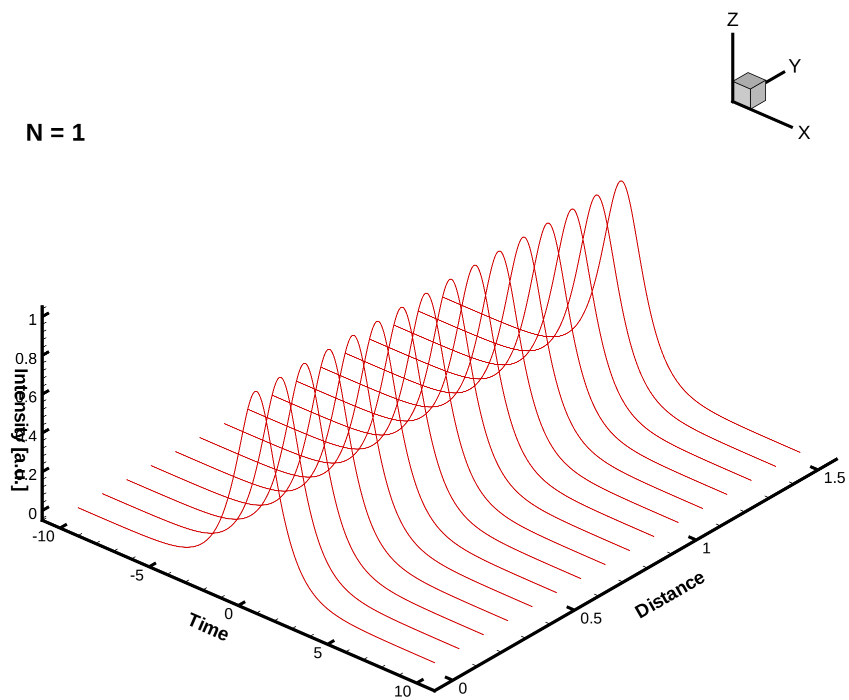
$$\frac{\partial^2 u}{\partial x^2} - j \frac{\partial u}{\partial q} + \kappa |u|^2 u = 0.$$

Solitons in optical fibers

Nonlinear Schrödinger Equations: Hermitian System

$$iU_z = -\frac{D}{2}U_{tt} - |U|^2U \qquad \text{, i.e.}$$

$$i\hbar\Psi_t = -\frac{\hbar^2}{2m}\nabla^2\Psi + \mathcal{V}\Psi = \mathcal{H}\Psi$$



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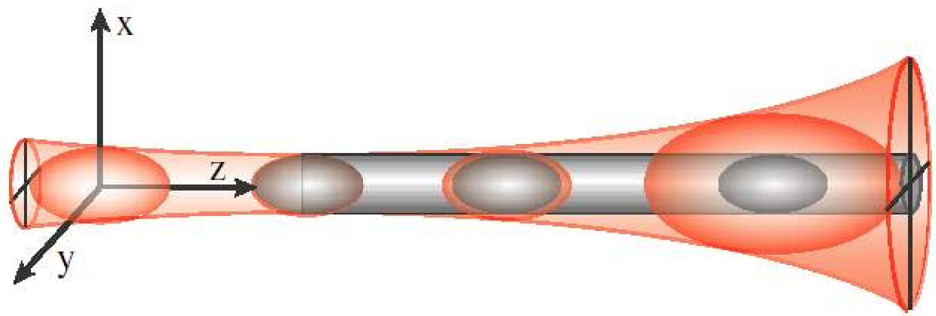
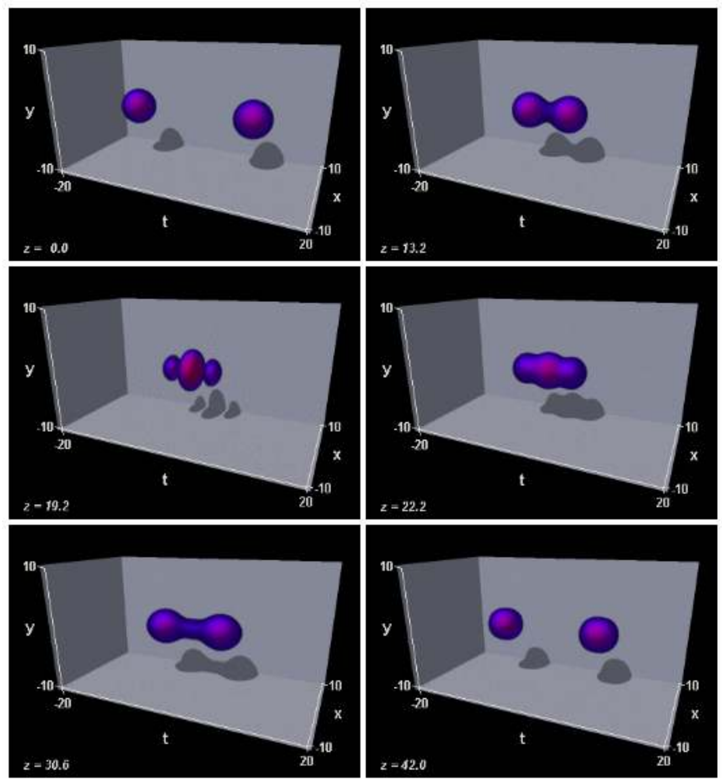
Spatio-temporal solitons: light bullet





