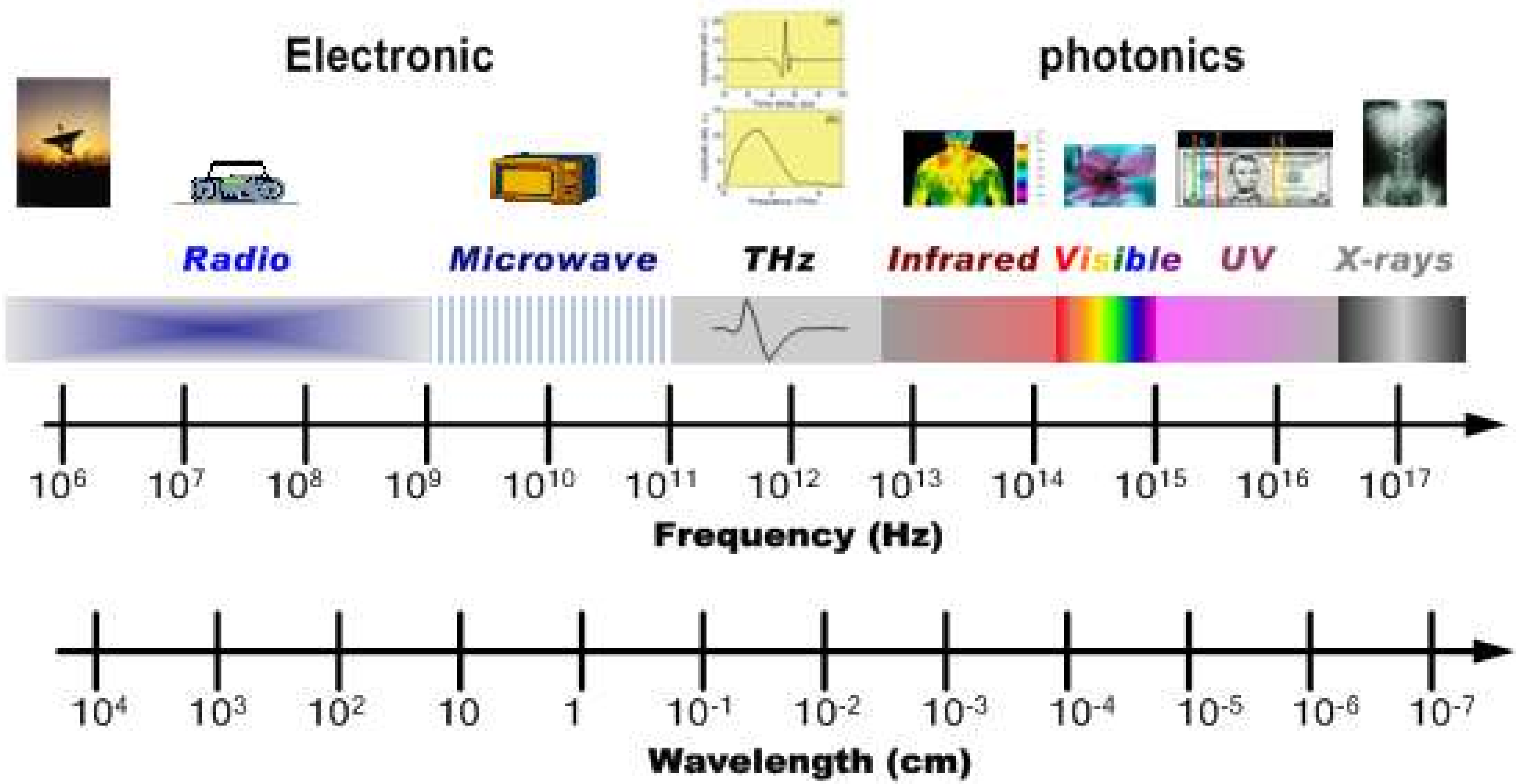
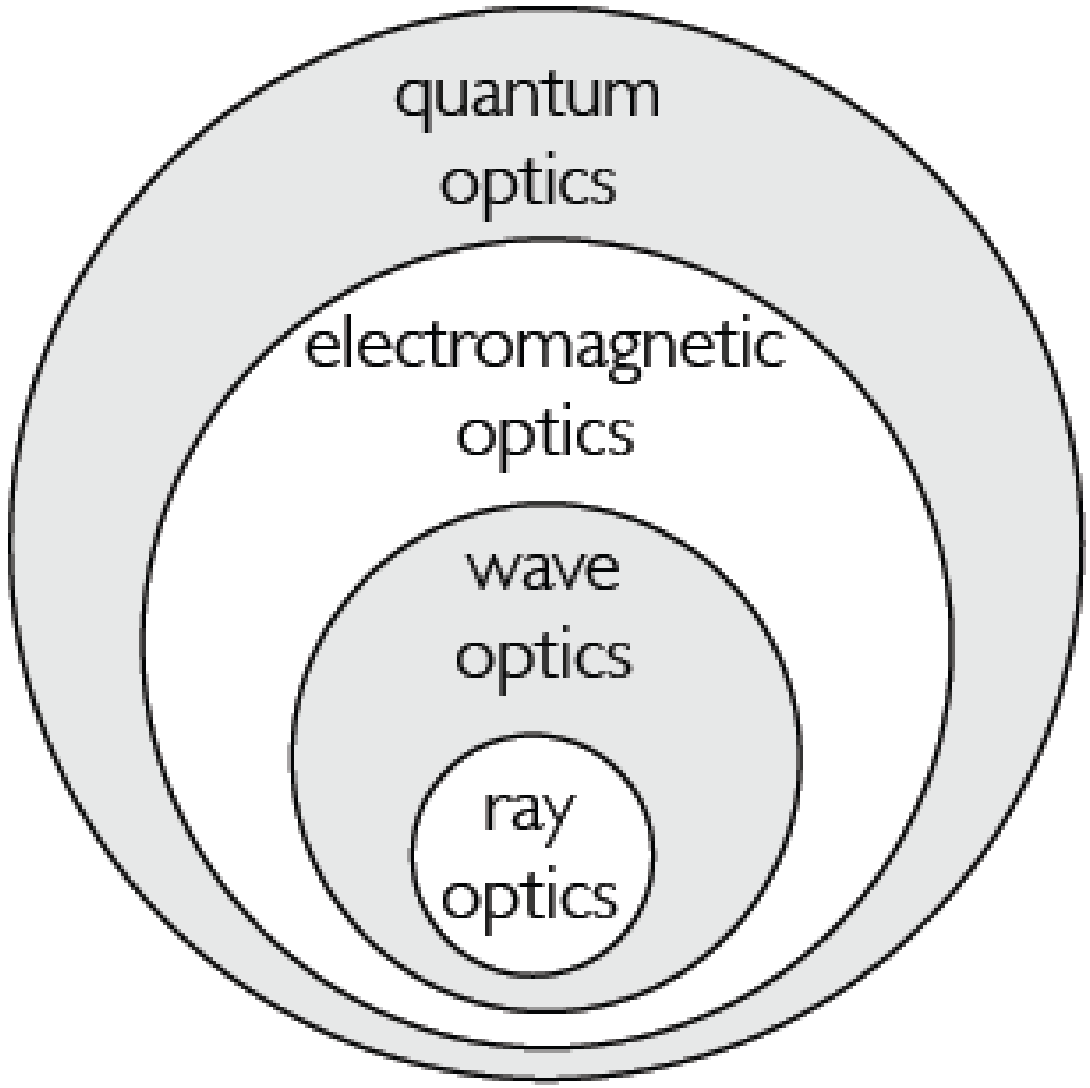


