IPT 5260

Computational Methods for Optoelectronics

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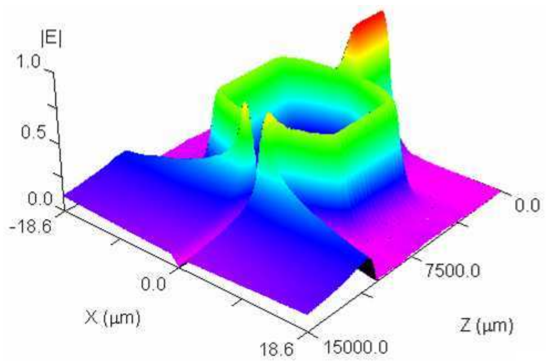
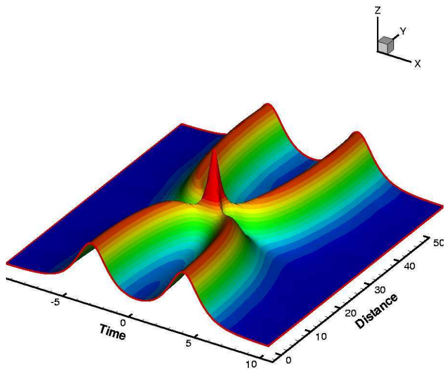
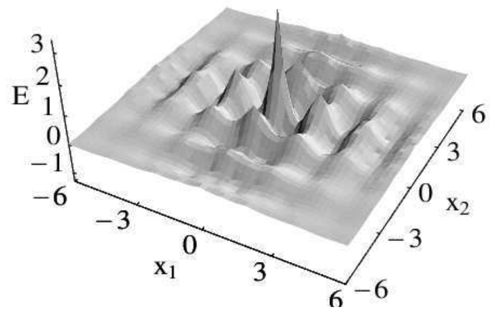
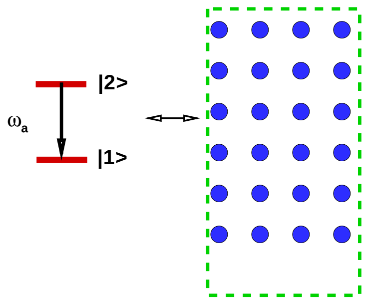
Course info: http://mx.nthu.edu.tw/∼rklee

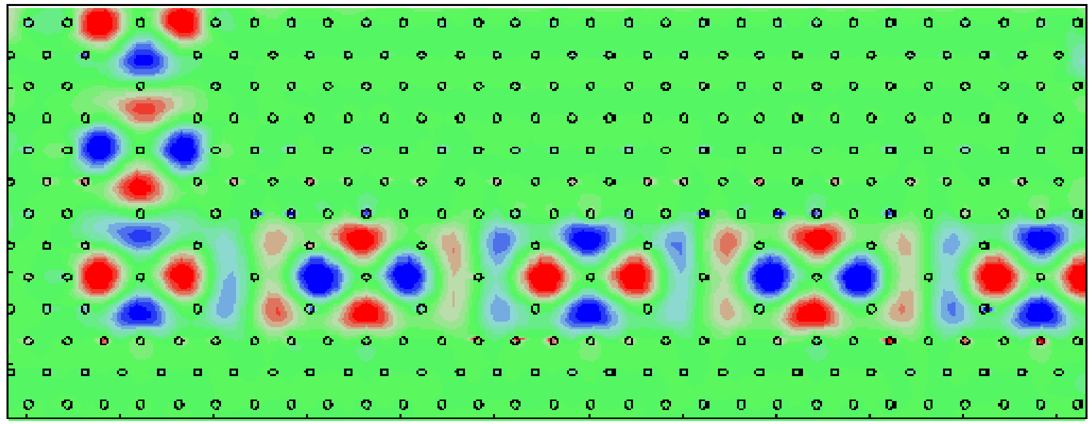
Computational Optoelectronics

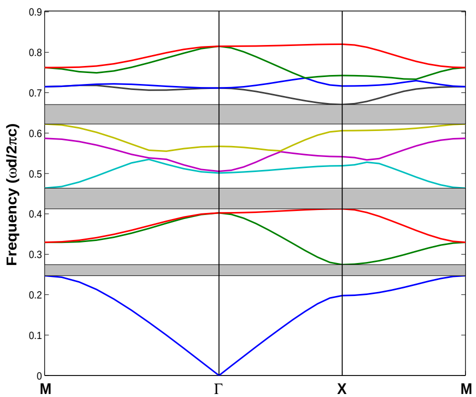


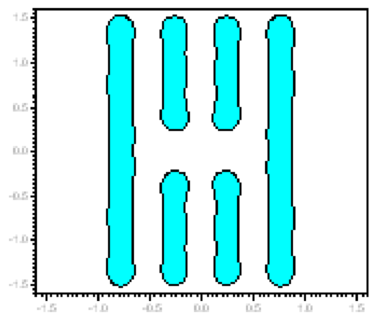


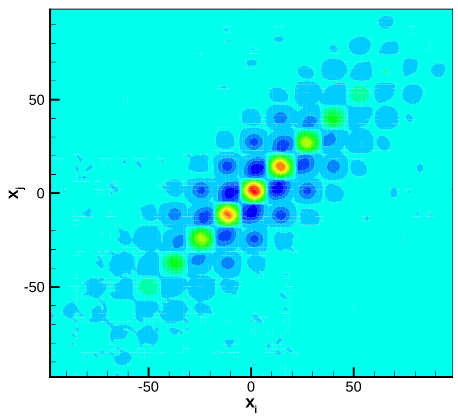












IPT 52600

Time: M2M3M4 (09:30-12:30 AM, Monday)

Course Description:

- Fundamental numerical simulation techniques for solving Optoelectronics related problems.
- Taking this course you are asked to program by yourself.
- Although this course is given primarily for the first year graduate students, those who are undergraduates or senior graduates are encouraged to take this course.
- Background: No required but you must learn how to program in C/C++, Fortran, Matlab, Mathematica, or Mapple (at least one of these programming languages).
- Remark: This course is more focused on numerical methods than *numerical analysis*.

Teaching Method: in-class lectures with examples and projects studies.

Evaluation:

1. Two Homeworks, 70%;
2. One Project, 30%.

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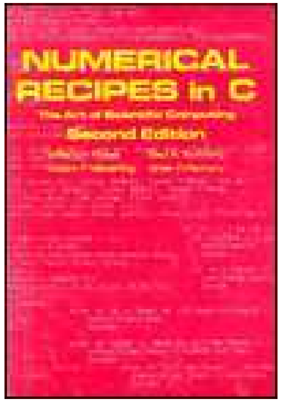
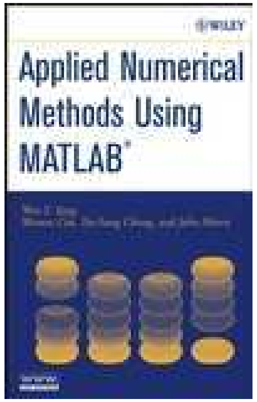
Reference Books

- W. H. Press et al., "Numerical Recipes (in C, C++, or Fortran)," Cambridge University Press (1992).
- W. Y. Yang et al., "Applied Numerical Methods Using MATLAB," Wiley (2005).





http://www.nr.com/

ftp://ftp.wiley.com/public/sci_tech_med/applied_numerical/

Related courses

- Data Structures
- Algorithm
- Numerical Analysis
- Numerical Partial Differential Equations
- Scientific Computation
- Computational Physics
- Computational Fluid Dynamics, CFD
- First Principle calculation, ab-initio
- Finite Element Method, FEM
- Monte Carlo/Molecular Dynamics calculation, MC/MD
- Computer-Aided Design, CAD

1, Linear Algebraic Equations

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b},$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ & \cdots & & \\ a_{M1} & a_{M2} & \cdots & a_{MN} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}, \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}.$$

- Gauss-Jordan elimination
- LU decomposition
- Jacobi iteration
- Gauss-Seidel iteration

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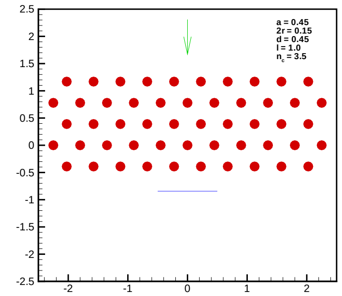
Multiple scattering method for PBG

Helmholtz equation:

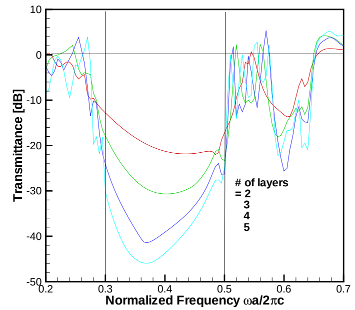


$$\bigtriangledown^2 E + \tilde{k}^2(M)E = 0$$

with

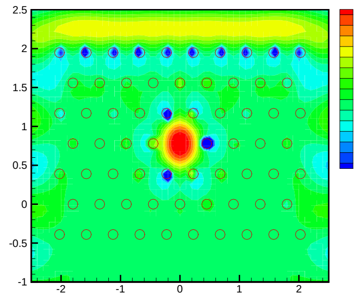
$$\tilde{k}^2(M) = k^2\tilde{\epsilon} = \begin{cases} k^2\epsilon_j & \text{if} \quad M \in C_j(j = 1, 2, \ldots, N) \\ k^2 & \text{if} \quad M \notin C_j(j = 1, 2, \ldots, N) \end{cases}$$

write down the total field in decomposition,



$$\begin{aligned} E(P) &= \sum_{m=-\infty}^{\infty} a_{l,m} J_m[kr_l(P)] e^{im\theta_l(P)} \\ &+ \sum_{m=-\infty}^{\infty} b_{l,m} H_m^{(1)}[kr_l(P)] e^{im\theta_l(P)}, \end{aligned}$$

then all we need is to solve



$$\hat{\mathbf{b}}_l - \sum_{j \neq l} \mathbf{S}_l \mathbf{T}_{l,j} \hat{\mathbf{b}}_j = \mathbf{S}_l \mathbf{Q}_l.$$

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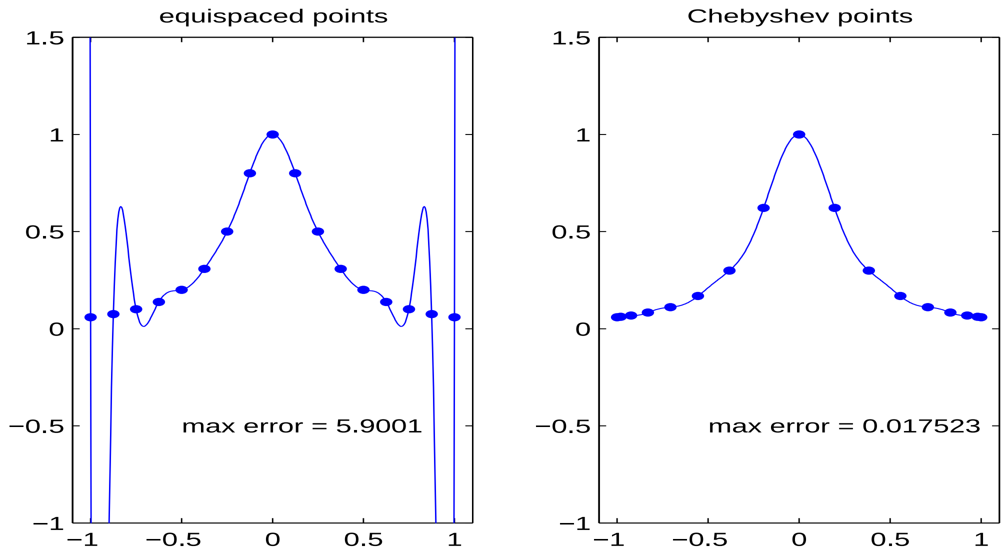
2, Interpolation, Curve Fitting, and Integration



- Polynomial interpolation and extrapolation
- Rational function interpolation
- Chebyshev approximation
- Padé approximation
- Fast Fourier Transform and Discrete Fourier Transform
- Trapezoidal and Simpson method for integration

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Finite difference approximation

Second-order FD approximation for $u'(x_j)$

$$u'(x_j) \approx \frac{u_{j+1} - u_{j-1}}{2h}$$

in the matrix-vector form (with periodic boundary)

$$\begin{pmatrix} u'_1 \\ \vdots \\ \\ u'_j \\ \vdots \\ \\ u'_N \end{pmatrix} = h^{-1} \begin{pmatrix} 0 & \frac{1}{2} & & & & & & -\frac{1}{2} \\ -\frac{1}{2} & 0 & & & & & & \\ & & & \ddots & & & & \\ & & -\frac{1}{2} & 0 & \frac{1}{2} & & & \\ & & & \ddots & & & & \\ & & & & & & 0 & \frac{1}{2} \\ \frac{1}{2} & & & & & & -\frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ \vdots \\ \\ u_j \\ \vdots \\ \\ u_N \end{pmatrix}$$

3, Ordinary Differential Equations

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + q(x)\frac{\mathrm{d}y}{\mathrm{d}x} = r(x),$$

- Euler's method
- Runge-Kutta method
- Adaptive stepsize control
- Predictor-corrector method
- Boundary value problems
- Relaxation method

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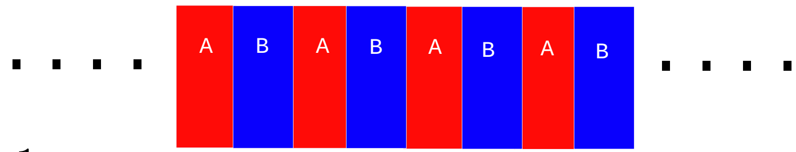
ODE with B. C.: 1D Bragg reflector

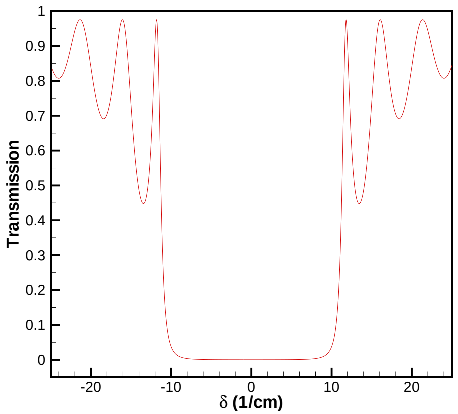
coupled-mode equation:

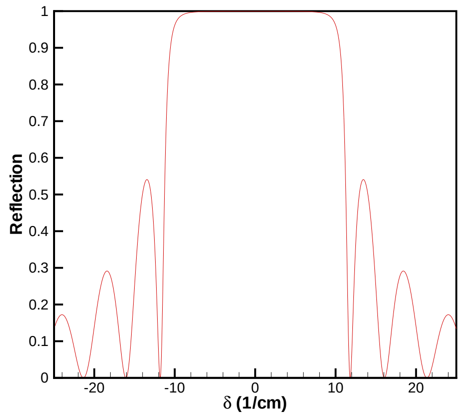
$$\frac{dE_+(z)}{dz} = i\delta E_+(z) + i\kappa E_-(z)$$
$$\frac{dE_-(z)}{dz} = i\delta E_-(z) + i\kappa E_+(z)$$

with the Boundary Condition:



$$E_+(z=0) = 1$$
$$E_-(z=L) = 0$$





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4, Partial Differential Equations

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} = f(x,y,u,\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}),$$

- Euler method
- Crank-Nicholson method
- Beam Propagation Method
- Slit-Step Fast Fourier Transform
- Finite Difference Time Domain method
- Absorption Boundary Condition
- Periodic Boundary Condition
- Perfect Matching Layers

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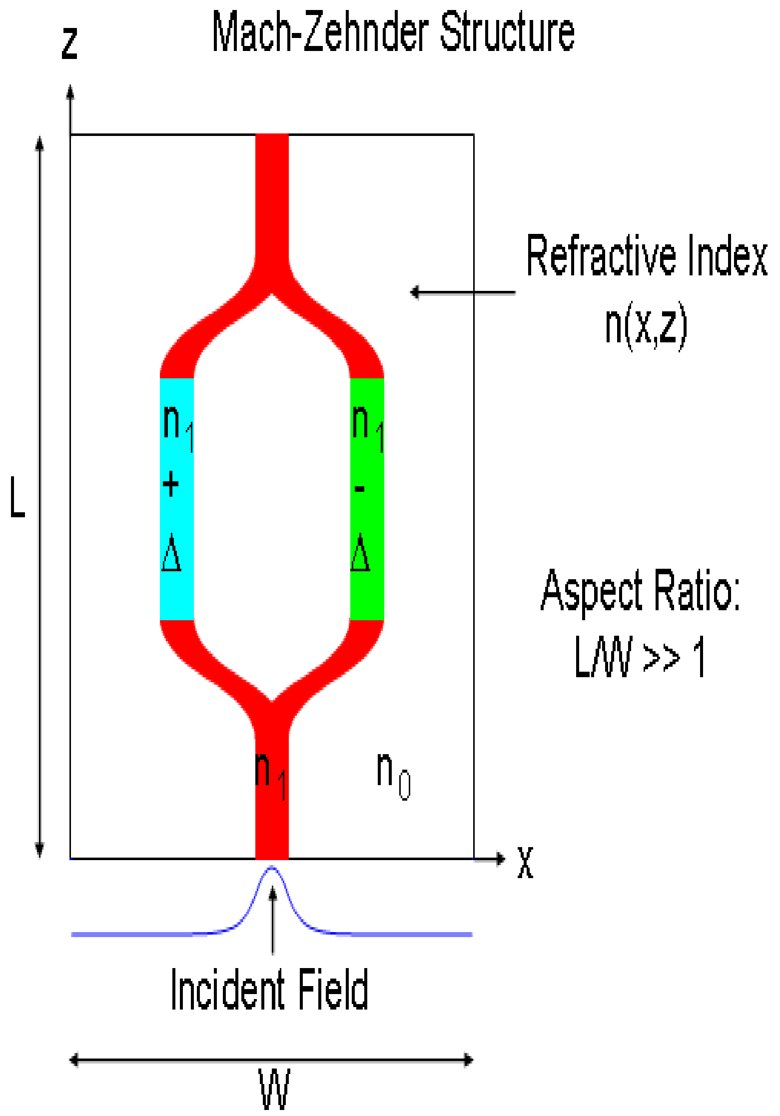
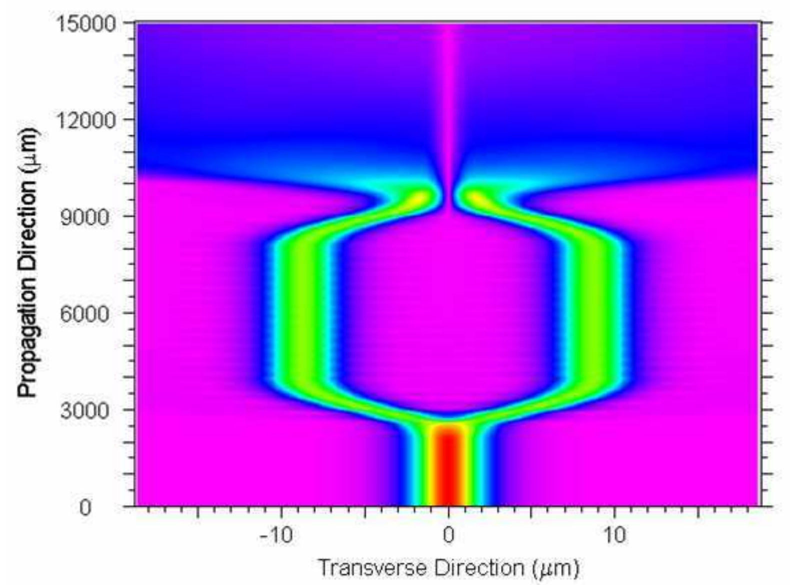
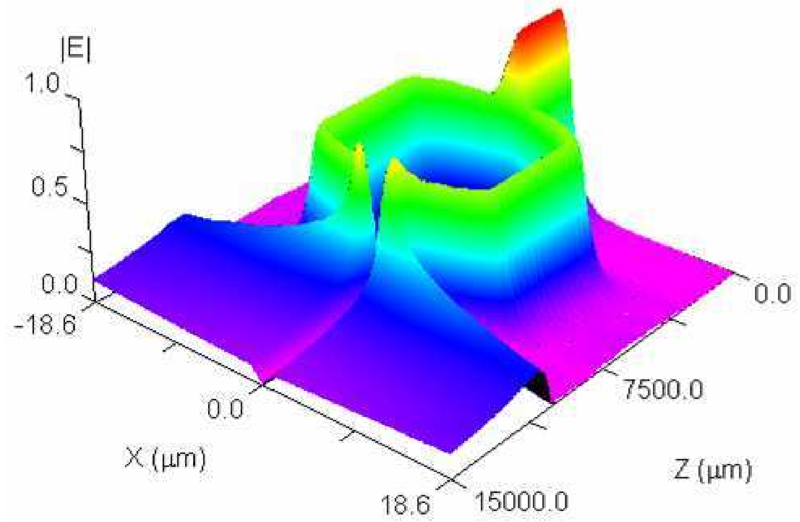
Mach-Zehnder structure







by BeamProp

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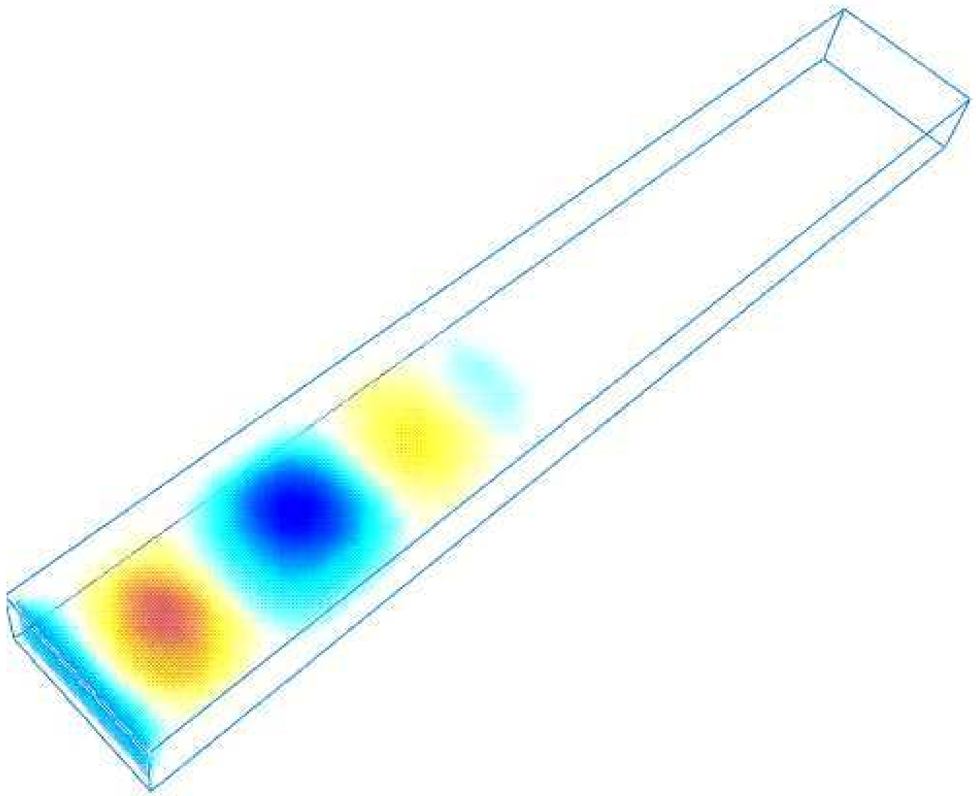
Metallic Waveguide



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by ToyFDTD

Nonlinear equation:

$$f(x) = x^3 + x^2 = 5,$$

- Iterative method
- Newton-Raphson method
- Secant Method

Nonlinear Schrödinger equation:

$$i\frac{\partial U}{\partial t} + \frac{\partial^2 U}{\partial x^2} + |U|^2 U = 0$$

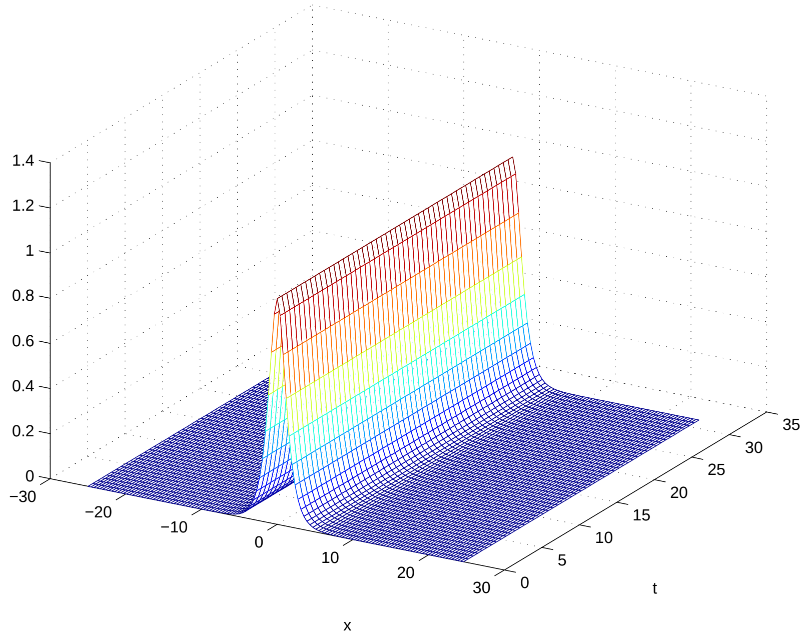
- Crank-Nicholson method
- Slit-Step Fast Fourier Transform
- Pseudospectral method

Propagation of solitons

Nonlinear Schrödinger equation:

$$i\frac{\partial U}{\partial t} + \frac{\partial^2 U}{\partial x^2} + |U|^2 U = 0$$

supports soliton solutions, $U(t = 0, x) = sech(x)$.



simulated by Fourier spectral + 4th-order explicit Runge-Kutta methods,

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N_x = 128, N_t = 50, error = 10^{-6}, indep. of N_t; nlse.m.