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B. A. Malomed, D. Mihalache, F. Wise, and L. Torner, *J. Op. B* **7**, R53-R72 (2005).

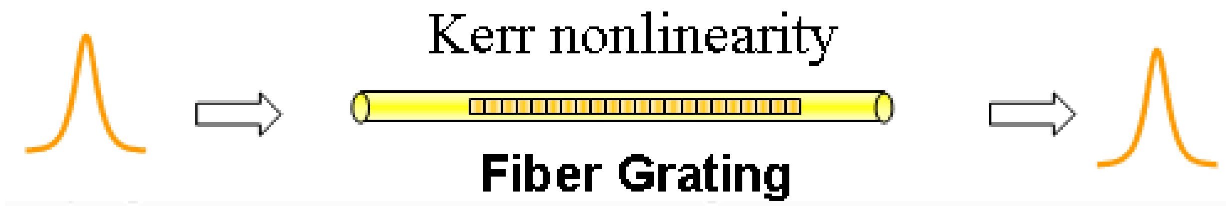
Universal Solitons

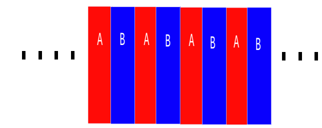
A Universal phenomenon of self-trapped wave packets.

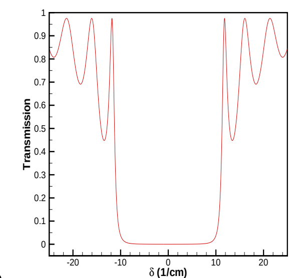
- EM waves in nonlinear optical materials;
- shallow- and deep-water waves;
- charge-density waves in plasmas;
- sound waves in liquid ^{3}He;
- matter waves in Bose-Einstein condensates;
- excitations on DNA chains;
- domain walls in supergravity, and
- "branes" at the end of open strings in superstring theory; to name only a few.

M. Segev, *Optics & Photonics News*, pp. 27 (Feb. 2002).

Fiber Bragg Grating Solitons





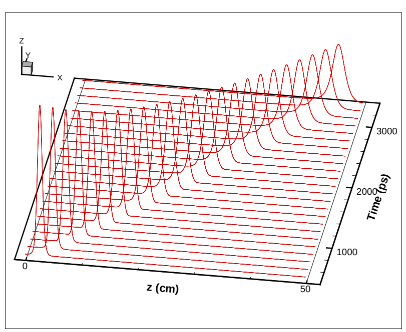


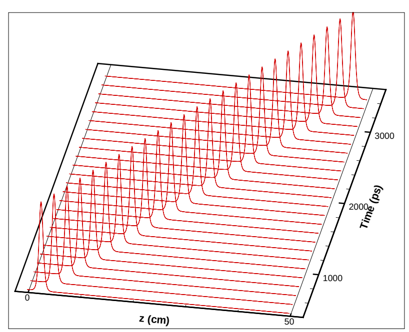
Nonlinear Coupled-Mode Equations:

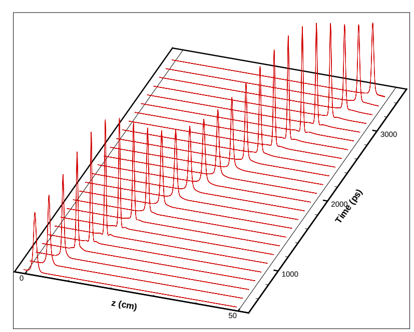
$$\frac{1}{v_g}\frac{\partial}{\partial t}U_a(z,t) + \frac{\partial}{\partial z}U_a = i\delta U_a + i\kappa U_b + i\Gamma|U_a|^2 U_a + 2i\Gamma|U_b|^2 U_a$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_b(z,t) - \frac{\partial}{\partial z}U_b = i\delta U_b + i\kappa U_a + i\Gamma|U_b|^2 U_b + 2i\Gamma|U_a|^2 U_b$$

decay stationary oscillate







A. Aceves and S. Wabnitz, *Phys. Lett. A* **141**, 37 (1986).

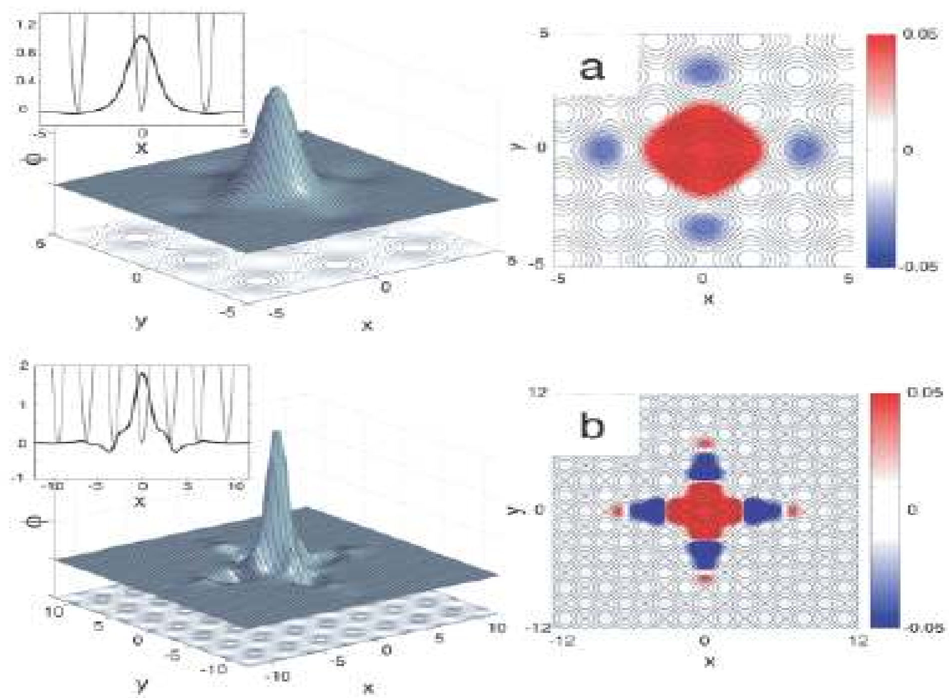
B. J. Eggleton, C. de Sterke, P. A. Krug, and J. E. Sipe, *Phys. Rev. Lett.* **76**, 1627 (1996).

BEC in optical lattices

Gross-Pitaevskii equation with periodic potentials,

$$i\hbar\frac{\partial}{\partial t}\Phi = -\frac{1}{2}\nabla^2\Phi + V(t)\Phi + g|\phi|^2\phi$$

which has gap soliton solutions in 1D, 2D, and 3D.



E. A. Ostrovskaya and Yu. S. Kivshar, *Phys. Rev. Lett.* **90**, 160407 (2003).

self-phase modulation, self focusing

phase shift of an optical beam:

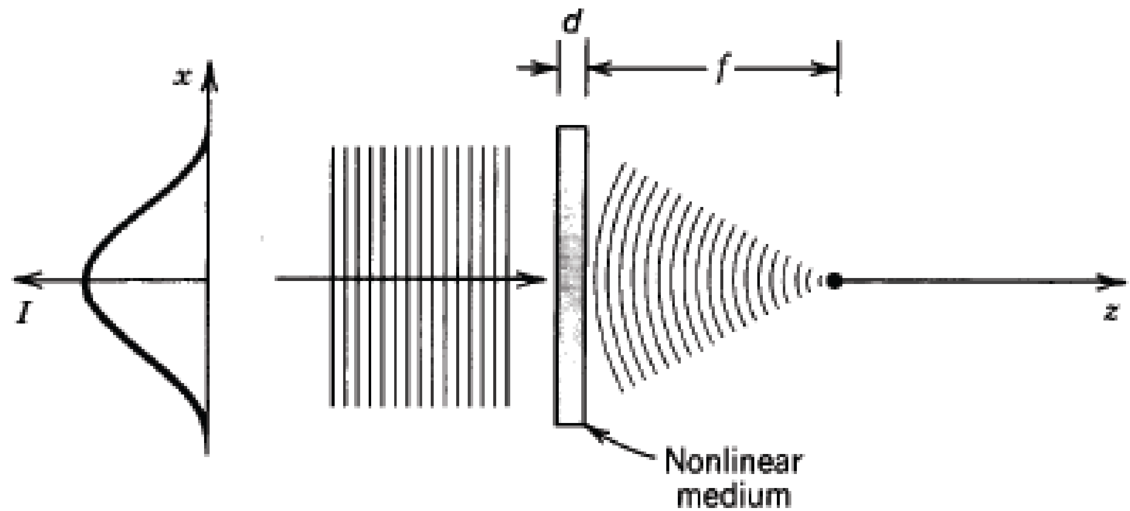
$$\phi = 2\pi n(I)L/\lambda_0 = 2\pi(n_0 + n_2 I)L/\lambda_0,$$

and

$$\Delta\phi = 2\pi n_2 I L/\lambda_0,$$

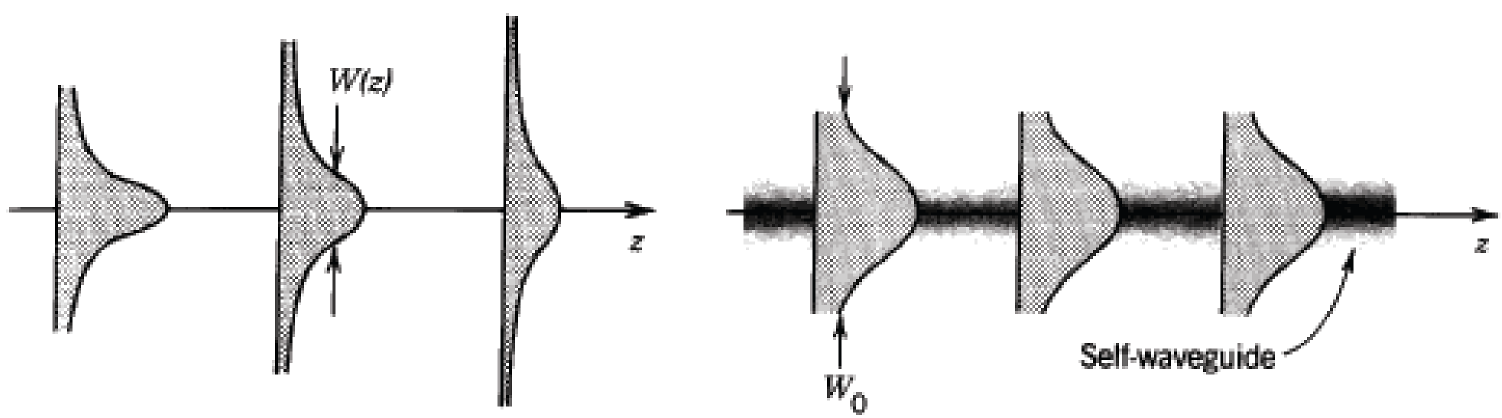
self-focusing:

refractive index change maps
the intensity pattern of the beam



self guiding

- intensity profile of the beam creates graded index waveguide,
- if the transverse intensity distribution matches the mode of the self-induced wave guide, the beam propagates self-consistently,
- self-guided optical beams are called *spatial solitons*,
- diffraction is compensated by nonlinear effect,



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spatial optical solitons

- Ansatz as usual,

$$E = A\text{exp}(-ikz), \qquad A = A(x, z),$$

amplitude A is slowly varying with z, so neglect second order parts,

- Helmholtz-equation becomes,

$$\frac{\partial^2 A}{\partial x^2} - 2ik\frac{\partial A}{\partial z} + k_0^2[n^2(I) - n^2]A = 0,$$

- with $n_2 I \ll n$, $n^2(I) - n^2 \approx 2nn_2 I$,

$$\frac{\partial^2 A}{\partial x^2} - 2ik\frac{\partial A}{\partial z} + 2n_2 k^2 |A|^2 A = 0,$$

- which has solutions of,

$$A(x, z) = A_0 \text{sech}(\frac{x}{x_0})\text{exp}(-i\frac{z}{z_0}),$$