國立清華大學
National Tsing Hua University

General Optics



Wave Optics

- Ray optics is wave optics for infinitly small wavelength
- Wave optics:
  - Plane waves
  - Spherical waves
  - Interference
- Diffraction
- Gaussian beams

Postulates of ray optics

- light travels in form of waves
- optical medium is characterized by a quantity n

$$c = \frac{c_0}{n},$$

- optical wave satisfies wave equation of type

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = 0,$$

- wave equation is linear - superposition
  if u_1 and u_2 is solution then also $a_1 u_1 + a_2 u_2$,
- optical intensity, $I(r,t) = 2\langle u^2(r,t)\rangle$,
  i.e. averaging over times longer than 1 optical cycle,
- optical power, $P(t) = \int_A I(r,t)\mathrm{d}A$,

國立清華大學
National Tsing Hua University

Wave equations

- For a *source-free* medium, $\rho = \mathbf{J} = 0$,

$$\nabla \times (\nabla \times E) = -\mu\epsilon \frac{\partial^2}{\partial t^2} E,$$

$$\Rightarrow \qquad \nabla(\nabla \cdot E) - \nabla^2 E = -\mu\epsilon \frac{\partial^2}{\partial t^2} E.$$

- When $\nabla \cdot E = 0$, one has *wave equation*,

$$\nabla^2 E = \mu\epsilon \frac{\partial^2}{\partial t^2} E$$

- which has following expression of the solutions, in 1D,

$$\begin{aligned} E &= \hat{x}[f_+(z - vt) + f_-(z + vt)], \\ H &= \sqrt{\frac{\epsilon}{\mu}} \hat{y}[f_+(z - vt) - f_-(z + vt)], \end{aligned}$$

with

$$\mu\epsilon = \mu_0\epsilon_0(1 + \chi) = \frac{n^2}{c^2},$$

國立清華大學
National Tsing Hua University

Plane waves

- 1D *wave equation*,

$$\frac{\partial^2}{\partial z^2}E = \mu\epsilon\frac{\partial^2}{\partial t^2}E,$$

