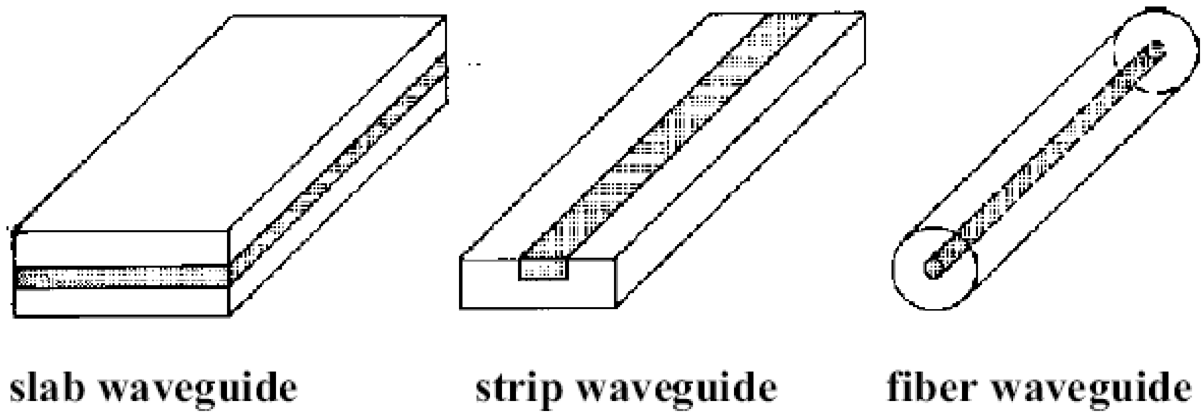
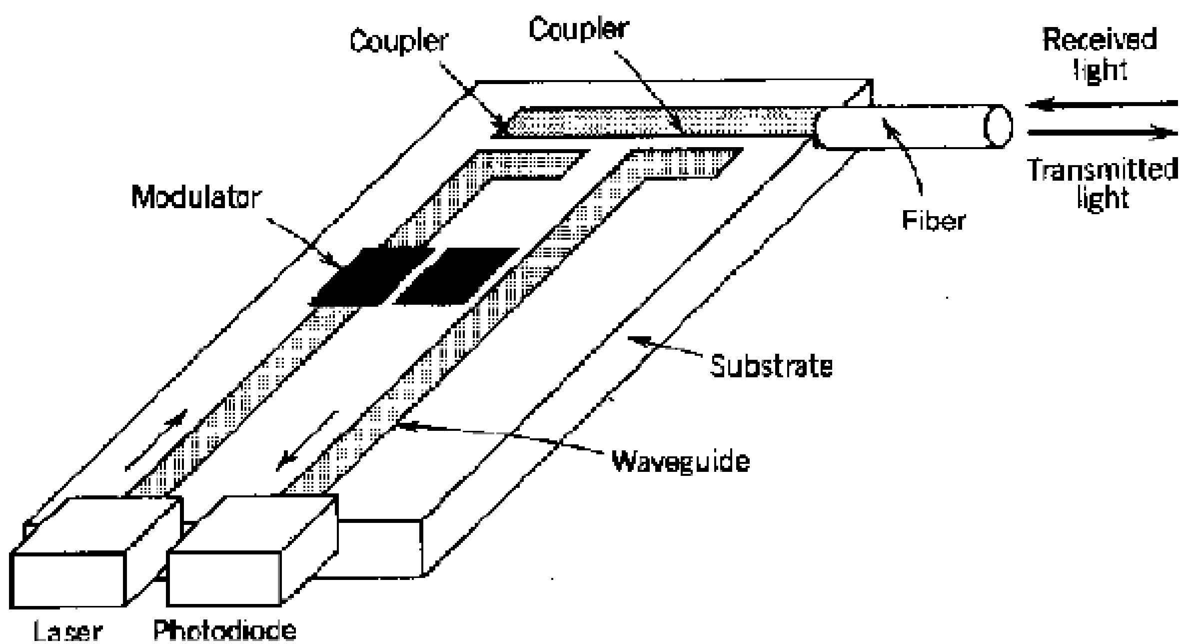
