EE 3130

Introduction to Optoelectronic Engineering,

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EE 3130

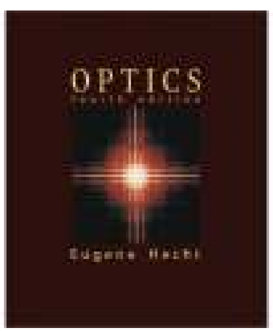
Time: M5M6W5W6 (01:10-03:00 PM, Monday; 01:10-03:00 PM, Wednesday)

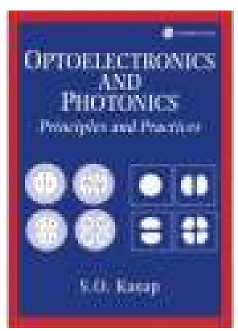
Course Description:

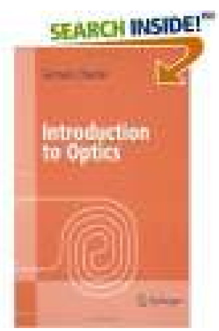
- This course is designed for the beginners who are interested in Optoelectronics and Photonics.
- Modern optics, from EM-waves, geometric optics, interference, diffraction, birefringence, liquid crystals, waveguides, displays, lasers, and nonlinear optics, would be involved.
- No background is required.
- **Teaching Method**: in-class lectures with discussion and project studies.
- TA: Chin-Ming Wu, 1st Ph.D. student of IPT, u8814013@msg.ndhu.edu.tw

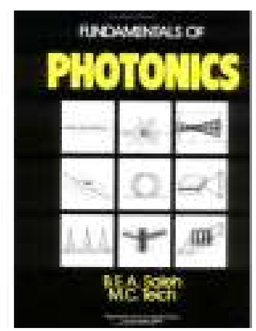
Reference Books

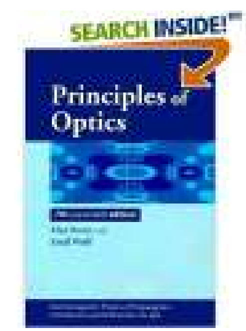
- In-class handouts.
- E. Hecht, "Optics," 4th edition, Addison Wesley (2001).
- S. O. Kasap, "Optoelectronics and Photonics," Prentice Hall (2001).
- G. Chartier, "Introduction to Optics," (2004).
- B. E. A. Saleh and M. C. Teich, "Fundamentals of Photonics," Wiley (1991).
- M. Born and E. Wolf, "Principles of Optics," 7th edition, Cambridge (1999).







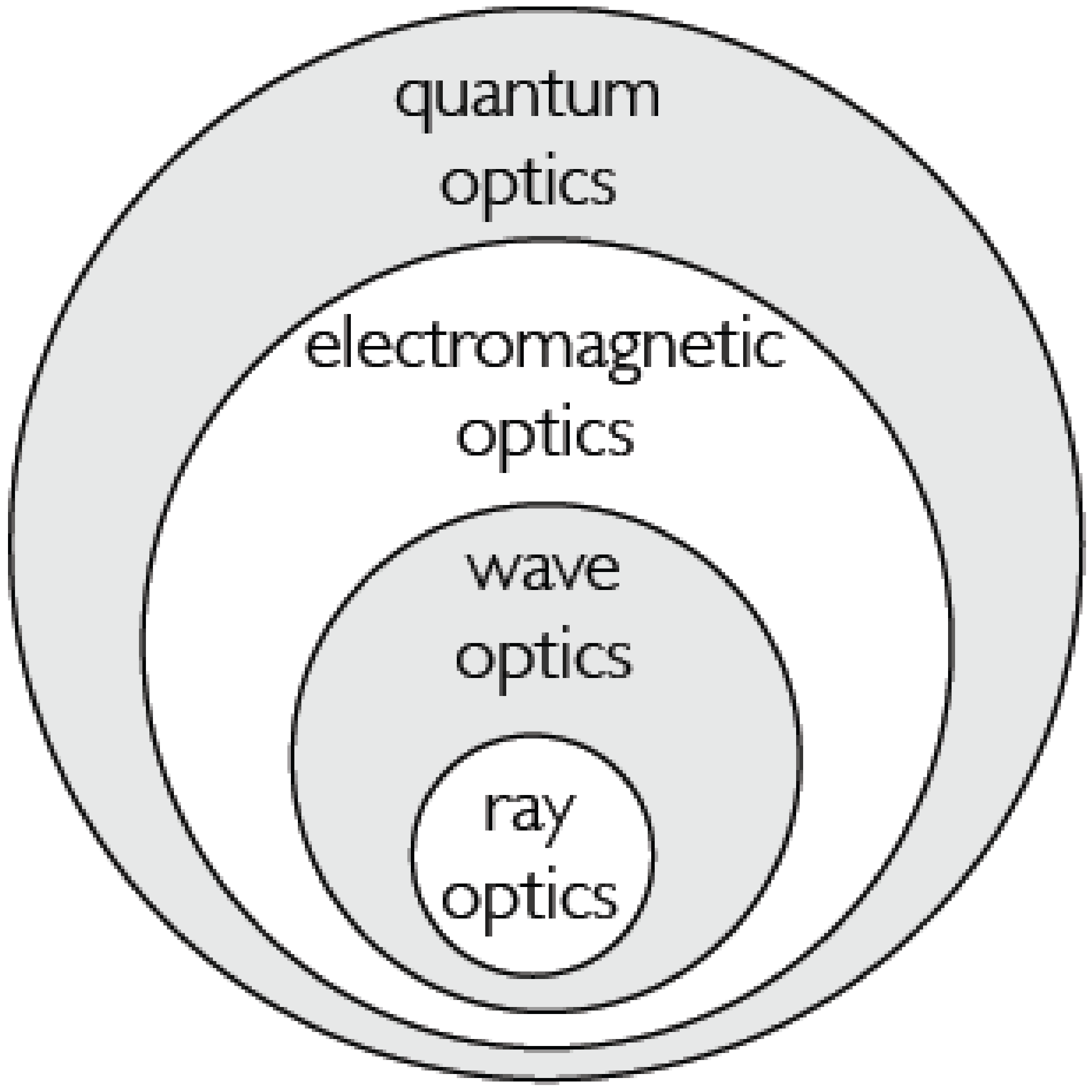




Syllabus

1. Introduction to modern photonics (Feb. 26),
2. Ray optics (lens, mirrors, prisms, et al.) (Mar. 7, 12, 14),
3. Wave optics (plane waves and interference) (Mar. 19, 26),
4. Beam optics (Gaussian beam and resonators) (Apr. 9, 11, 16),
5. Electromagnetic optics (reflection and refraction) (Apr. 18, 23, 25),
6. Fourier optics (diffraction and holography) (Apr. 30, May 2),
   Midterm (May 7-th),
7. Crystal optics (birefringence and LCDs) (May 9, 14),
8. Waveguide optics (waveguides and optical fibers) (May 16, 21),
9. Photon optics (light quanta and atoms) (May 23, 28),
10. Laser optics (spontaneous and stimulated emissions) (May 30, June 4),
11. Semiconductor optics (LEDs and LDs) (June 6),
12. Nonlinear optics (June 18),
13. Quantum optics (June 20),
    Final exam (June 27),
14. Semester oral report (July 4),

General Optics



General Optics

- Ray Optics: Fermat's principle, ABCD matrix
  diffraction free optics, $\lambda \to 0$,
- Wave Optics: Wave equaiton
  scalar field theory
- Electromagnetic Optics: Maxwell's equations
  provide the explanation of classical (continuous) optics, i.e. classical electrodynamics,
- Quantum Optics: Shrödinger equaiton
  allow the explanations of all optical phenomena, i.e. quantum field theory,

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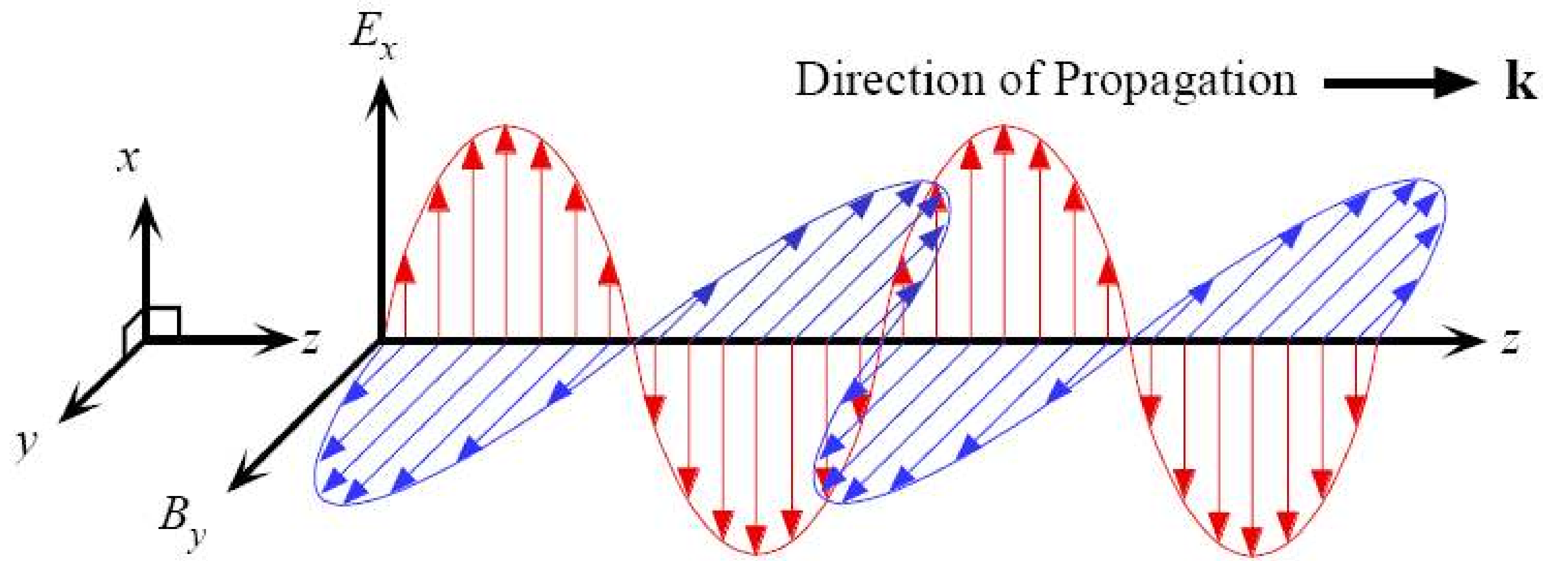
Ray Optics

- Postulates of Ray optics and the principle of Fermat,
- Ray optics v.s. Classical mechanics,
- Reflection, Refraction, and Snell's law,
- Refraction at spherical surfaces,
- Thin lenses, imaging equations,
- Stops, Mirrors, and Prisms,
- Fiber optics,
- Matrix optics for optical system, ABCD matrix,

ElectroMagnetic waves

An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, z.



Maxwell's equations

- Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B},$$

- Ampére's law:

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} + \mathbf{J},$$

- Gauss's law for the electric field:

$$\nabla \cdot \mathbf{D} = \rho,$$

- Gauss's law for the magnetic field:

$$\nabla \cdot \mathbf{B} = 0,$$

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Simple media

Constitutive relation: $\mathbf{B} = \mu\mathbf{H}$ and $\mathbf{D} = \epsilon$E.

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E},$$

where D is the electric flux density (C/m^2), E is the electric field strength (V/m), and P is the *dipole moment density* (C/m^2).

- source-free: $\mathbf{J} = \rho = 0$,
- linear: $\mathbf{P} = \epsilon_0\chi\mathbf{E}$, where ϵ is the permittivity (F/m), χ is the electric susceptibility,
- isotropic: $\chi(x) = \chi(y) = \chi(z)$,
- homogenerous: $\chi(r)$ is independent of r,
- dispersion-free media: $\chi(\omega)$ is independent of ω

Material equations: $\mathbf{D} = \epsilon$E, where

$$\mu\epsilon = \mu_0\epsilon_0(1 + \chi) = \frac{n^2}{c^2},$$

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Maxwell-Schrödinger equations

- the equations for the two-level atomic medium coupled to the field **E** are

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho_{aa} &= \frac{i}{\hbar}[\mathbf{p}_{ab}\mathbf{E}\rho_{ba} - \text{c.c}] - \gamma_a\rho_{aa}, \\
\frac{\partial}{\partial t}\rho_{bb} &= -\frac{i}{\hbar}[\mathbf{p}_{ab}\mathbf{E}\rho_{ba} - \text{c.c}] - \gamma_b\rho_{bb}, \\
\frac{\partial}{\partial t}\rho_{ab} &= -\frac{i}{\hbar}\mathbf{p}_{ab}\mathbf{E}(\rho_{aa} - \rho_{bb}) - (i\omega + \frac{\gamma_a + \gamma_b}{2})\rho_{ab},
\end{aligned}
$$

- the condition of *self-consistency* requires that the equation of motion for the field **E** is driven by the atomic population matrix elements,
-
- the field is described by the Maxwell's equation,

$$
\nabla \cdot \mathbf{D} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},
$$

$$
\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = J + \frac{\partial \mathbf{D}}{\partial t},
$$

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Ref: Quantum Optics

Wave equations

- For a *source-free* medium, $\rho = \mathbf{J} = 0$,

$$\nabla \times (\nabla \times E) = -\mu\epsilon \frac{\partial^2}{\partial t^2} E,$$

$$\Rightarrow \qquad \nabla(\nabla \cdot E) - \nabla^2 E = -\mu\epsilon \frac{\partial^2}{\partial t^2} E.$$

- When $\nabla \cdot E = 0$, one has *wave equation*,

$$\nabla^2 E = \mu\epsilon \frac{\partial^2}{\partial t^2} E$$

- which has following expression of the solutions, in 1D,

$$\begin{aligned} E &= \hat{x}[f_+(z - vt) + f_-(z + vt)], \\ H &= \sqrt{\frac{\epsilon}{\mu}}\hat{y}[f_+(z - vt) - f_-(z + vt)], \end{aligned}$$

with

$$\mu\epsilon = \mu_0\epsilon_0(1 + \chi) = \frac{n^2}{c^2},$$

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Plane waves

- 1D *wave equation*,

$$\frac{\partial^2}{\partial z^2}E = \mu\epsilon\frac{\partial^2}{\partial t^2}E,$$

which has the solutions of

$E = \hat{x}[f_+(z - vt) + f_-(z + vt)]$, with

$$v^2 = \frac{1}{\mu\epsilon} = \frac{n^2}{c_0^2},$$

- plane wave solutions:

$$E_+ = E_0\cos(kz - \omega t),$$

where $\frac{\omega}{k} = \frac{c_0}{n}$.