which has the solutions of

$E = \hat{x}[f_+(z - vt) + f_-(z + vt)]$, with

$$v^2 = \frac{1}{\mu\epsilon} = \frac{n^2}{c_0^2},$$

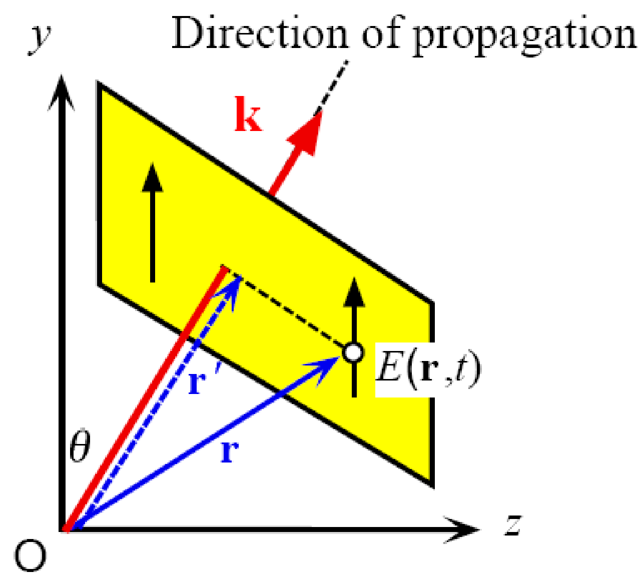
- plane wave solutions:

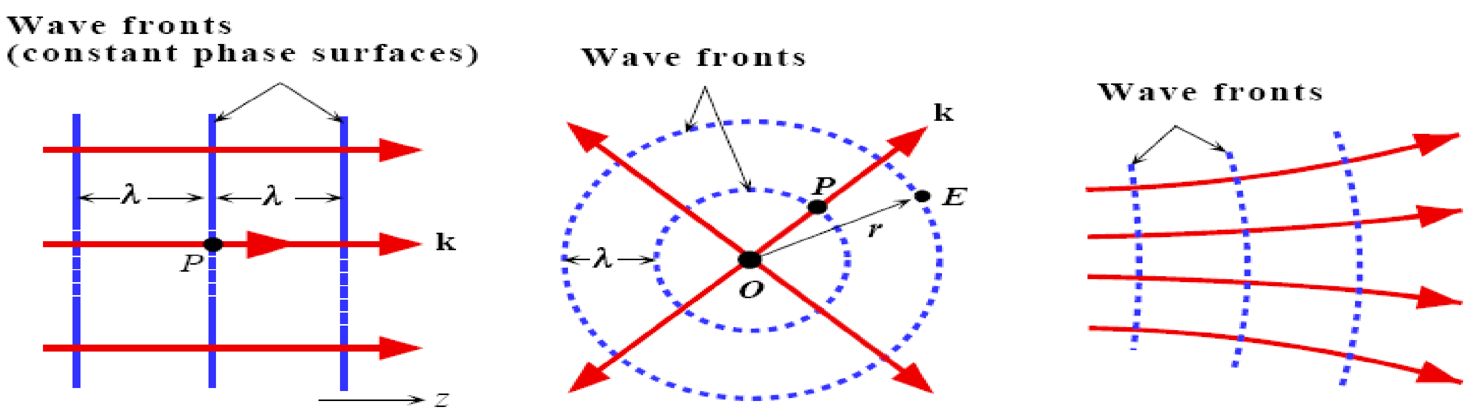
$$E_+ = E_0\cos(kz - \omega t),$$

where $\frac{\omega}{k} = \frac{c_0}{n}$.

Travelling waves

A travelling plane EM wave along a direction $\mathbf{k}$.





A perfect plane wave (a) A perfect spherical wave (b) A divergent beam (c)

solution of wave equation

- monochromatic wave is solution of wave equation

$$u(r,t) = a(r)\cos[\omega t + \phi(r)],$$

where $a(r)$ is the *amplitude*, $\omega = 2\pi\mu$ is the *frequency*, and $\phi(r)$ is the phase.

- complex representation:

$$u(r,t) = \textbf{Re}\{U(r,t)\} = \frac{1}{2}[U(r,t) + U^*(r,t)],$$

where

$$U(r,t) = a(r)\text{exp}[i\phi(r)]\text{exp}(i\omega t) = U(r)\text{exp}(i\omega t),$$

has to satisfy

$$\nabla^2 U - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}U = 0,$$

國立清華大學
National Tsing Hua University

complex representation and Helmholtz Equation

- separation time/space

$$U(r,t) = a(r)\text{exp}[i\phi(r)]\text{exp}(i\omega t) = U(r)\text{exp}(i\omega t),$$