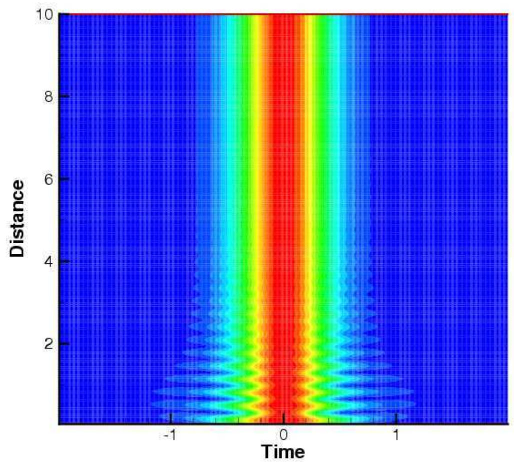
Bound solitons in CGLE

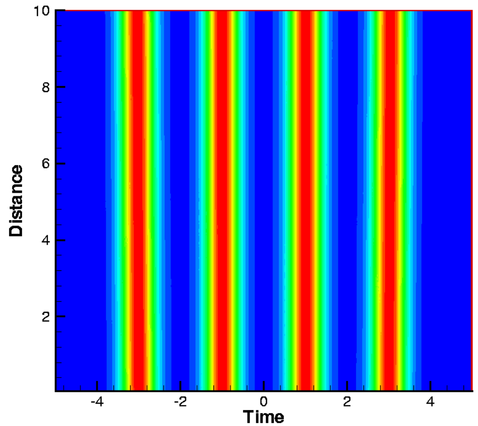
Complex Ginzburg-Landau Equation:

$$iU_z + \frac{D}{2}U_{tt} + |U|^2 U = i\delta U + i\epsilon|U|^2 U + i\beta U_{tt} + i\mu|U|^4 U - v|U|^4 U,$$

seek for bound-state solutions by propagation method.

$$U(z,t) = \sum^{N} U_0(z, t+\rho_j)e^{i\theta_j}$$





R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004).

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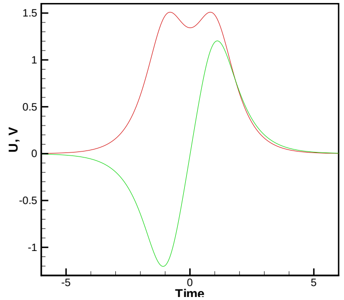
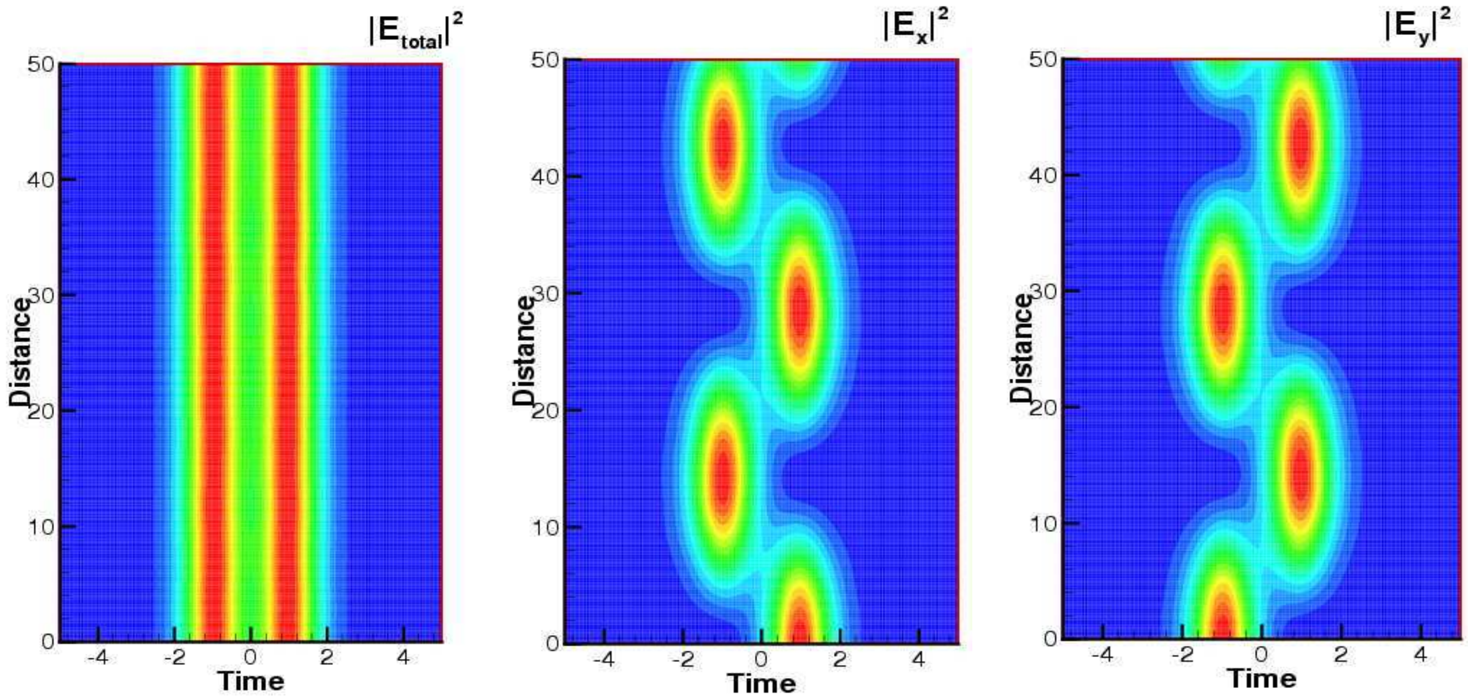
Vector bound solitons

Coupled Nonlinear Schrödinger Equations:

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2 U + B|V|^2 U = 0$$
$$i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2 V + B|U|^2 V = 0$$



where $A = 1/3$, $B = 2/3$; and U, V are circular polarization fields.



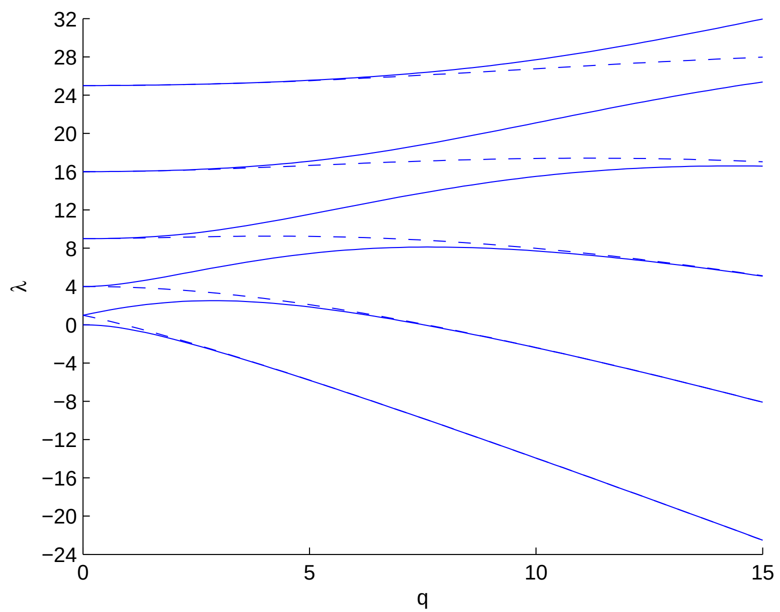
M. Haelterman, A. P. Sheppard, and A. W. Snyder, *Opt. Lett.* **18**, 1406 (1993).

6, Eigenvalues and Eigenvectors

$$\mathbf{A} \cdot \mathbf{x} = \lambda \mathbf{x},$$

- Similarity transformation and Diagonalization
- Jacobi method
- The QR algorithm

Mathieu equation: $-u_{xx} + 2q\cos(2x)u = \lambda x,$



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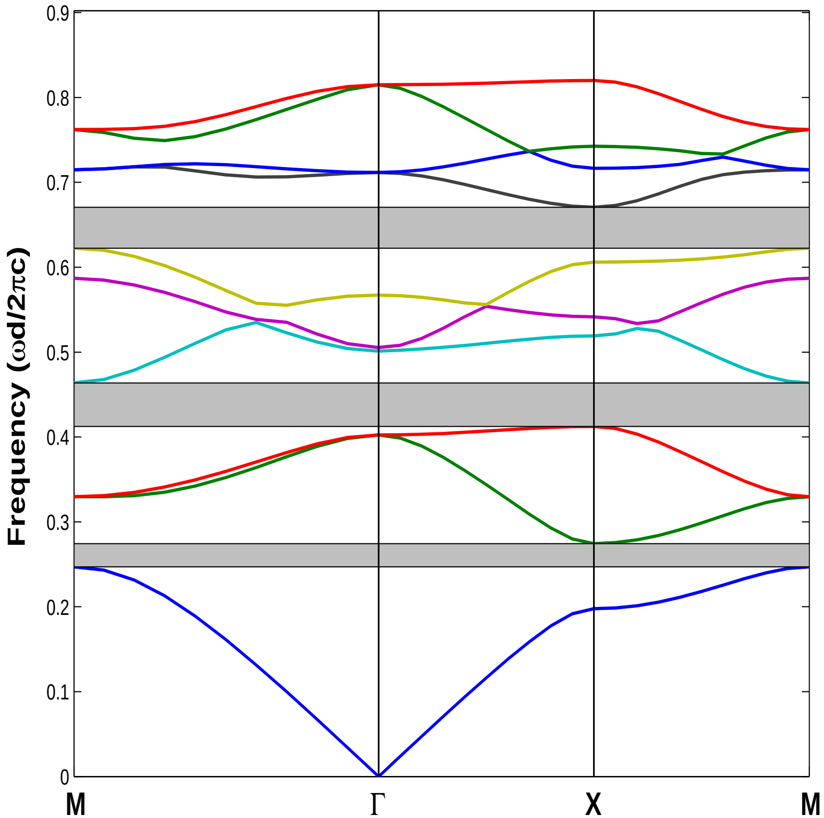
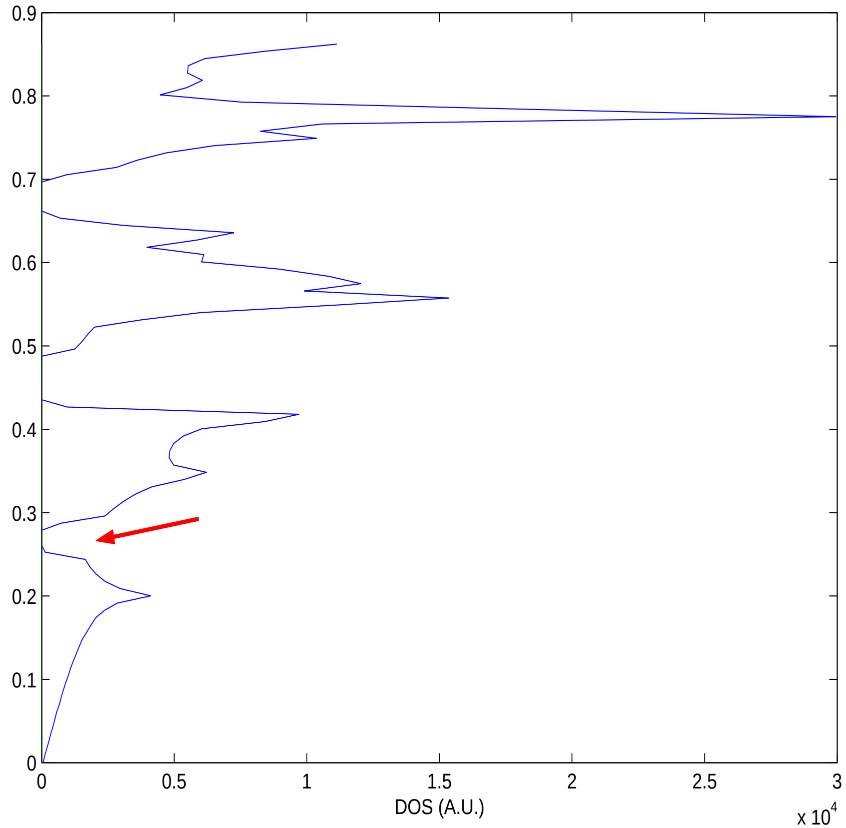
Band diagram and Density of States

$$\frac{1}{\epsilon(\mathbf{r})}\nabla \times \{\nabla \times \mathbf{E}(\mathbf{r})\} = \frac{\omega^2}{c^2}\mathbf{E}(\mathbf{r}),$$





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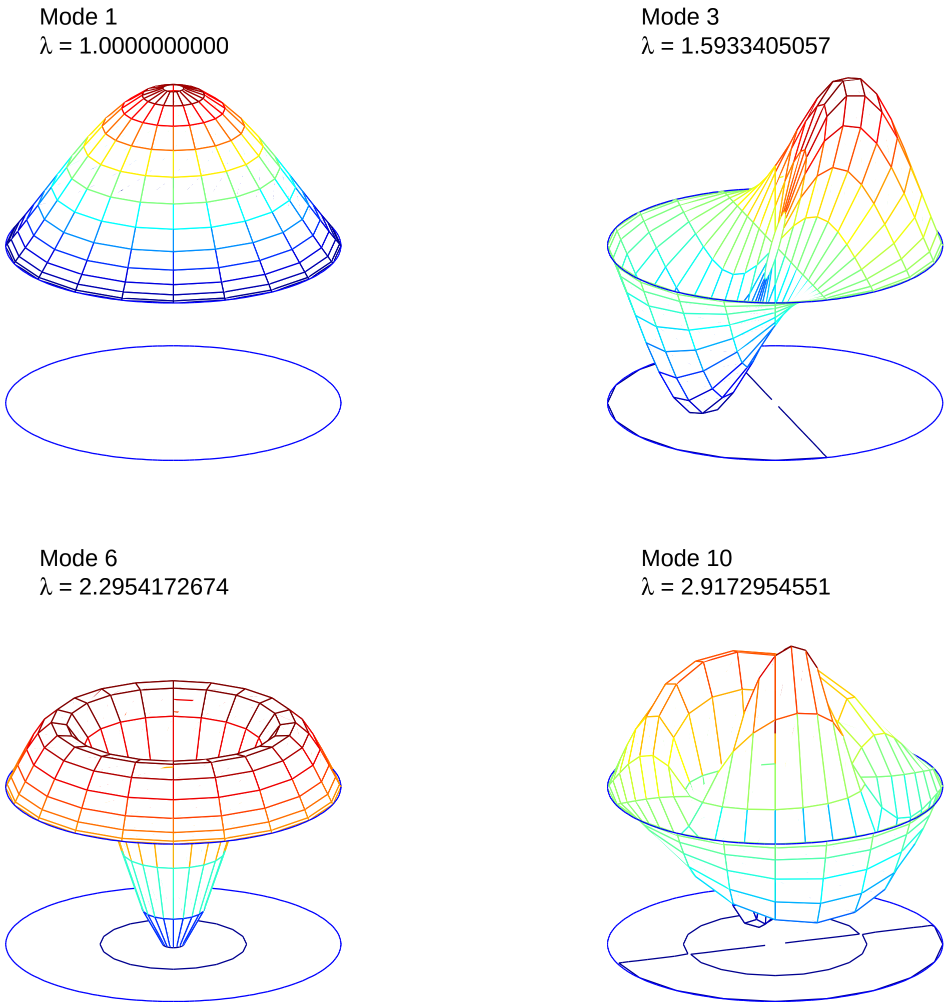
Laplacian equation in a disk

Eigenmodes of Laplacian equations, $[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}]u(x,y) = -\lambda f(x,y)$.



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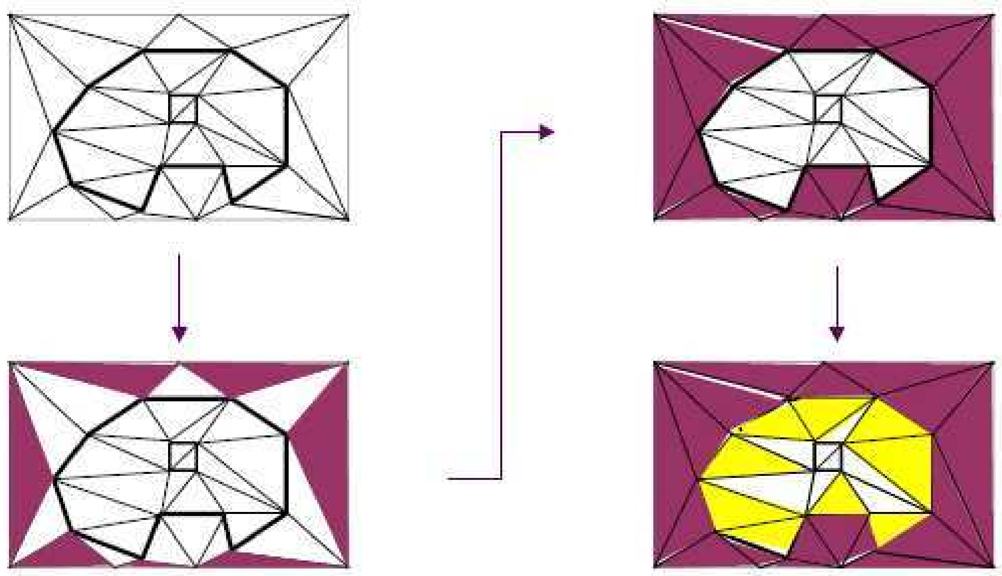
L. N. Trefethen, *"Spectral methods in Matlab,"* SIAM (2000).