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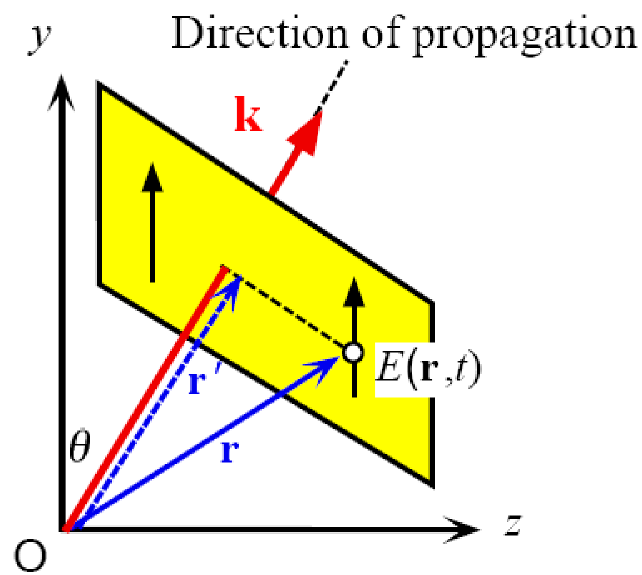
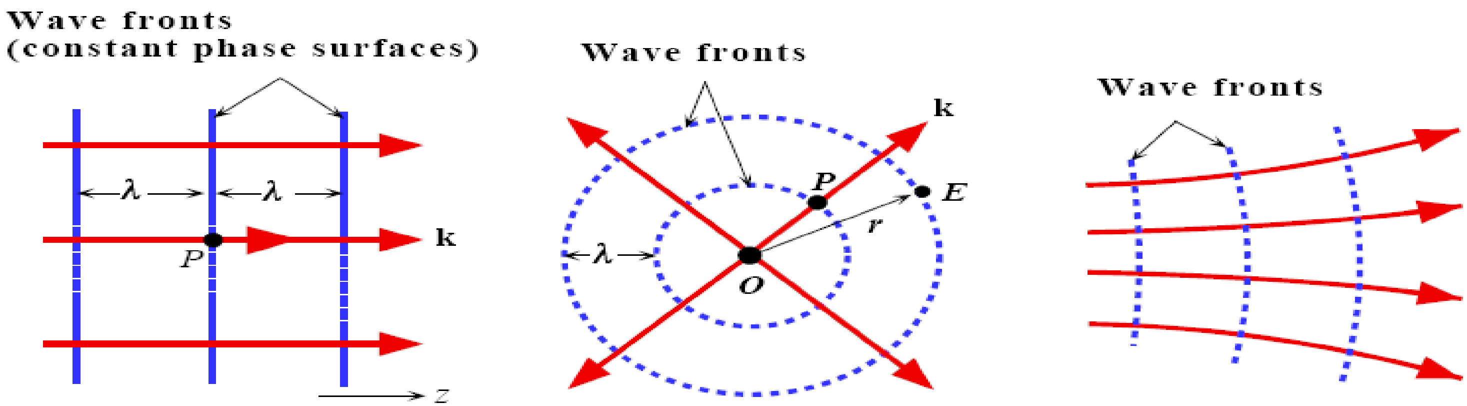
Travelling waves

A travelling plane EM wave along a direction $\mathbf{k}$.





A perfect plane wave

(a)

A perfect spherical wave

(b)

A divergent beam

(c)

Postulates of ray optics

- light travels in form of diffraction-free ray,
  - emitted by light sources,
  - detected by optical detector.

  The arrow points toward the direction of energy flow, and the density is proportional to the optical energy.

- optical medium is characterized by a quantity n

$$n = \frac{c_0}{c}, \qquad n \geq 1,$$

- time to travel distance d in a *homogeneous* medium is

$$t = \frac{d}{c} = \frac{nd}{c_0},$$

  where nd is optical path length.

- Fermat's principle: An optical rays always chooses an optical path that is an extremum. Mathematically

$$\delta \int n(r) \mathrm{d}s = 0.$$

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Principle of Fermat

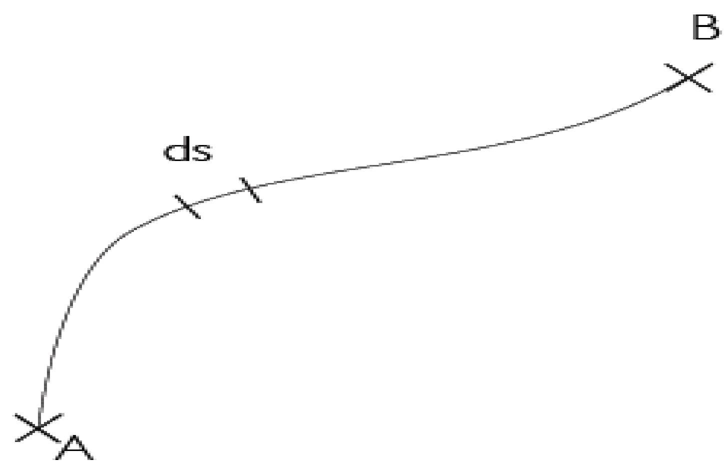
- n is function of r in an inhomogeneous medium,

$$\text{optical path} = \int_A^B n(r)\mathrm{d}s,$$

- ray takes path of shortest time,

$$\delta \int_A^B n(r)\mathrm{d}s = 0,$$

- the optical path in an medium is an extremum compared to neighboring paths,



Ray optics v.s. Classical mechanics

- $\lambda \rightarrow 0 \leftrightarrow \hbar \rightarrow 0$
- ray optics $\leftrightarrow$ classical mechanics
- light travels in form of diffraction-free ray $\leftrightarrow$ classical particle
- Fermat's principle $\leftrightarrow$ Hamilton principle

$$\delta \int n(r) \mathrm{d}s = 0 \leftrightarrow \delta \int L \mathrm{d}t = 0,$$

- in the differential formulation

$$\frac{\mathrm{d}}{\mathrm{d}s}(n \frac{\mathrm{d}r}{\mathrm{d}s}) = \nabla n \leftrightarrow \frac{\mathrm{d}}{\mathrm{d}t} \frac{\mathrm{d}L}{\mathrm{d}\dot{q}_i} = \frac{\mathrm{d}L}{\mathrm{d}q_i},$$

- minimize the *optical path* $\leftrightarrow$ minimize the *energy*,
- represented by $y, \theta \leftrightarrow$ represented by q, p,
- Feynman's path integral for Quantum Electrodyanmics, QED,

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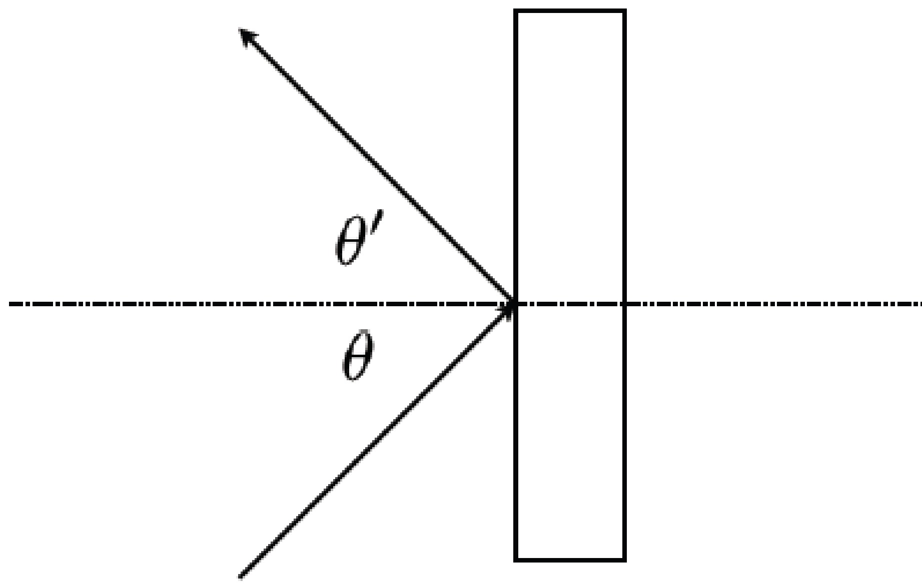
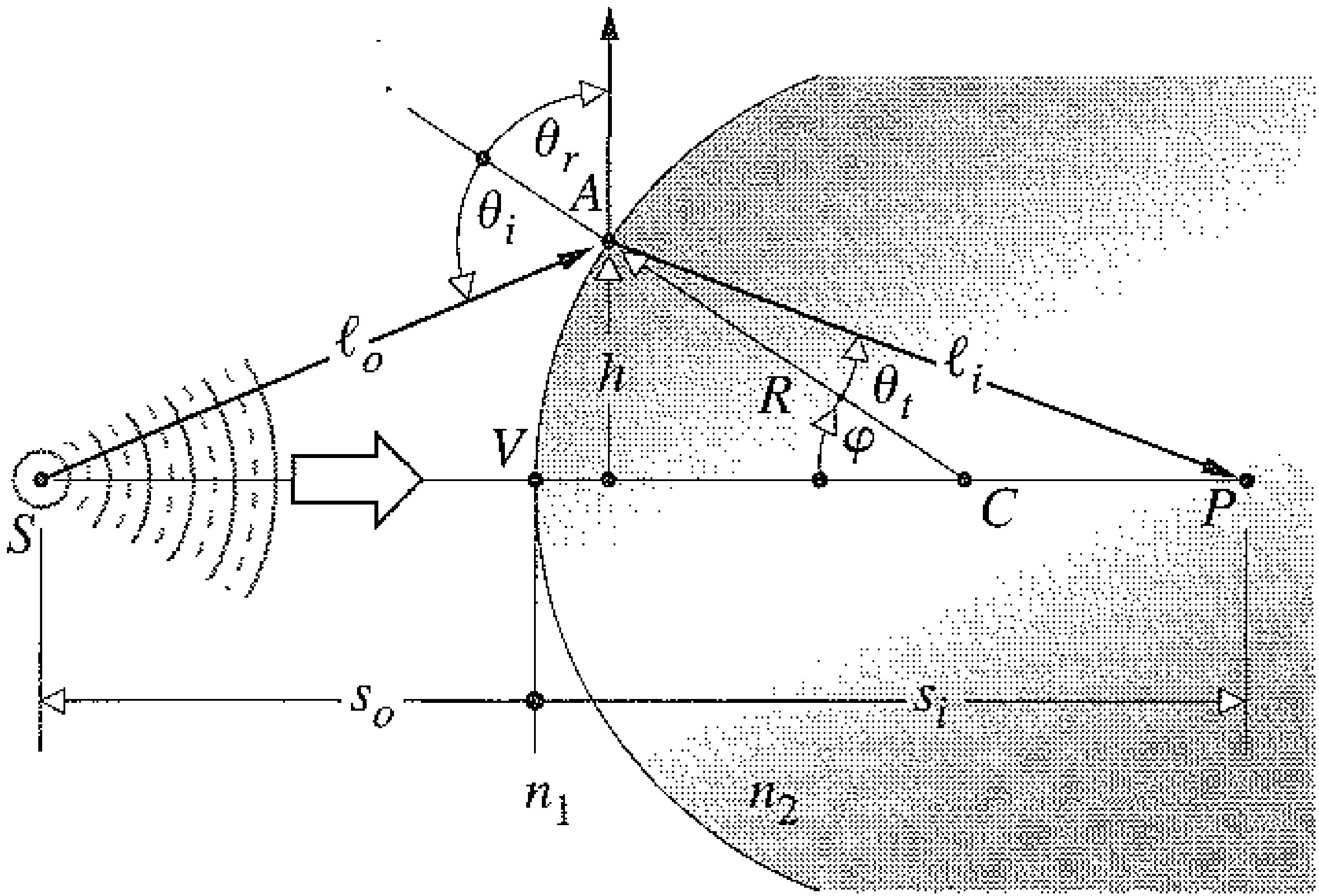
Reflection

- reflected ray lies in the plane of incidence,
- angle of reflection θ' equals the angle of incidence θ,



v.s. infinite potential well,

Refraction at Spherical Surfaces



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Thin-lens equaitons

- optical path: **OPL** $= n_1 l_0 + n_2 l_i$,

$$
\begin{aligned}
l_0 &= [R^2 + (s_0 + R)^2 - 2R(s_0 + R)\cos\phi]^{1/2}, \\
l_i &= [R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\phi]^{1/2},
\end{aligned}
$$

  where the identity $a^2 = b^2 + c^2 - 2bc\cos\theta$ is used.

- the optical path,

$$
\begin{aligned}
\textbf{OPL} &= \int_A^B n(r)\mathrm{d}s, \\
&= n_1[R^2 + (s_0 + R)^2 - 2R(s_0 + R)\cos\phi]^{1/2} \\
&+ n_2[R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\phi]^{1/2},
\end{aligned}
$$

- Fermat's principle:

$$
\delta \int n(r)\mathrm{d}s = \frac{\mathrm{dOPL}}{\mathrm{d}\phi} = 0,
$$

  then

$$
\frac{n_1 R(s_0 + R)\sin\phi}{2l_0} - \frac{n_2 R(s_i - R)\sin\phi}{2l_i} = 0,
$$

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Thin-lens equaitons

- from Fermat's principle,

$$\frac{n_1 R(s_0 + R)\sin\phi}{2l_0} - \frac{n_2 R(s_i - R)\sin\phi}{2l_i} = 0,$$

- re-arrange,

$$\frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R}(\frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0}),$$

