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Synthesis of long-period fiber gratings with a Lagrange multiplier optimization method

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Abstract

Based on the Lagrange multiplier optimization (LMO) method, a new synthesis approach for designing complex long-period fiber grating (LPG) filters is developed and demonstrated. The proposed synthesis method is a simple and direct approach. It was used to efficiently search for optimal solutions and constrain various parameters of the designed LPG filters according to practical requirements. The inverse scattering algorithm is a good tool for designing FBG filters; as well as for designing transmission-type fiber grating filters like LPGs. Compared to the results of the discrete layer-peeling (DLP) inverse scattering algorithm for LPGs, the synthesized LPGs, using the LMO approach, are more flexible and workable for the constraint conditions can be easily set in the user-defined cost functional, such as the limitation on the maximum value of the refractive index modulation and the parameters of the initial guess in LMO algorithm. Linear transmission LPG filters and EDFA gain flattening filters are synthesized and analyzed systematically by using the proposed method with different parameters. Moreover, as a variation-based method, we find that the convergence and the synthesized results of the LMO method are strongly dependent on the initial guess parameters.

Keywords: Long-period fiber grating (LPG); Lagrange multiplier optimization (LMO); Grating synthesis

1. Introduction

Long-period fiber gratings (LPGs) are transmission-type fiber grating devices in which the guided core mode is coupled to the forward propagating cladding modes through the periodic photo-induced index modulation. LPGs are important key components in fiber communication and fiber sensing systems, acting as EDFA gain flattening filters, band-rejection filters, mode converters, and high sensitivity fiber sensors [1]. Due to lots of applications, a simple and efficient approach to design and synthesize LPGs with complex properties is highly demanded. From literatures, the existing synthesis methods for LPGs can be classified into two categories: (1) the inverse scattering method-based synthesis approaches [2–8]; and (2) the stochastic-based optimization approaches [9–11]. By using the discrete layer-peeling (DLP) inverse scattering algorithm, grating assisted co-directional couplers (GACCs) and LPG filters can be synthesized, in which the coupling coefficient corresponding to a given spectral transmission response is uniquely determined if additional assumptions about the filter properties are used (such as the under-coupled assumption) [6]. Even though LPG filters with required spectral transmission shapes can be inversely synthesized with DLP inverse scattering algorithm, but there are still a number of disadvantages in using this method for some LPG filter syntheses. These disadvantages include: the required grating length is typically too long to fabricate, the spatial grating profiles

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(including amplitude and phase) are usually too complicated, and the phase of the target transmission spectrum of the designed LPG should be undetermined. Due to these disadvantages, the practical application of the LPG devices is severely limited. Therefore, several LPG synthesized optimization methods based on stochastic methods or Monte–Carlo optimization approaches are proposed to directly synthesize the LPG with some possible constraint conditions imposed on the grating parameters [9–11]. These optimization methods directly minimize the difference between the targeted and the synthesized spectra. The minimization can be performed by using popular optimization algorithms or in particular the evolutionary algorithms (EA). Compared to the inverse scattering DLP methods, the optimization approaches do not have an ambiguity problem caused by the unknown phase spectrum. Moreover, optimization methods have the potential capability to obtain an index profile that can be more practically implemented by properly imposing additional constraints on the solution to be found. Although the optimization method has the superiorities mentioned above, however, the calculation time in finding an optimization solution is typically too long and the designed coupling coefficient profiles are usually piecewise-defined functions [9,10]. It is not very practical because the UV beam output from the exposure laser used for fiber grating writing is normally a Gaussian profile with a certain width, which is not suitable for writing the coupling coefficients with uniformly leveled profiles [11]. Therefore, in this study, a new and simple approach to the solution of LPG synthesis problem was developed. The proposed method is based on the Lagrange multiplier optimization (LMO) method, which is a variation-based approach and various parameters of the designed devices can be embedded through a user-defined cost functional. The LMO method has been proven to be very useful in designing optical pulse shapes in linear and non-linear materials [12–15]. We also have demonstrated that LMO optimization algorithm can be implemented for designing multi-channel FBGs (reflection-type) in the DWDM applications [16]. An extension of LMO optimization method is to synthesize transmission-type gratings, in such a way that another branch of fiber grating-based devices such as LPGs or directional couplers can also be synthesized by the proposed simple and fast method.

In general, compared to layer-peering method, our proposed method can be easily embedded with various constraint parameters. And compared to Monte–Carlo based approach, such as evolutionary algorithm, our method is a direct synthesis method with fast convergence rate and without using any random number generators [8–11]. For the first time, transmission-type LPGs including a linear LPG and an EDFA gain flattening LPG filters are successfully synthesized and analyzed by using LMO method. The designed results also prove that the presented algorithm is an effective method both for reflection-type and transmission-type fiber grating devices.