7, Finite Element Method



- Elements
- Mesh generation
- Element assembly
- Boundary condition

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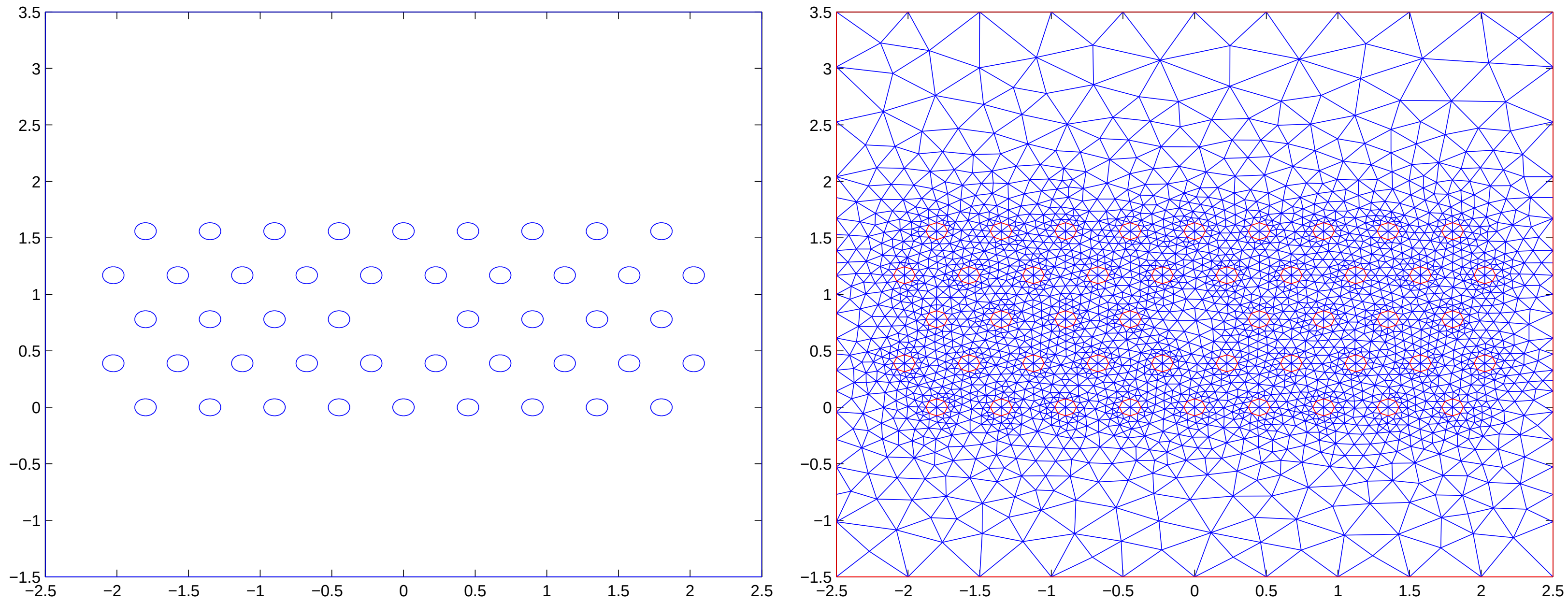
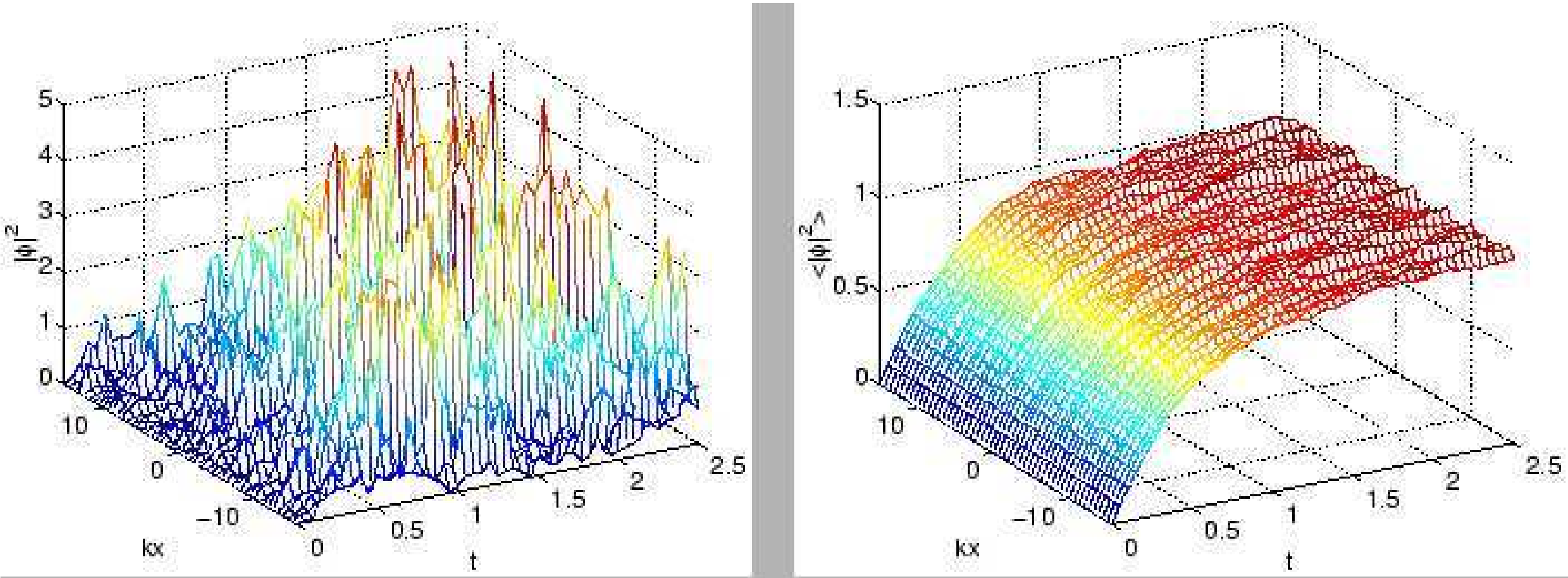
Photonic Crystals



8, Monte Carlo Method



a) Single path   b) 1024 path mean

- Random numbers with uniform deviates
- Transformation method
- Rejection method
- Random bits
- Monte Carlo methods

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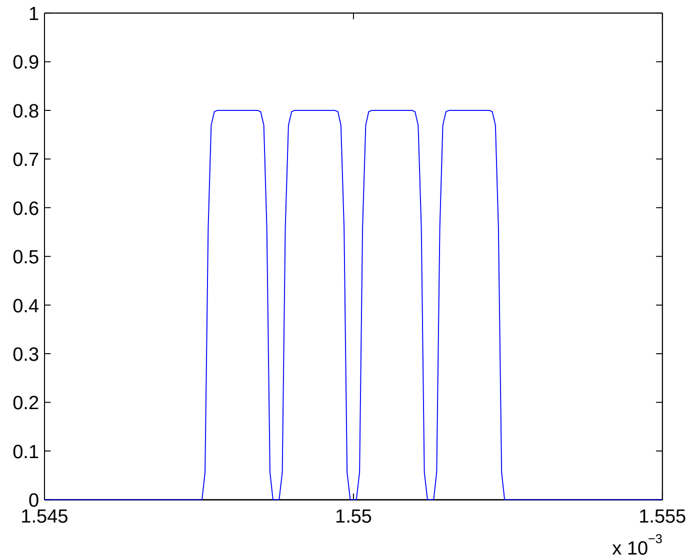
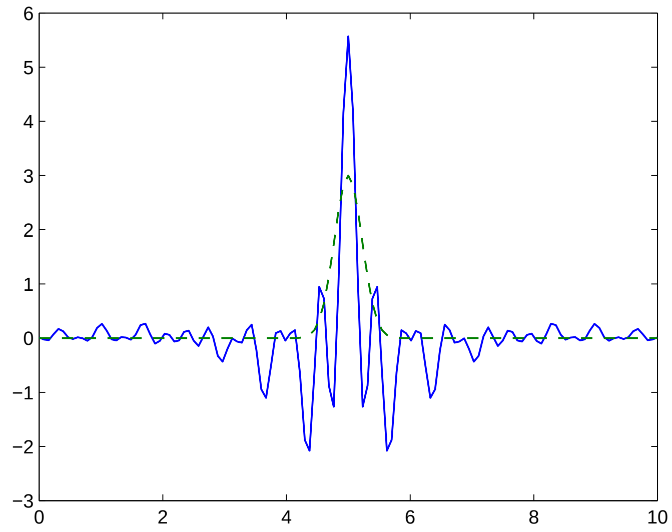
9, Optimization





- Simulated annealing
- Genetic algorithm
- Penalty function
- Optimal control method
- Matlab built-in routines

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10, Case studies

- 2D Gross-Pitaevskii equation
- Moment method
- . . . your project
- . . . your project
- Numerical Libraries
  - Matlab
  - IMSL
- Programming in Unix
- GNU Make and Concurrent Version System (CVS)
- Parallel programming

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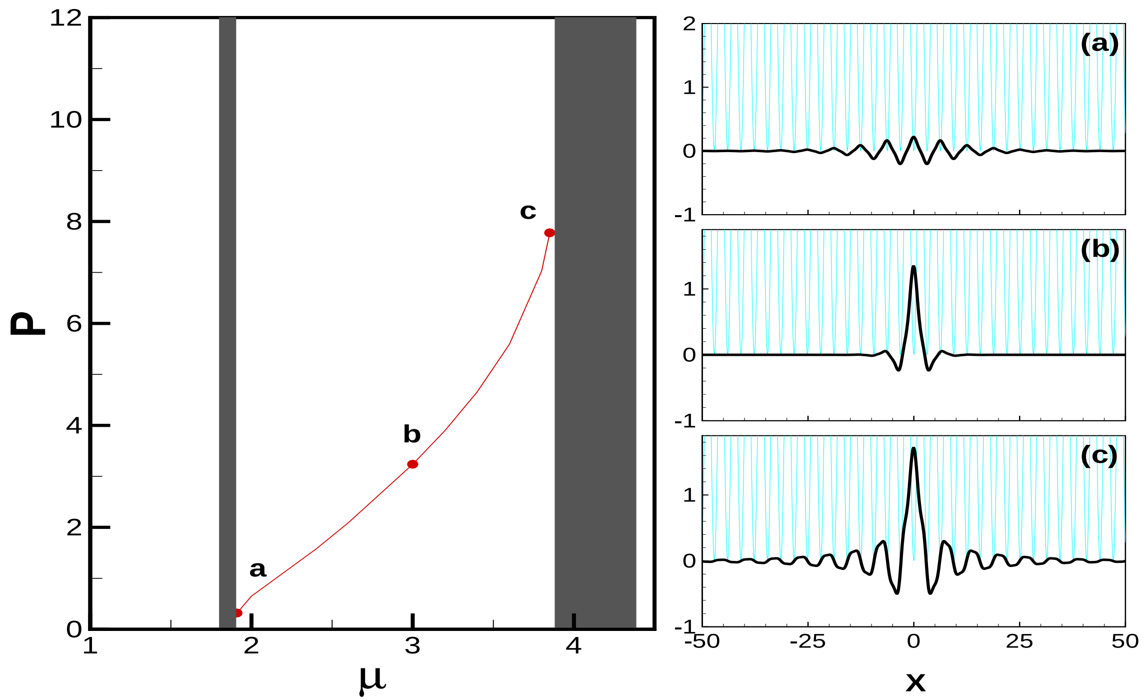
Eigenfunction of Nonlinear PDF

Gap solitons in optical lattices,

$$i\hbar\frac{\partial}{\partial}\Phi_0(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi_0(t,x) + V(x)\Phi_0(t,x) + g_{1D}|\phi_0(t,x)|^2\phi_0(t,x)$$

which has gap soliton solutions.



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by FS + NK methods, N_x = 512, no. of iterations < 10; gpeol.m.

R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72**, 033607 (2005).

Syllabus

1. Linear Algebraic Equations (Sep. 17th)
2. Interpolation, Curve Fitting, and Integration (Oct. 1st)
3. Ordinary Differential Equations (Oct. 8th, 15th)
   Homework # 1: two-weeks to finish, (deadline: Oct. 22th)
4. Partial Differential Equations (OCt. 22th, 29th)
5. Nonlinear Equations and Nonlinear PDE (Nov. 5th, 12th)
   Homework # 2: two-weeks to finish (deadline: Nov. 19th),
6. Eigenvalues and Eigenvectors (Nov. 19th)
7. Finite Element Method (Nov. 26th)
8. Monte Carlo Method (Dec. 10th)
9. Optimization (Dec. 17th)
   Project: one-month to finish (deadline: Dec. 31th),
10. Case studies (Dec. 24th, 31th)

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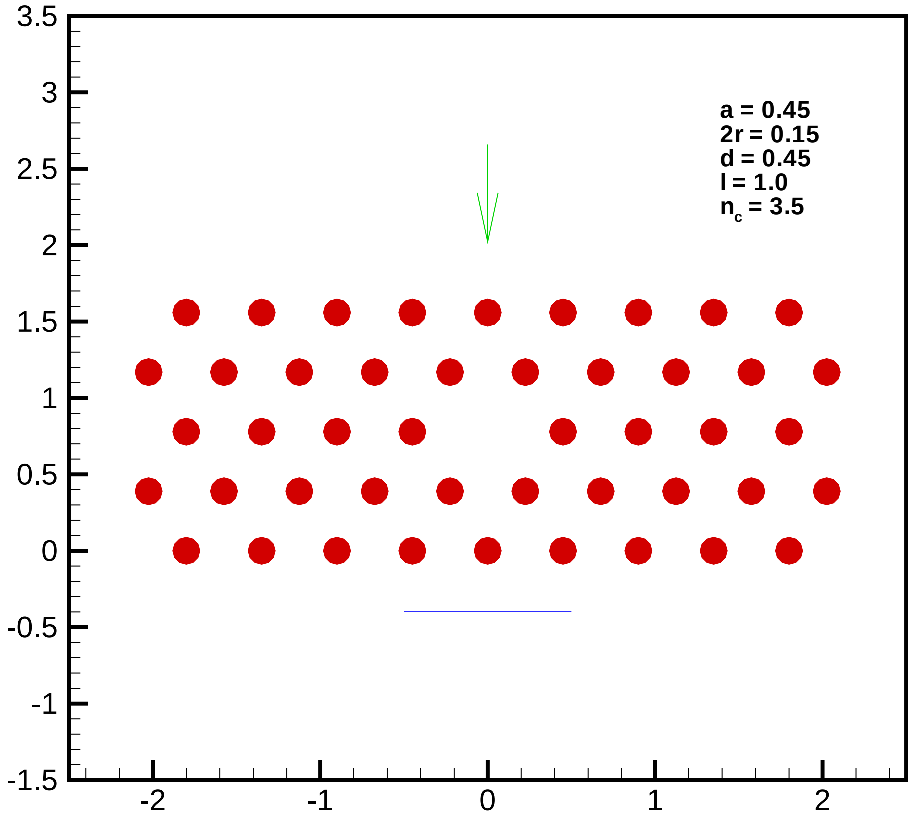
Evaluation

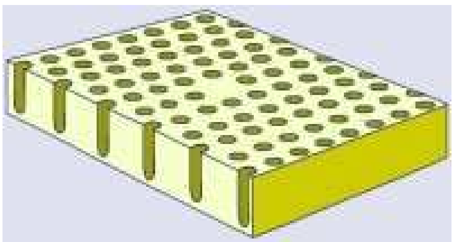
1. Two Homeworks (assigned), 70%
   - HW1, Ordinary Differential Equations
   - HW2, Nonlinear Partial Differential Equations
2. One Project (chosen one that is related to your research), 30%
   - Two/Three-dimensional PDE
   - Nonlinear/Coupled ODE/PDE
   - Finite Element Method
   - Monte Carlo Method
   - Optimization
   - . . .
   - other suggestions

Course Projects

- 2D/3D Finite-Difference Time-Domain mehtod
- Band-spectrum for 1D nonlinear Schrödinger equation
- Band-spectrum for 2D/3D Maxwell equation
- Coupled nonlinear PDE in 2+1 dimensions
- Soliton solutions for nonlinear PDE
- Finite Element method
- Optimization problem
- Monte-Carlo simulation

One-defect in photonic crystal





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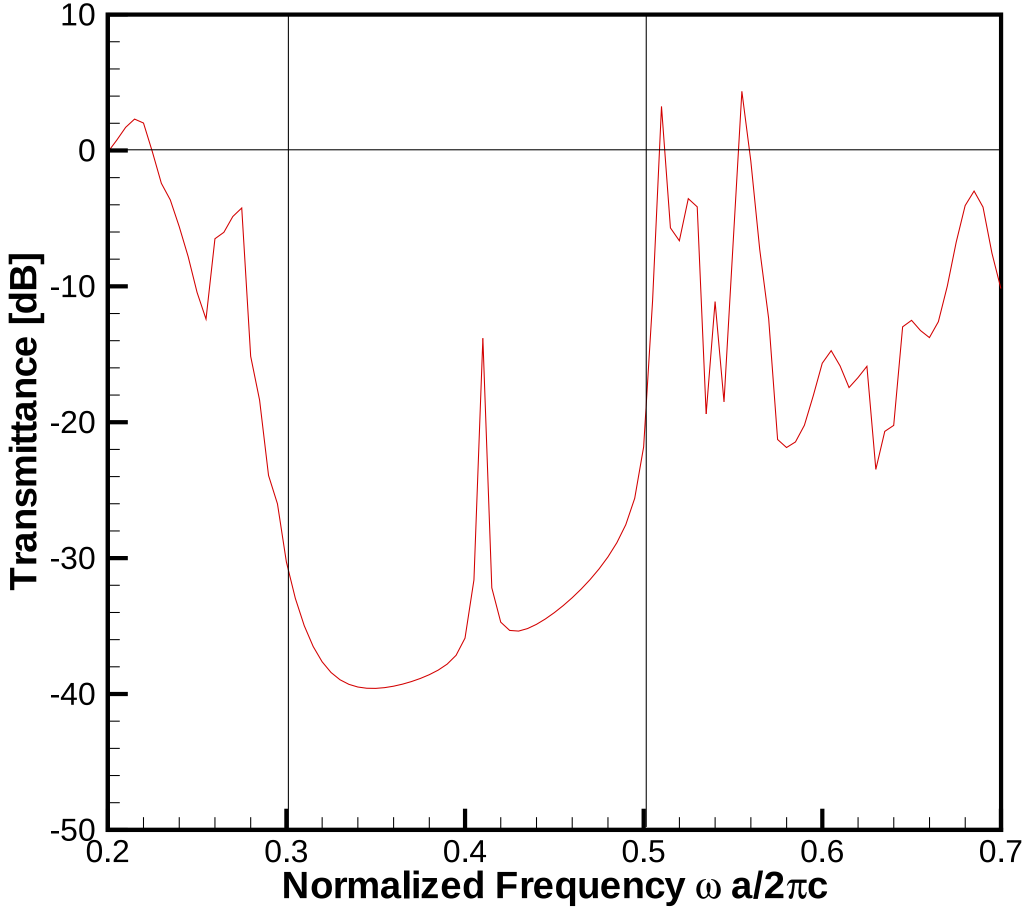
Transmittance: one-defect



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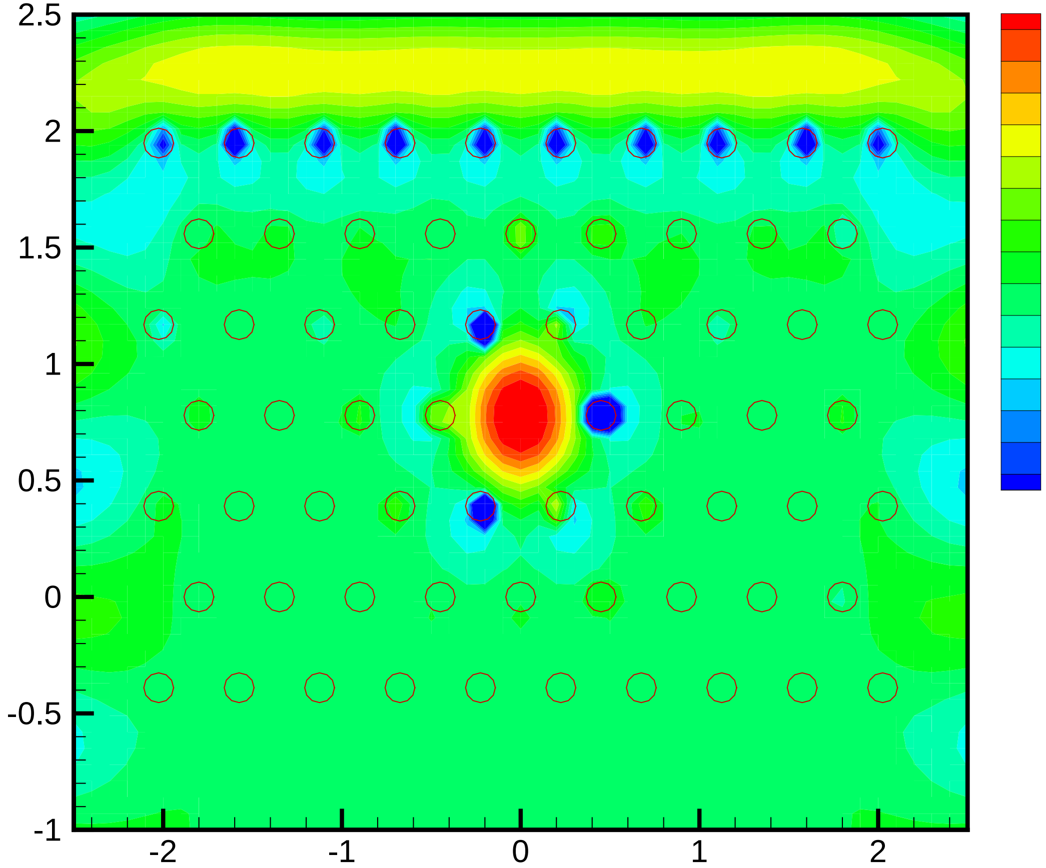
Localized field: one-defect



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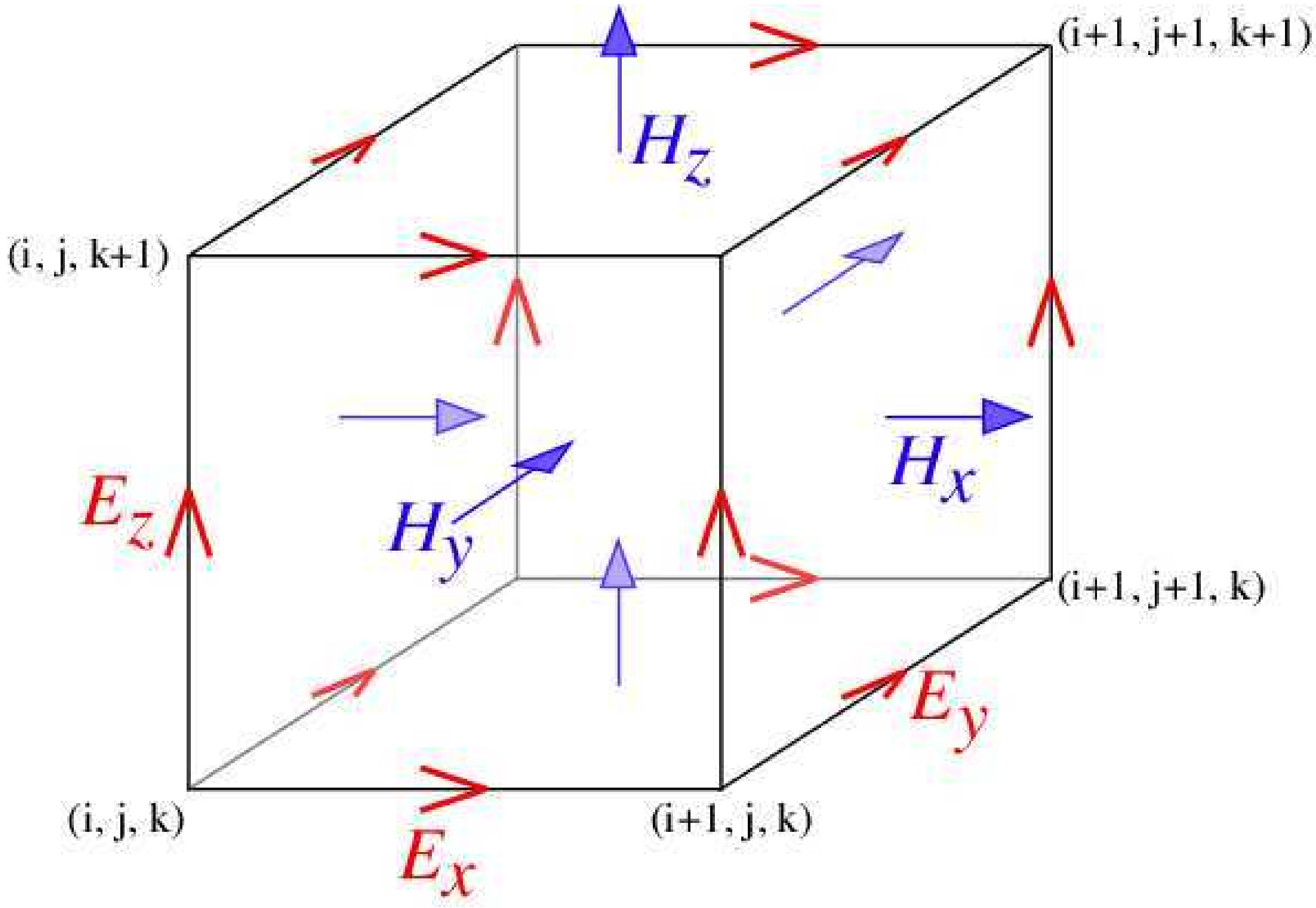
Finite-Difference Time-Domain, FDTD: Yee's algorithm