where

$$U(r) iscomplex,$$

with the amplitude $|U(r)|$ and the phase arg$\{U(r)\}$,

- Helmholtz wave equation

$$(\nabla^2 + k^2)U(r) = 0,$$

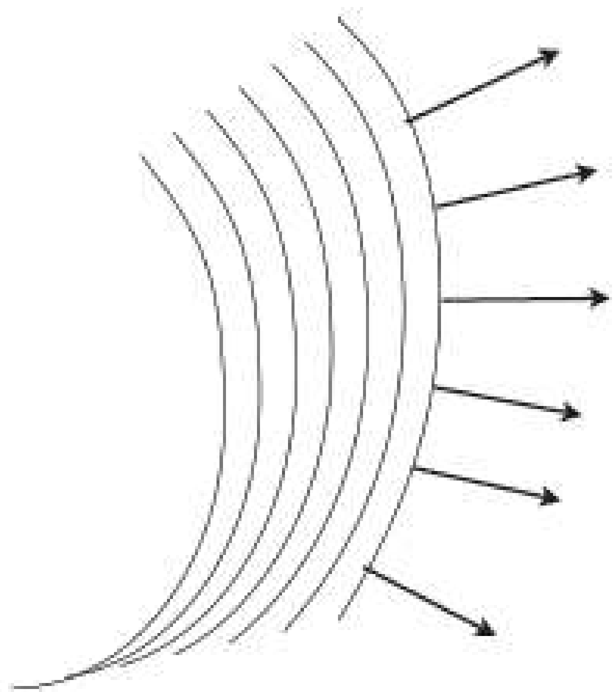
where

$$k = \frac{\omega}{c}, \qquad \text{wavenumber},$$

國立清華大學
National Tsing Hua University

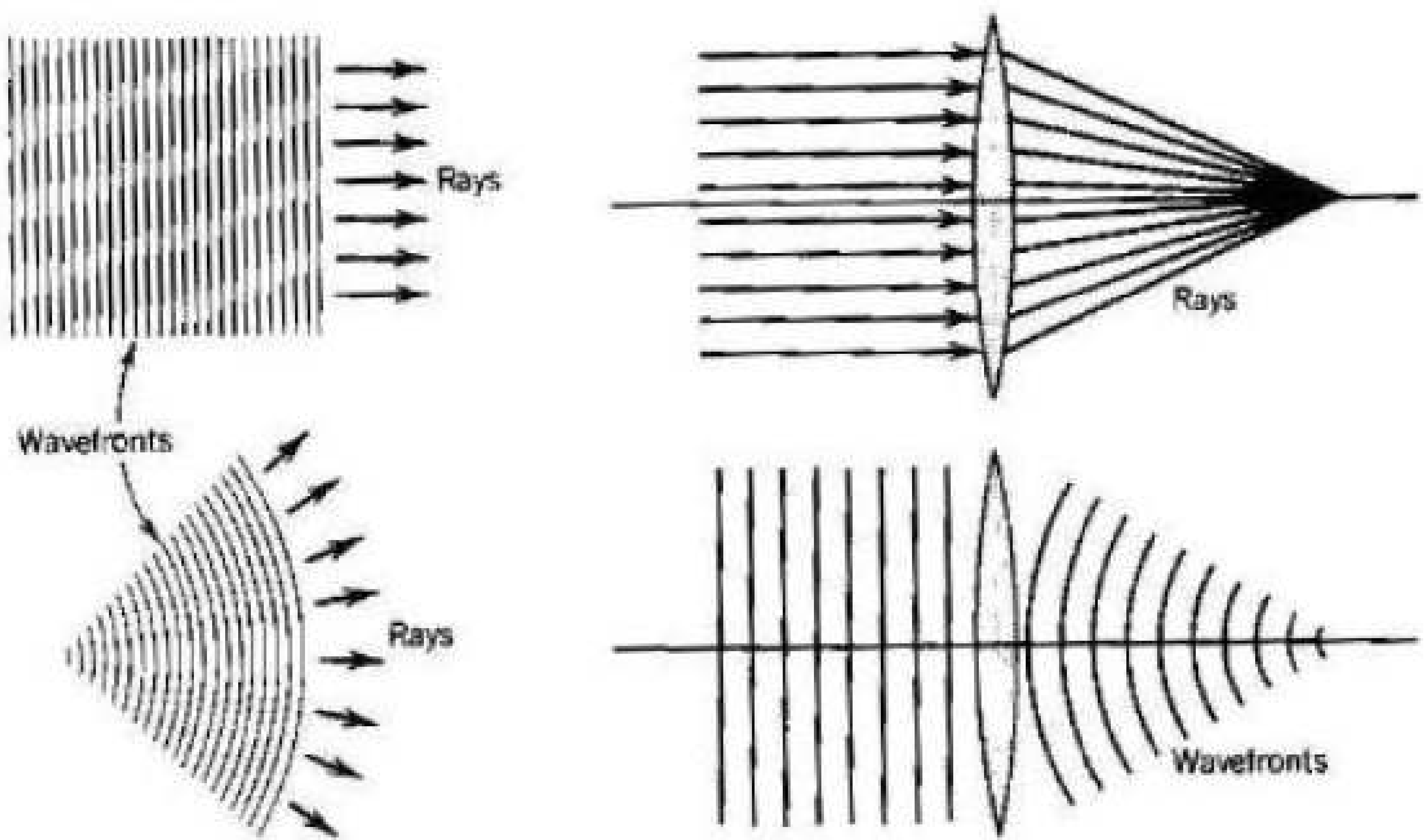
relation to ray optics

- intensity: $I(r) = |U(r)|^2$,
- wavefront: $\phi(r) =$ constant,
- rays are normals to the wavefronts change in the curvature of wavefronts bends rays