- for paraxial rays, i.e. small values of ϕ,

$$l_0 \approx s_o, \qquad l_i \approx s_i,$$

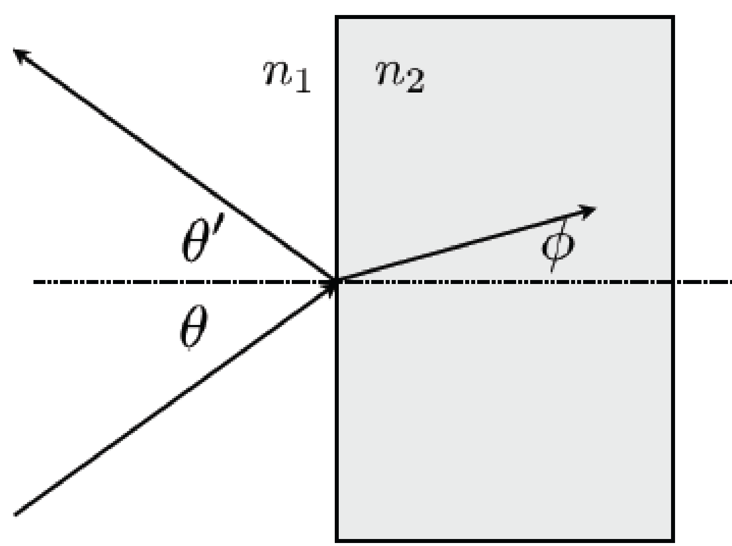
then

$$\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_!}{R},$$

Refraction

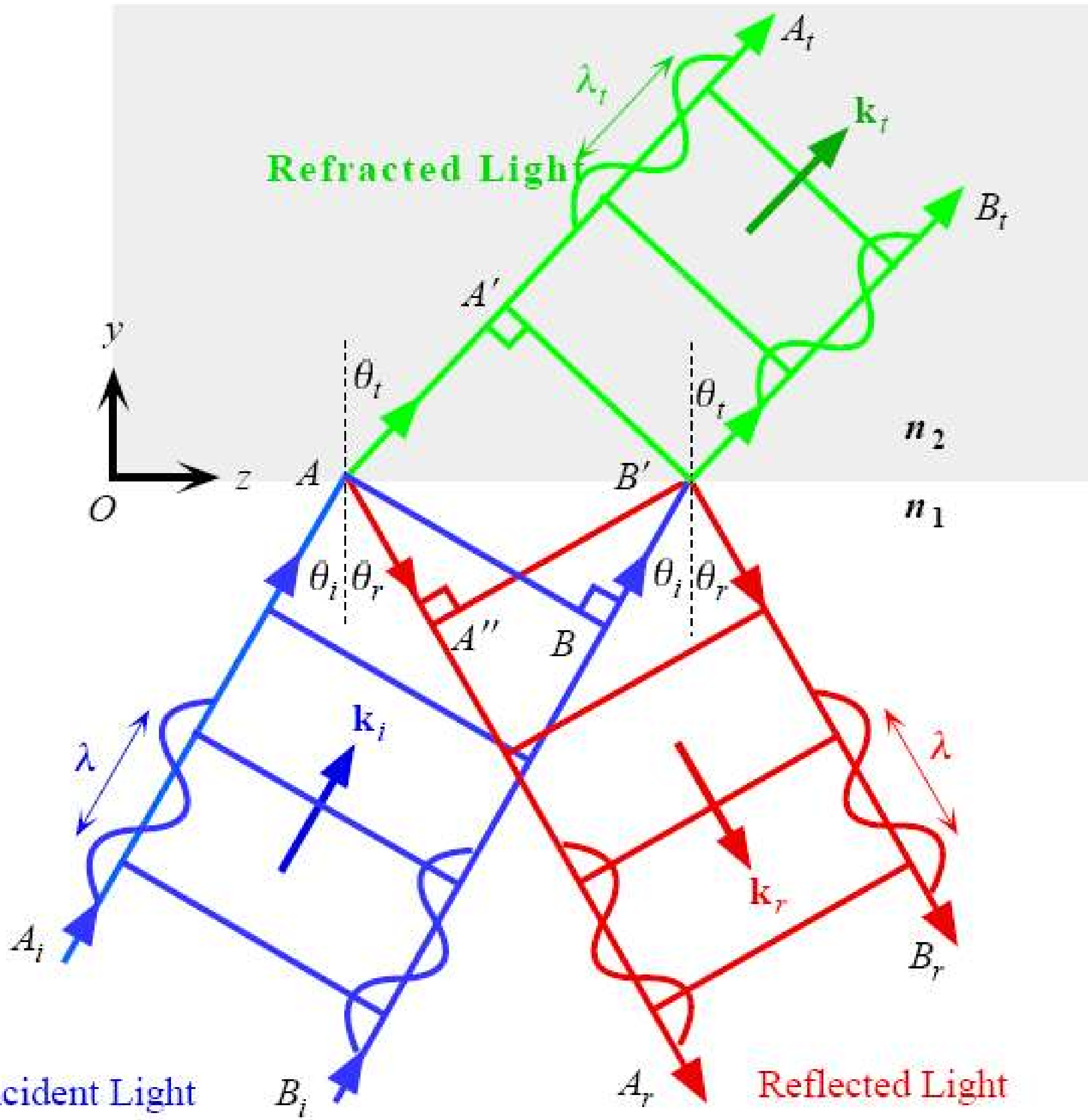
- refracted ray lies in the plane of incidence
- angle of refraction ϕ is related to angle of incidence θ by the Snell's law

$$n_1 \sin\theta = n_2 \sin\phi,$$



v.s. ???

Refraction



A_t
λ_t
$\mathbf{k}_t$
Refracted Light
B_t
A'
y
θ_t
θ_t
n_2
O
z
A
B'
n_1
θ_i θ_r
θ_i θ_r
A''
B
$\mathbf{k}_i$
λ
λ
$\mathbf{k}_r$
A_i
B_r
Incident Light
B_i
A_r
Reflected Light

Snell's law

- plane wave solutions:

$$E_+ = E_0 \cos(kz - \omega t),$$

where

$$\frac{\omega}{k} = \frac{c_0}{n},$$

- for 3D waves,

$$|\mathbf{k}|^2 = k_x^2 + k_y^2 + k_z^2 = n^2\omega^2/c_0^2,$$

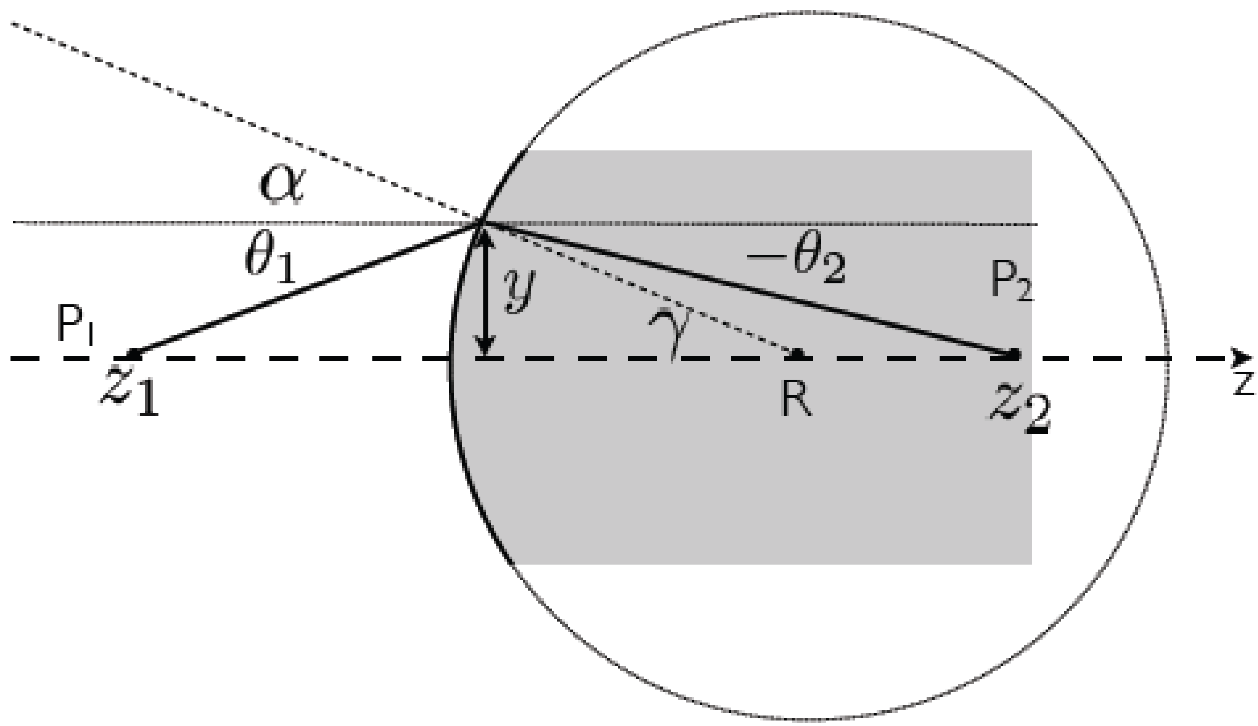
- in the transverse plane (x,z),

$$k_x^{(1)} = |\mathbf{k}^{(1)}|\sin\theta = k_x^{(2)} = |\mathbf{k}^{(2)}|\sin\phi,$$

$$\rightarrow n_1 \sin\theta = n_2 \sin\phi,$$

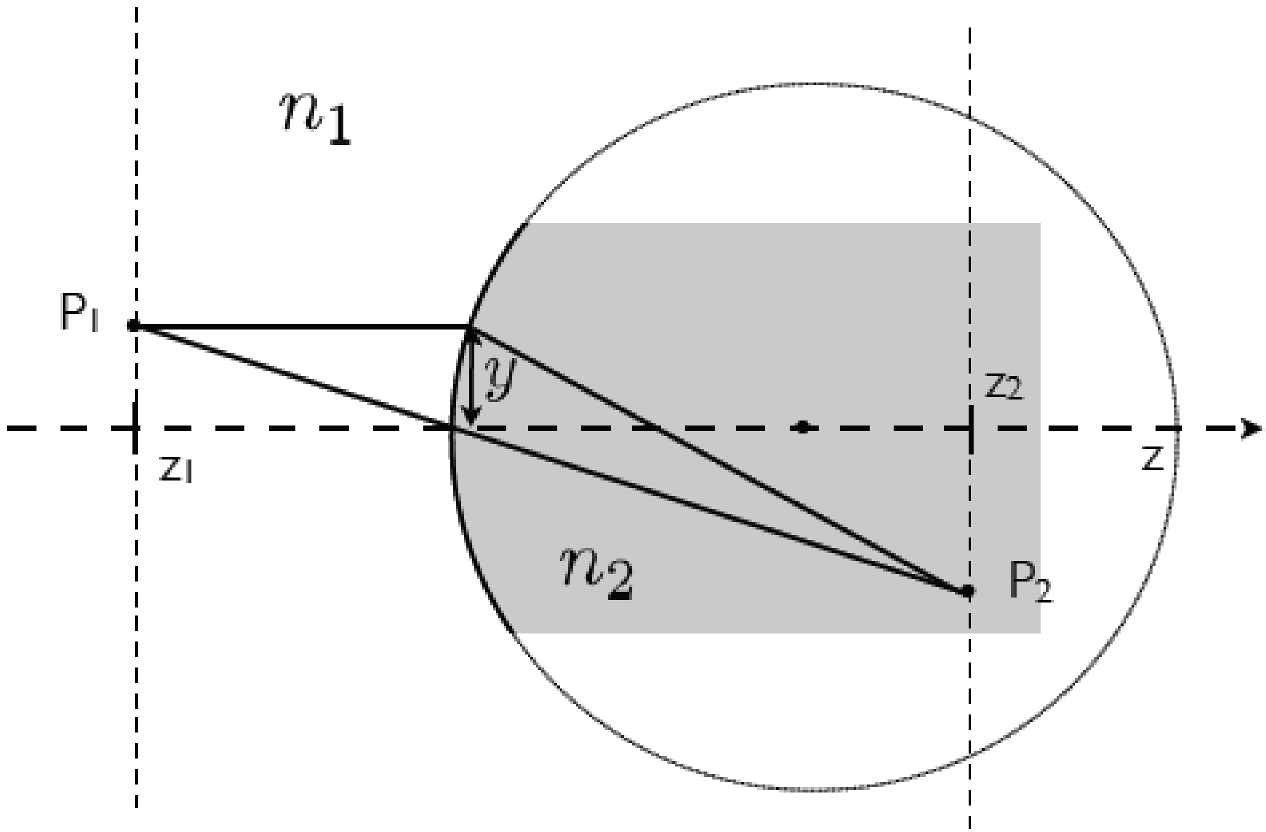
Refraction at spherical boundaries

- derivation for paraxial rays
- paraxial means close to the optical axis



$$\theta_2 \approx \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y,$$

Conjugated planes



$$\frac{n_1}{z_1} + \frac{n_2}{z_2} \approx \frac{n_2 - n_1}{R},$$

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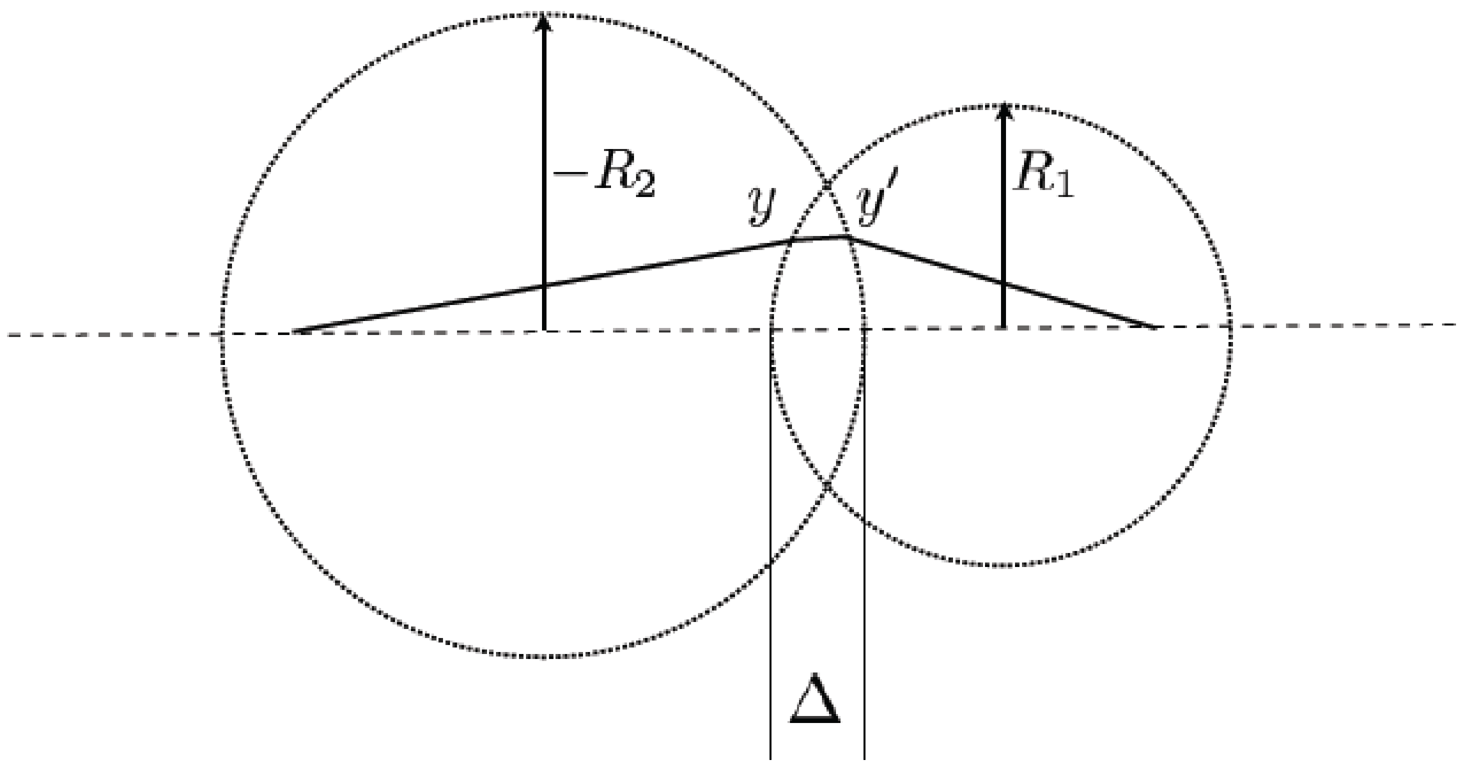
Refraction at sperical lenses

- lens includes two spherical surfaces with different radii
- biconvex lens
- lens is thin if $y = y'$,