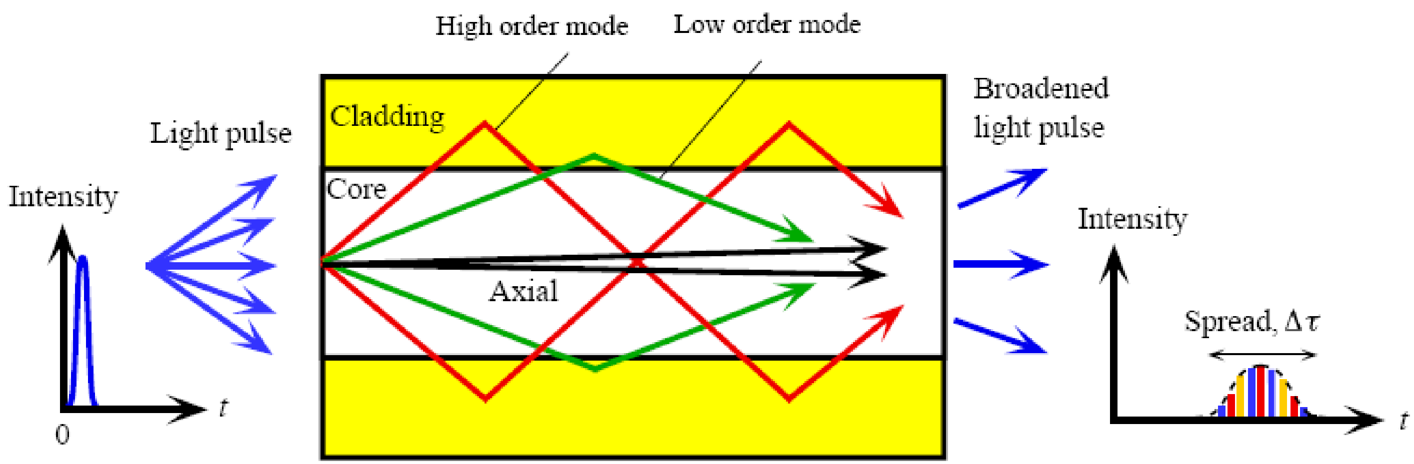
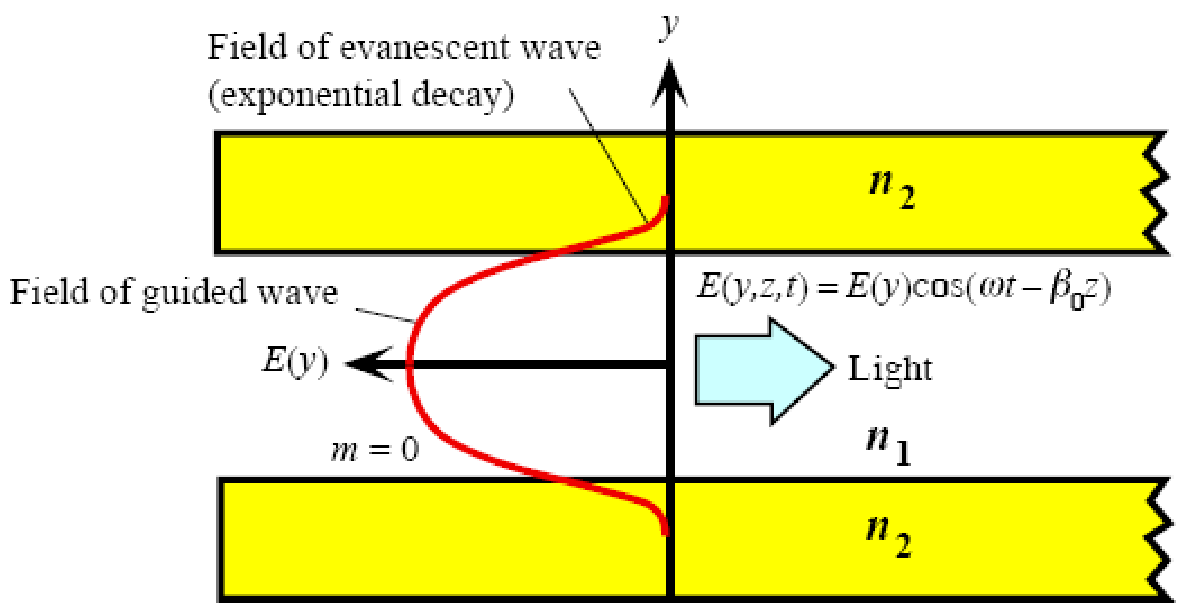
# Integrated Optics