國立清華大學
National Tsing Hua University

Link between wave optics and ray optics



plane waves

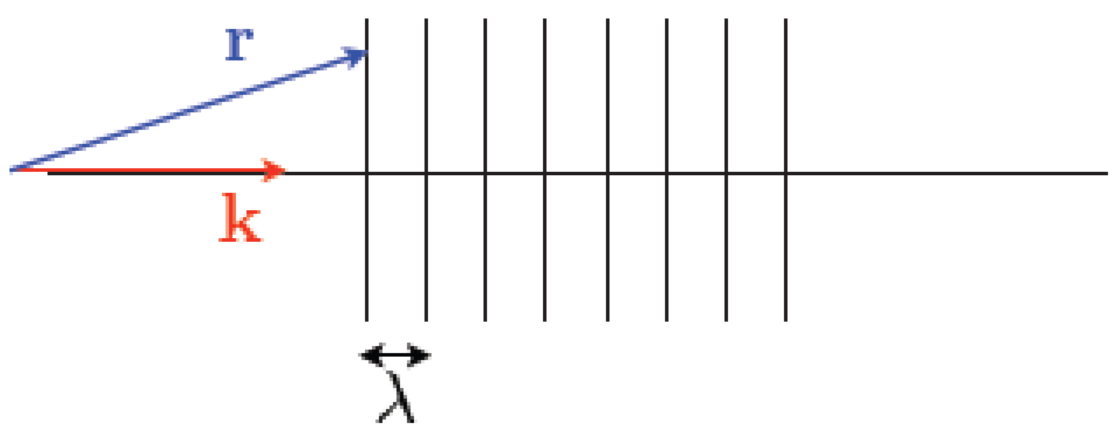
- plane wave

$$U(r) = A\exp(-i\mathbf{k}\cdot\mathbf{r}),$$

where **k** is the wavevector, defines propagation direction,

$$\mathbf{k}\cdot\mathbf{r} = 2\pi n, \qquad n \quad \text{is an integer},$$

- distance between neighboring wavefronts, $\lambda = \frac{2\pi}{k} = \frac{c}{\mu}$,
- in a medium with refractive index n, $\lambda = \frac{c}{\mu} = \frac{c_0}{n\mu} = \frac{\lambda_0}{n}$, and $k = nk_0$,
- intensity, $I(r) = |A|^2$



As a monochromatic wave propagates through media of different refractive indices its **frequency remains the same** but its **velocity, wavelength and wavenumber are altered**.

spherical waves

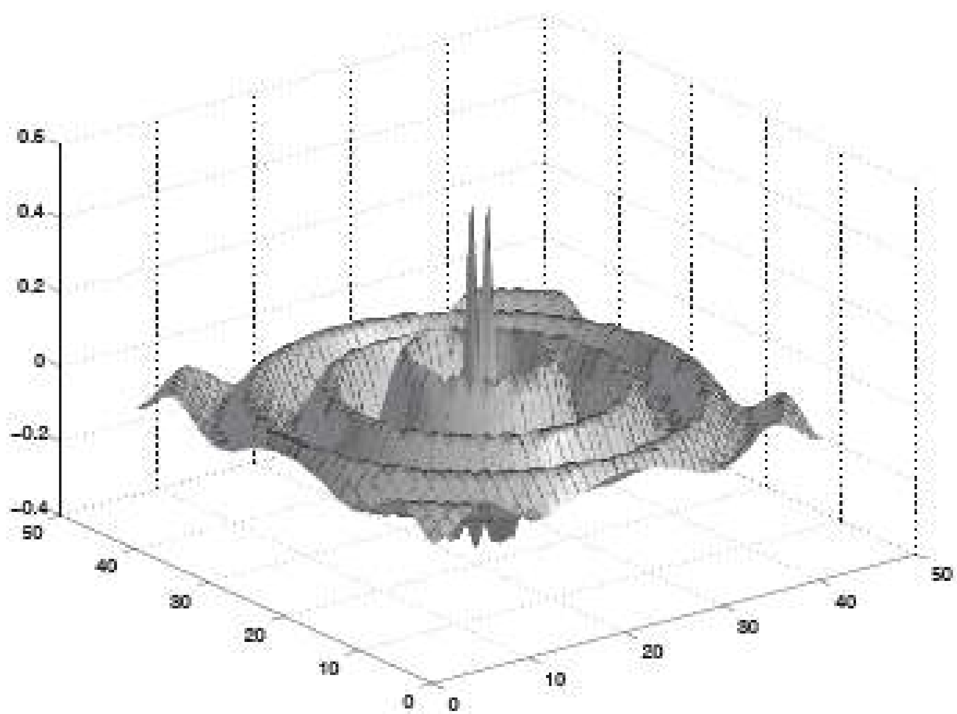
- spherical wave:

$$U(r) = \frac{A}{|r - r_0|}\exp(-ik|r - r_0|),$$

where $k|r - r_0| =$ constant, wavefronts resemble sphere surfaces,

- intensity:

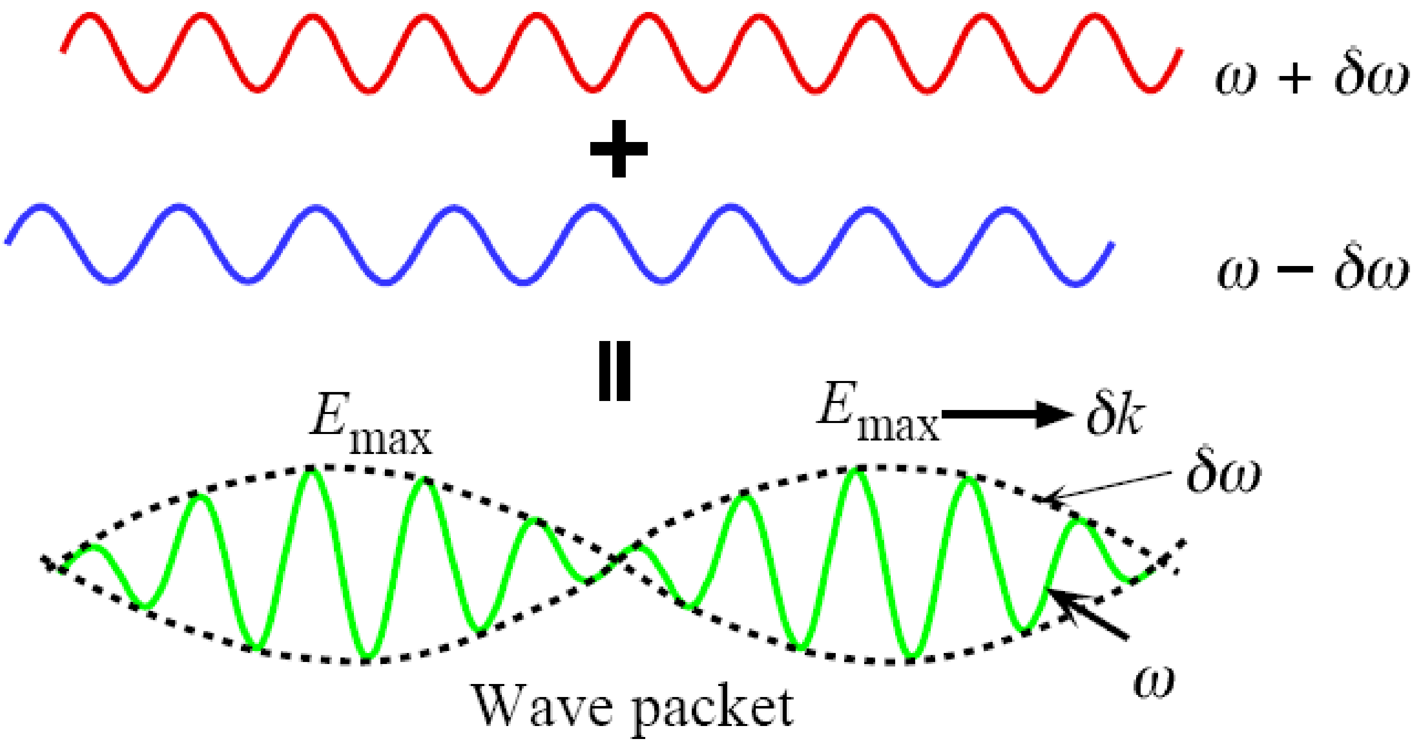
$$I(r) = \frac{|A|^2}{r^2},$$



國立清華大學
National Tsing Hua University

Phase velocity and Group velocity

- phase velocity: $v_p = c = \frac{c_0}{n} = \frac{\omega}{k}$,
- group velocity: $v_G = \frac{\mathrm{d}\omega}{\mathrm{d}k}$,



Interference (spatial)