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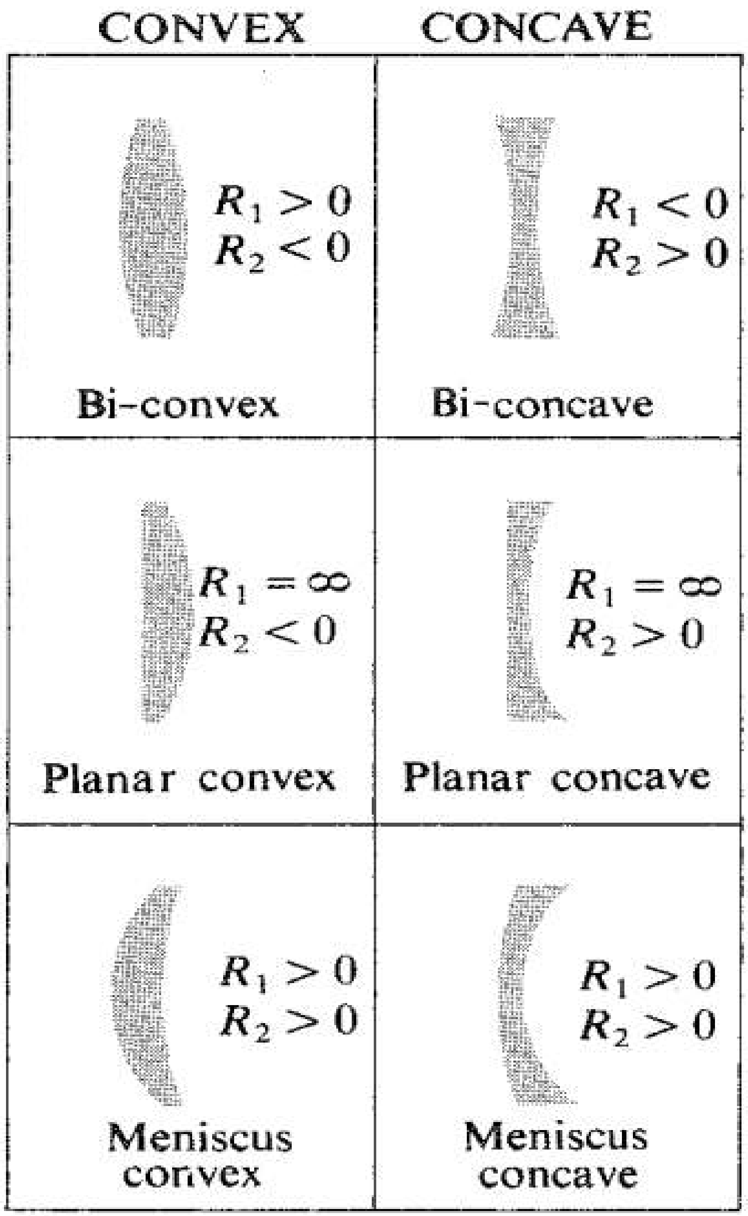
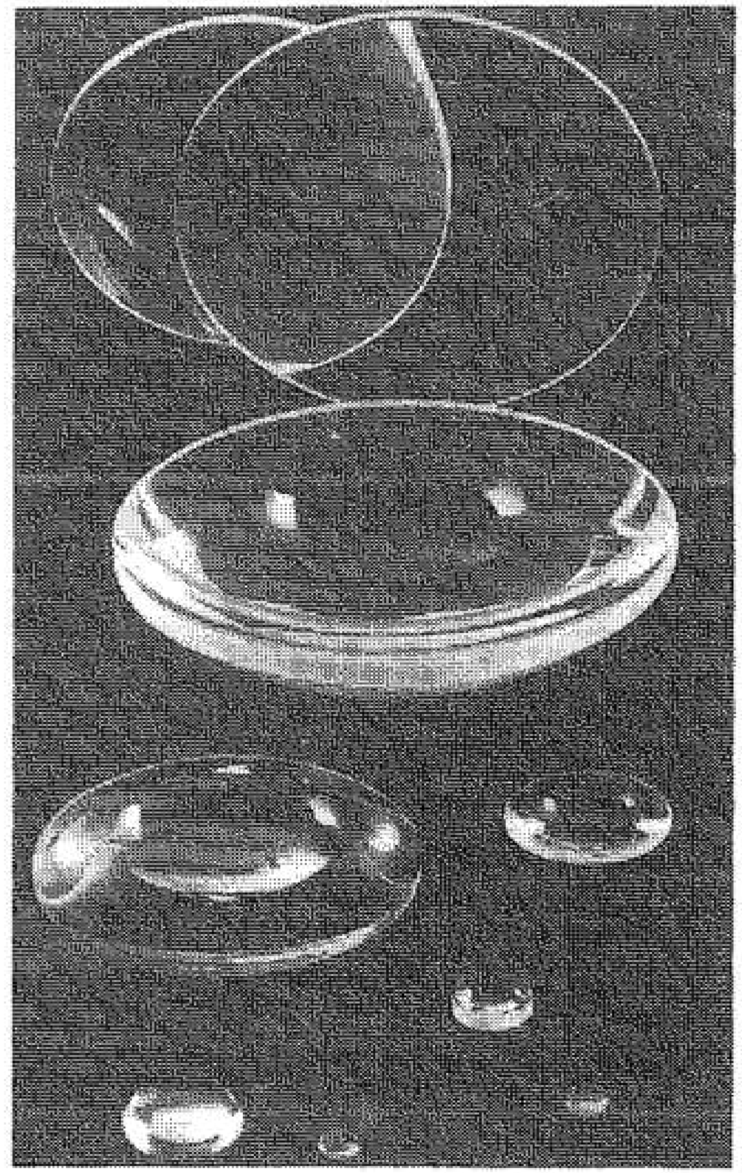
Lens





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Refraction at a thin lens

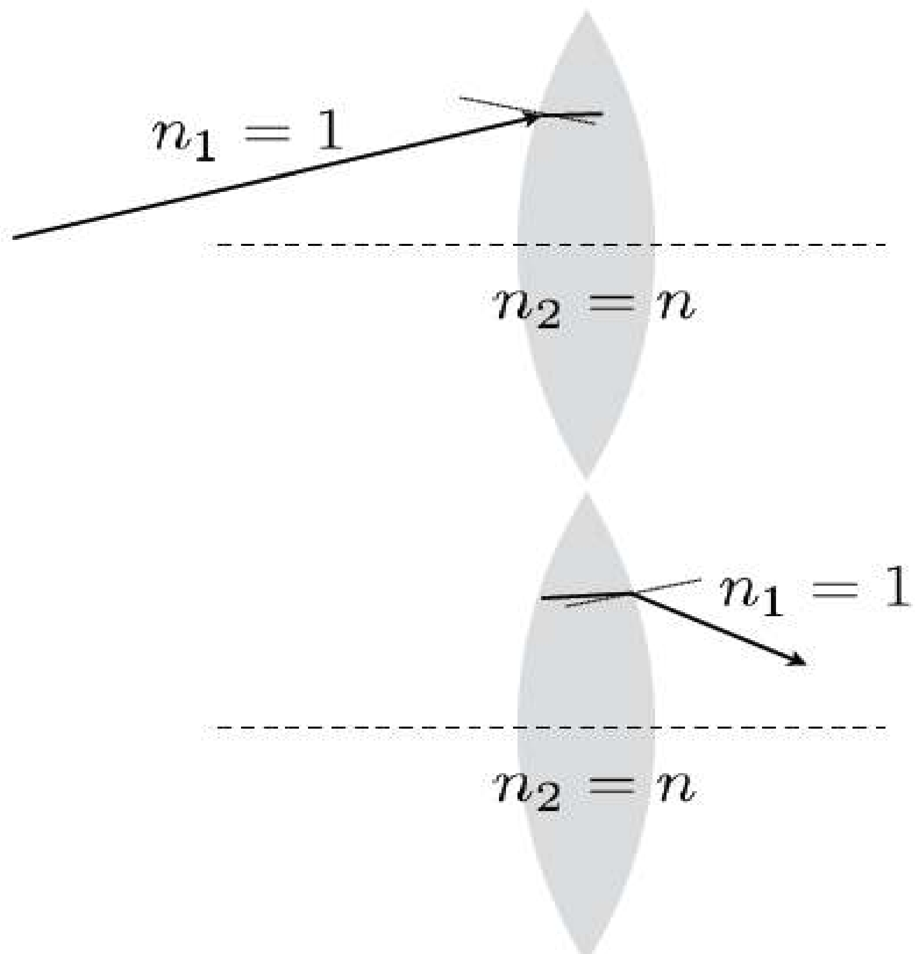
$$\theta_2 \approx \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y,$$

- first refraction

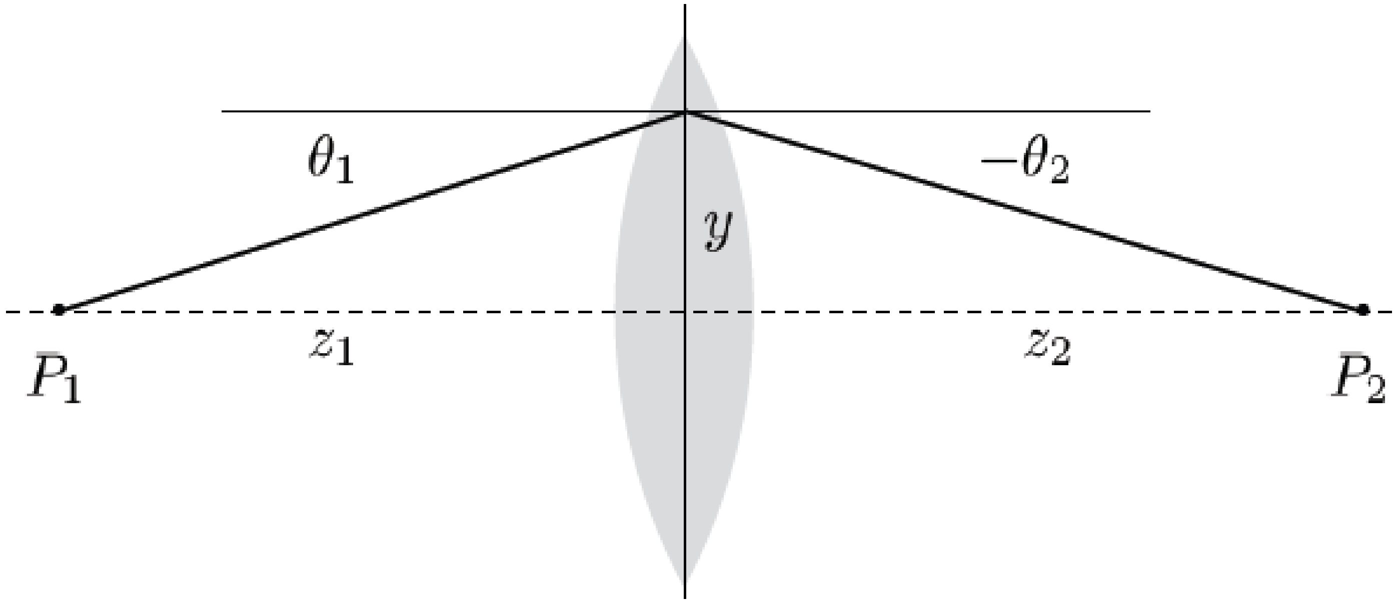
  $\theta_t \approx \frac{1}{n}\theta_1 - \frac{n-1}{nR_1} y,$

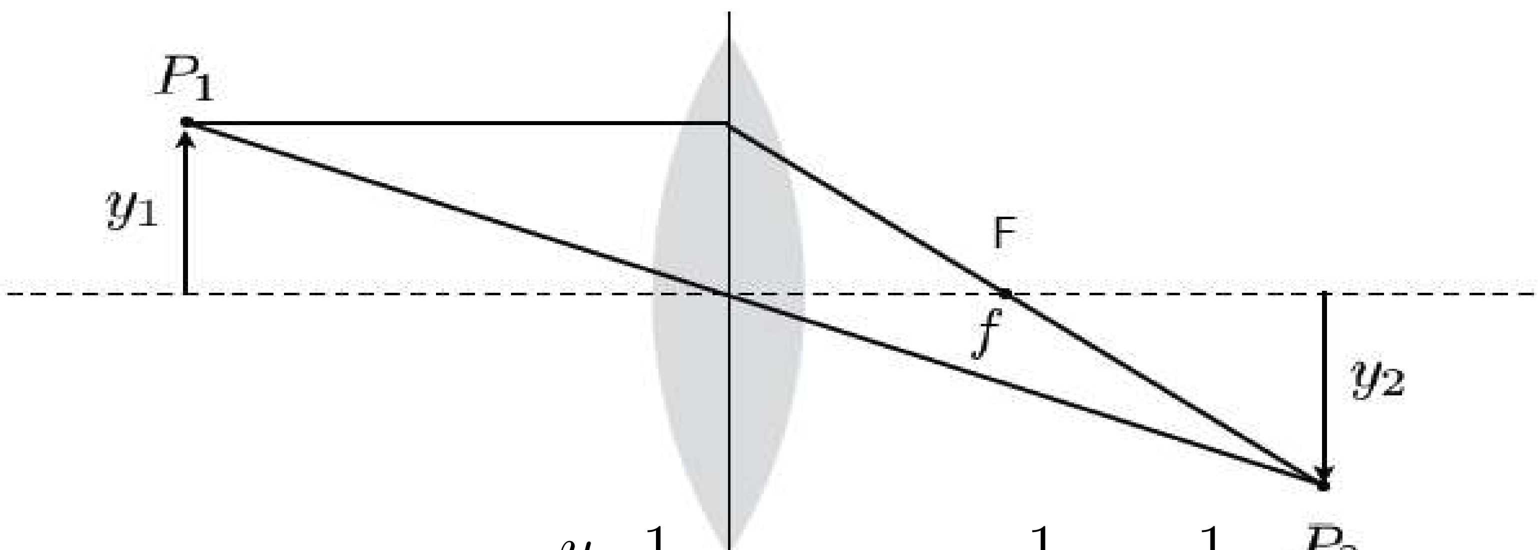
- second refraction

  $\theta_2 \approx \frac{n}{1}\theta_t - \frac{1-n}{R_2} y,$



Refraction at a spherical lens





$$\theta_2 = \theta_1 - \frac{y}{f}, \frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2})$$

Imaging with a lens

- imaging equation:

$$\frac{1}{f} = \frac{1}{z_1} + \frac{1}{z_2},$$

  where

z_1	object distance
z_2	image distance

- magnification:

$$y_2 = -\frac{z_2}{z_1} y_1,$$

- focal length f completely defines the effect of the lens on paraxial ray.