2. LMO algorithm for the synthesis of LPGs

For weakly index modulation,the characteristics of LPGs can be well described by the coupled mode equations [9–11]. If only the coupling between two modes (core mode and one cladding mode) in the LPG is considered, the coupled mode equations can be written as

$$\frac{\mathrm{d}A^{\mathrm{co}}(\delta,z)}{\mathrm{d}z} = \mathrm{i}\delta A^{\mathrm{co}}(\delta,z) + \mathrm{i}\kappa(z)A^{\mathrm{cl}}(\delta,z), \tag{1a}$$

$$\frac{\mathrm{d}A^{\mathrm{cl}}(\delta,z)}{\mathrm{d}z} = -\mathrm{i}\delta A^{\mathrm{cl}}(\delta,z) + \mathrm{i}\kappa^*(z)A^{\mathrm{co}}(\delta,z). \tag{1b}$$

Here $A^{\mathrm{co}}(z)$ and $A^{\mathrm{cl}}(z)$ represent the core and the cladding modes respectively, $\delta = (1/2)[\beta^{\mathrm{co}} - \beta^{\mathrm{cl}} - 2\pi/\Lambda] = \pi\Delta n_{\mathrm{eff}}(1/\lambda - 1/\lambda_{\mathrm{D}})$ is the detuning parameter for LPG, Λ is the grating period, $\lambda_{\mathrm{D}} = \Delta n_{\mathrm{eff}} \cdot \Lambda$ is the designed wavelength, Δn_{eff} is the difference of the effective indices for the core and cladding modes, β^{co} and β^{cl} are the propagation constants, and $\kappa(z) = \eta\pi\Delta n(z)/\lambda_{\mathrm{D}}$ is the designed coupling coefficient function with $\Delta n(z)$ being the envelope function of the grating index modulation and η the overlapping factor.

The main idea of the LMO algorithm to synthesize LPGs is to find the complex spatially coupling coefficient; $\kappa(z)$, of the grating with that the corresponding transmission spectrum, $T_{\mathrm{co}}(\lambda)$, can meet a given target spectrum, $T_{\mathrm{d}}(\lambda)$. Therefore, the objective functional needed to be minimized can be defined by the users, such as,

$$\Phi = \frac{1}{2}\int_{-\infty}^{\infty} [T_{\mathrm{co}}(\lambda) - T_{\mathrm{d}}(\lambda)]^2 \mathrm{d}\lambda + \frac{\beta}{2}\int_0^L [\kappa(z)]^2 \mathrm{d}z, \tag{2}$$

where $T_{\mathrm{co}}(\lambda) = |A^{\mathrm{co}}(L)/A^{\mathrm{co}}(0)|^2$is the calculated transmission spectrum of output core mode for LPG, $T_{\mathrm{d}}(\lambda)$ is the target transmission spectrum, L is the total length of the grating, and β is a positive number acting as a weighting parameter for the constraint control on the spatial coupling coefficient, $\kappa(z)$. In the defined cost functional, Eq. (2), the spatially coupling coefficient $\kappa(z)$ is used to shape an output transmission spectrum in the core mode $T_{\mathrm{co}}(\lambda)$ for a given transmission spectrum $T_{\mathrm{d}}(\lambda)$, meanwhile both the spectra discrepancy and the norm of the coupling coefficient profiles are minimized simultaneously. In the proposed LMO algorithm to synthesize LPGs, the Lagrange multipliers μ^{co} and μ^{cl} were introduced for the core and cladding modes of LPG, respectively. With the objective functional for LPG inverse design in Eq. (2), a specific cost functional is expressed as follows:

$$\begin{aligned} J = \Phi &+ \int_0^L \int_{-\infty}^{\infty} \mu_{\mathrm{R}}^{\mathrm{co}} \cdot Re\left[\frac{\mathrm{d}A^{\mathrm{co}}}{\mathrm{d}z} - \mathrm{i}\delta A^{\mathrm{co}} - \mathrm{i}\kappa A^{\mathrm{cl}}\right]\mathrm{d}\lambda\mathrm{d}z \\ &+ \int_0^L \int_{-\infty}^{\infty} \mu_{\mathrm{I}}^{\mathrm{co}} \cdot Im\left[\frac{\mathrm{d}A^{\mathrm{co}}}{\mathrm{d}z} - \mathrm{i}\delta A^{\mathrm{co}} - \mathrm{i}\kappa A^{\mathrm{cl}}\right]\mathrm{d}\lambda\mathrm{d}z \\ &+ \int_0^L \int_{-\infty}^{\infty} \mu_{\mathrm{R}}^{\mathrm{cl}} \cdot Re\left[\frac{\mathrm{d}A^{\mathrm{cl}}}{\mathrm{d}z} + \mathrm{i}\delta A^{\mathrm{cl}} - \mathrm{i}\kappa^* A^{\mathrm{co}}\right]\mathrm{d}\lambda\mathrm{d}z \\ &+ \int_0^L \int_{-\infty}^{\infty} \mu_{\mathrm{I}}^{\mathrm{cl}} \cdot Im\left[\frac{\mathrm{d}A^{\mathrm{cl}}}{\mathrm{d}z} + \mathrm{i}\delta A^{\mathrm{cl}} - \mathrm{i}\kappa^* A^{\mathrm{co}}\right]\mathrm{d}\lambda\mathrm{d}z \end{aligned} \tag{3}$$