- superposition of two monochromatic waves of the same frequency,

$$U(r) = U_1(r) + U_2(r),$$

- itensity:

$$I = |U(r)|^2 = |U_1 + U_2|^2 = |U_1|^2 + |U_2|^2 + U_1^* U_2 + U_1 U_2^*,$$

- define:

$$U_1 = I_1^{1/2} \text{exp}(i\phi_1), \qquad U_2 = I_2^{1/2} \text{exp}(i\phi_2),$$

then

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos\phi,$$

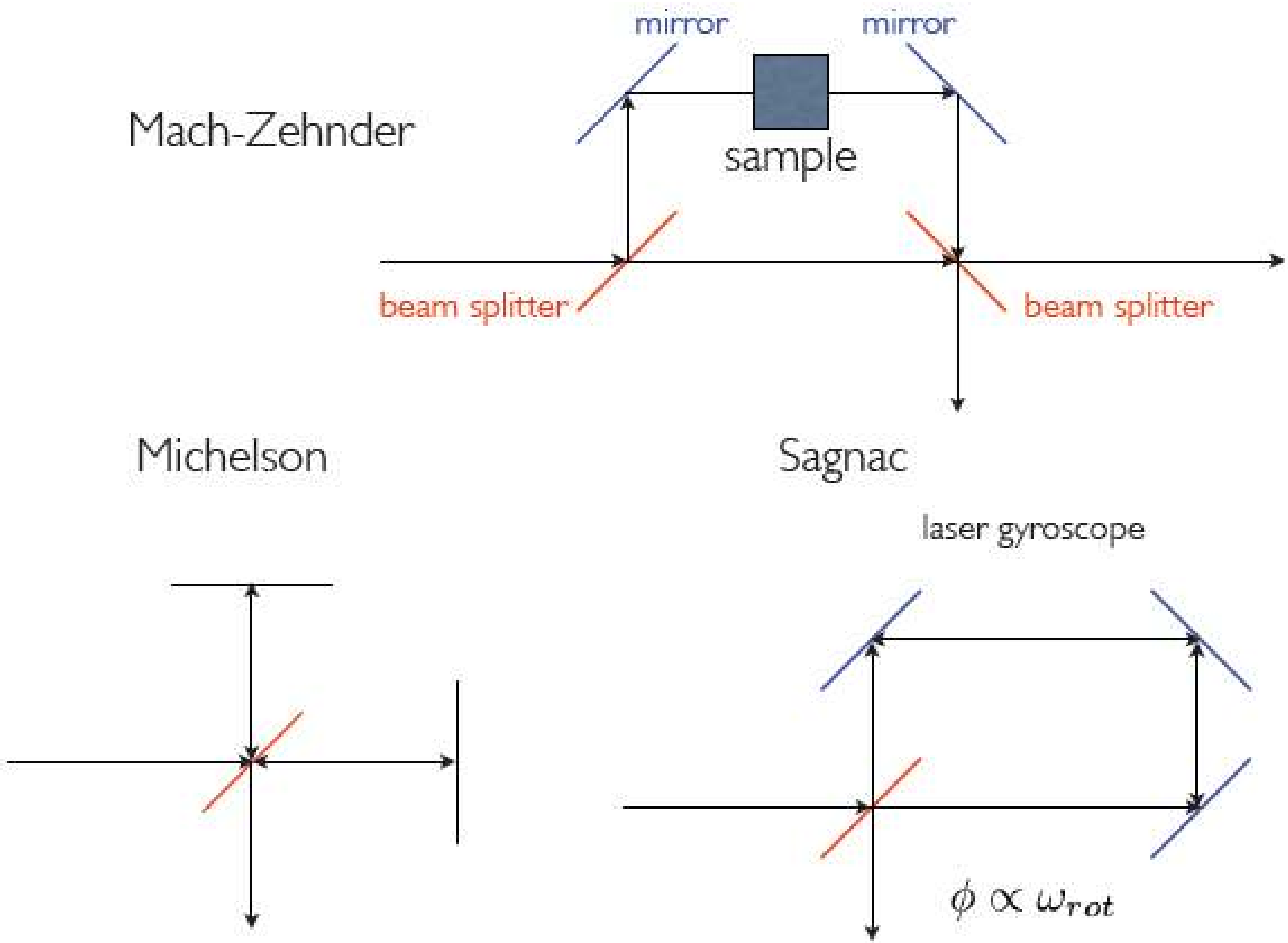
where

$$\phi = \phi_2 - \phi_1,$$

the phase can be measured by interference.

國立清華大學
National Tsing Hua University

Interferometers



laser gyro in F16

國立清華大學
National Tsing Hua University

Interference (temporal)

- superposition of two monochromatic waves of different frequency,

$$\begin{aligned} U_1(\mathbf{r},t) &= A_1 \text{exp}(-i\mathbf{k}_1 \cdot \mathbf{r})\text{exp}(i\omega_1 t), \\ U_2(\mathbf{r},t) &= A_2 \text{exp}(-i\mathbf{k}_2 \cdot \mathbf{r})\text{exp}(i\omega_2 t), \end{aligned}$$