# Waveguides

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# Wave equation in nonuniform dielectric

$$\nabla^2 A - \mu_0 \epsilon \frac{\partial^2 A}{\partial t^2} \approx 0,$$

- here $\epsilon(\rho)$ is not a constant, which represents an axially uniform medium, but with radial variation.
- We write

$$A = \hat{y} u(x, y) e^{-j\beta z},$$

where $\beta$ is an unknown propagation constant.
- The wave equation for $u(x, y)$ is,

$$\nabla_T^2 u + [\omega^2 \mu_0 \epsilon(\rho) - \beta^2] u = 0,$$

where $\rho = \hat{x}x + \hat{y}y$.

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# Wave equation in nonuniform dielectric

- The wave equation for $u(x, y)$ is,

$$\nabla_T^2 u + [\omega^2 \mu_0 \epsilon(\rho) - \beta^2]u = 0,$$

- Compared to the Schrödinger equation of a particle in a two-dimensional potential $V$, i.e.

$$\frac{-\hbar^2}{2m}\nabla_T^2 \Psi + (V - E)\Psi = 0,$$

then the solutions are bounded, if and only if, there are local negative values of the function,

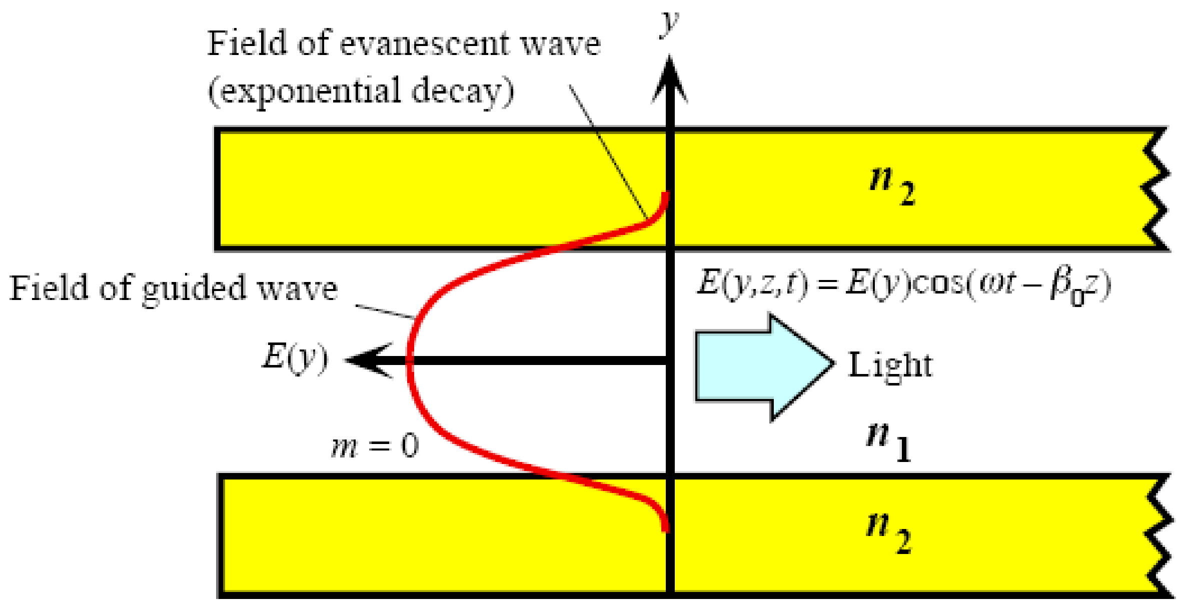
$$|V| - |E| > 0,$$

with

$$V = -\omega^2 \mu_0 \epsilon(\rho), \qquad E = -\beta^2.$$

Bounded solutions correspond to guided waves and are found only for specific values of the eigenvalues $\beta^2$.

# Slab-waveguides




- transverse Electric modes:

$$
\begin{aligned}
E_x &= A\cos(k_y y)e^{-j\beta_0 z}, \qquad |y| < d, \\
&= Be^{-j\beta_0 z}e^{-\alpha_y y}, \qquad y > d, \\
&= Be^{-j\beta_0 z}e^{\alpha_y y}, \qquad y < -d,
\end{aligned}
$$

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# Slab-waveguides

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&= Be^{-j\beta_0 z}e^{\alpha_y y}, \qquad y < -d,
\end{aligned}
$$

- the magnetic field follows from Faraday's law,

$$
\begin{aligned}
H_z &= \frac{jk_y}{\omega\mu_0}A\sin(k_y y)e^{-j\beta_0 z}, \qquad |y| < d, \\
&= \frac{j\alpha_y}{\omega\mu_0}Be^{-j\beta_0 z}e^{-\alpha_y y}, \qquad y > d, \\
&= -\frac{j\alpha_y}{\omega\mu_0}Be^{-j\beta_0 z}e^{\alpha_y y}, \qquad y < -d.
\end{aligned}
$$

- Continuity of $E_x/H_z$ at $y = d$ gives,

$$
\tan(k_y d) = \frac{\alpha_y}{k_y}.
$$

# Slab-waveguides

- From the wave equation, we have

$$\begin{aligned} \beta_0^2 - \alpha_y^2 &= \omega^2 \mu_0 \epsilon_2, \\ \beta_0^2 + k_y^2 &= \omega^2 \mu_0 \epsilon_1, \end{aligned}$$

- and combine these two equations,

$$\frac{\alpha_y}{k_y} = \sqrt{\frac{\omega^2 \mu_0 (\epsilon_1 - \epsilon_2)}{k_y^2} - 1},$$