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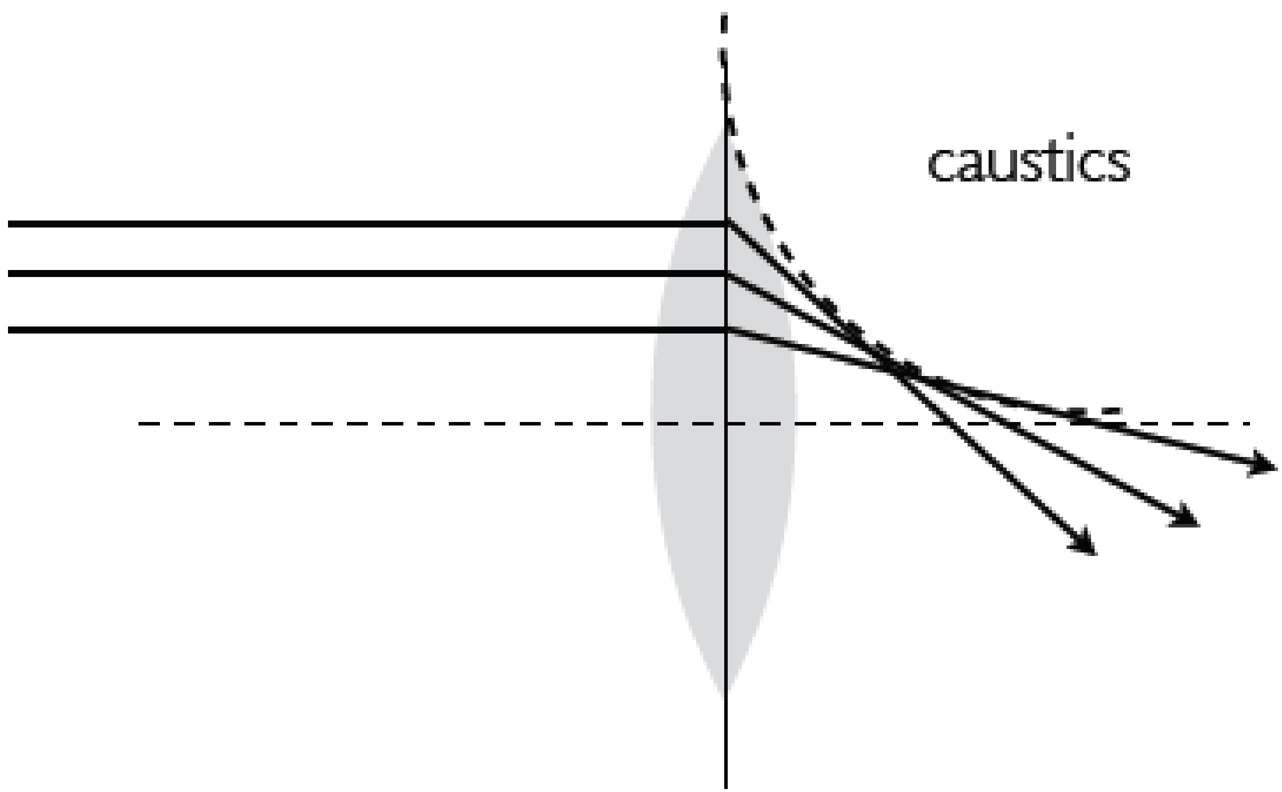
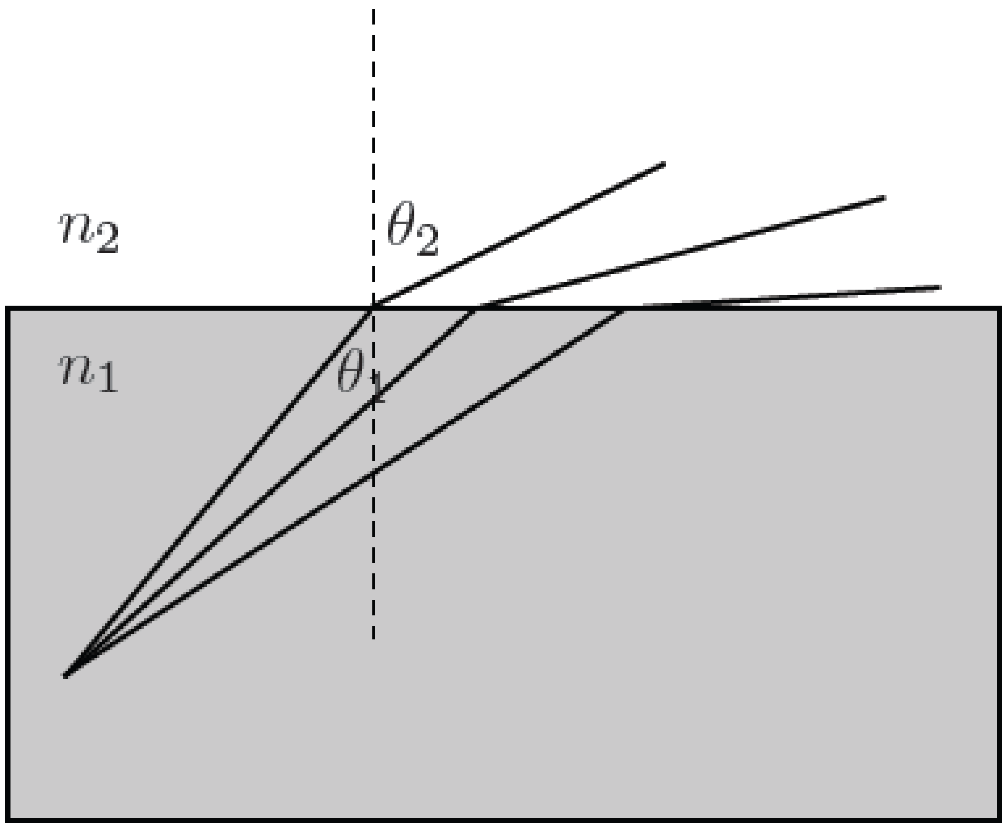
Imaging errors

- spherical optics only for paraxial beams,
- spherical aberration $\rightarrow$ aspheric lenses,
- chromatic aberration $n = n(\lambda) \rightarrow$ achromatic lenses,



Total Internal Reflection

- Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$,
- $n_1 > n_2$, $\theta_2 > 90^o$,
- critical angle, $\theta_c = \sin^{-1}\frac{n_2}{n_1}$, i.e. $\theta_c \approx 42^o$ for glass (n=1.5)



Snell's law for total internal reflection

- in the transverse plane (x,z),

$$k_x^{(1)} = |\mathbf{k}^{(1)}| \sin\theta = k_x^{(2)} = |\mathbf{k}^{(2)}| \sin\phi,$$
$$\rightarrow n_1 \sin\theta = n_2 \sin\phi,$$

- if $n_1 > n_2$, then

$$k_x^{(2)} = k_x^{(1)} = |\mathbf{k}^{(1)}| \sin\theta > |\mathbf{k}^{(2)}|,$$

- then define $k_z^{(2)} \equiv j\alpha$,

$$|\mathbf{k}^{(2)}|^2 = (k_x^{(2)})^2 + (k_z^{(1)})^2 = (k_x^{(2)})^2 - \alpha^2,$$

- in the n_2 medium, the wave is an *evanescence* wave, decaying along z-direction.

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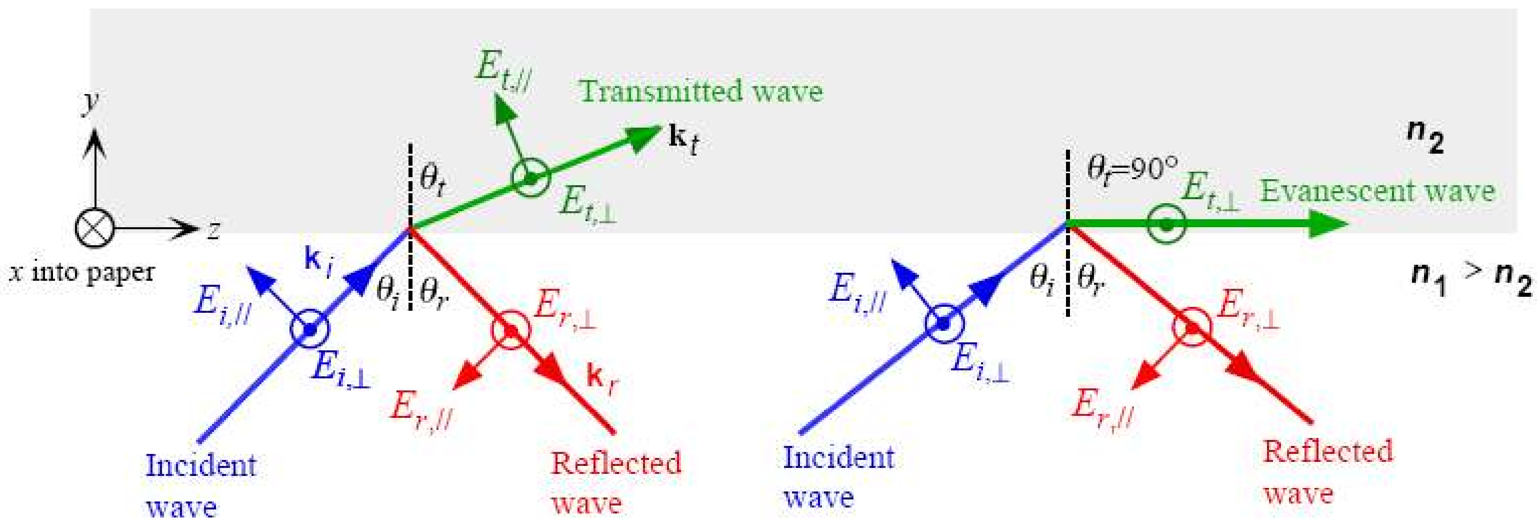
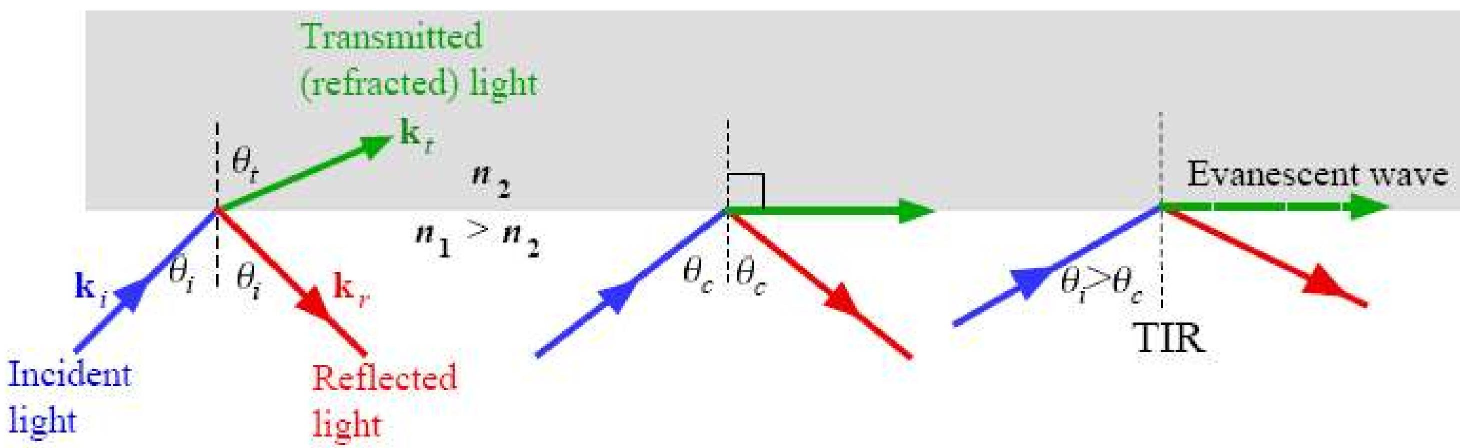
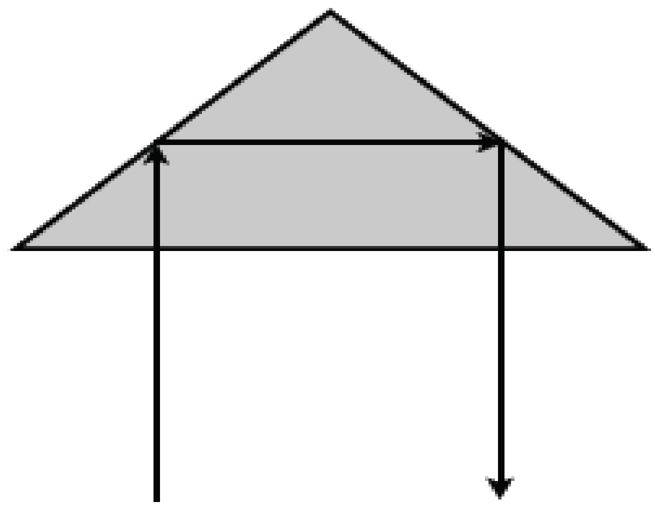
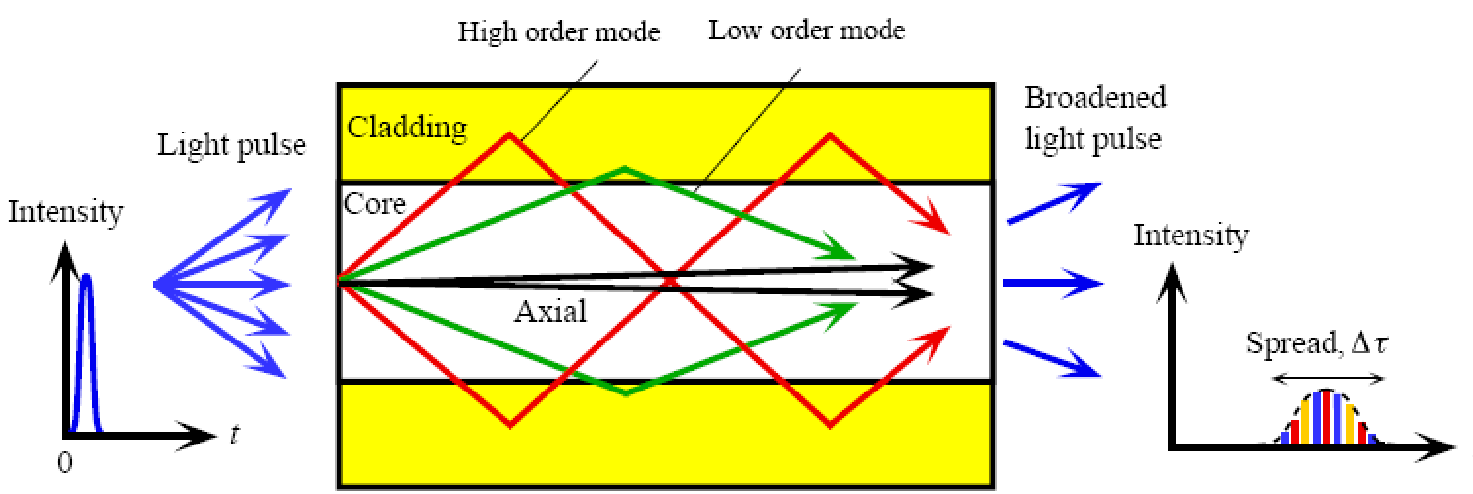
Total internal reflection





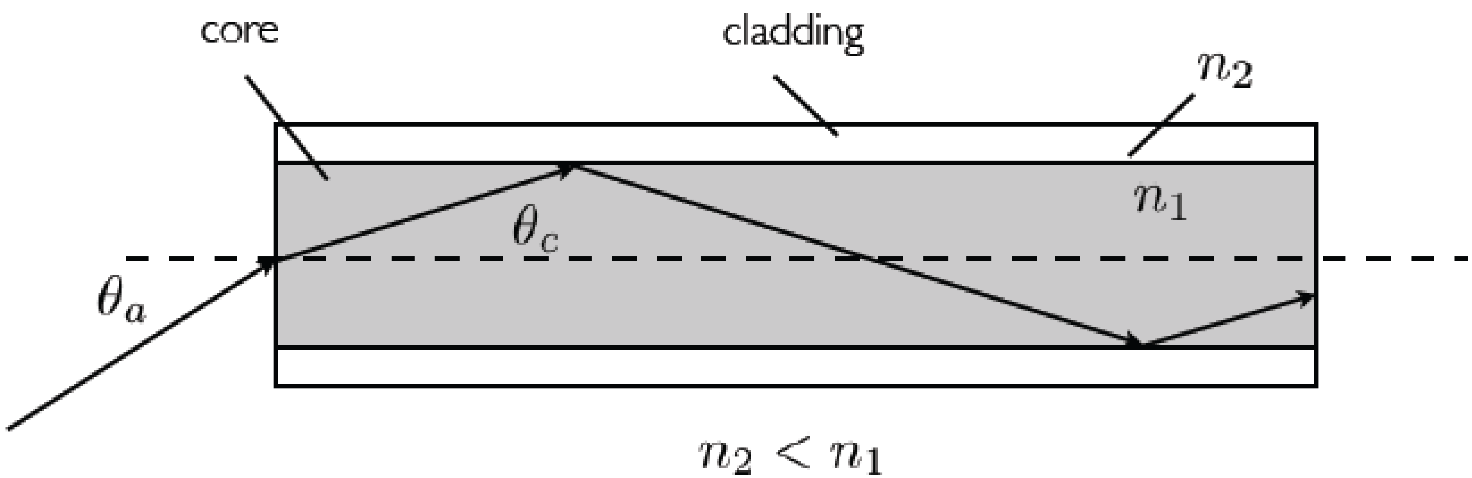
Prisms and Optical fibers





Guiding light

optical fiber (step index fiber)