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Course Project, Spring 2006

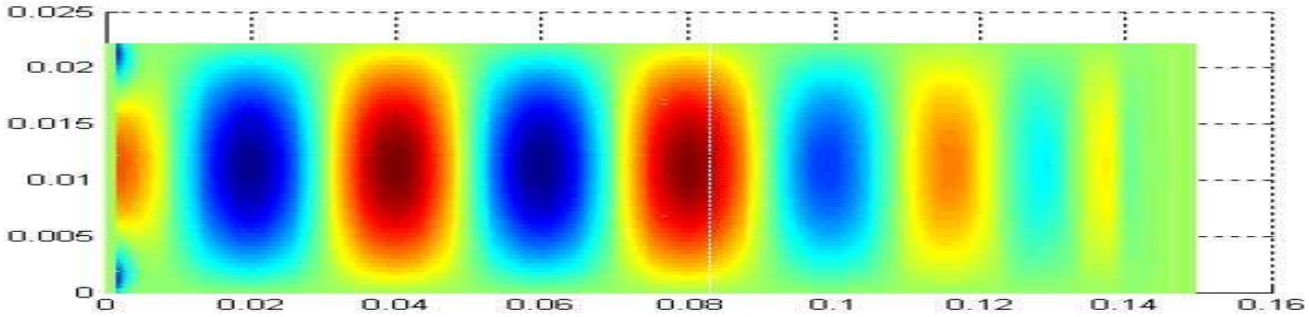

Figure 1: Hx 在 xz 平面

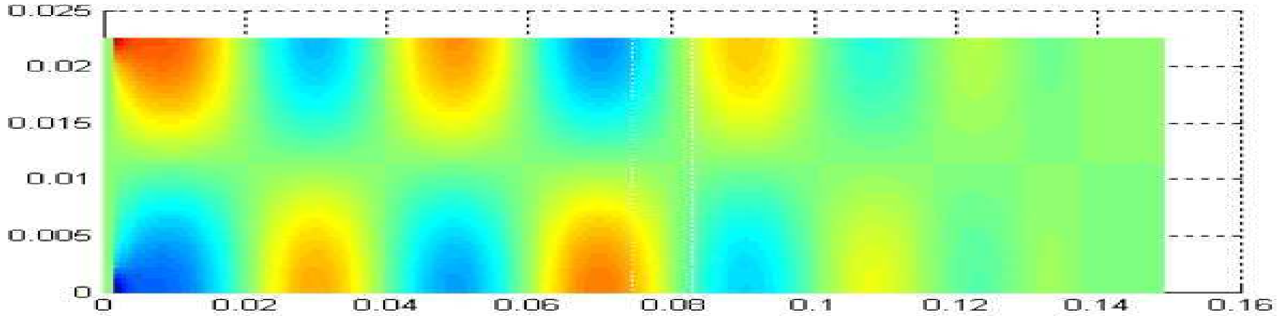

Figure 2: Hz 在 xz 平面

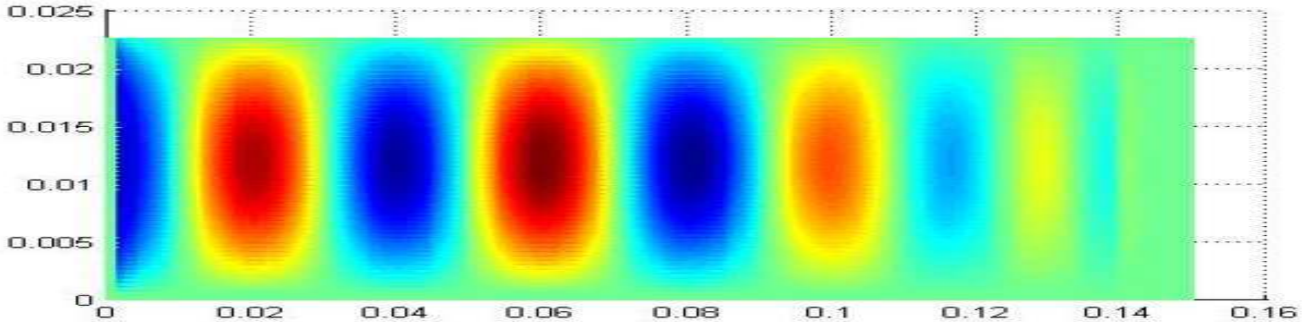

Figure 3: Ey 在 xz 平面

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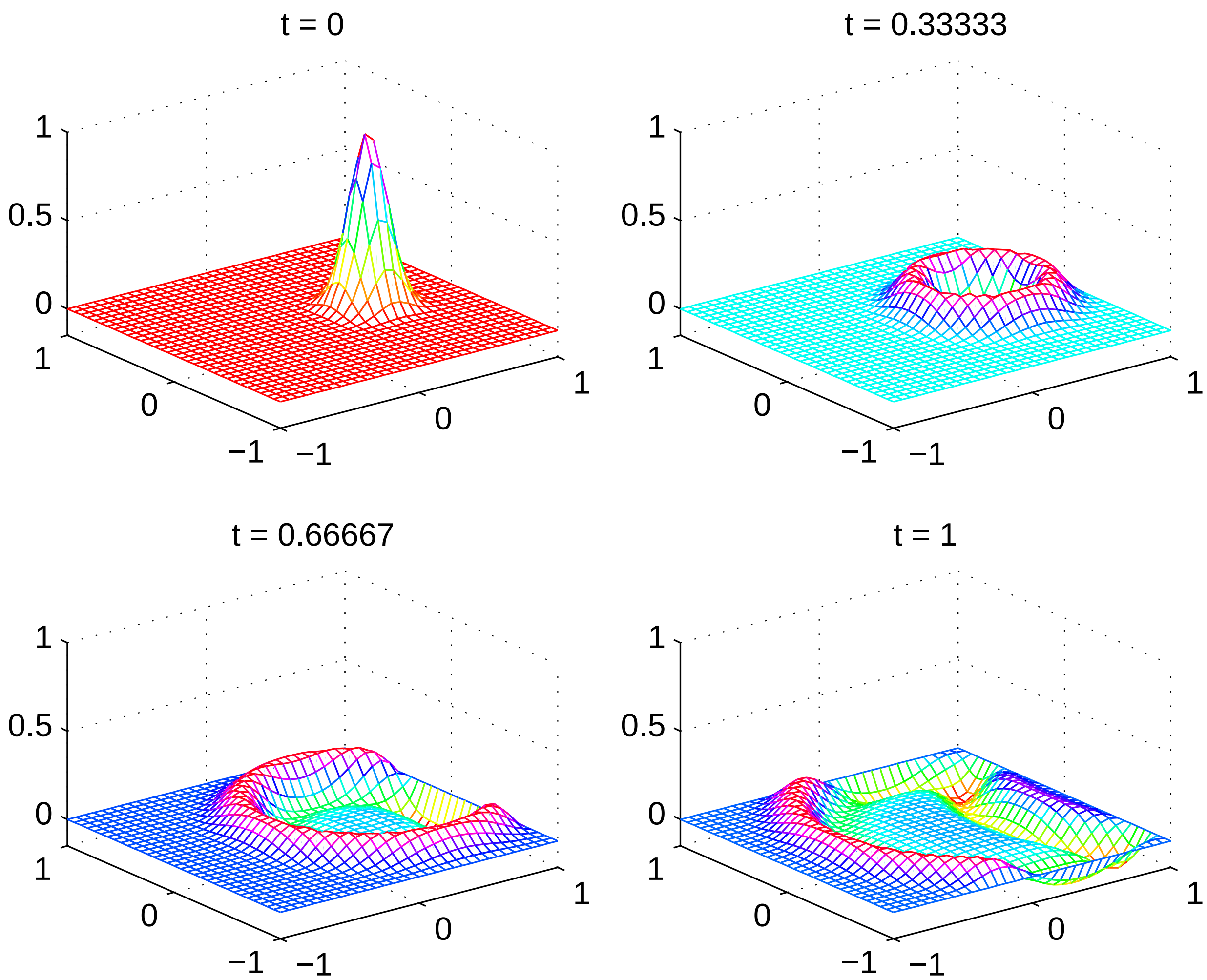
g933326,

FFT method for wave equation

$$u_{tt} = u_{xx} + u_{yy}, \qquad -1 < x, y < 1, \quad t > 0, \quad u = 0 \quad \text{on the boundary}$$



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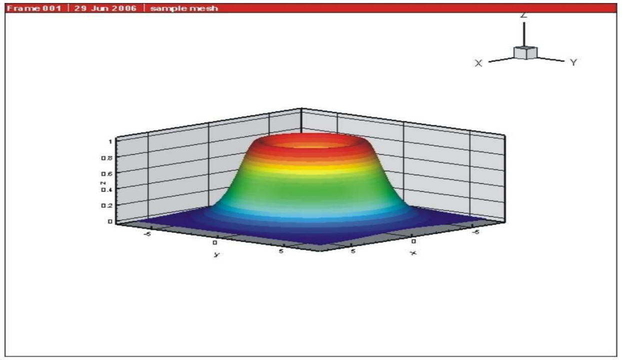
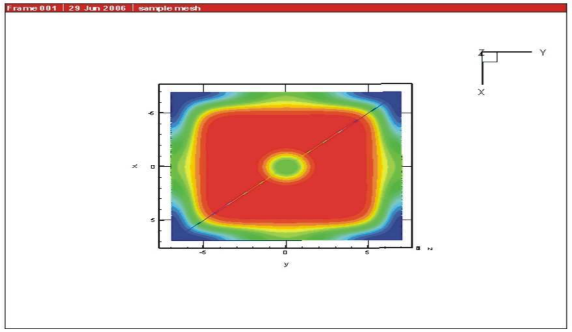
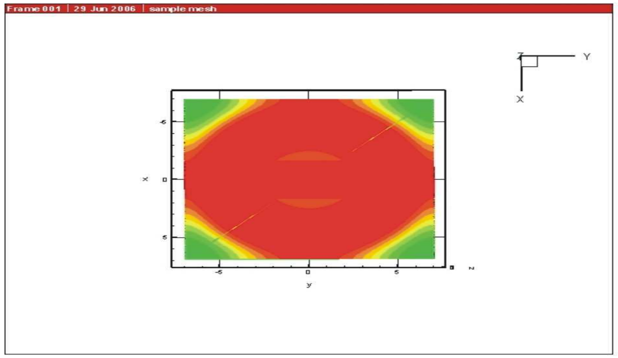
2-D sine-Gordon solitons,



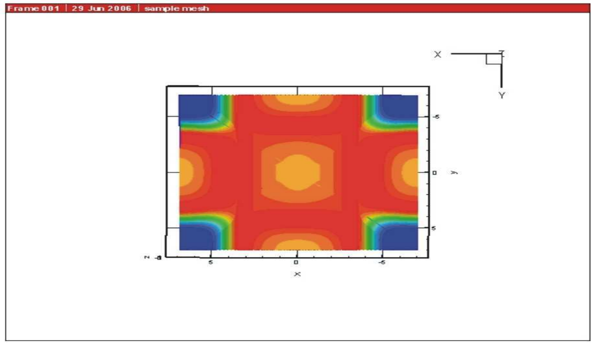
T=0



T=5.6



T=2.8



T=8.1

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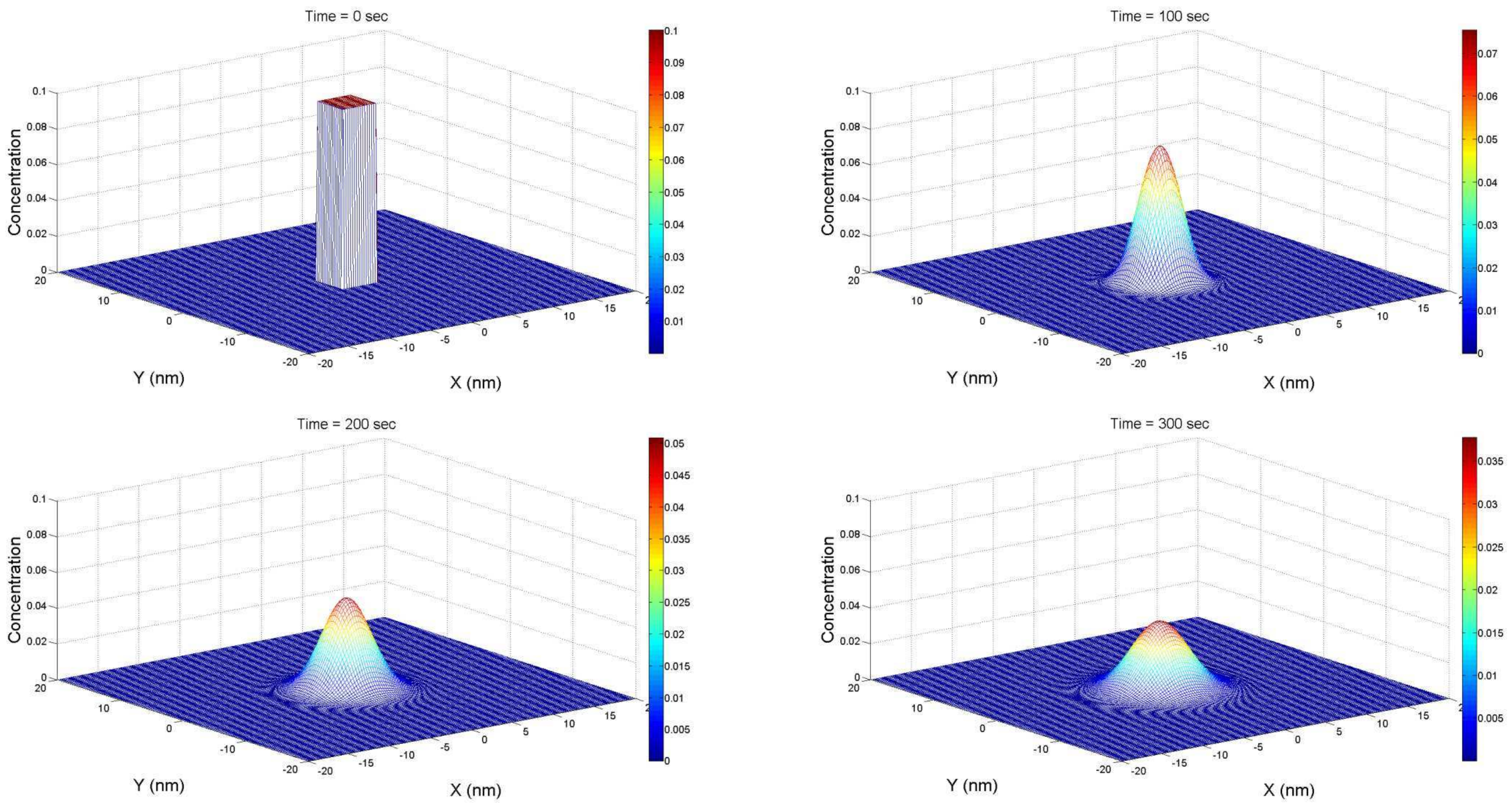
Yu-Sheng Hung, g933357,

Diffussion of dopant, Spring 2007



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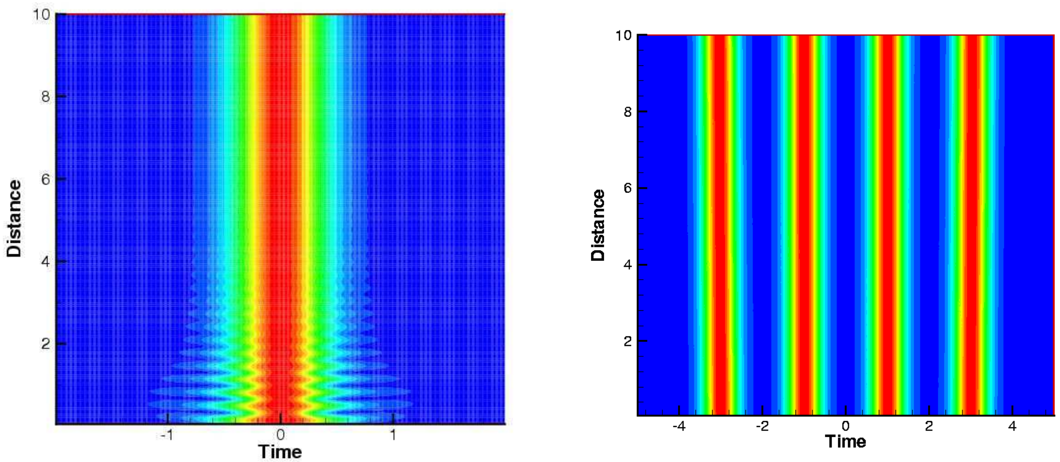
Bound solitons in CGLE

Complex Ginzburg-Landau Equation:

$$iU_z + \frac{D}{2}U_{tt} + |U|^2U = i\delta U + i\epsilon|U|^2U + i\beta U_{tt} + i\mu|U|^4U - v|U|^4U,$$

seek for bound-state solutions by propagation method.

$$U(z,t) = \sum^{N} U_0(z, t+\rho_j)e^{i\theta_j}$$



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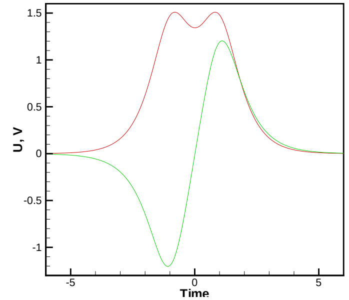
R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004).

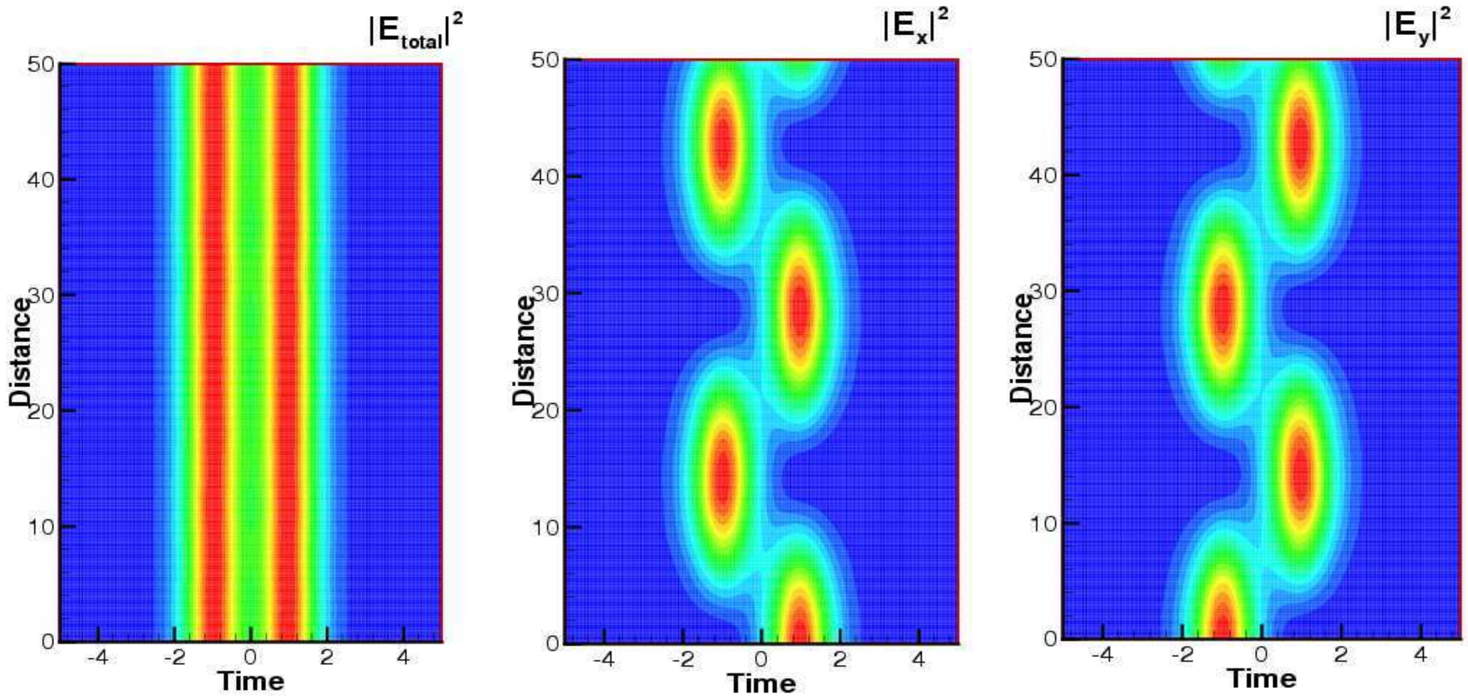
Vector bound solitons

Coupled Nonlinear Schrödinger Equations:

$$
\begin{aligned}
i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2U + B|V|^2U &= 0 \\
i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2V + B|U|^2V &= 0
\end{aligned}
$$



where $A = 1/3$, $B = 2/3$; and U, V are circular polarization fields.



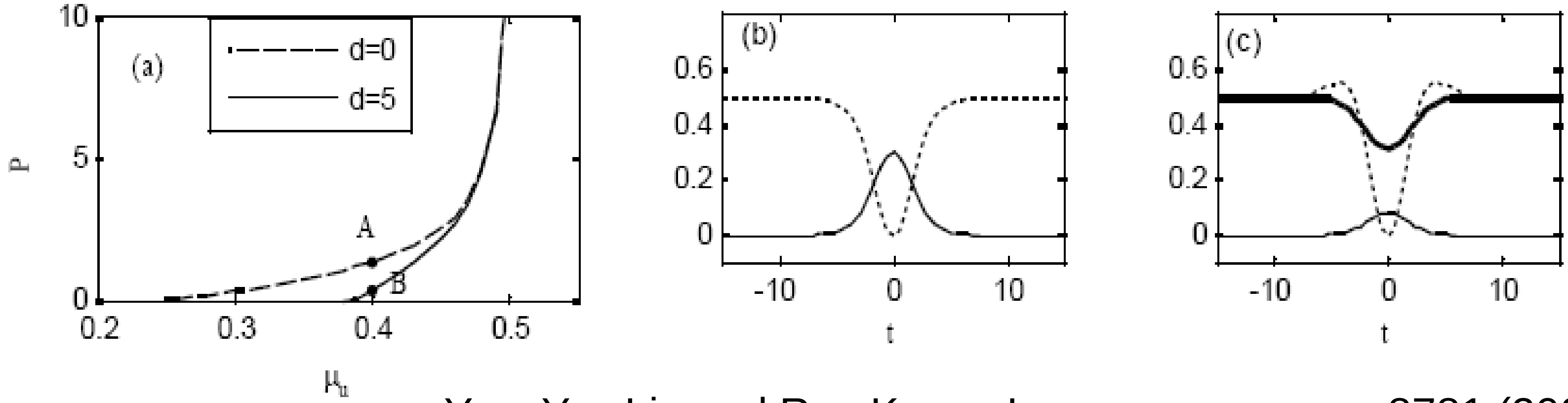
M. Haelterman, A. P. Sheppard, and A. W. Snyder, *Opt. Lett.* **18**, 1406 (1993).