- one can find the dispersion diagram, the dependence of the propagation constant $\beta$ on frequency,

$$\tan(k_y d) = \sqrt{\frac{\omega^2 \mu_0 (\epsilon_1 - \epsilon_2)}{k_y^2} - 1},$$

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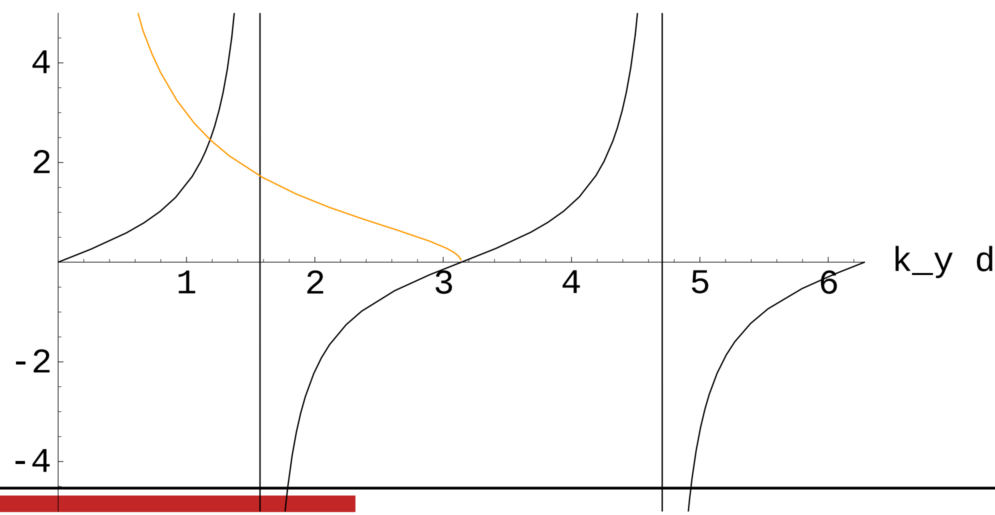
# Slab-Waveguides

$$\tan(k_y d) = \sqrt{\frac{\omega^2 \mu_0 (\epsilon_1 - \epsilon_2)}{k_y^2} - 1},$$

- For decreasing $\omega$, $\alpha_y / k_y$ moves toward the origin and intersections are lost, except for the first branch of the $\tan$ function.
- This corresponds to the dominant mode, $m = 0$, with no *cutoff*.
- At low frequency, the fundamental mode acquires a small $k_y$, $\tan(k_y d) \approx k_y d$,