The core and cladding modes, $A^{co}(z)$ and $A^{cl}(z)$, coupling coefficient, $\kappa(z)$, and the Lagrange multipliers, μ^{co} and μ^{cl}, used in LMO methods are all complex numbers here. We separate these complex variables into real and imaginary parts, respectively, i.e. $A^{co}=A^{co}_{R}+iA^{co}_{I}$, $A^{cl}=A^{cl}_{R}+\ iA^{cl}_{I}$, $\kappa=\kappa_{R}+i\kappa_{I}$, $\mu^{co}=\mu^{co}_{R}+i\mu^{co}_{I}$ and $\mu^{cl}=\mu^{cl}_{R}+i\mu^{cl}_{I}$. To minimize the cost functional, J, in Eq. (3) a variational method was used with respect to A^{co}_{R},A^{co}_{I} of the core mode, and A^{cl}_{R},A^{cl}_{I} of the cladding mode through the Lagrange multipliers $\mu^{co}(z),\mu^{cl}(z)$. The resulting equations of motion for the Lagrange multipliers can be derived as follows:

$$\frac{\partial\mu^{co}(z)}{\partial z}=i\delta\cdot\mu^{co}(z)+i\kappa(z)\mu^{cl}(z), \tag{4a}$$

$$\frac{\partial\mu^{cl}(z)}{\partial z}=-i\delta\cdot\mu^{cl}(z)+i\kappa^{*}(z)\mu^{co}(z). \tag{4b}$$

The boundary conditions for the coupled equations of the Lagrange multipliers, Eq. (4), can be obtained by using the variational method on the cost functional J with respect to A^{co} and A^{cl} at $z=L$, i.e. $\partial J/\partial A^{cl}(L)=0$, $\partial J/\partial A^{co}(L)=0$. The corresponding boundary conditions for Eq. (4) at $z=L$ are shown as below:

$$\mu^{co}(L)=-2\cdot A^{co}(L)\cdot\Delta_{t}, \tag{5a}$$

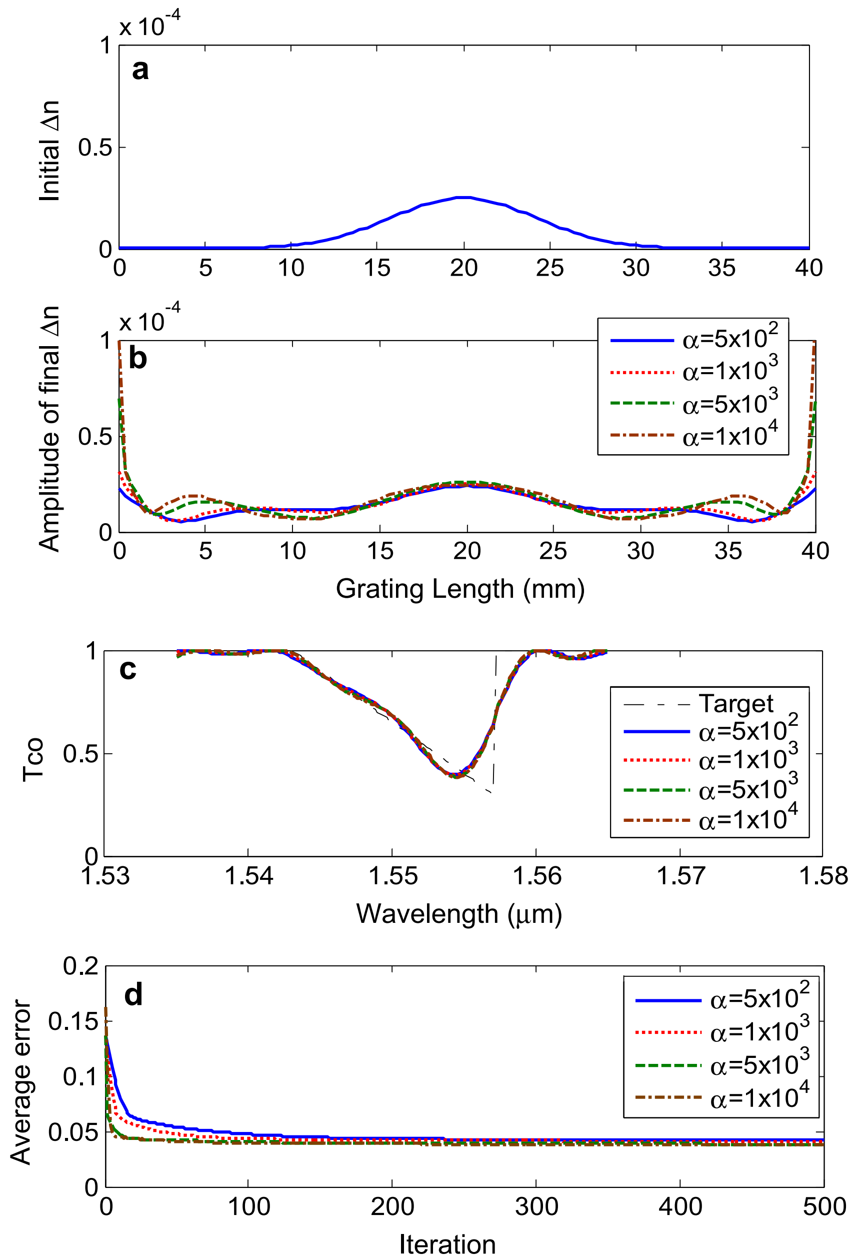

Fig. 1. (a) The initial guess of the Gaussian apodization coupling coefficient for the proposed algorithm, (b) final designed amplitude of the apodization profiles, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the absolute value of average error (average $|\Delta_t|$) for the designed linear LPGs with different *ad hoc* parameter α in the LMO method.