Nonlocal vector dark-bright soliton pairs, Spring 2006

- Nonlocal response of materials can stabilize solitons due to the diffusion of the Kerr nonlinearity.
- In this situation, solitons also need to increase their formation power to compensate the diffusion effect in nonlocal materials.
- We show that in normal dispersive media with Kerr-type nonlocal nonlinearity, one can stabilize dark-bright vector soliton pairs as well as reduce the forming threshold power for guided bright solitons.

$$i\frac{\partial\Psi}{\partial z} - \frac{1}{2}\frac{\partial^2}{\partial t^2}\Psi + n(t,z)\Psi = 0, \quad \Psi = U, V,$$

$$n(t,z) = \int_{-\infty}^{\infty} dt'\, R(t-t')(|U|^2 + |V|^2), \quad \text{and} \quad R(t) = \frac{1}{2\sqrt{d}}\exp\left(-\frac{|t|}{\sqrt{d}}\right),$$



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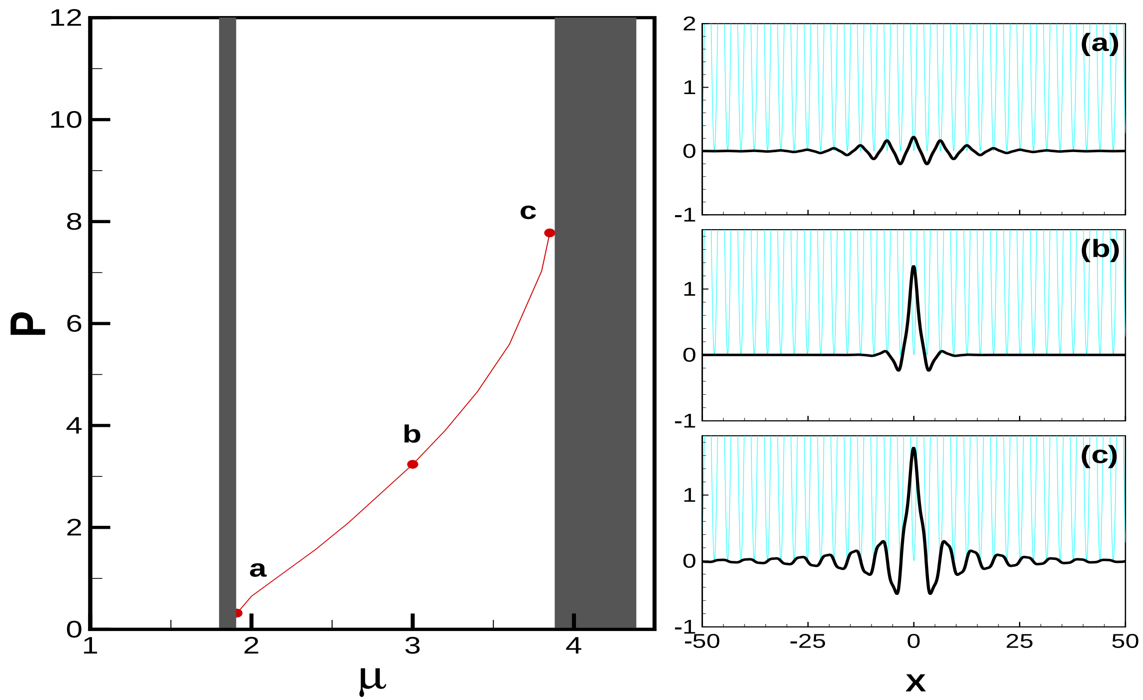
YuanYao Lin and Ray-Kuang Lee, *Opt. Express* **15**, 8781 (2007).

Gap solitons in optical lattices

1-D Gross-Pitaevskii equation with periodic potentials, $V(x) = V_0 \sin^2(k_0 x)$,

$$i\hbar\frac{\partial}{\partial t}\Phi_0(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi_0(t,x) + V(x)\Phi_0(t,x) + g_{1D}|\phi_0(t,x)|^2\phi_0(t,x)$$

which has gap soliton solutions.



by FS + NK methods, N_x = 512, no. of iterations < 10; gpeol.m.

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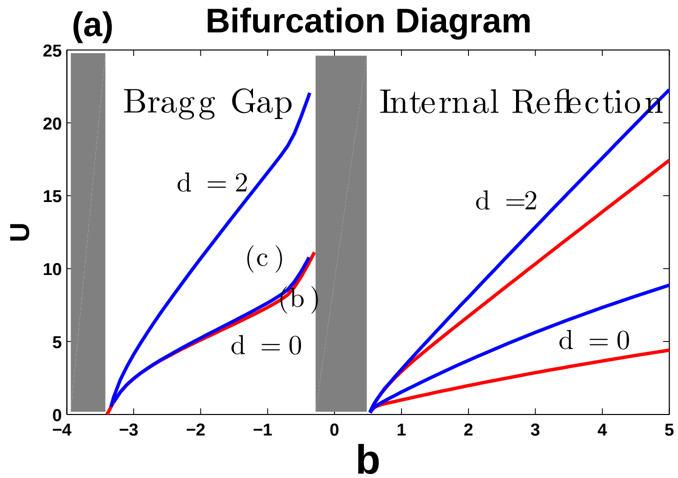
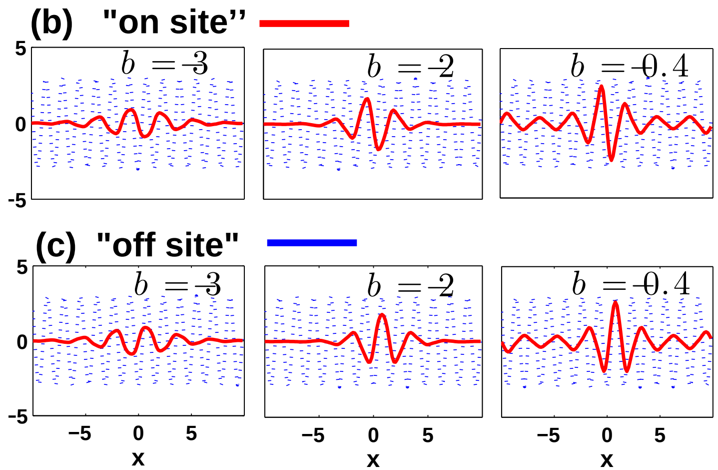
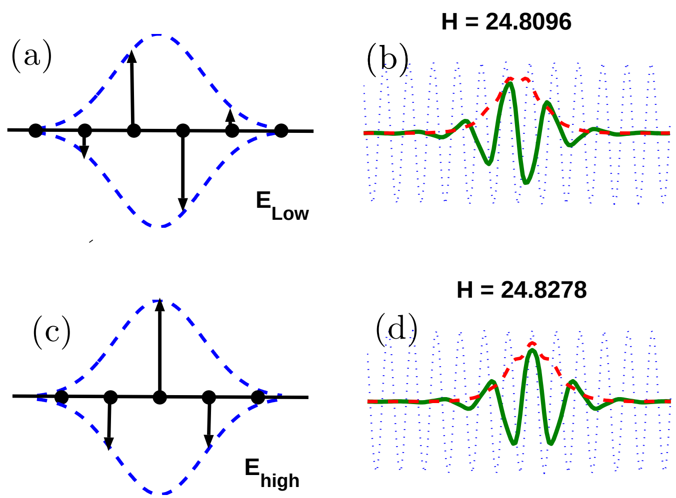
R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72**, 033607 (2005).

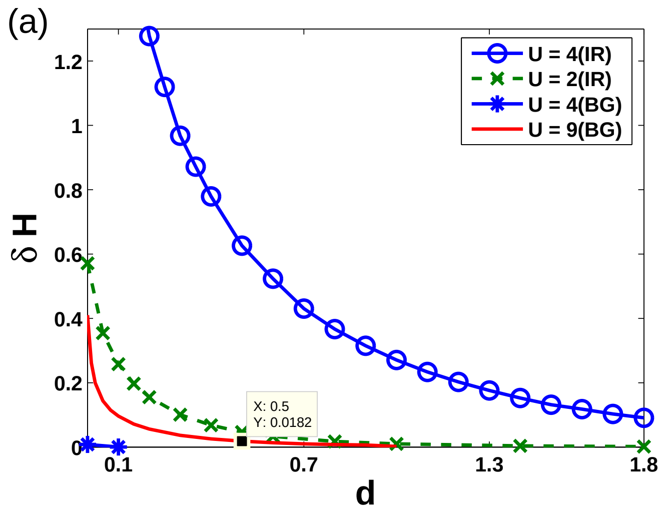
Nonlocal gap solitons in optical lattices, Spring 2006

$$i\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U}{\partial \eta^2} + V(\eta)U + n(\xi,\eta)U = 0, \quad n - d\frac{\partial^2 n}{\partial \eta^2} = |U|^2,$$









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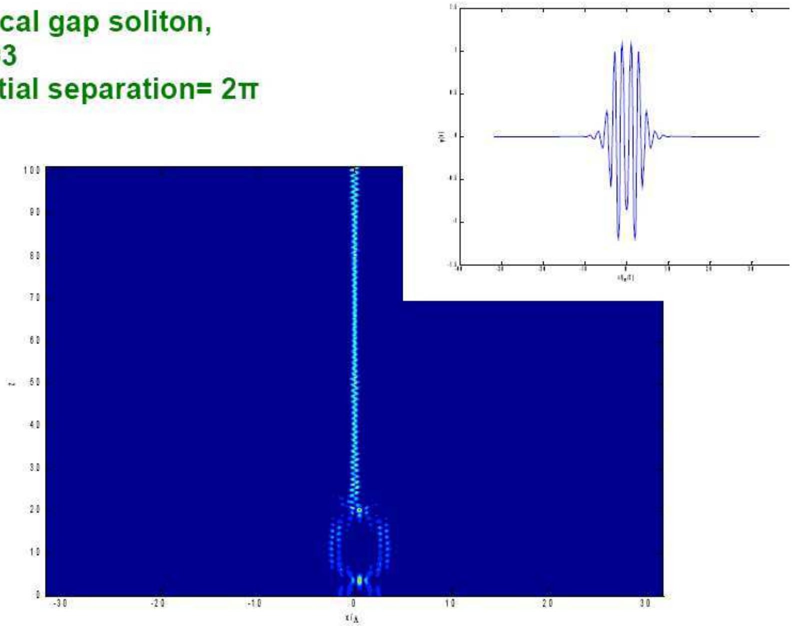
I-Hong Chen

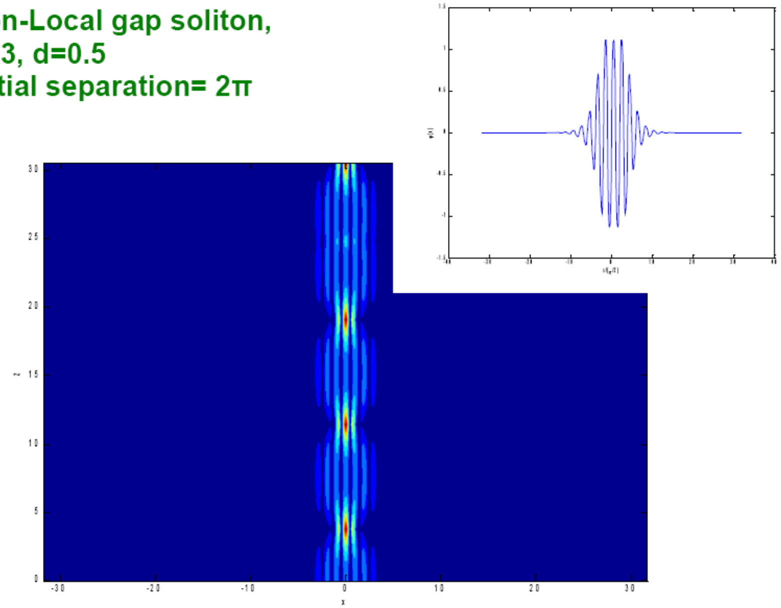
Collisions of Bragg gap soliton within nonlocal lattices

Collisions between two gap solitons in Bragg gap, with/without non-locality.

Local gap soliton,
U=3
Initial separation= 2π



Non-Local gap soliton,
U=3, d=0.5
Initial separation= 2π



- Only with a small degree of non-locality, solitons in Bragg gaps can be stabilized as well as movable.
- In such situation, the collision between two solitons behaves like the case without optical lattices.

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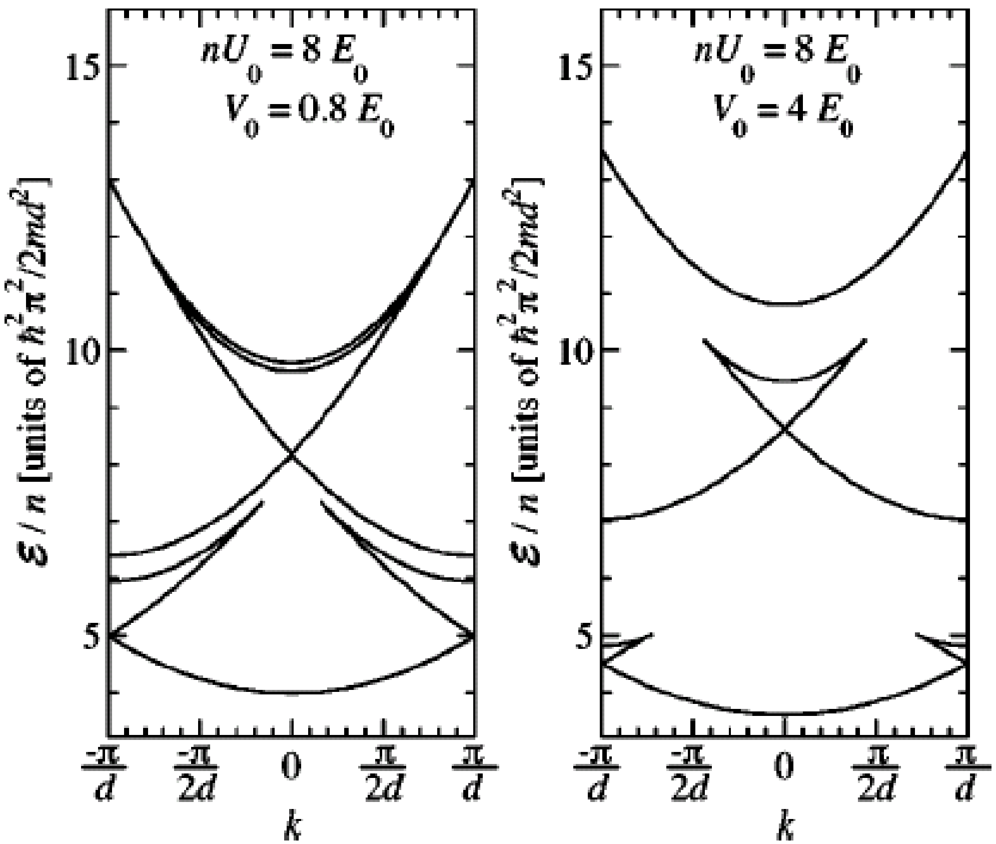
Y.Y. Lin, I.H. Chen, and R.-K. Lee,
IEEE/LEOS Optical MEMS and Nanophotoincs (2007); to *special issue in J. Phys. A*.

Nonlinear band structure for BEC in OL

$$\frac{1}{2}\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} - [V_0 \sin^2(Kx)\phi - \mu\phi] - \sigma|\phi|^2\phi = 0.$$



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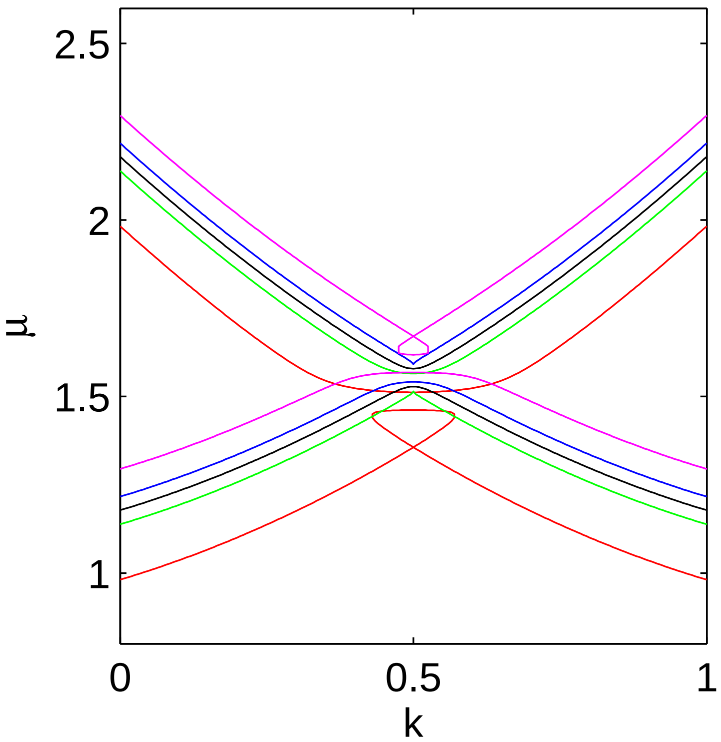
M. Machholm, C. J. Pethick, and H. Smith, *Phys. Rev. A* **67**, 053613 (2003).

Band structure with dipolar BEC, Spring 2006

$$i\frac{\partial u}{\partial t} = \frac{-1}{2}\frac{\partial^2 u}{\partial x^2} + c|u|^2 u + d^2 n(x,t)u + \nu\cos(x)u,$$

$$n(x,t) = \int_b^\infty d\eta\, V(x-\eta)|u(\eta)|^2, \quad V(x) = \frac{1 - 3 * \cos^2(r)}{|x|^3}, \text{for} \quad x > b,$$

where ν: lattice strength, c: mean field effect, d: dipole moment, b: scattering length.



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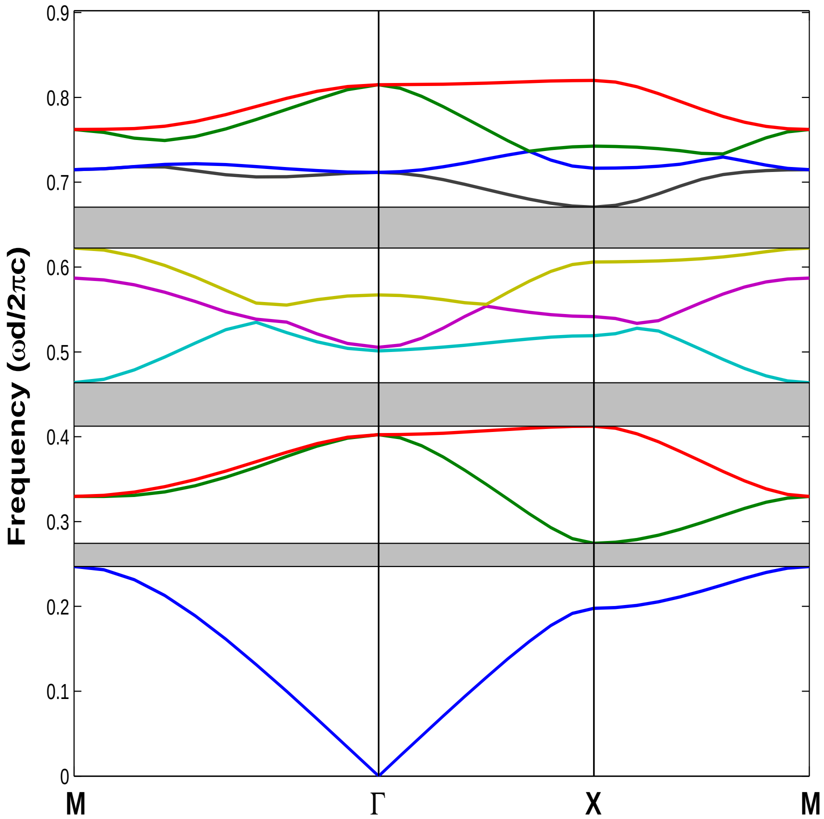
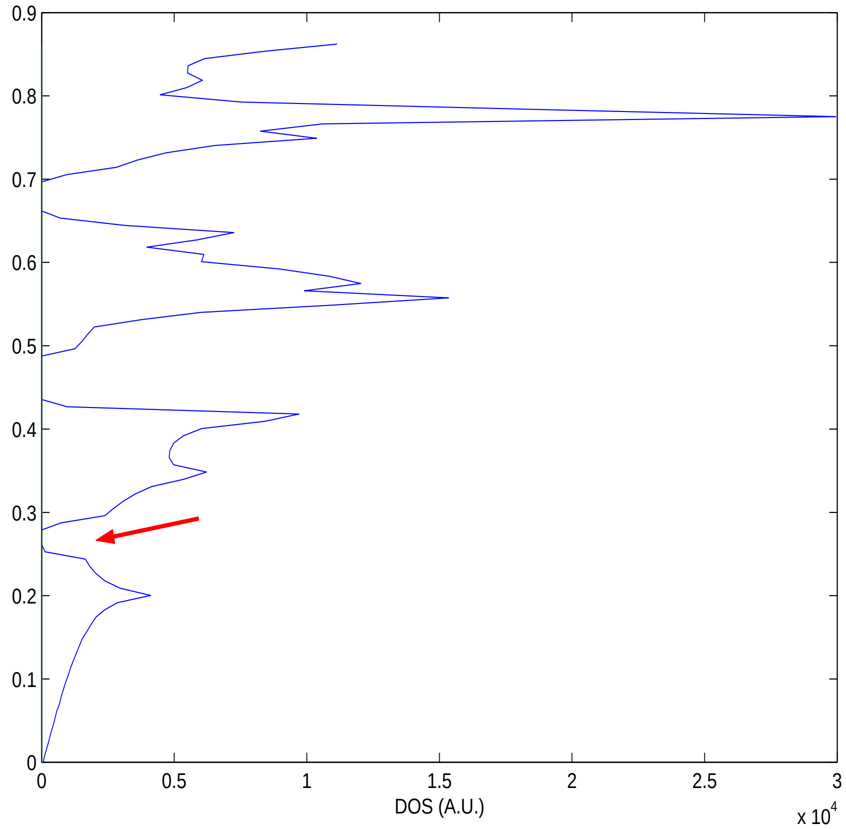
Y.Y. Lin, R.-K. Lee, and T.F. Jiang, *in preparation*.