$$\omega^2 \mu_0 (\epsilon_1 - \epsilon_2) d^2 - k_y^2 d^2 \approx k_y^4 d^4.$$

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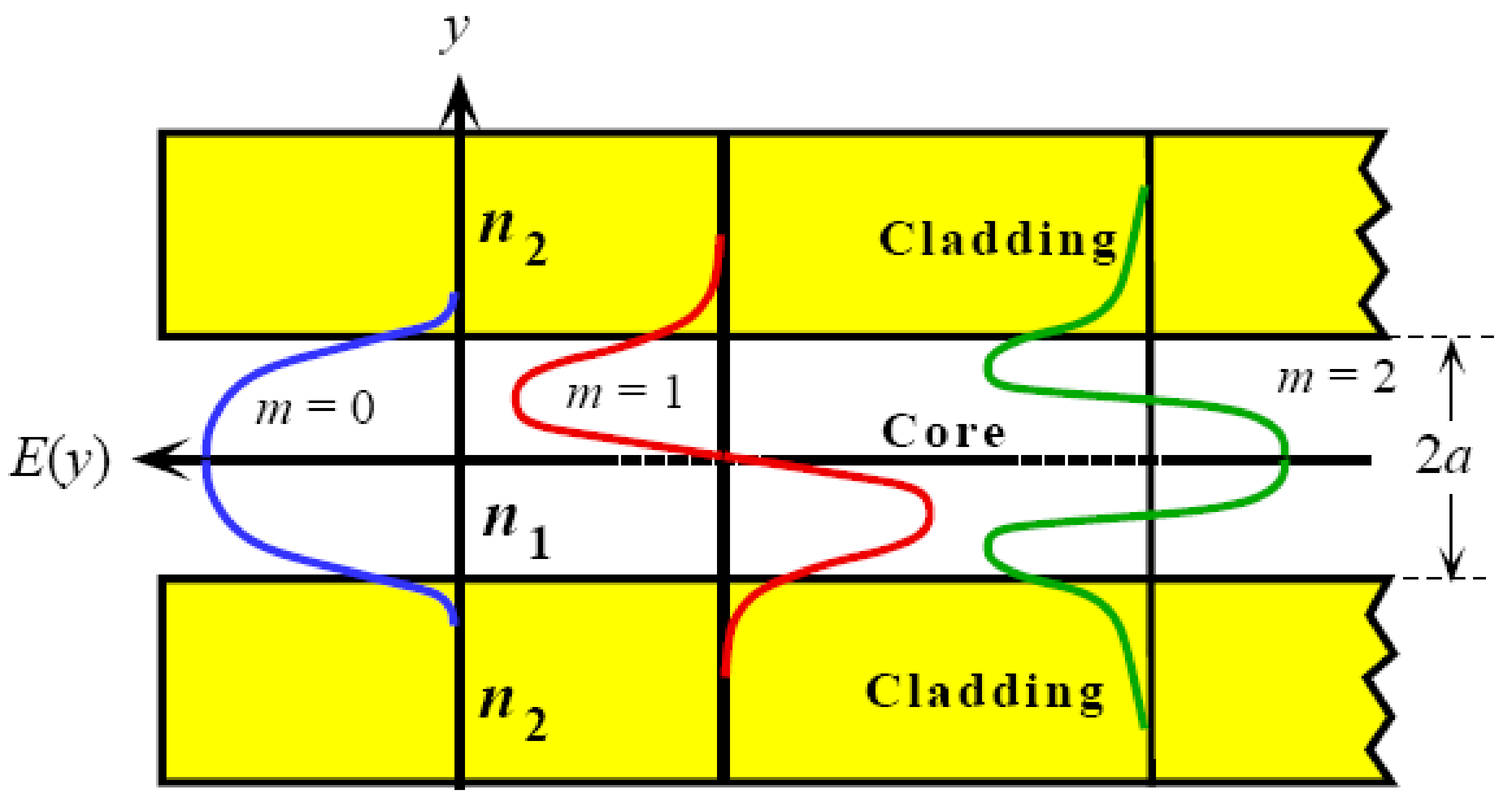
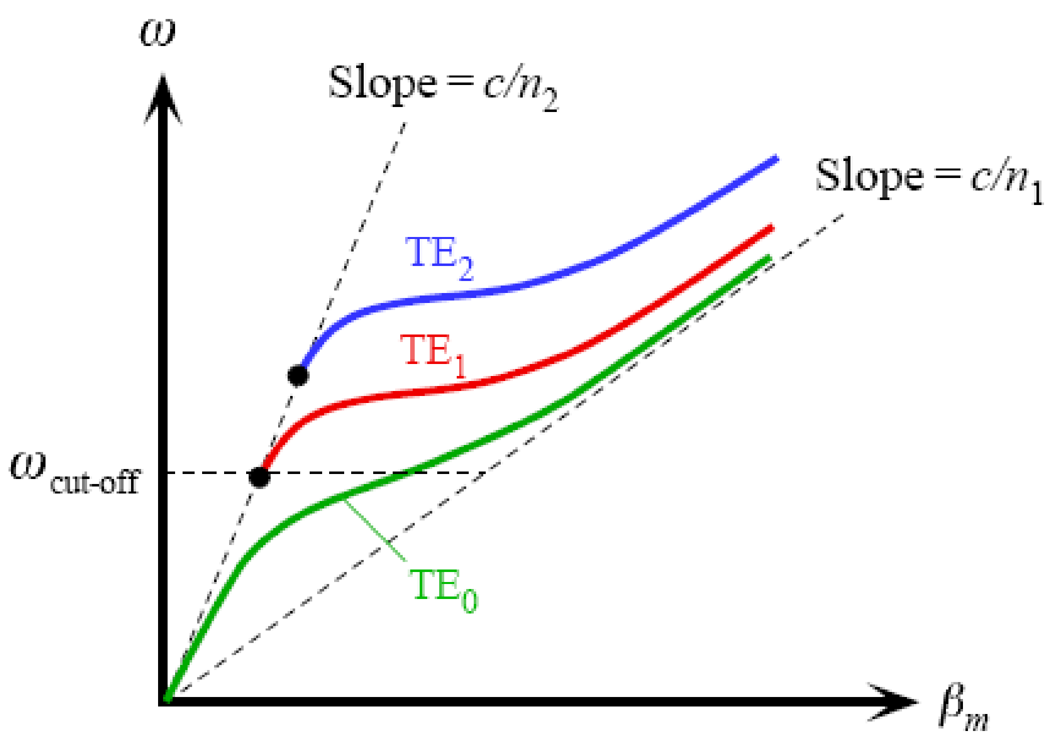
# Slab-Waveguides



# Slab-Waveguides

- Neglecting $k_y^4 d^4$ compared with $k_y^2 d^2$, we have $k_y^2 = \omega^2 \mu_0 (\epsilon_1 - \epsilon_2)$ and $\beta_0 \approx \omega \sqrt{\mu_0 \epsilon_2}$.
- The wave propagates at the speed characteristic of the external region.
- In the other hand, when $\omega \to \infty$, $k_y d$ approaches $\pi/2$ and we find that $\beta_0 \approx \omega \sqrt{\mu_0 \epsilon_1}$.