- $n_2 < n_1$,
- acceptance angle θ_a
- numerical aperture: $NA = sin\theta_{max} = \sqrt{n_1^2 - n_2^2}$,
- typical value: $NA = 0.2$ for $n_1 = 1.475$ and $n_2 = 1.46$,

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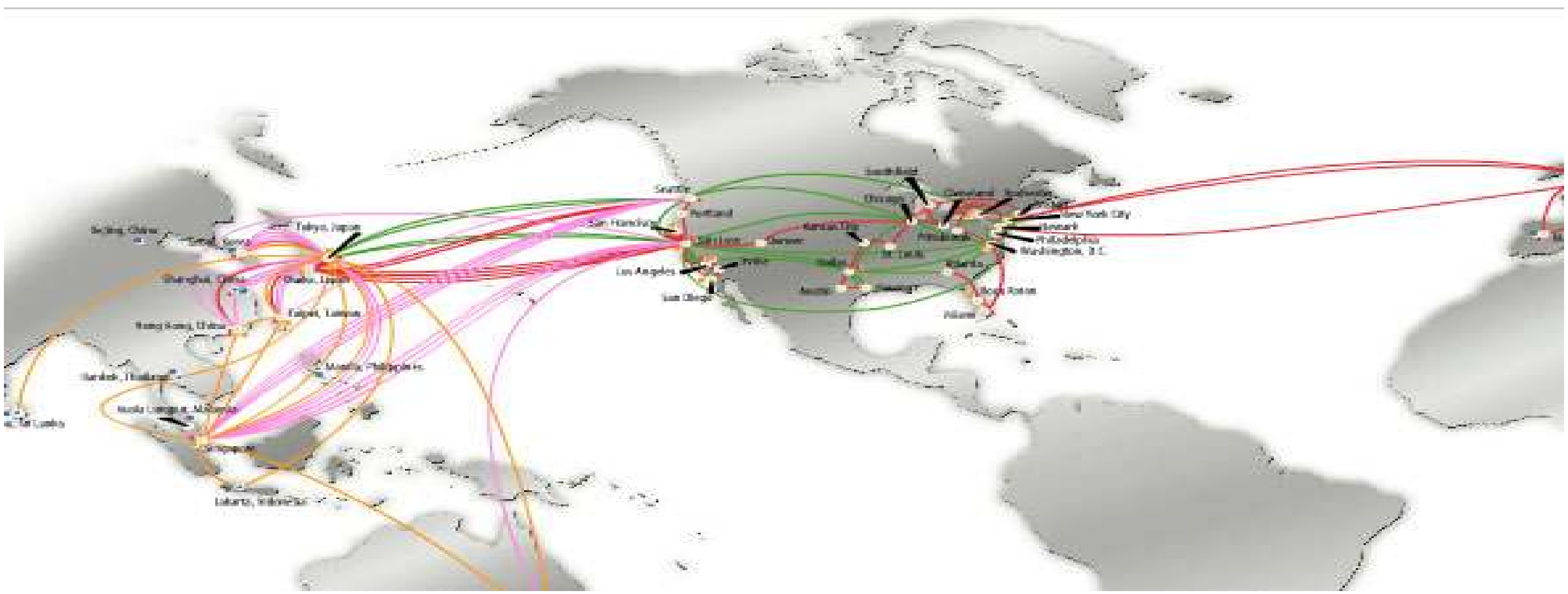
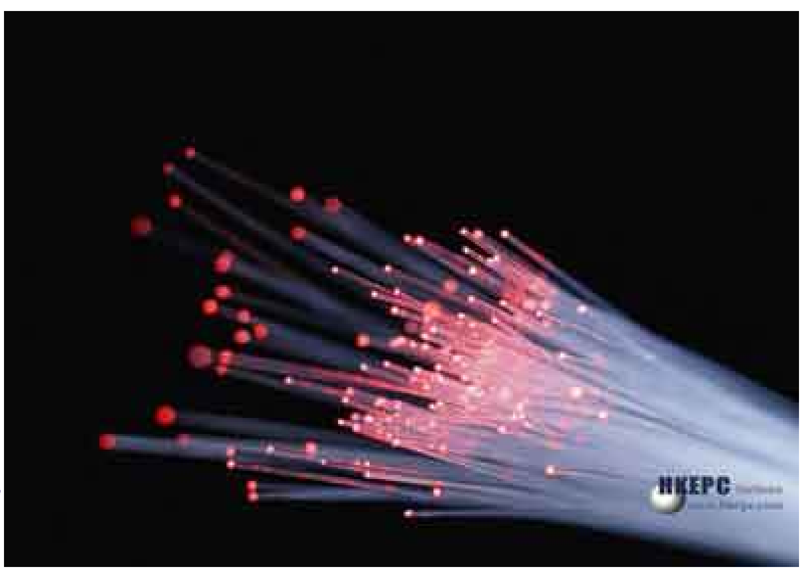
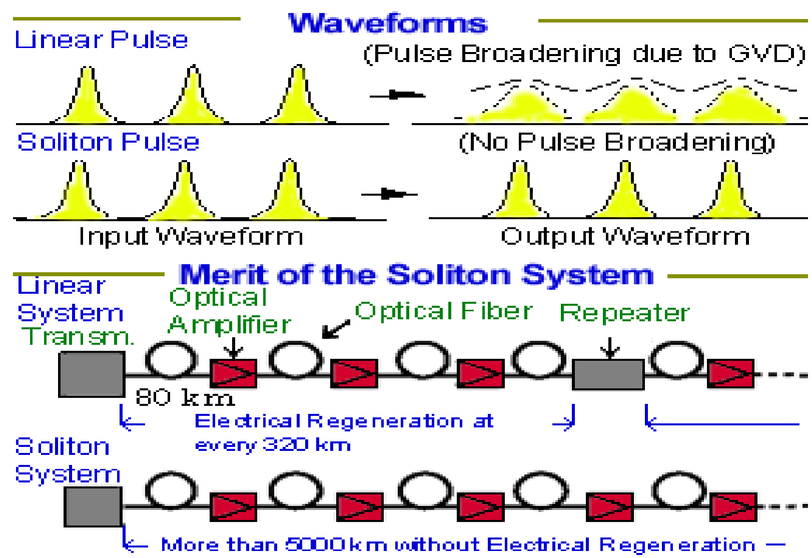
Fiber optics







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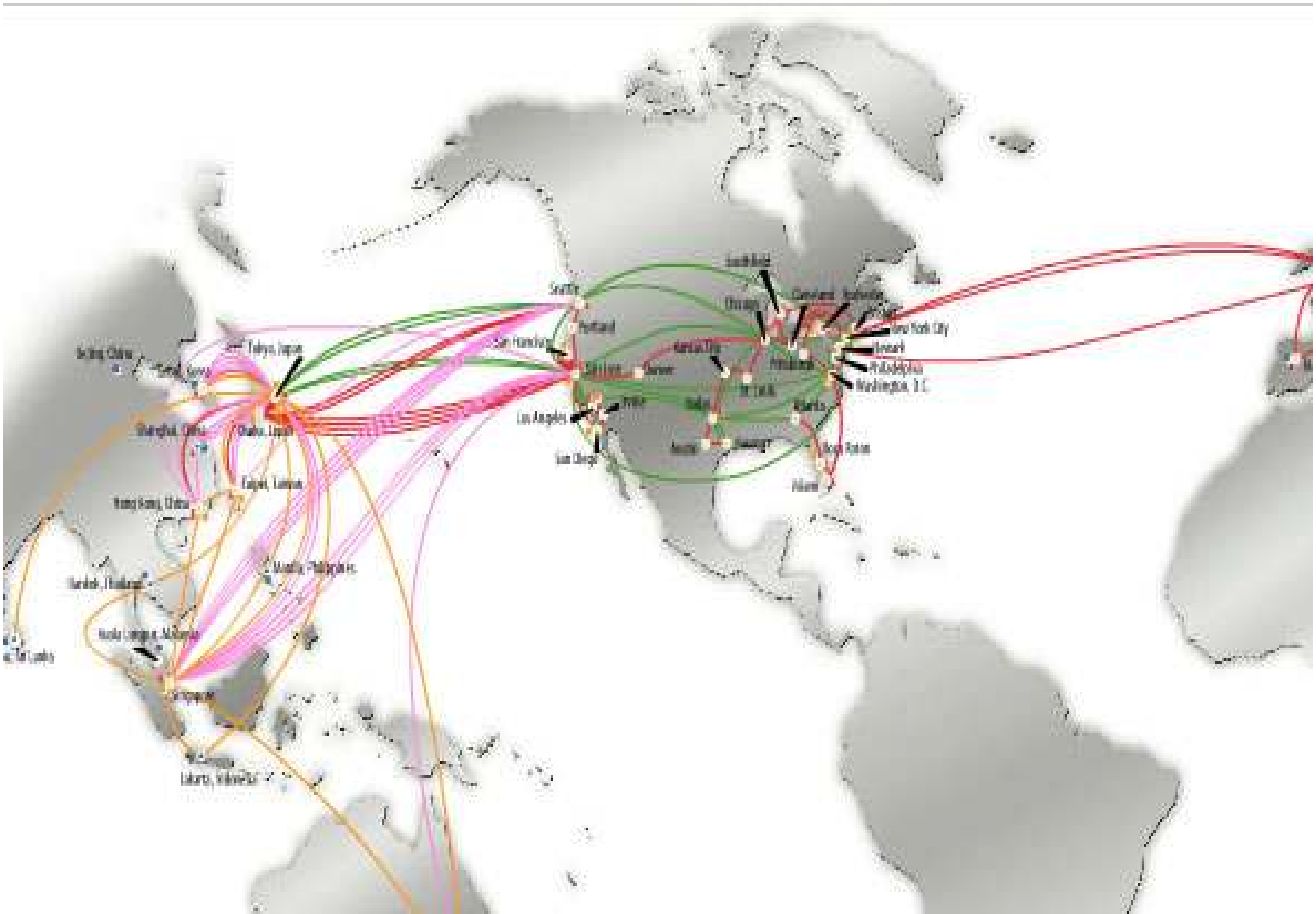
Global overseas fiber network



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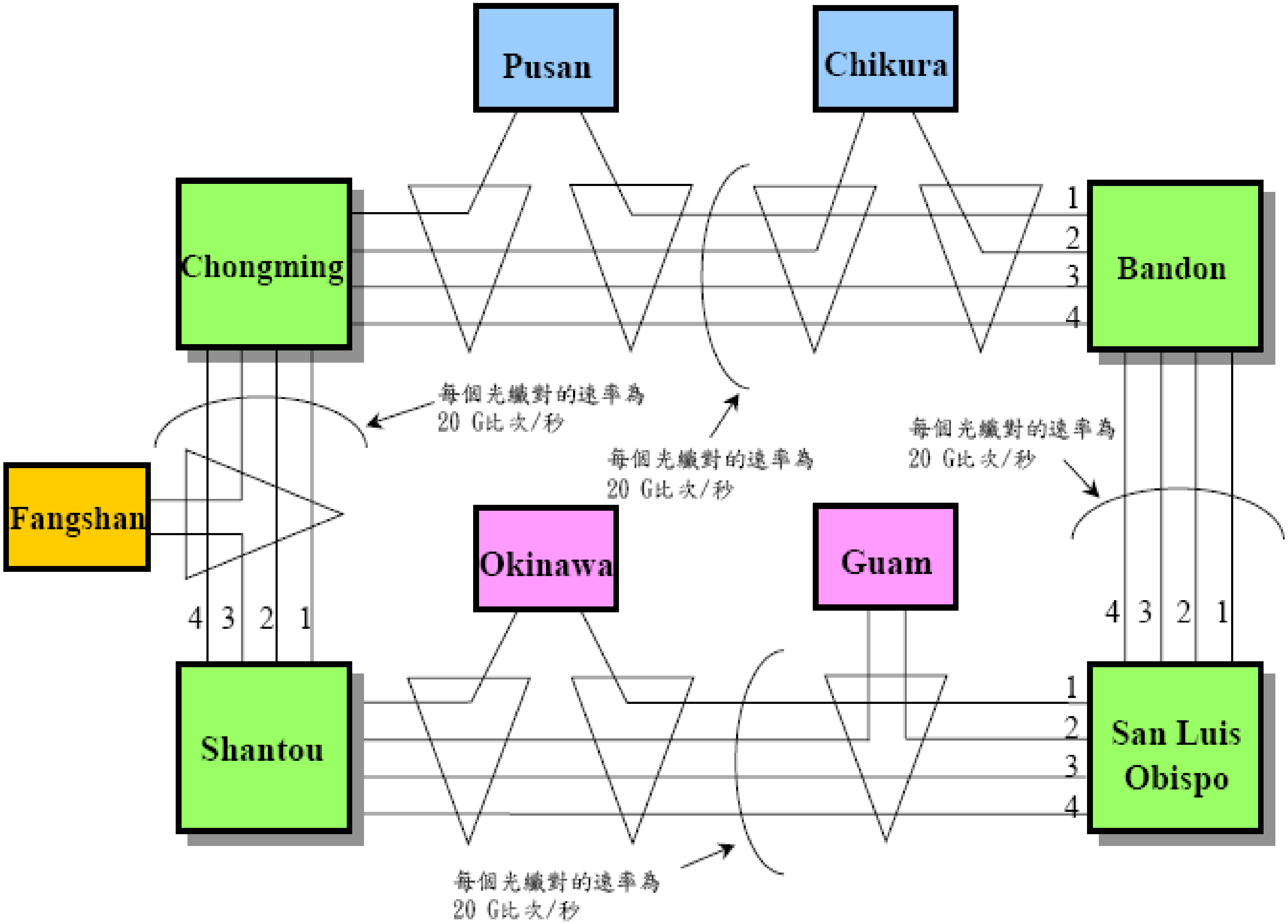
Taiwan-US overseas fiber network