$$\mu^{\mathrm{cl}}(L) = 0, \tag{5b}$$

where $\Delta_t = T_{co}(\lambda) - T_d(\lambda)$ is the discrepancy between the output and target transmission spectra. Again, we use the variation method on the cost functional J with respect to the real and imaginary parts of the coupling coefficient function κ_R and κ_I, respectively, i.e.

$$\frac{\delta J}{\delta \kappa_R} = \beta \cdot \kappa_R + \int_{\infty}^{-\infty} \left(\mu_R^{co} \cdot A_I^{cl} - \mu_I^{co} \cdot A_R^{cl}\right) d\lambda + \int_{-\infty}^{\infty} \left(\mu_R^{co} \cdot A_I^{co} - \mu_I^{cl} \cdot A_R^{co}\right) d\lambda, \tag{6a}$$

$$\frac{\delta J}{\delta \kappa_I} = \beta \cdot \kappa_I + \int_{\infty}^{-\infty} \left(\mu_R^{co} \cdot A_R^{cl} + \mu_I^{co} \cdot A_I^{cl}\right) d\lambda + \int_{-\infty}^{\infty} \left(-\mu_R^{co} \cdot A_R^{co} - \mu_I^{cl} \cdot A_I^{co}\right) d\lambda. \tag{6b}$$

With an initial guess for the coupling coefficient profile, $\kappa(z) = \kappa_R(z) + i\kappa_I(z)$, the equations of motion for the multipliers are solved with the help of Eq. (4) and the corresponding boundary conditions in Eq. (5). The optimization of the LMO method progressed until the convergence requirement was satisfied through following iteration procedure:

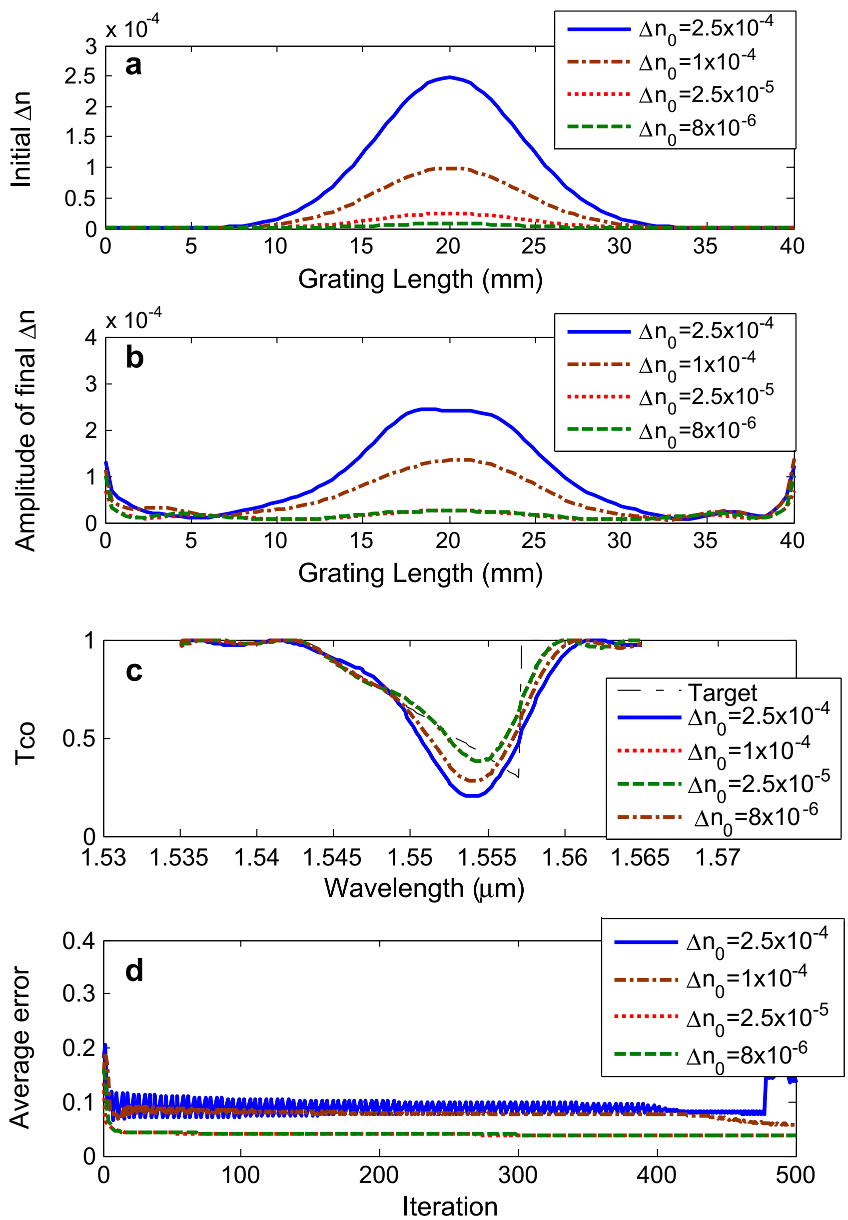

Fig. 2. (a) The initial guess of the Gaussian apodization functions with different maximum values of index modulation Δn_0, (b) final designed amplitude of the apodization profiles, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for synthesized linear LPGs.