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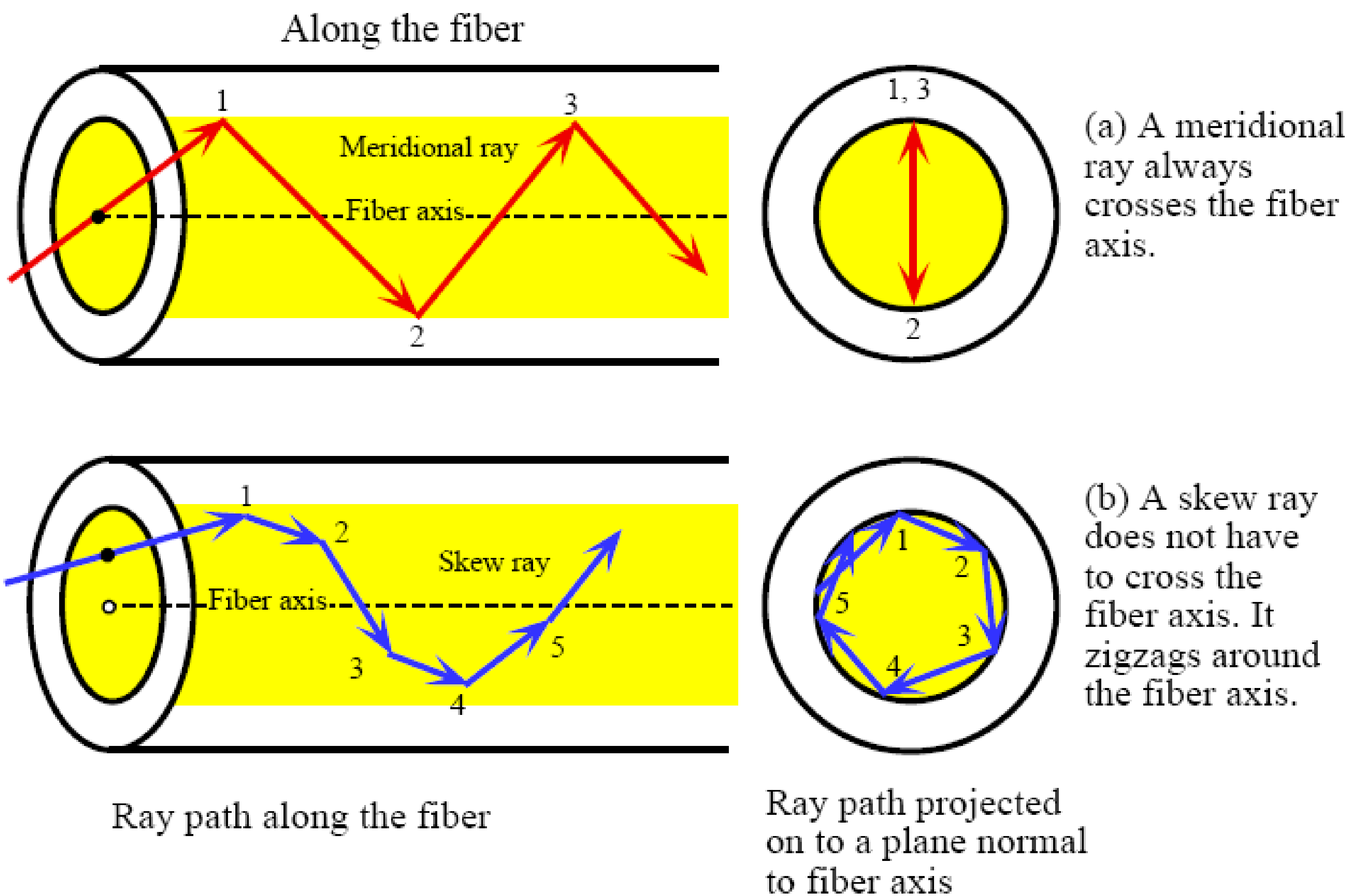
# Optical fibers