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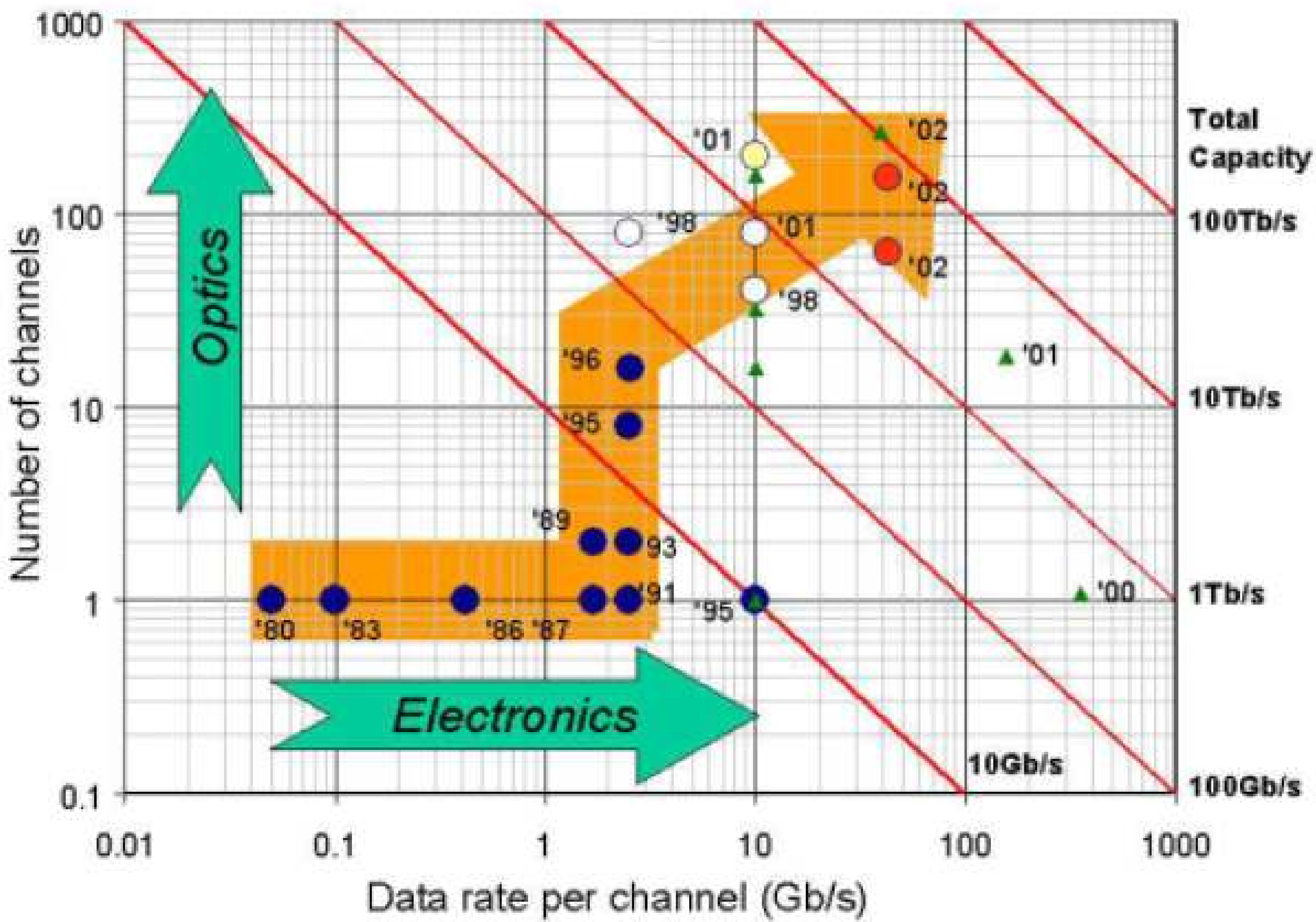
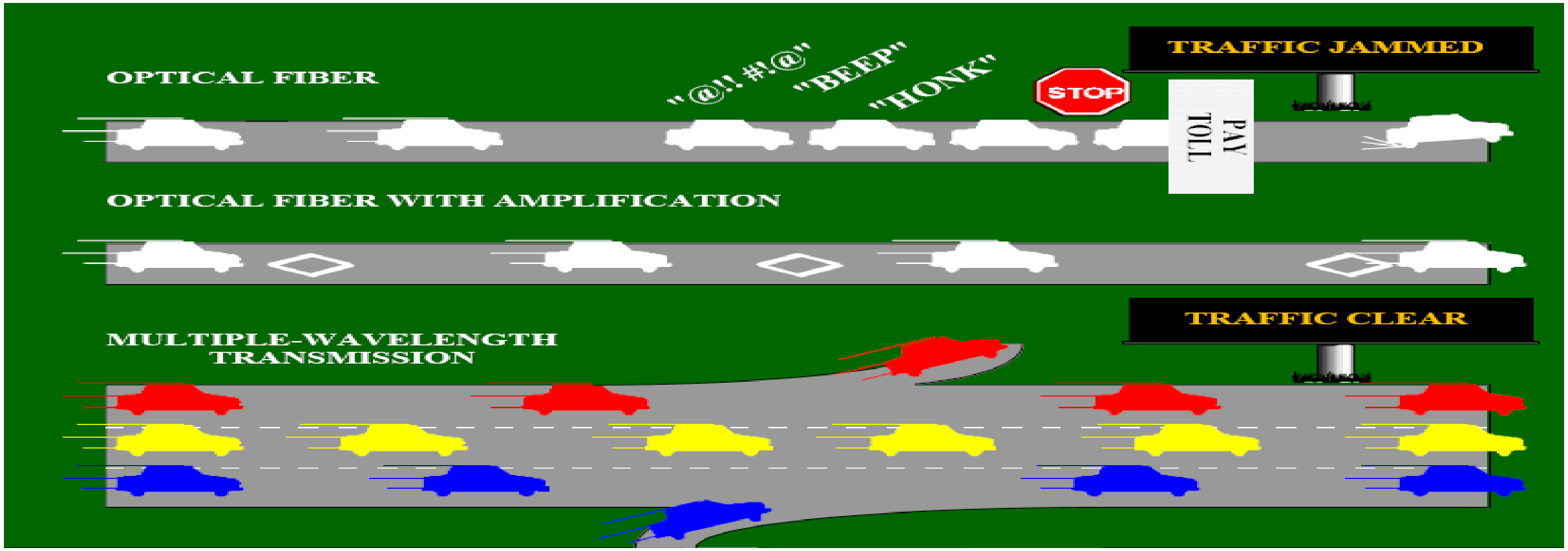
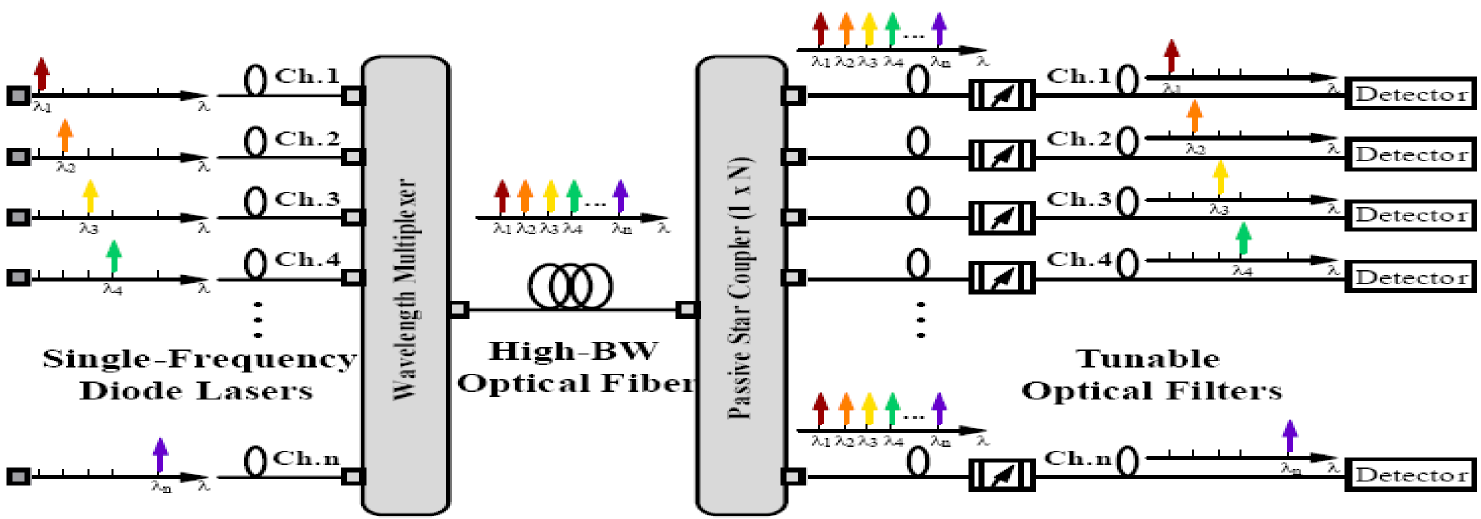
Capacities of optical network



Wavelength-Division-Multiplex





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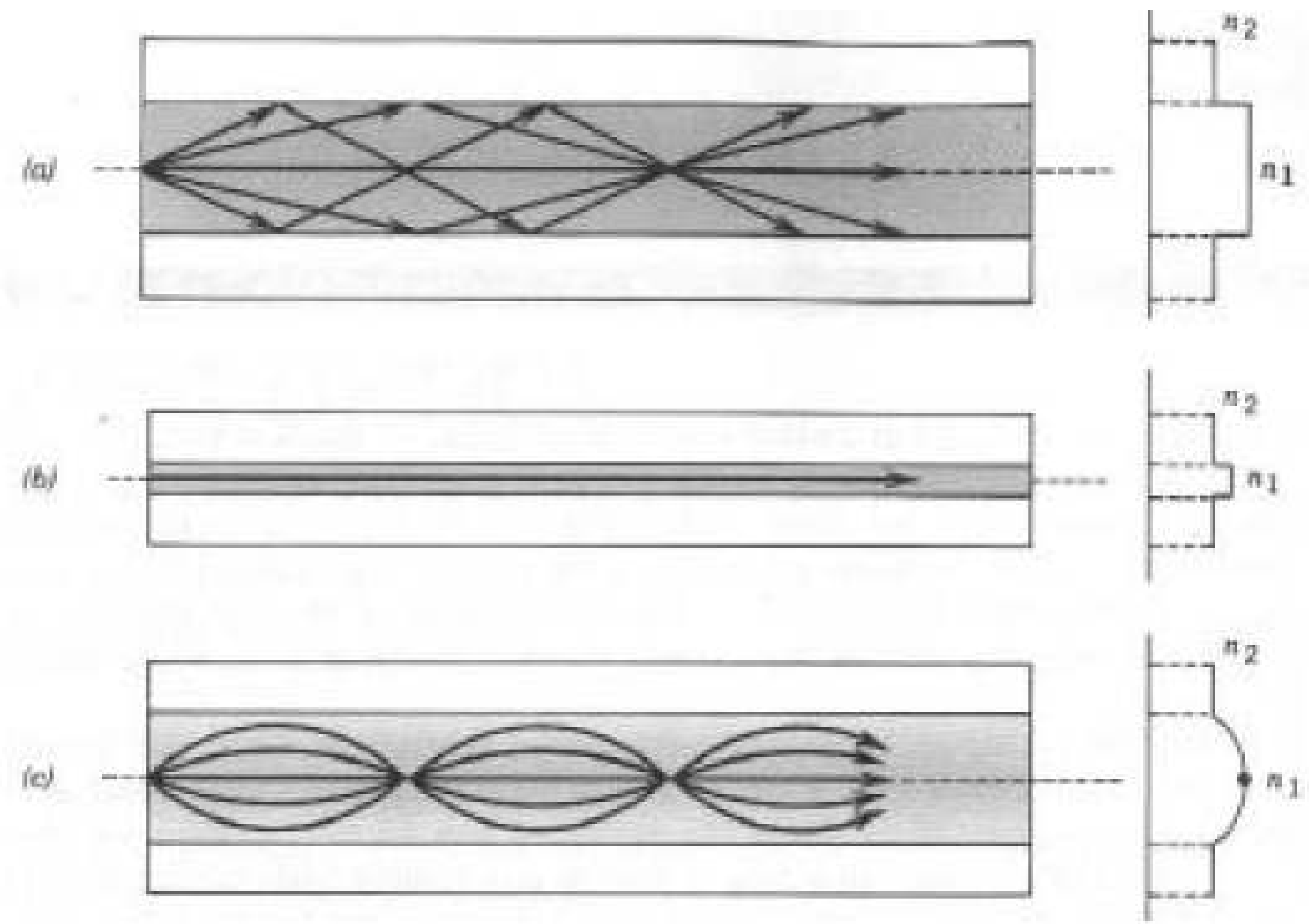
Multi-mode and Single-mode fibers



(a) multimode step index fiber; (b) single-mode step index fiber; (c) multimode graded index fiber;

Fermat's principle

- integral formulation:

$$\delta \int n(r)\mathrm{d}s = 0,$$

- differential formulation:

$$\frac{\mathrm{d}}{\mathrm{d}s}(n\frac{\mathrm{d}r}{\mathrm{d}s}) = \nabla n,$$

$$\frac{\mathrm{d}}{\mathrm{d}s}(n\frac{\mathrm{d}x}{\mathrm{d}s}) = \frac{\partial n}{\partial x}, \frac{\mathrm{d}}{\mathrm{d}s}(n\frac{\mathrm{d}y}{\mathrm{d}s}) = \frac{\partial n}{\partial y}, \frac{\mathrm{d}}{\mathrm{d}s}(n\frac{\mathrm{d}z}{\mathrm{d}s}) = \frac{\partial n}{\partial z},$$

$$x(s), y(s), z(s)$$

- parametrize x, y as function of z, i.e. $x = x(z)$, $y = y(z)$,

$$\mathrm{d}s = \mathrm{d}z\sqrt{1 + (\mathrm{d}x/\mathrm{d}z)^2 + (\mathrm{d}y/\mathrm{d}z)^2},$$

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Paraxial ray equation

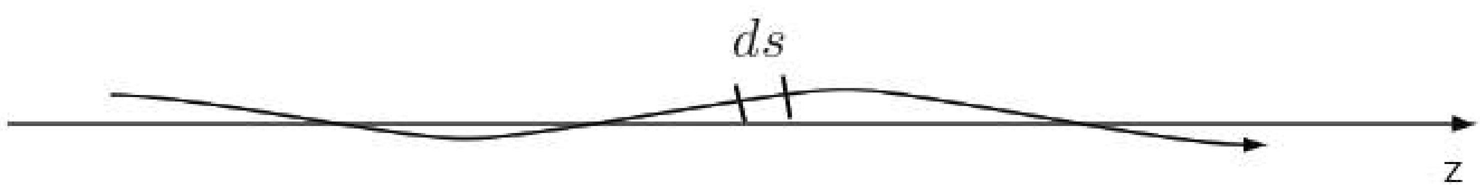
- simplification for paraxial rays,
- $\mathrm{d}s \approx \mathrm{d}z$,

$$\frac{\mathrm{d}}{\mathrm{d}z}(n\frac{\mathrm{d}x}{\mathrm{d}z}) \approx \frac{\partial n}{\partial x}, \frac{\mathrm{d}}{\mathrm{d}z}(n\frac{\mathrm{d}y}{\mathrm{d}z}) \approx \frac{\partial n}{\partial y},$$

- homogeneous medium: $n =$ constant,

$$\frac{\mathrm{d}^2 x}{\mathrm{d}z^2} = \frac{\mathrm{d}^2 y}{\mathrm{d}z^2} = 0,$$