- at fixed **r**,

$$U(t) = I_1^{1/2}\text{exp}(i\omega_1 t) + I_2^{1/2}\text{exp}(i\omega_2 t),$$

- the intensity,

$$I(t) = I_1 + I_2 + 2(I_1 I_2)^{1/2}\cos[(\omega_2 - \omega_1)t],$$

- light beating at the frequency,

$$\mu = \frac{\omega_2 - \omega_1}{2\pi},$$

國立清華大學
National Tsing Hua University

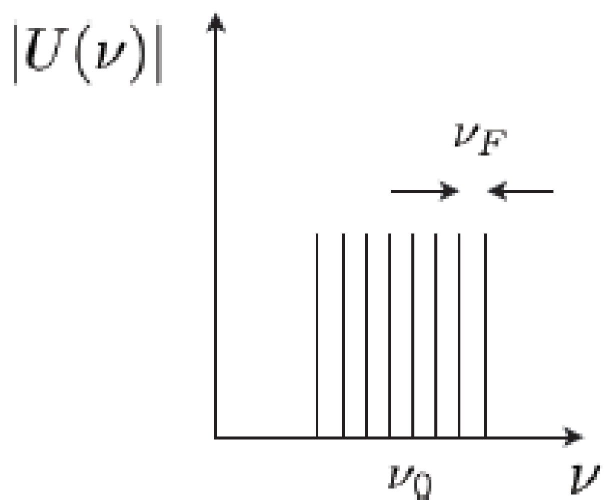
multiple-wave interference (temporal with M waves)

- equal amplitudes and equal phase differences, such as Fabry-Perot filter, Bragg filter,
- the total scalar field is thus the summation

$$U(t) = I_0^{1/2} \sum_{q=-M}^{M} \exp(i\omega_q t),$$

where

$$\omega_q = 2\pi\mu_q = 2\pi(\mu_0 + q\mu_F),$$



國立清華大學
National Tsing Hua University

multiple-wave interference (temporal with M waves)

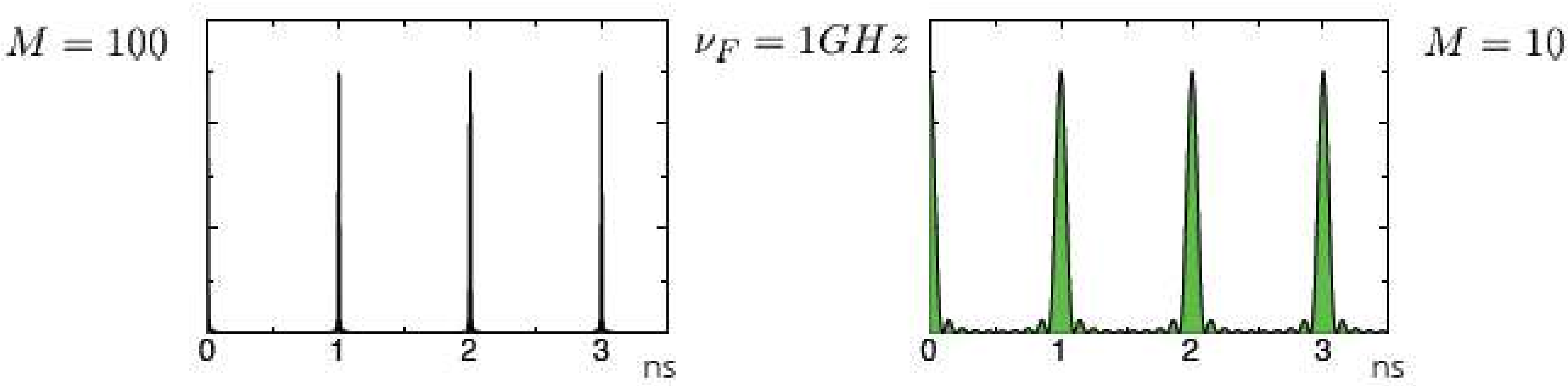
- multiple-wave interference,

$$U(t) = I_0^{1/2} \sum_{q=-M}^{M} \exp(i\omega_q t),$$

- intensity:

$$I(t) = |U(t)|^2 = I_0 \frac{\sin^2(M\pi\mu_F t)}{\sin^2 \pi\mu_F t)},$$

- acts as a high-finess/high-Q filter,



National Tsing Hua University

Summary of Wave Optics

- light propagates in form of waves
- wave equation in its simplest form is linear, which gives rise to superposition and separation of time and space dependence (interference, diffraction)
- waves are characterized by wavelength and frequency
- propagation through media is characterized by refractive index n, which describes the change in phase velocity
- media with refractive index n alter velocity, wavelength and wavenumber but not frequency
- lenses alter the curvature of wavefronts

paraxial wave approximation

- paraxial wave = wavefronts normals are paraxial rays

$$U(r) = A(r)\text{exp}(-ikz),$$

- $A(r)$ slowly varying with at a distance of λ,
- paraxial Helmholtz equation

$$(\nabla^2 + k^2)U(r) = 0,$$
$$\rightarrow (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - 2ik\frac{\partial}{\partial z})A(r) = 0,$$

- solution of the paraxial Helmholtz equation is the $Gaussian$ beams,

國立清華大學
National Tsing Hua University