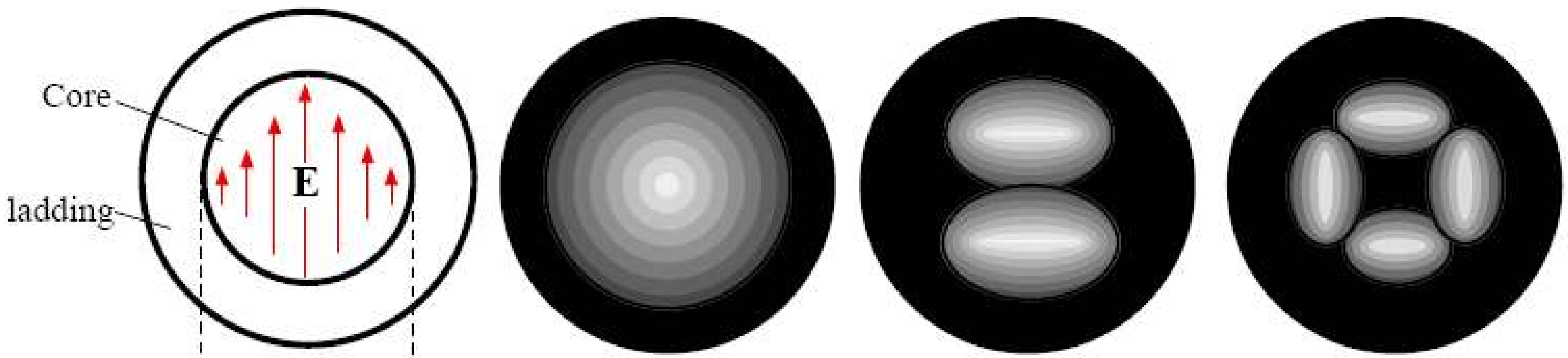
# Optical fibers

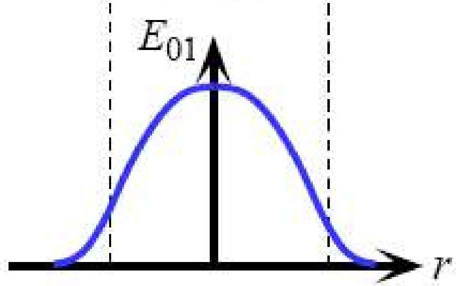
(a) The electric field of the fundamental mode

(b) The intensity in the fundamental mode $LP_{01}$

(c) The intensity in $LP_{11}$

(d) The intensity in $LP_{21}$




The electric field distribution of the fundamental mode in the transverse plane to the fiber axis z. The light intensity is greatest at the center of the fiber. Intensity patterns in $LP_{01}$, $LP_{11}$ and $LP_{21}$ modes.

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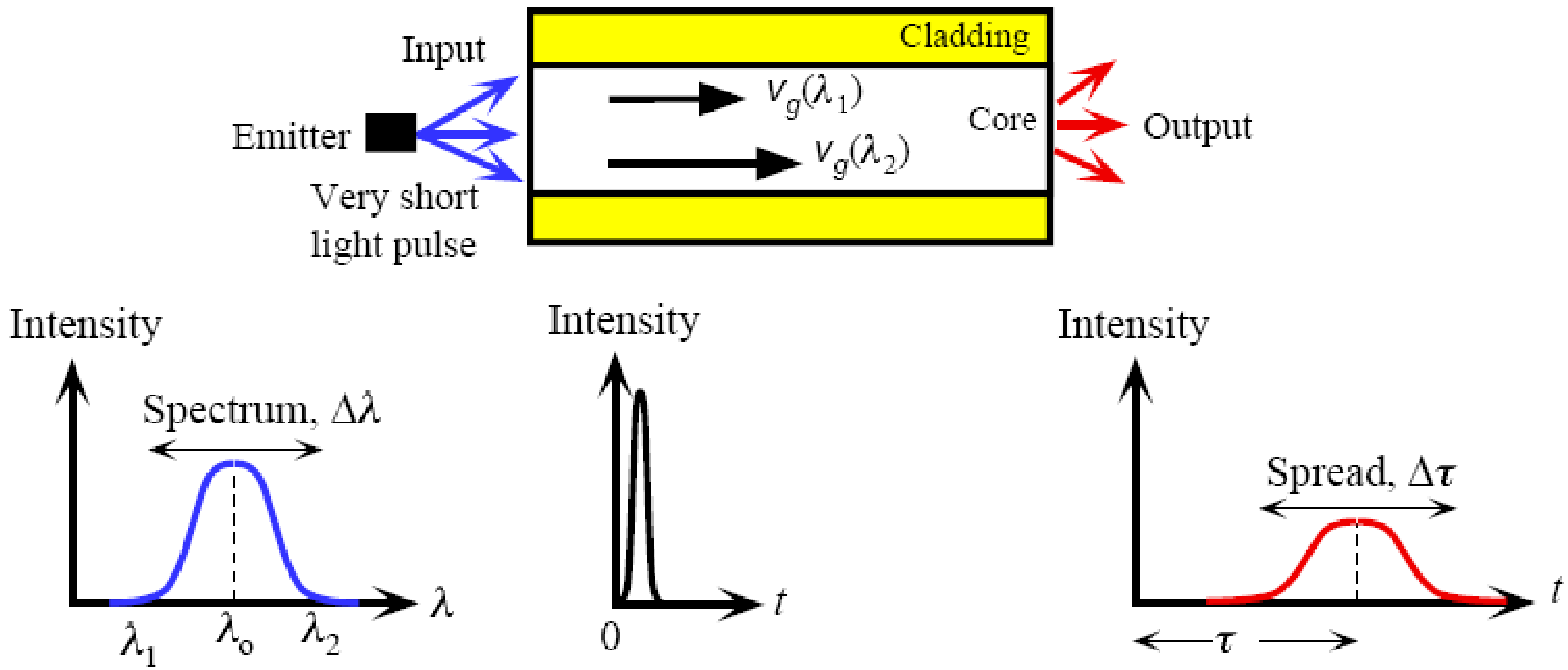
# Optical fibers