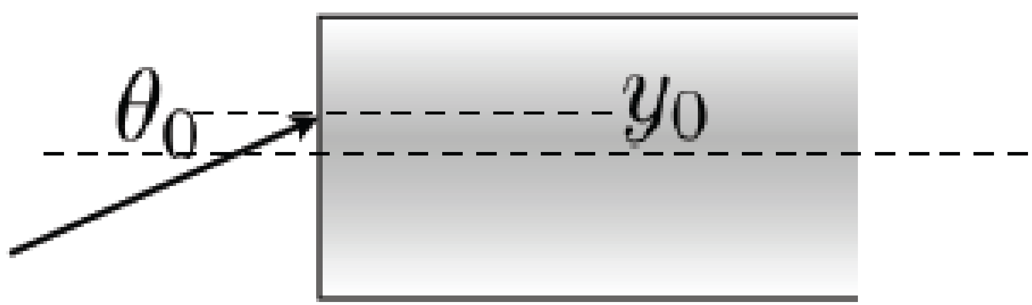
optical trajectory is a line,



GRIN - graded index optics

$$n(y) = n_0\sqrt{(1-\alpha^2 y^2)} \approx n_0(1-\frac{1}{2}\alpha^2 y^2),$$

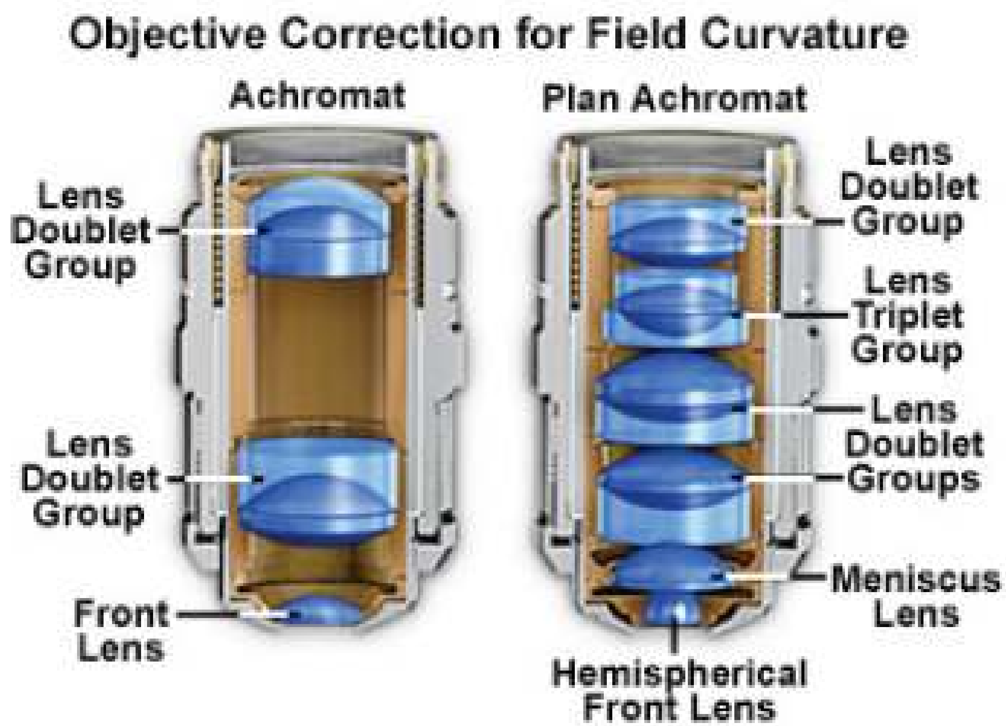
paraxial ray equation: $\frac{\mathrm{d}^2 y}{\mathrm{d}z^2} = -\alpha^2 y$, this is differential equation of oscillation,



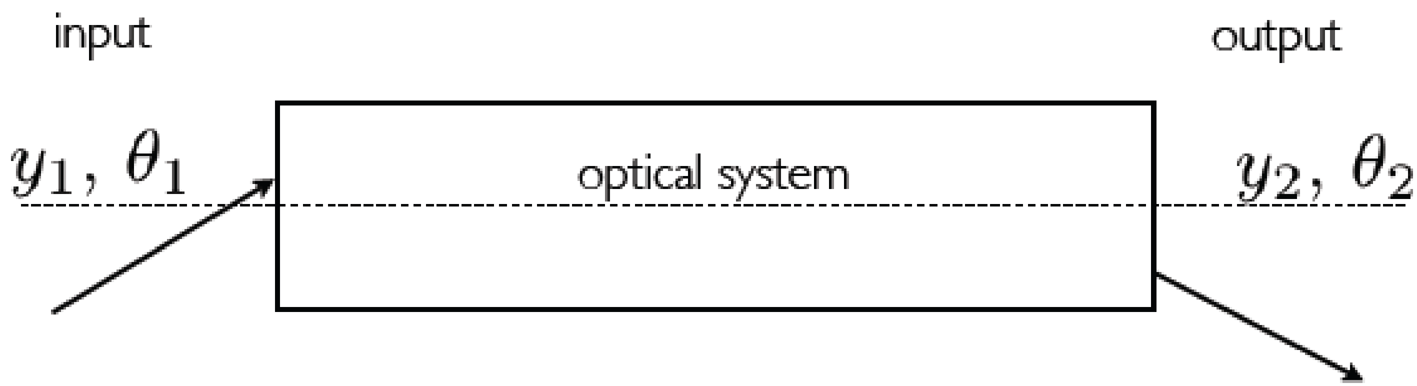
$$y(z) = y_0 \cos\alpha z + \frac{\theta_0}{\alpha}\sin\alpha z,$$

Real systems

- if the systems is more complex (lots of optical elements ...)
- we need fast algorithms to calculate ray propagation → matrix optics



Matrix optics



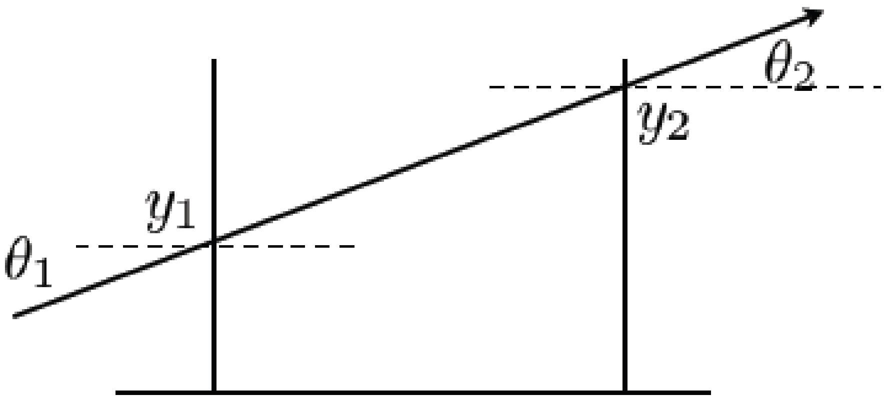
- for paraxial beams (small angles):

$$y_2 = Ay_1 + B\theta_1, \qquad \theta_2 = Cy_1 + D\theta_1,$$

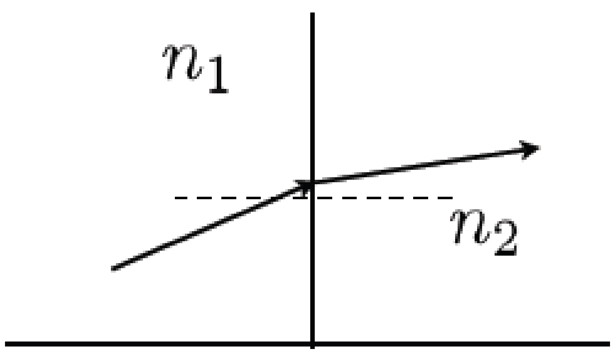
- matrix form, with a tranfer matrix **M**

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \mathbf{M} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix},$$

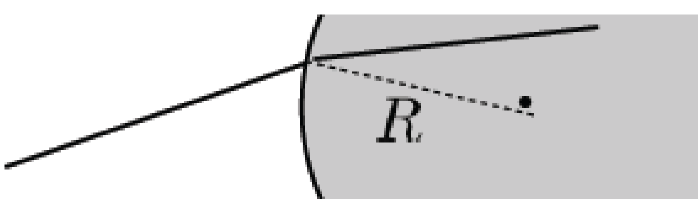
examples - matrix optics



$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix},$$

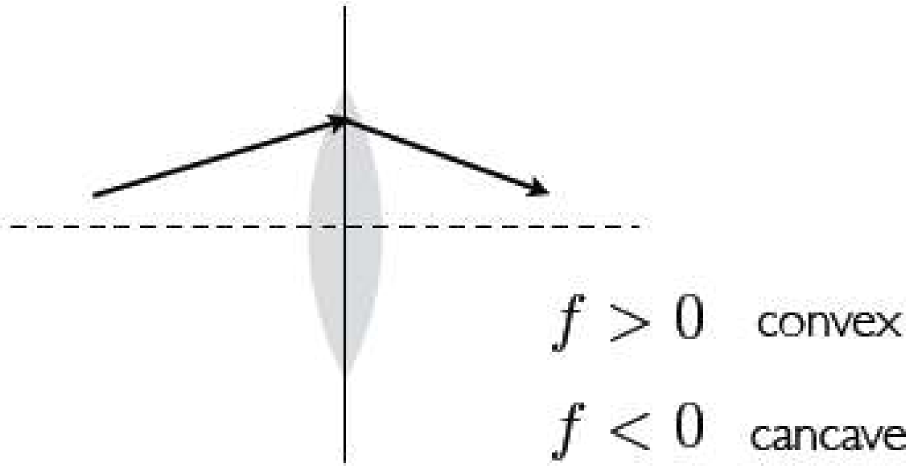


$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix},$$

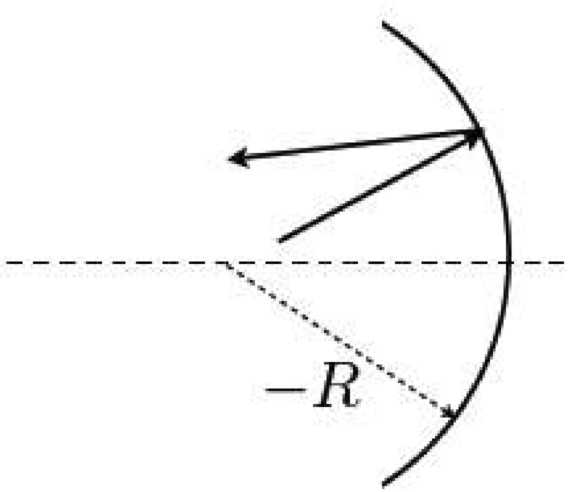


$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{n_2-n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix},$$

examples - matrix optics



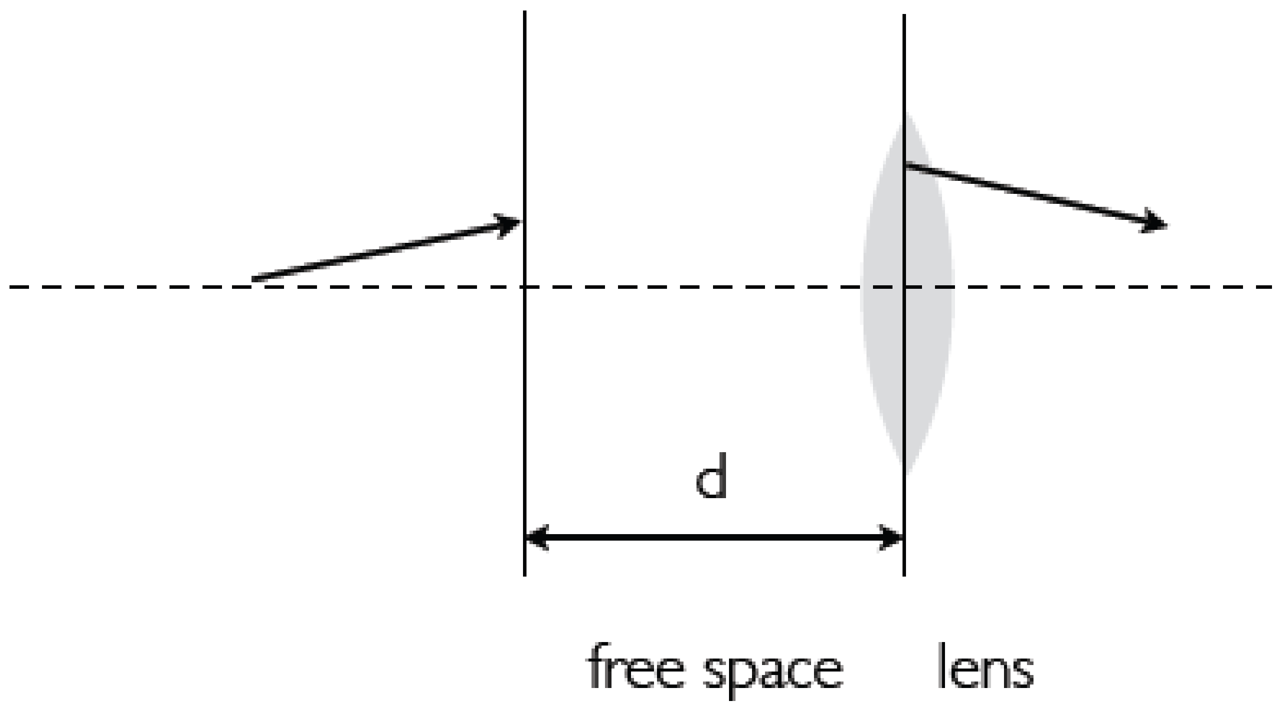
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 0 \end{bmatrix},$$



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix},$$

examples - matrix for an system of objects

$$\mathbf{M} = \mathbf{M}_N \mathbf{M}_{N-1} \cdots \mathbf{M}_1,$$



$$\mathbf{M}_1 = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \qquad \mathbf{M}_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 0 \end{bmatrix},$$

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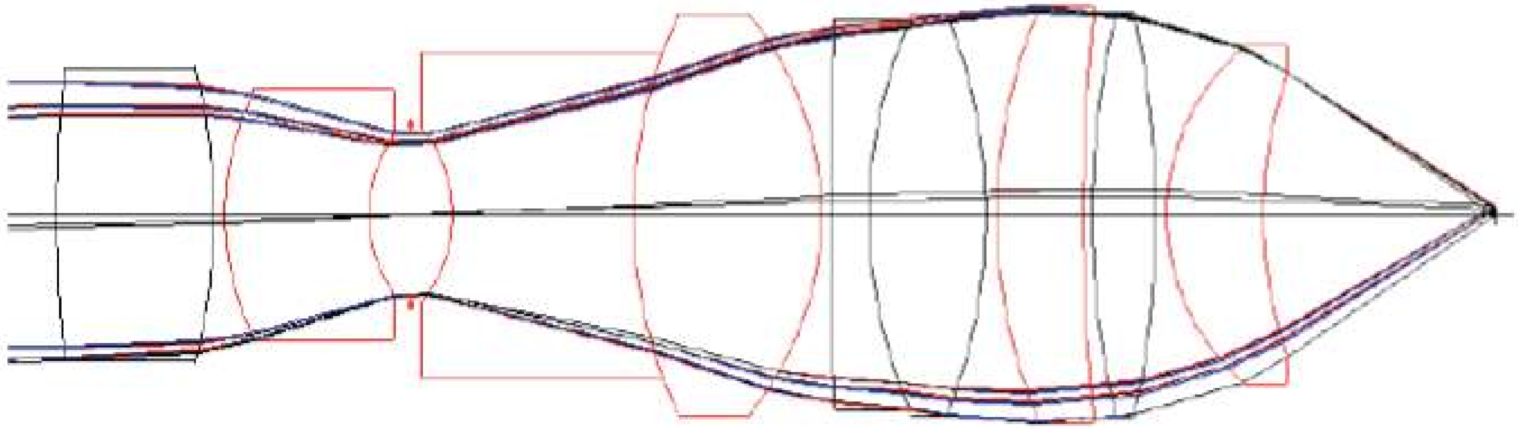
do it yourself matrix optics



Summary of Ray Optics

- Light propagates in rays
- Rays take path of shortest time (Fermat) straight in homogeneous media
- Rays are reflected and refracted at interfaces refraction
  - according to Snells law
  - spherical surfaces transform object points into image points for paraxial rays = imaging by lenses
  - total internal reflection for wave guiding
- matrix optics delivers fast way to evaluate complex optical systems

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