$$\kappa'_{\mathrm{R}}(z) = \kappa_{\mathrm{R}}(z) - \alpha_{\mathrm{R}} \frac{\delta J}{\delta \kappa_{\mathrm{R}}}, \tag{7a}$$

$$\kappa'_{\mathrm{I}}(z) = \kappa_{\mathrm{I}}(z) - \alpha_{\mathrm{I}} \frac{\delta J}{\delta \kappa_{\mathrm{I}}}, \tag{7b}$$

where α_{R} and α_{I} are *ad hoc* constants for real and imaginary parts of the coupling coefficient, respectively. The new coupling coefficient can be expressed as follows $\kappa'(z) = \kappa'_{\mathrm{R}}(z) + \mathrm{i}\kappa'_{\mathrm{I}}$. The algorithm we use to implement the LMO methods for designing LPGs can be summarized with following steps,

(a) Guess an initial coupling coefficient function κ_{R} (only real part is given for simplifying the algorithm).
(b) Calculate the core and cladding modes, $A^{\mathrm{co}}(z)$ and $A^{\mathrm{cl}}(z)$, with Eq. (1) from $z = 0$ to $z = L$ with the given initial conditions, $A^{\mathrm{co}}(0) = 1$, $A^{\mathrm{cl}}(0) = 0$ and obtain all values of $A^{\mathrm{co}}(z)$, $A^{\mathrm{cl}}(z)$ from $z = 0$ to $z = L$.
(c) If the $A^{\mathrm{co}}(L)$, $A^{\mathrm{cl}}(L)$ are calculated and known, then the boundary condition at $z = L$ for the Lagrange multipliers, μ^{co} and μ^{cl}, in Eq. (5) are known as well.
(d) Compute the propagation of the Lagrange-multiplier functions in Eq. (4) from $z = L$ to $z = 0$ with the boundary conditions in Eq. (5).
(e) Update the new coupling coefficient profile with a suitable *ad hoc* parameter by using the Eqs. (6) and (7).

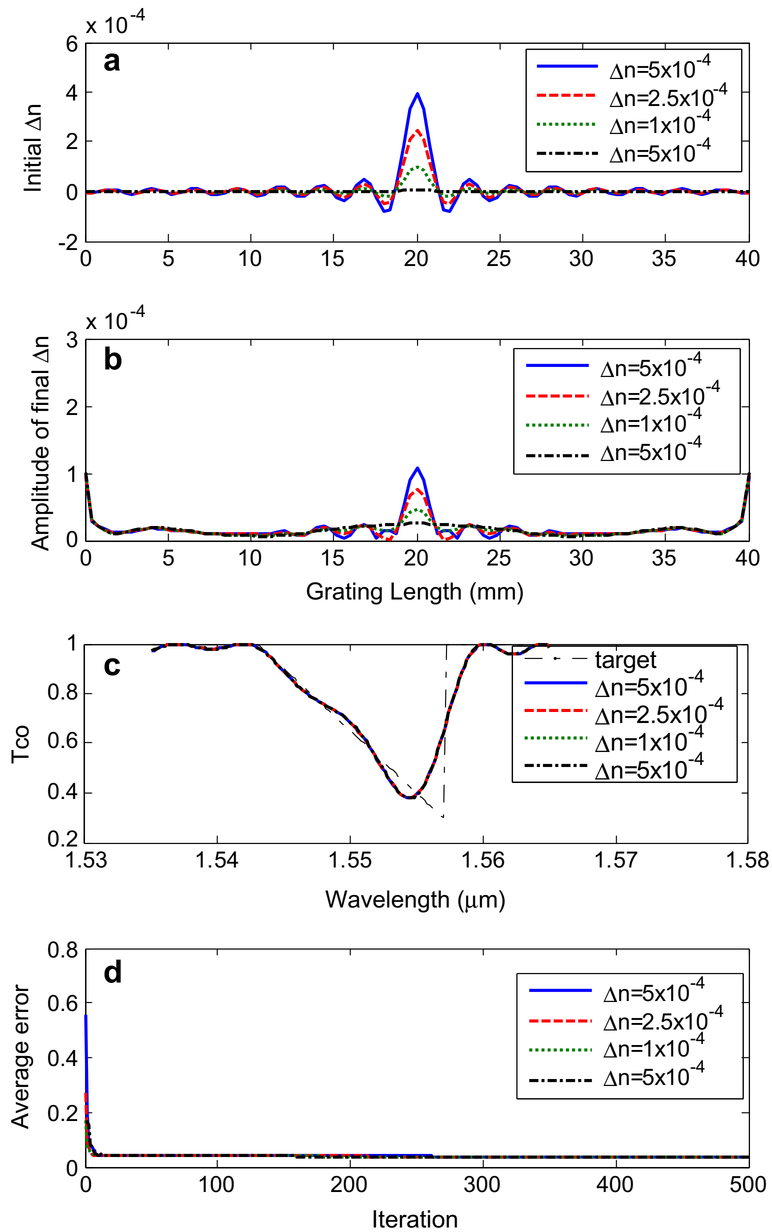

Fig. 3. (a) The initial guess of the sinc apodization functions with different maximum values of index modulation Δn_0, (b) final designed amplitude of apodization profiles, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for synthesized linear LPGs.