Band diagram and Density of States, Spring 2006

$$\frac{1}{\epsilon(\mathbf{r})}\nabla \times \{\nabla \times \mathbf{E}(\mathbf{r})\} = \frac{\omega^2}{c^2}\mathbf{E}(\mathbf{r}),$$





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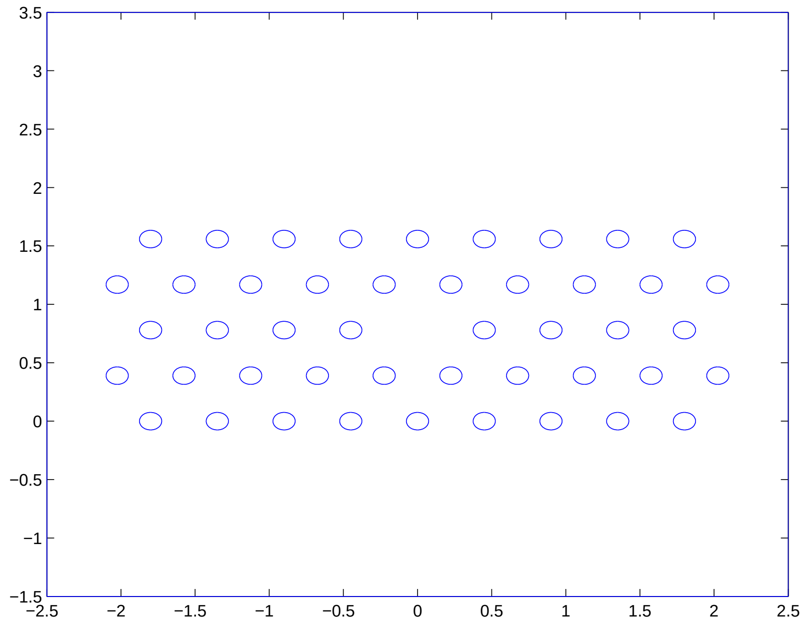
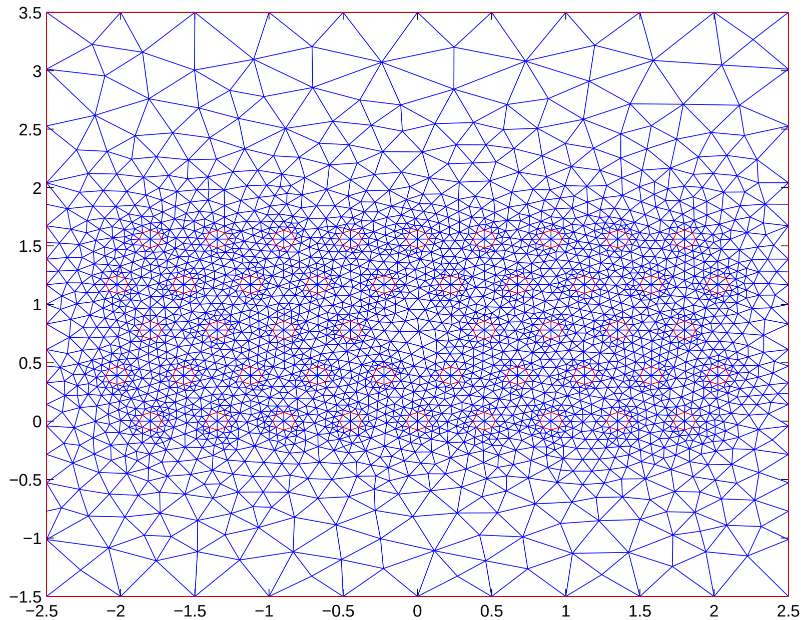
J.-P. Wu

Photonic Crystals





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Thermal analysis of Nd:YVO4 laser crystal in diode pump solid state laser by finite element method

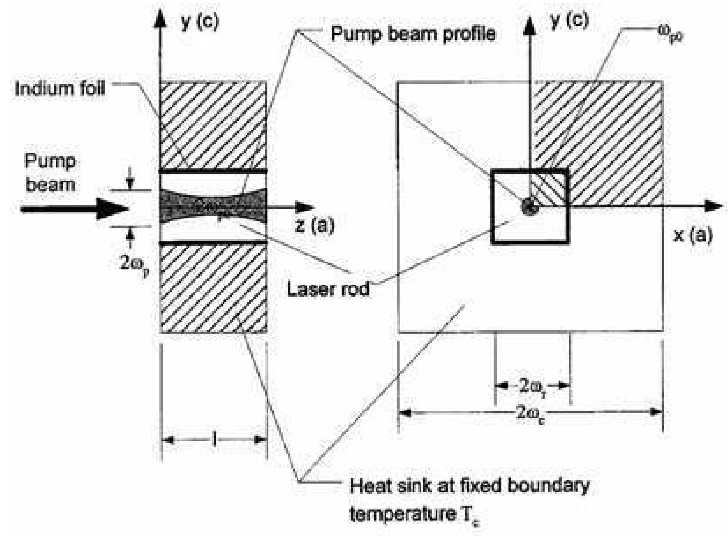
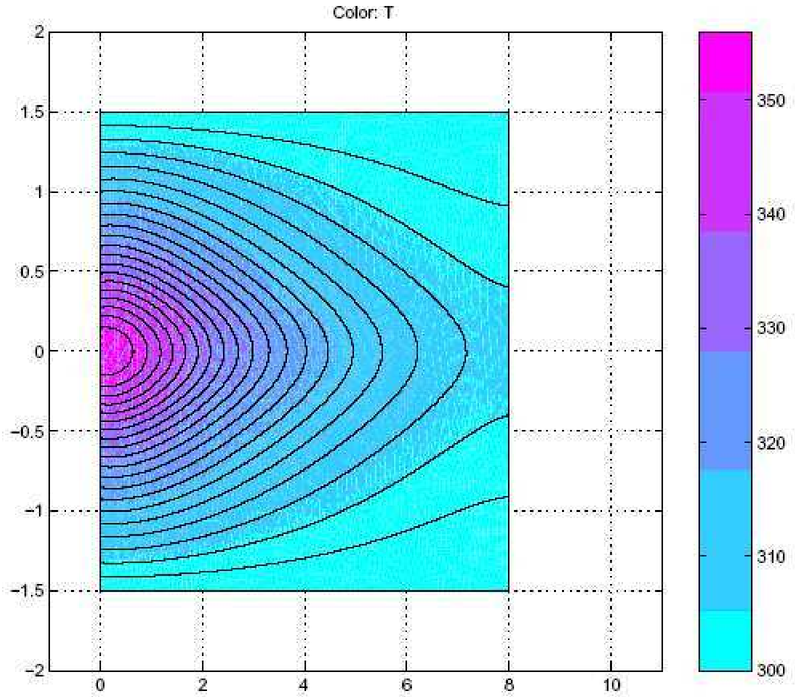


Figure 1. Side and end view of an end-pumped DPSS laser system



Shoutai Lin, 938106

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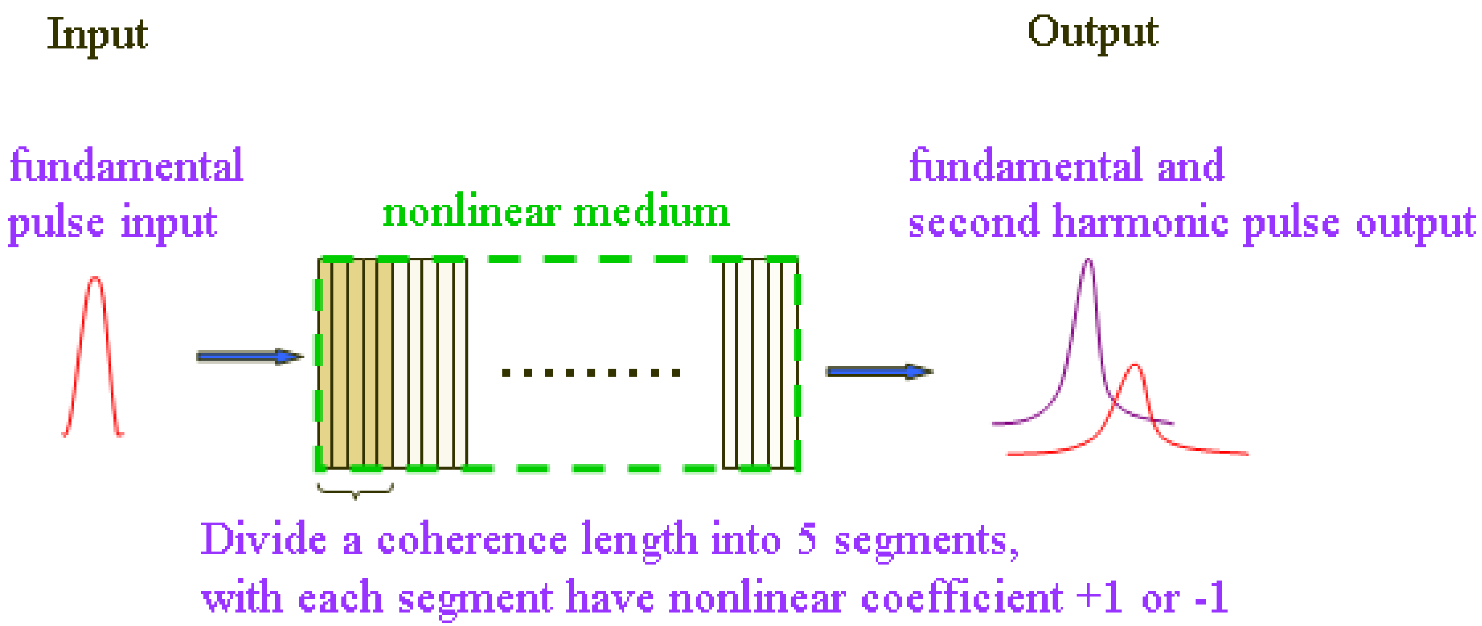
Optimization of SHG pulse

$$\frac{\partial A}{\partial z} = \frac{\eta}{2}\frac{\partial A}{\partial T} + i\xi_1 \frac{\partial^2 A}{\partial T^2} - i\rho_1 A^* B,$$

$$\frac{\partial B}{\partial z} = -\frac{\eta}{2}\frac{\partial B}{\partial T} + i\xi_2 \frac{\partial^2 A}{\partial T^2} - i\Delta k B - i\rho_1 A^2,$$



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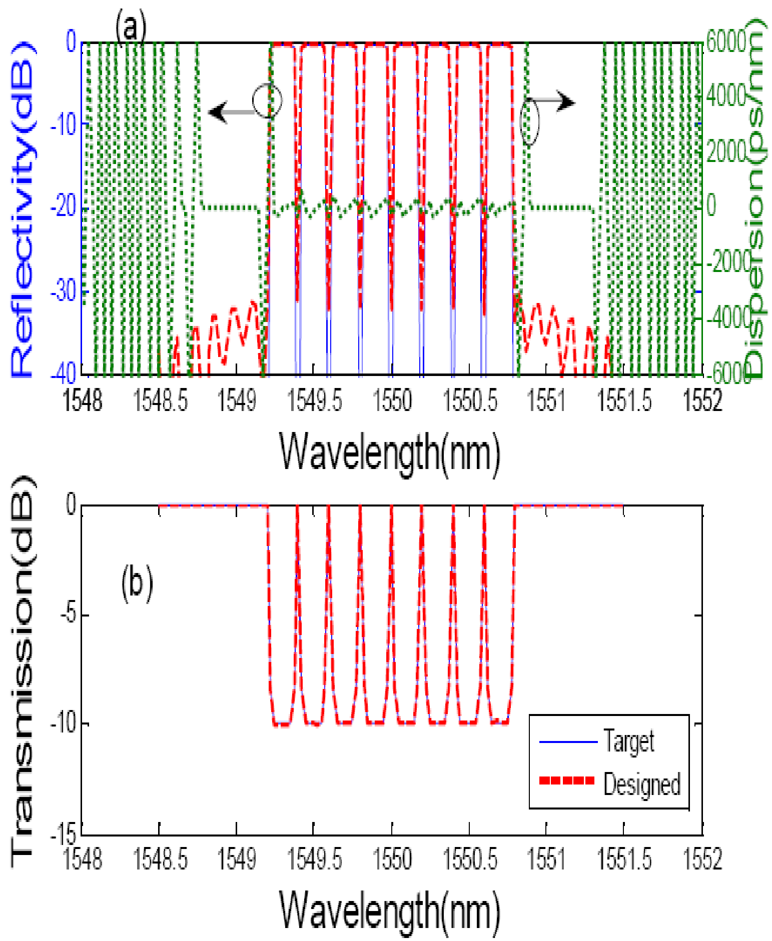
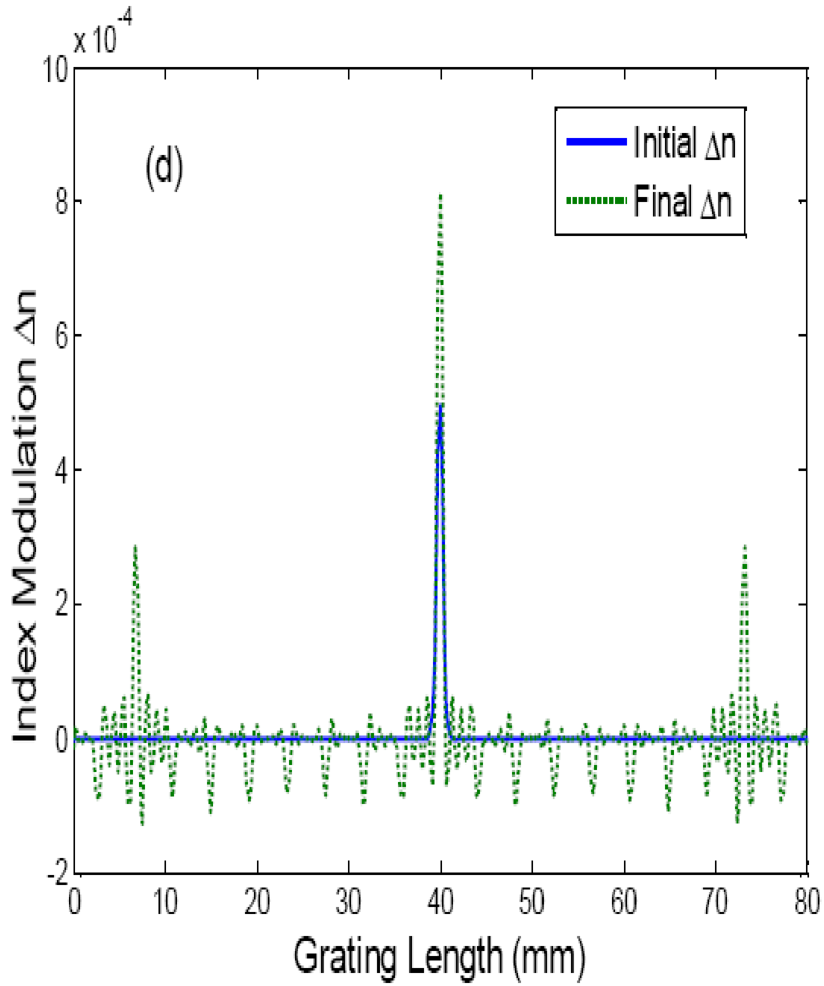
Lagrange Multiplier method for FBG





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C.L. Lee, R.-K. Lee, and Y.M. Kao, *Opt. Express* **14**, 11002 (2006).

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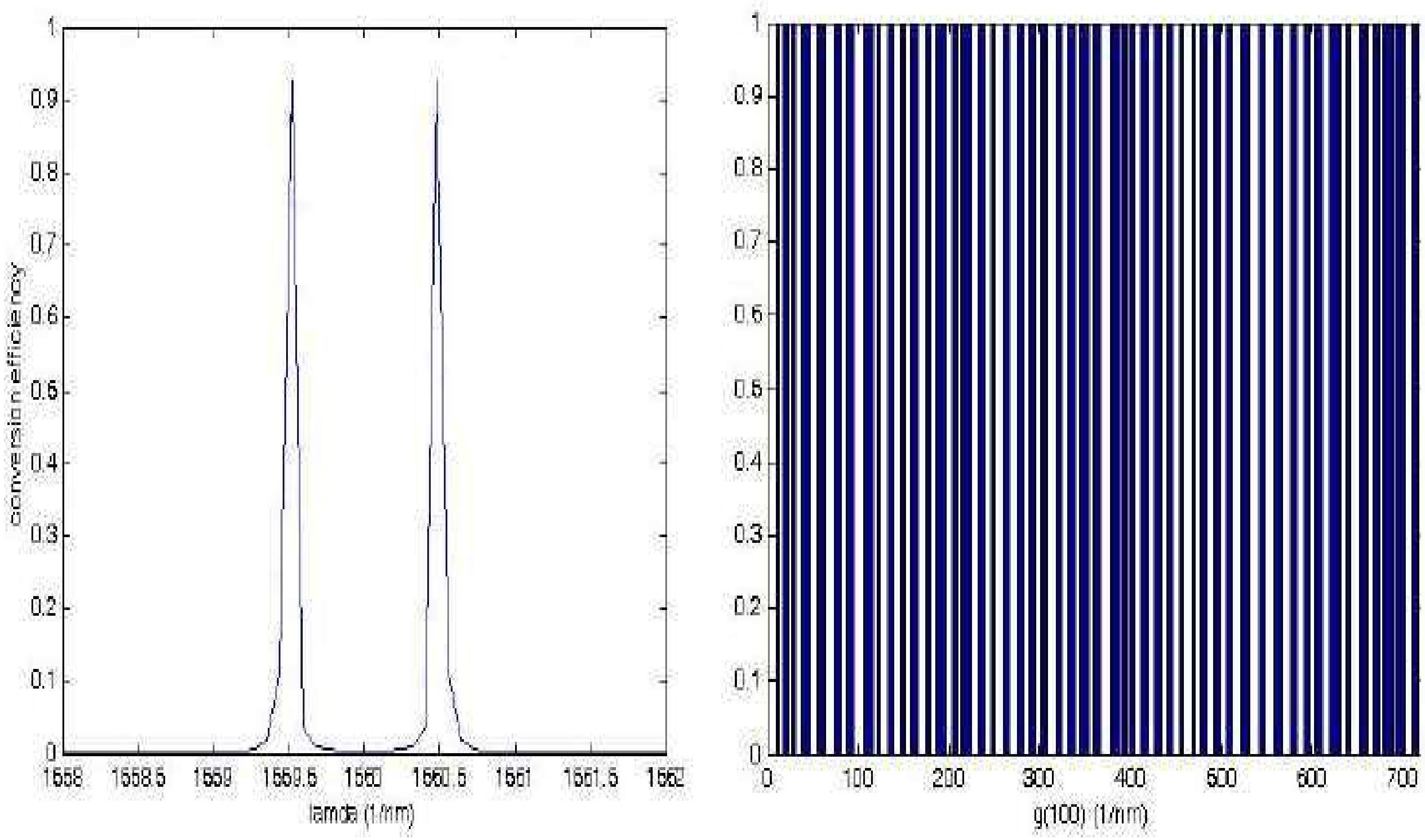

Ching-Jen Cheng, g936812,

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Feedback of VCSEL, Spring 2007

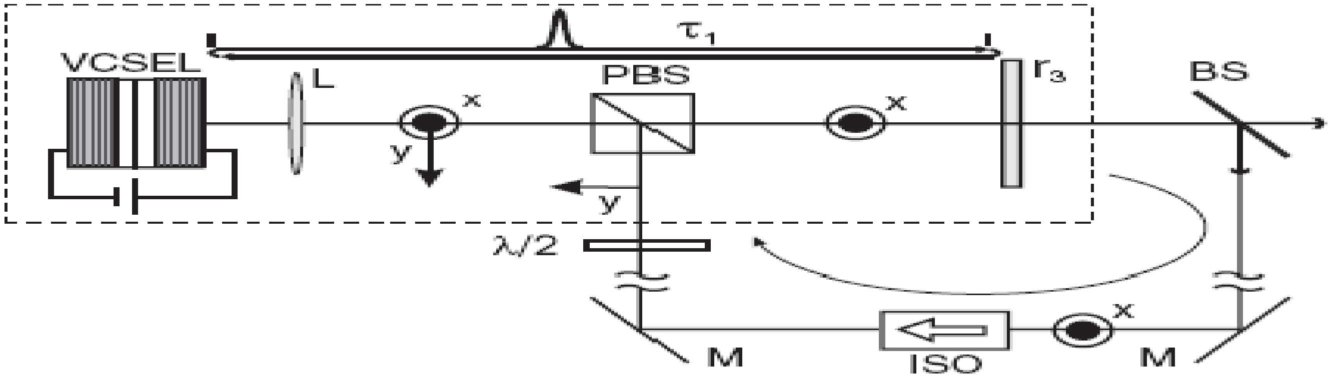
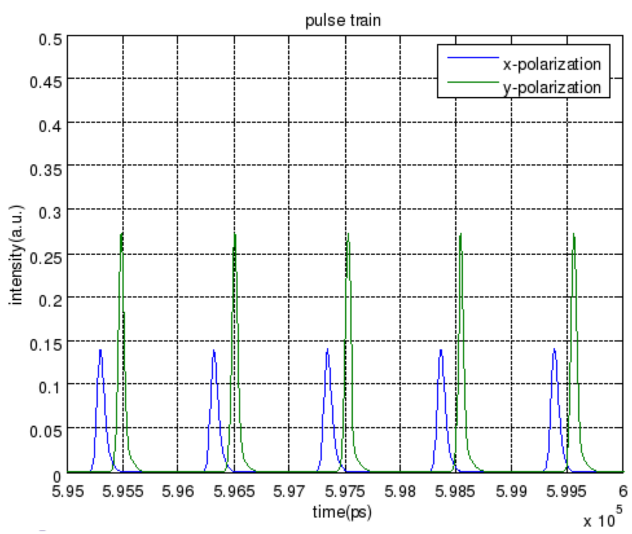
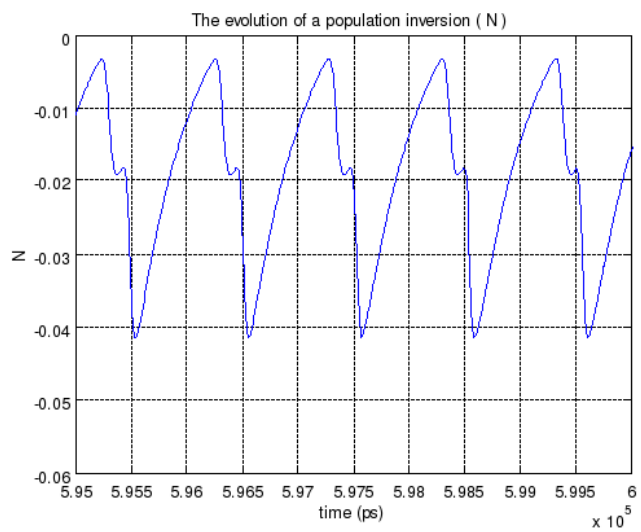


FIG. 1. Mode-locked laser setup comprising an external-cavity VCSEL (dashed box) and a reinjection arm. Symbols: polarizing beam splitter (PBS), optical isolator (ISO), and half-wavelength plate (λ/2).