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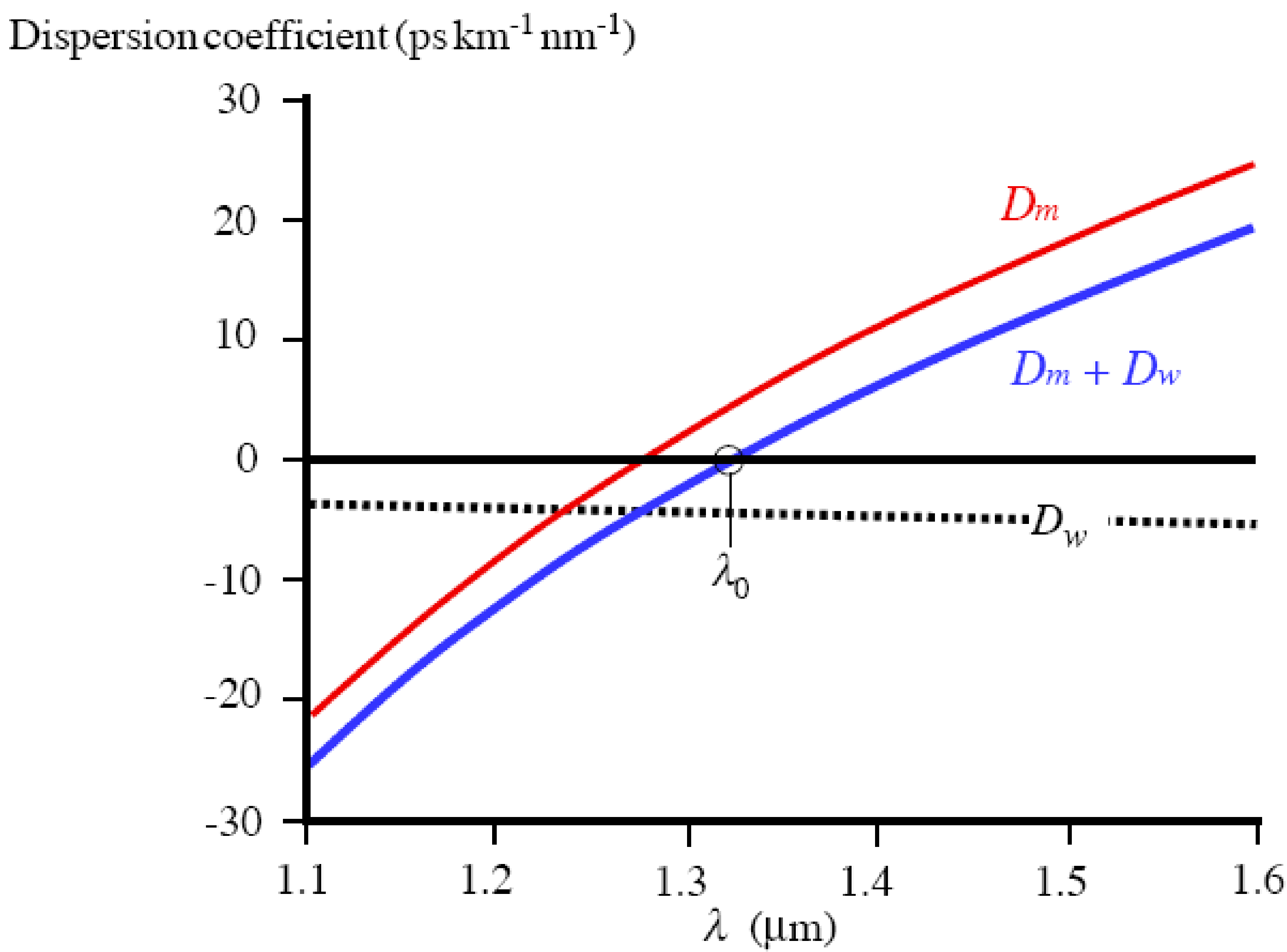

# Optical fibers