(f) Repeat step (b) to step (e) until $\frac{\delta J}{\delta \kappa_R} = 0$ and $\frac{\delta J}{\delta \kappa_I} = 0$, i.e. the LMO method converges.

3. Design results and discussion

In this section, the effectiveness and feasibility of the proposed synthesis algorithm for the LPG filters is demonstrated, the designs for both a linear filter and an EDFA gain flattening filter are given. In the following synthesis demonstrations, we set the *ad hoc* parameters $\alpha = \alpha_R = \alpha_I$, the parameter β to be zero, and the grating length fixed $L = 40$ mm for simplicity.

The first example is a LPG filter with a linear transmission response within a certain range of wavelength. Such linear transmission filters have applications in many fiber sensing systems in which the wavelength modulation of the optical signal is converted into the intensity modulation. After passing through the linear LPG, transmitted core mode can be readily detected by a photo-detector. In this example, the maximum transmission coefficient for the cladding mode $T_{max} = 0.7$ at the wavelength $\lambda_D = 1.5555 \times 10^{-3}$ mm. The results for linear LPGs are shown systematically in Figs. 1–7 for different *ad hoc* parameter α, different initial maximum index modulation, and different initial apodization function profiles. First of

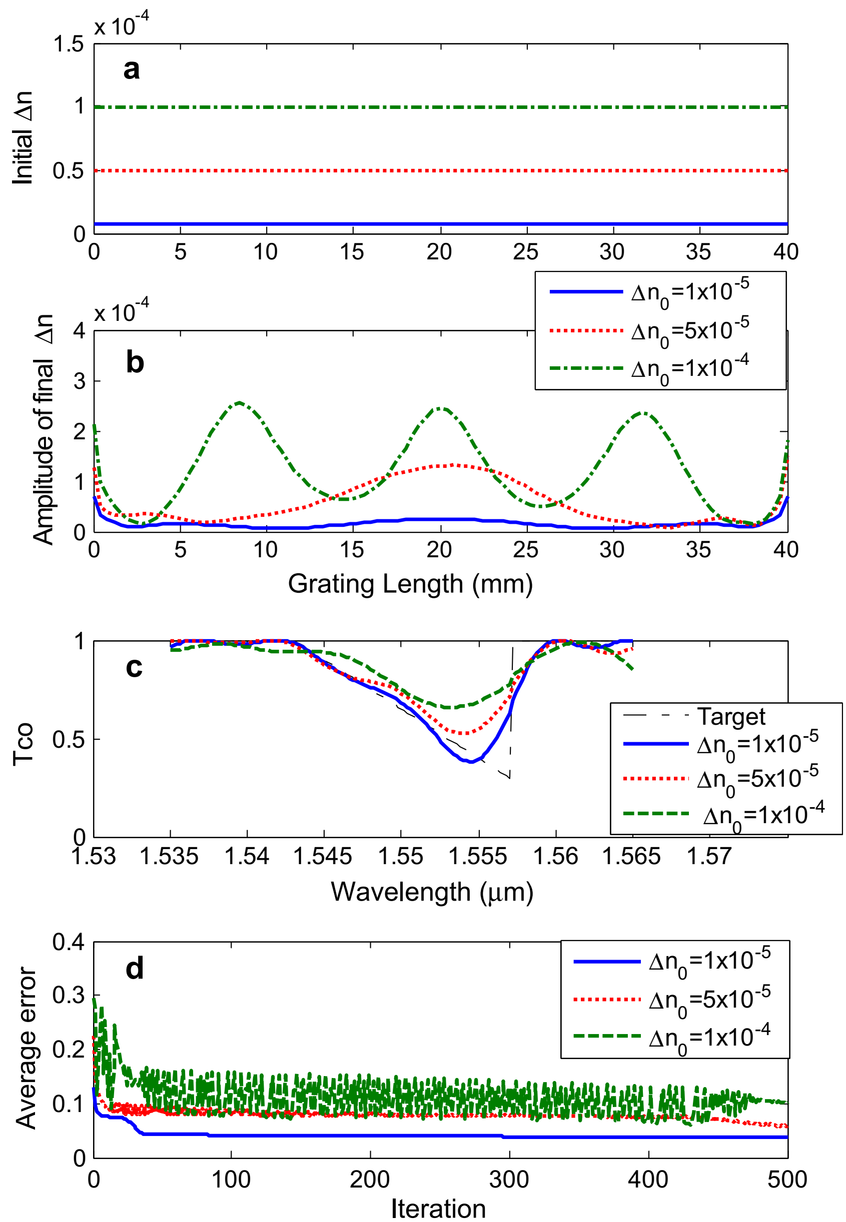

Fig. 4. (a) The initial guess of the uniform index modulation with different maximum value of index modulation Δn_0, (b) final designed amplitude of apodization profiles, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for synthesized linear LPGs.