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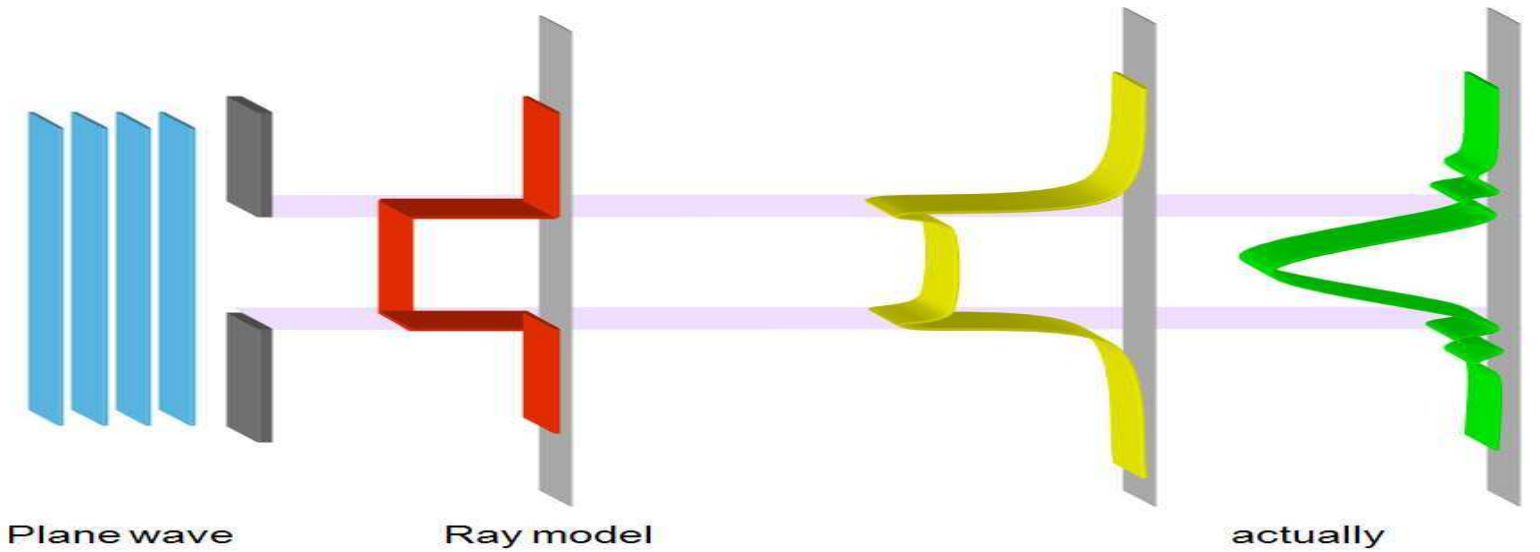

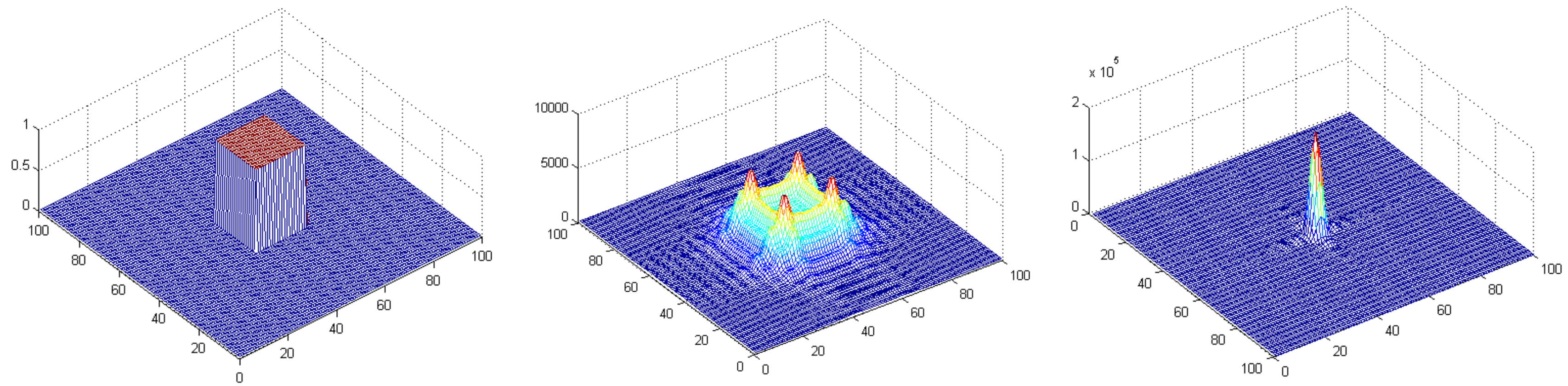

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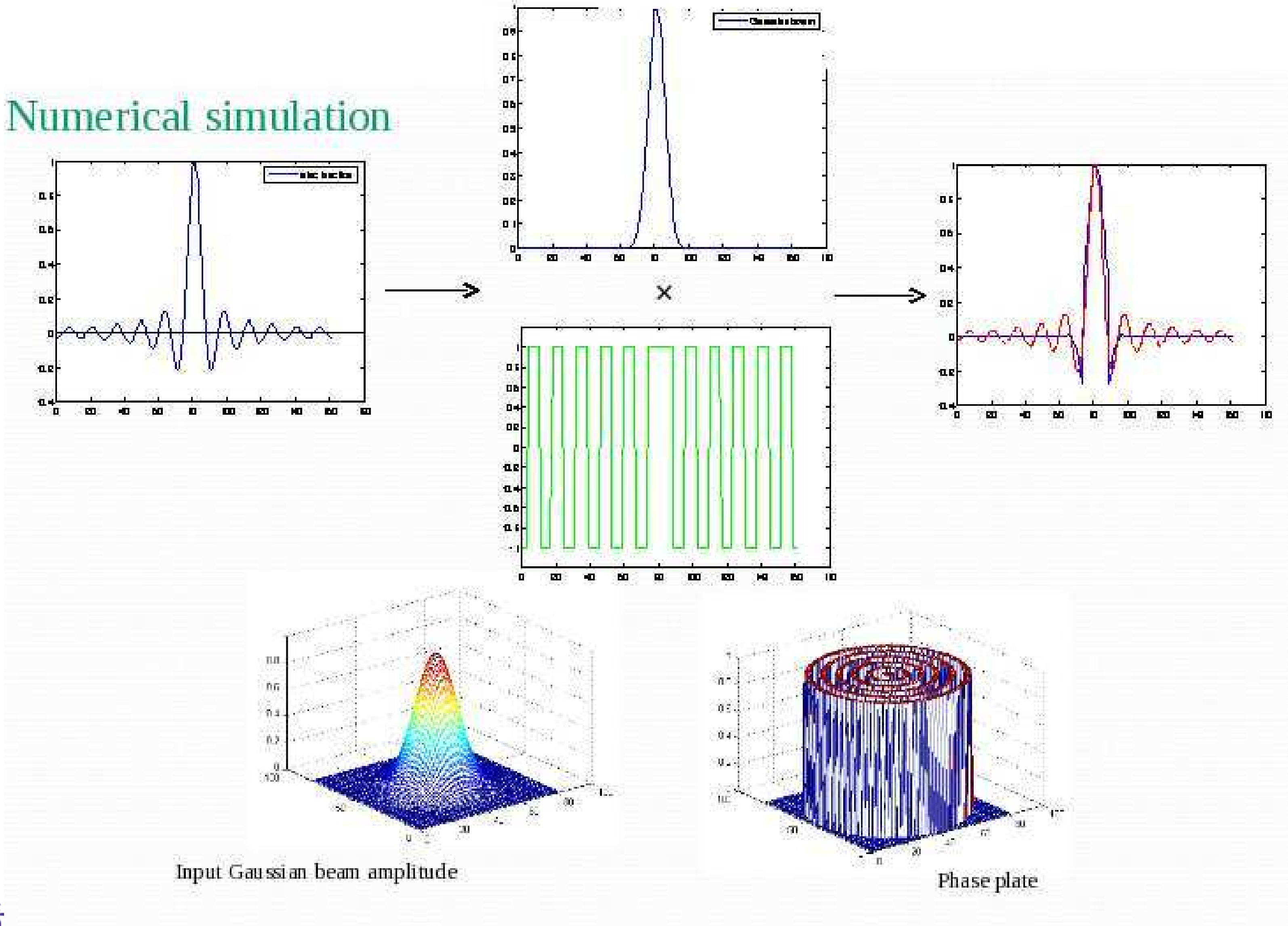

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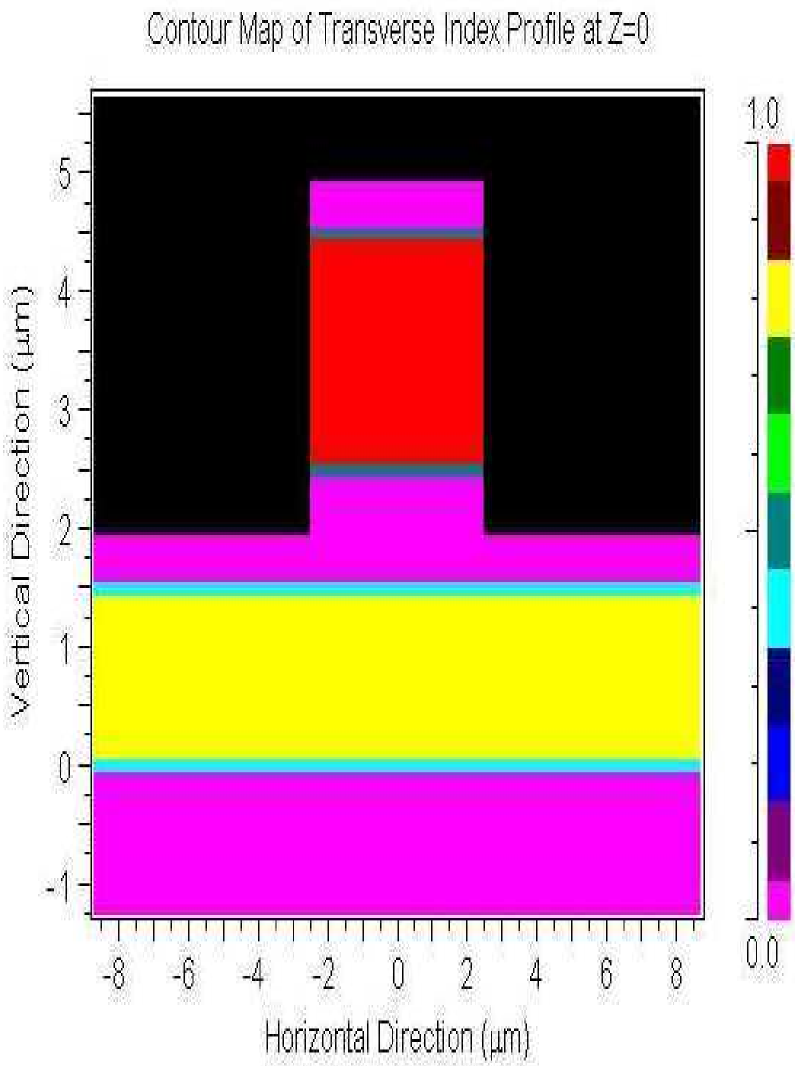
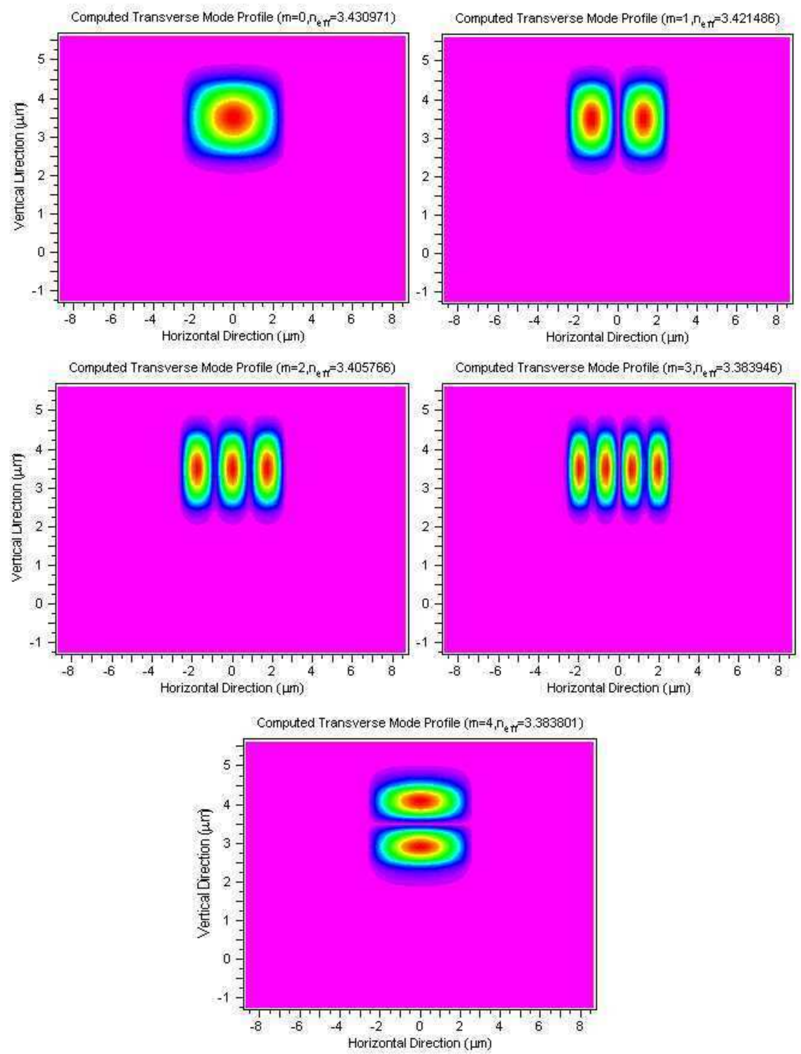
Waveguide structures





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by BeamProp

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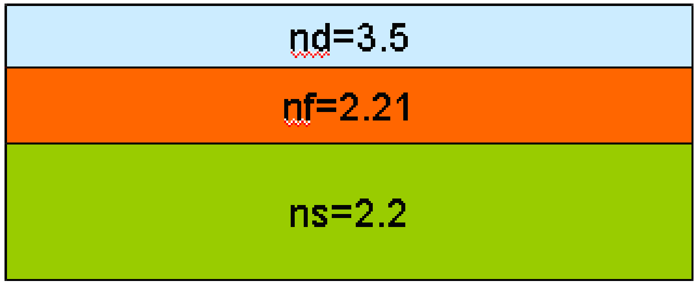
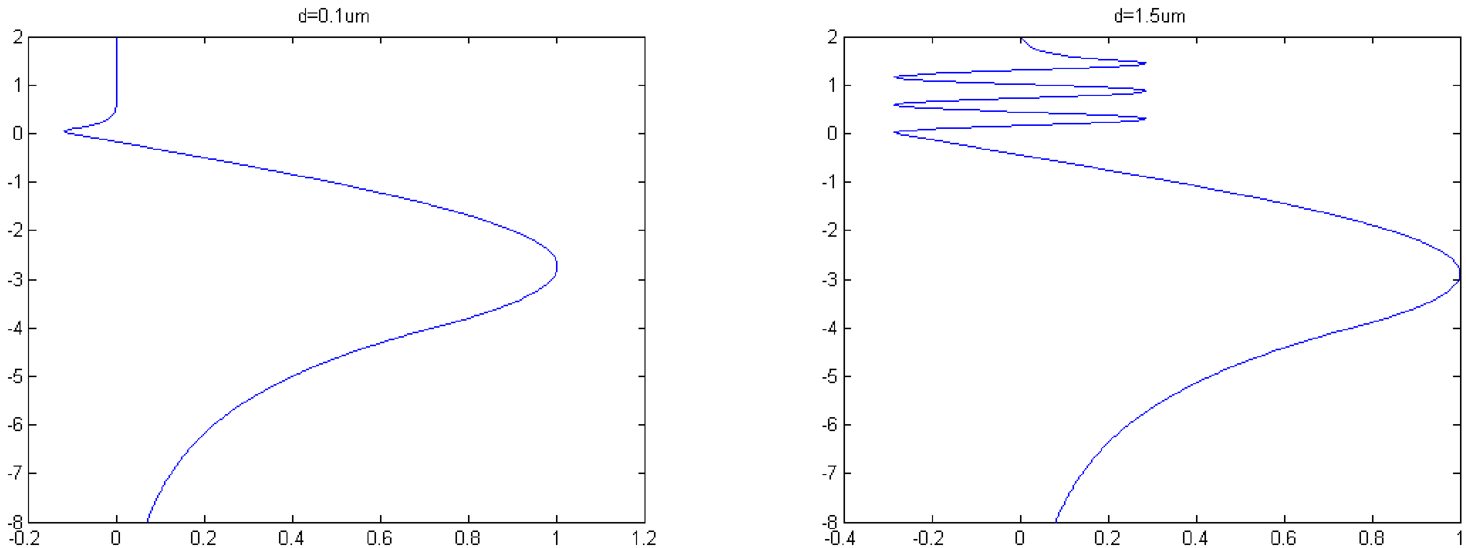
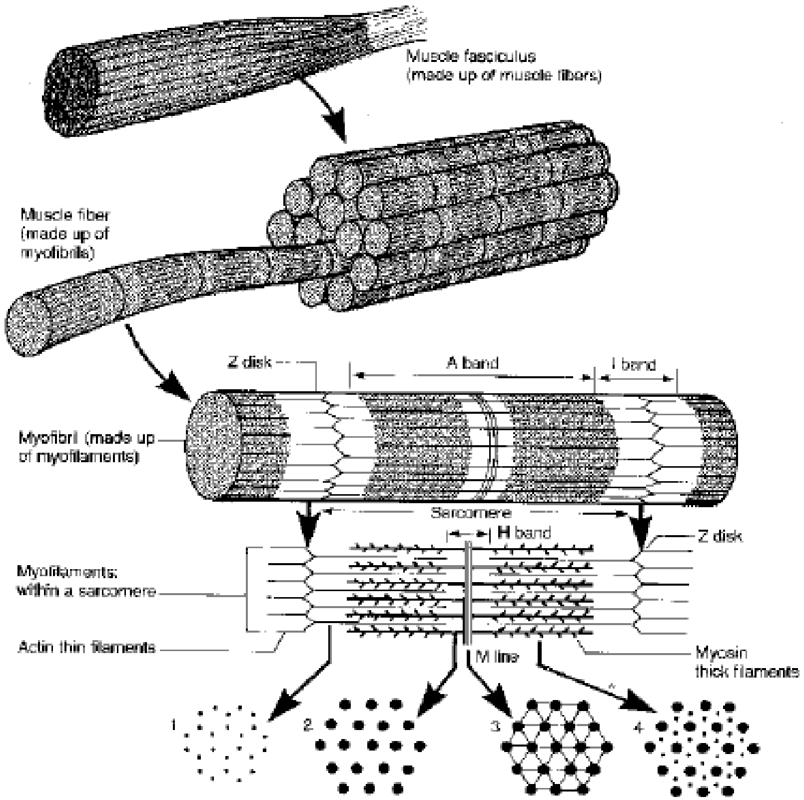
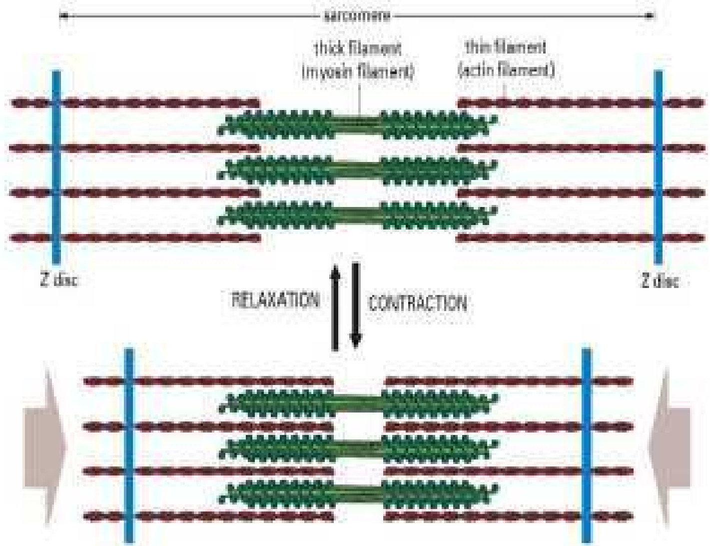
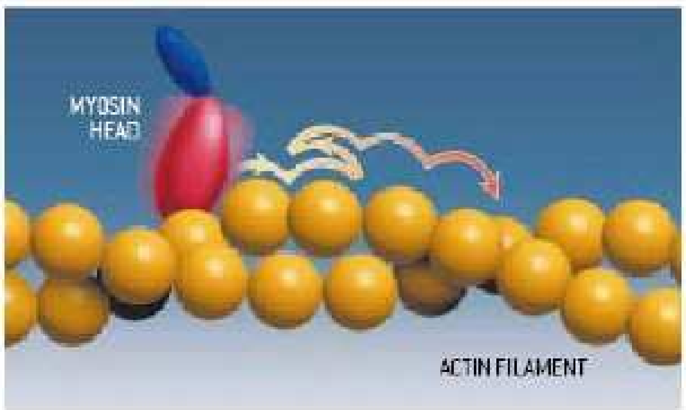




Muscle Contraction: Myosin







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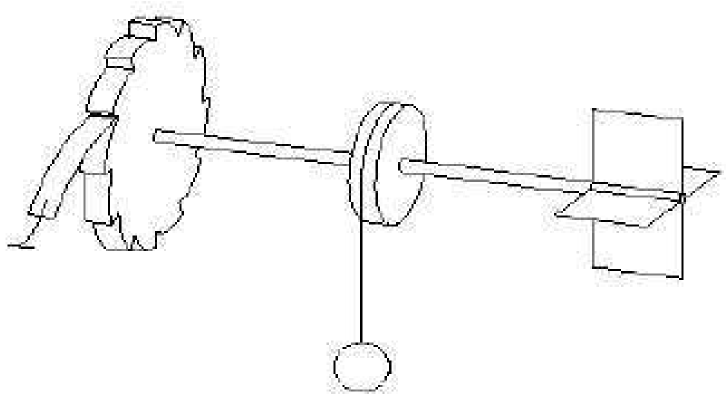
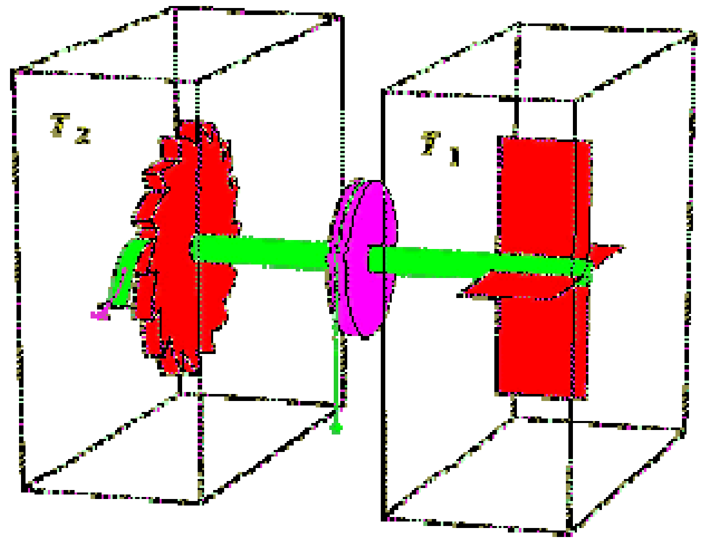
Ratchets





thought experiment of a perpetual mobile against the 2nd Law of Thermodynamics

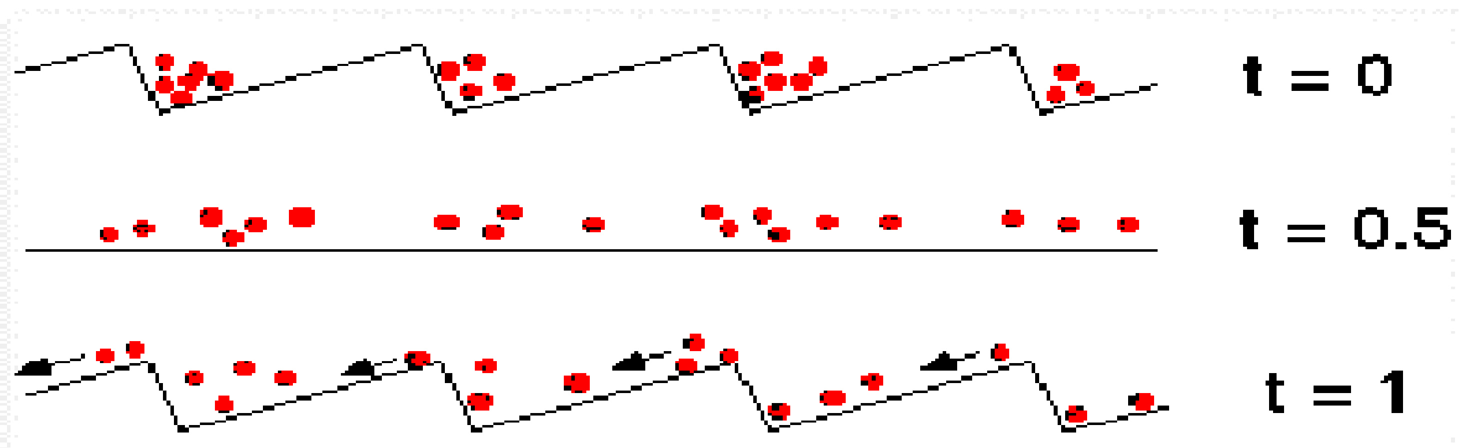
R. P. Feynman, *The Feynman Lectures on Physics*, Vol. 1, Chap. 46 (1963).