all, in Fig. 1a, an initial guess of Gaussian apodization function is used, i.e. $\kappa(z) = \Delta n_0 \cdot \exp\{-4\ \ln 2[(z - 0.5\mathrm{L})/\mathrm{FWHM}]^2\}$, where Δn_0 is maximum value of index modulation in the grating, and FWHM is the bandwidth of full width at half maximum of the Gaussian function (always set to be 10 mm). In our calculations, the units of the wavelength λ and the grating length L are in mm. With different *ad hoc* values, $\alpha = \alpha_\mathrm{R} = \alpha_\mathrm{I}$, the designed amplitude of apodization profiles of the index modulation and the synthesized core mode transmission spectra for linear LPGs are shown in Fig. 1b and c, respectively. Fig. 1d shows the typical evolution curves of the absolute value of average error: $|\Delta_t|$ with respect to the iteration times, i.e. $|\Delta_t|$ is the absolute value of total errors divided by the number of spectral points. In this linear LPG synthesis case, the best α value is chosen to be in the range 5×10^3–1×10^4. For these α values, all the simulations quickly converge only with small variations in the final apodization profiles, as shown in Fig. 1b. But for a larger value of the *ad hoc* parameter, $\alpha \geqslant 5 \times 10^4$, we find that the algorithm divergences for the designed case due to too strong modification in the variation method. Based on the knowledge of α value, in Fig. 2, we fix $\alpha = 1 \times 10^4$ and use the same initial guess profile with the Gaussian apodization function but different maximum values of the index modulation, Δn_0, as shown in Fig. 2a. From the resulted apodization profiles and the

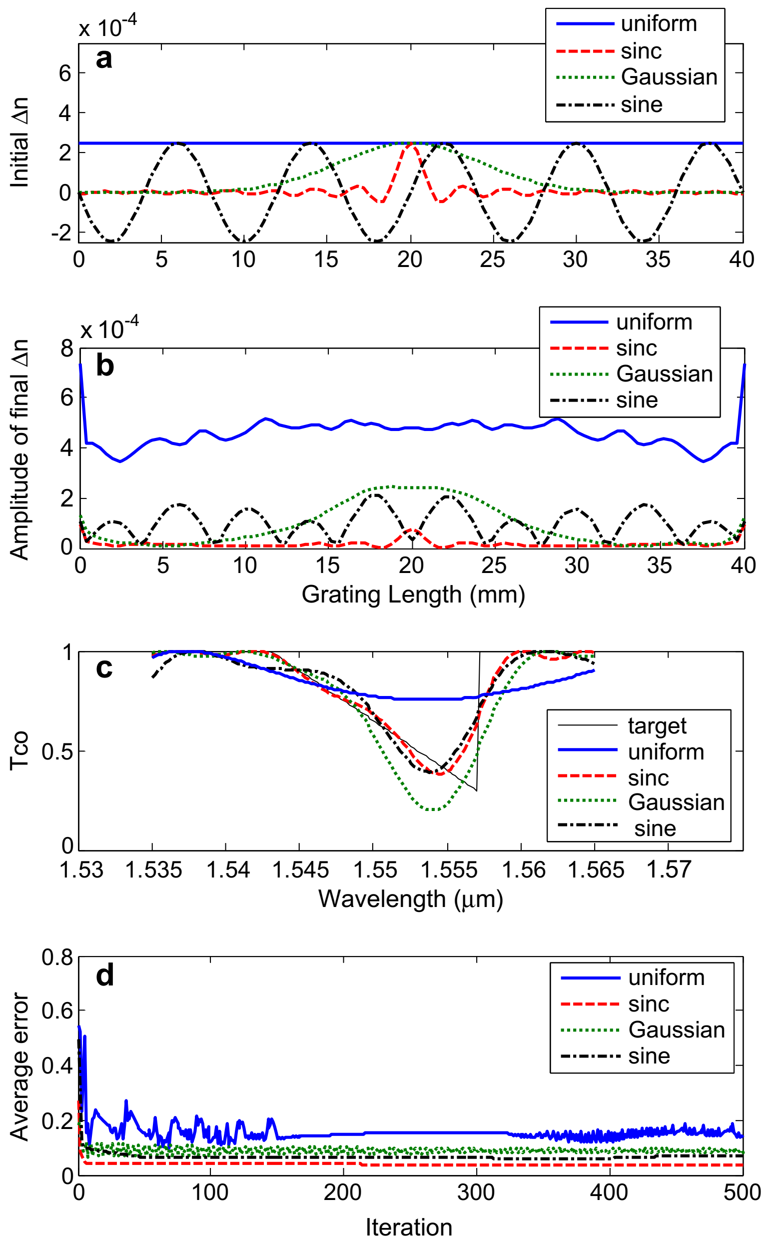

Fig. 5. (a) Different initial guesses of the apodization function, (b) final designed amplitude of apodization profiles of the index modulation, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for the designed linear LPGs with a fixed maximum value of index modulation $\Delta n_0 = 2.5 \times 10^{-4}$.

transmission spectra in Fig. 2b and c, we can clearly see that an appropriate maximum value of the initial index modulation helps to find a better synthesized result more efficiently. For a larger value of the maximum index modulation, the convergence curve of the average error declines quickly at the first state but oscillates in the long run, as shown in Fig. 2d.