Mathematical Model for Ratchets

Ratchet ingredients:

1. Brownian particle (mass m)
2. periodic asymmetric potential $V(x,t)$
3. zero mean driving forces $f(t)$, i.e. $\langle f(t) \rangle = 0$

Ratchet model: $m\ddot{x} + \gamma \dot{x} + \frac{d}{dx}V(x,t) = f(t)$ **Interesting Behavior:** $\langle x(t) \rangle \neq 0$ even when $\langle f(t) \rangle = 0$



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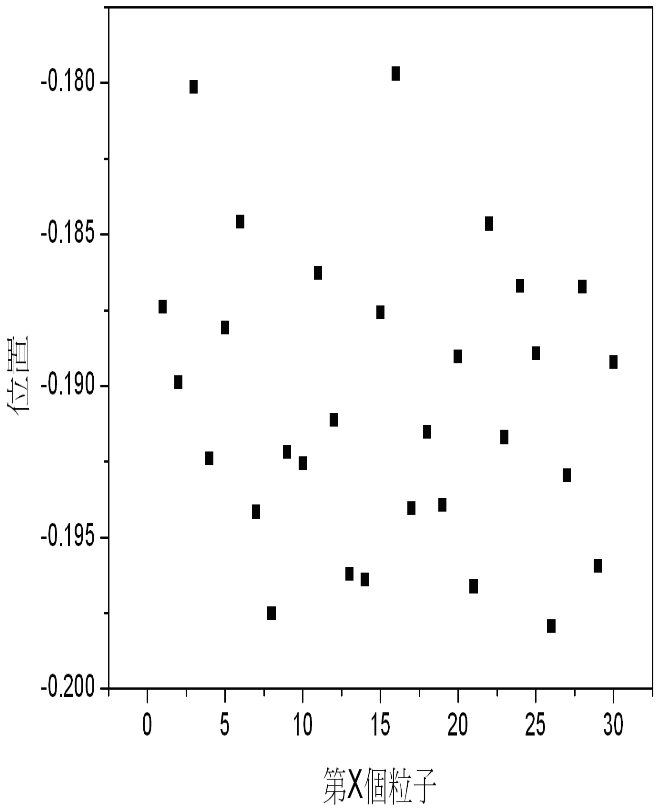
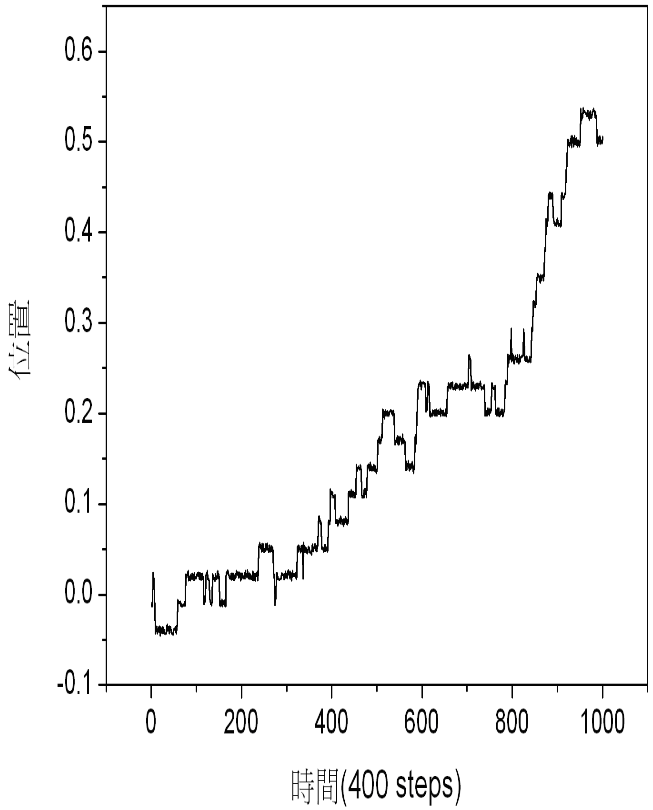
Brownian motor,





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933325,

Smith-Purcell radiation,

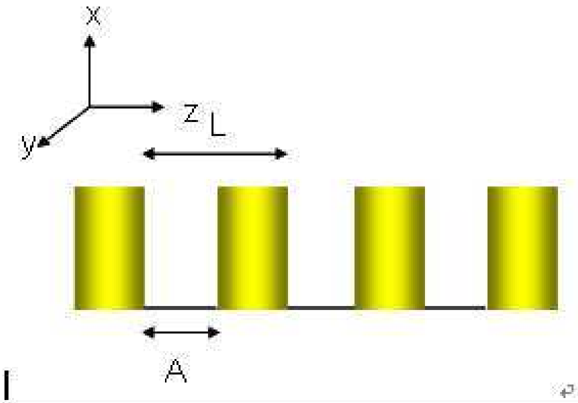

Fig.1

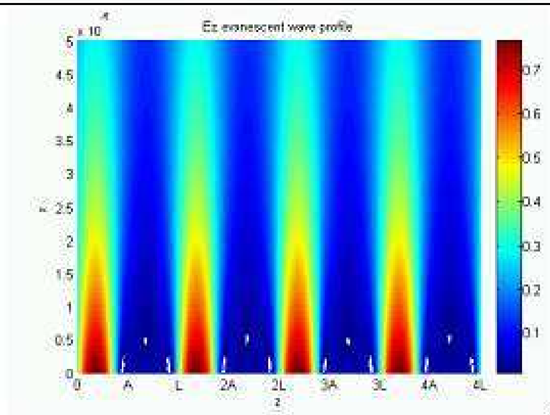

Fig.2

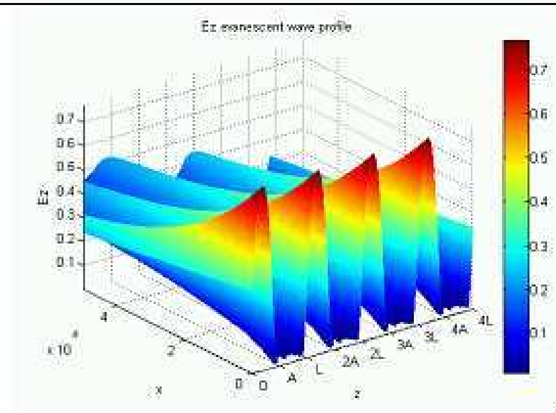

Fig.3

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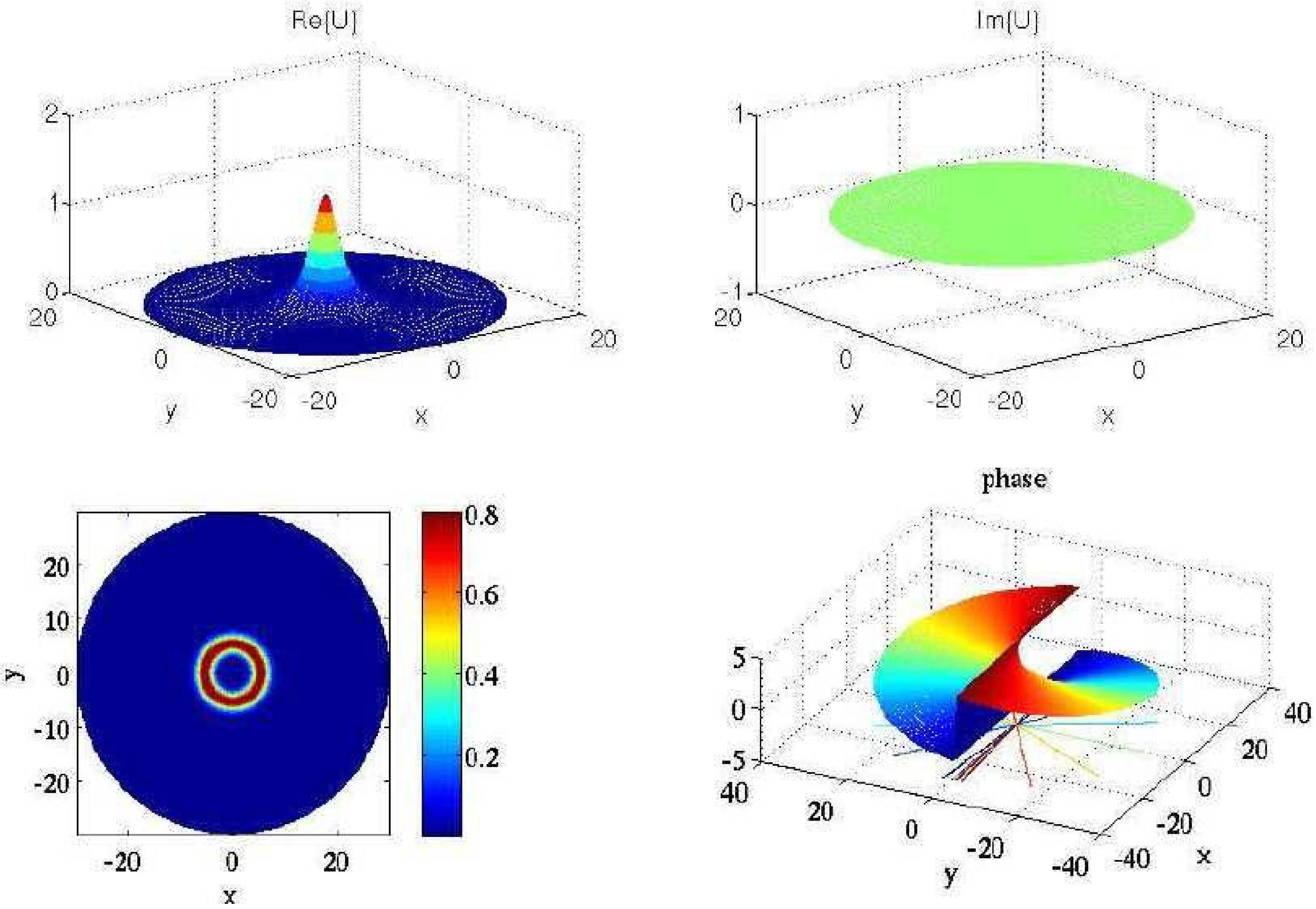
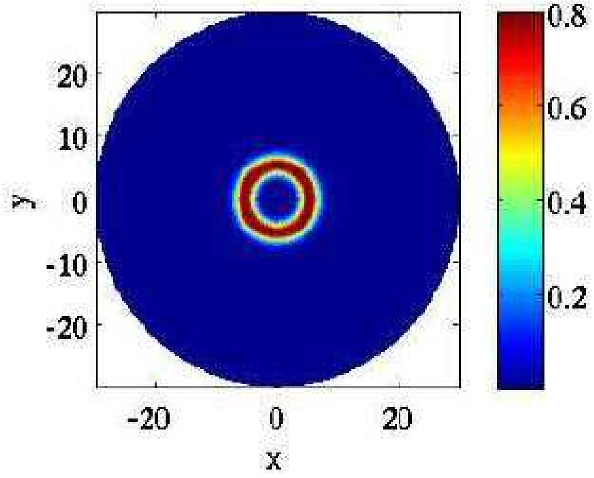
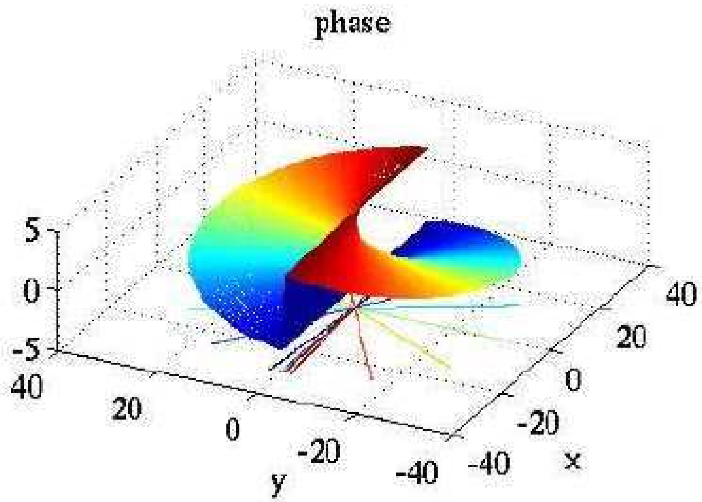
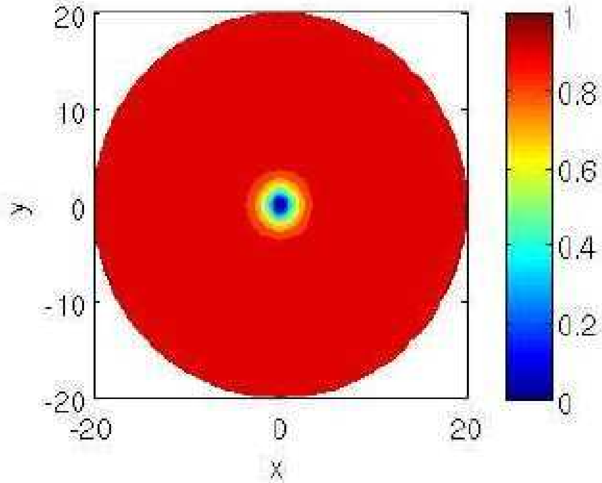
Vortex and Vortex solitons

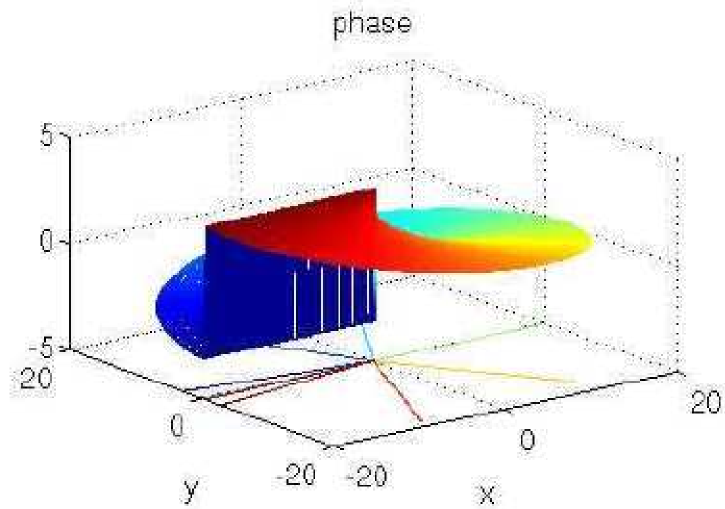


Vortex and Vortex solitons









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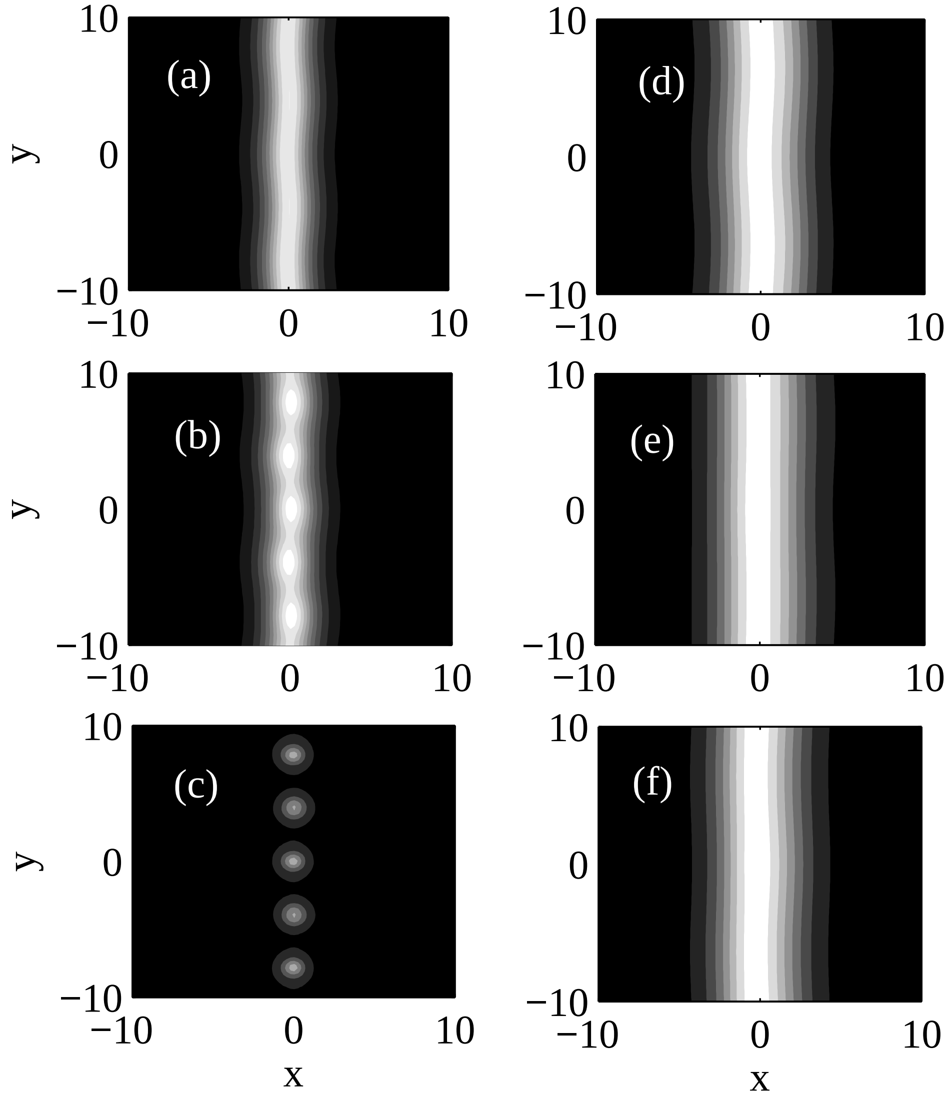
Soliton transverse instabilities in nonlocal nonlinear media

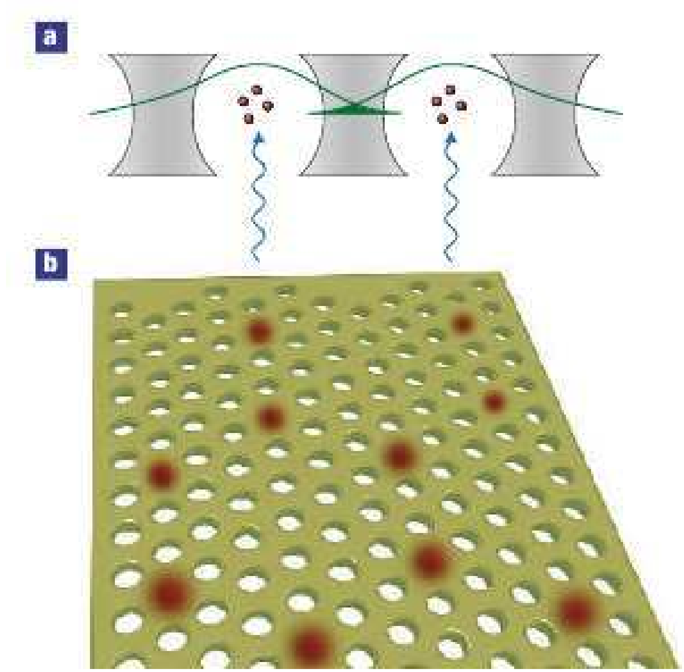


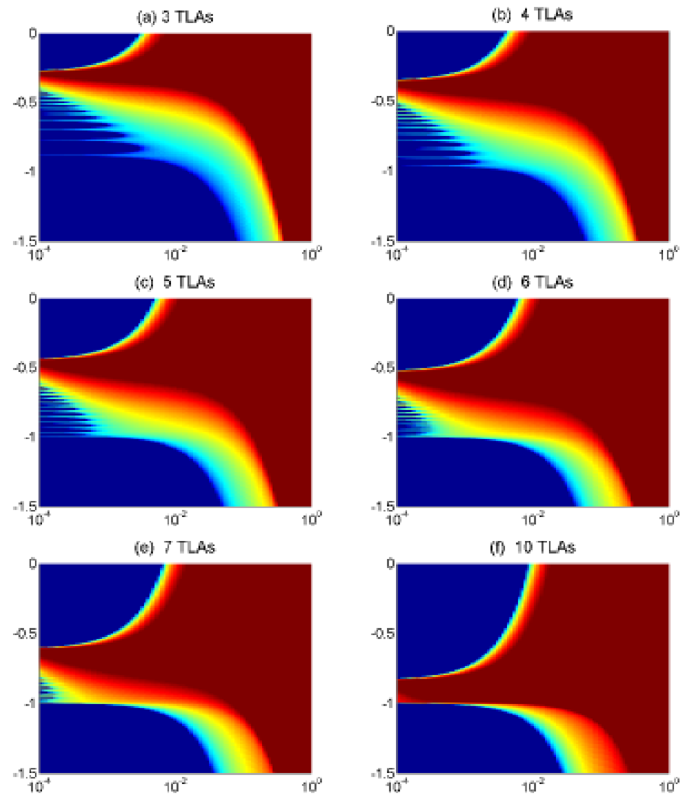
Y.Y. Lin, R.-K. Lee, and Yu. S. Kivshar *arXiv*:0708.2596v1(2007); to *Opt. Lett.*

Quantum Phase Transitions of Light in the Dicke-Bose-Hubbard model

$$\hat{H} = \sum_i H_i^{\text{DM}} - \kappa \sum_{ij} a_i^+ a_j - \mu \sum_i N_i,$$

$$H_i^{DM} = \varepsilon J_i^+ J_i^- + \omega a_i^+ a_i + \beta (a_i J_i^+ + a_i^+ J_i^-),$$





Soi-Chan Lai and Ray-Kuang Lee, *arXiv*:0709.1352 (quant-ph) (2007),

to *Phys. Rev. A*.

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