Moreover, due to the intrinsic variation-based property of the proposed LMO method, the final apodization profile of the index modulation has very similar shape with the initial guess one. To prove this point, in the following simulations, we compare the syntheses of linear LPGs with different maximum values of Δn_0 as well as different initial apodization functions, in Figs. 2–6. Except for the above mentioned Gaussian function, initial guess index modulations as sinc function ($SC(z)$, Fig. 3), uniform function (Fig. 4), and sinusoidal function $S(z)$ are also presented with the explicit formulations of $SC(z) = \Delta n_0 \cdot \mathrm{sinc}[10\pi(\frac{z-0.5L}{L})]$ and $S(z) = \Delta n_0 \cdot \sin[10\pi(\frac{z-0.5L}{L})]$. Due to the linear transmission spectrum of the target, one can see that the synthesized LPGs have better results when the initial guess functions are sinc functions. In this situation, a broad range of the initial guess value for the maximum value of the index modulation Δn_0, a fast and

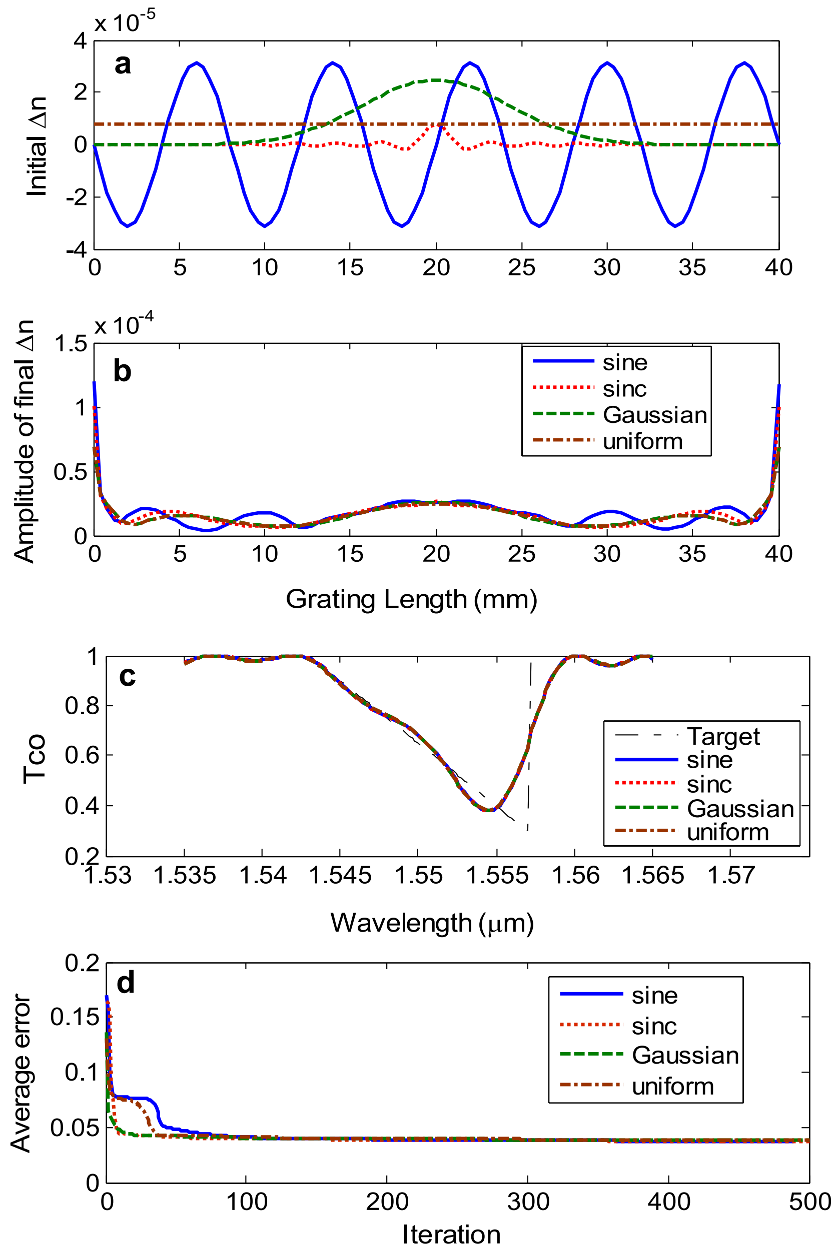

Fig. 6. (a) Different initial guess of the apodization functions, (b) final designed amplitude of apodization profiles of the index modulation, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for the designed linear LPGs with a maximum value of index modulation around $\Delta n_0 = 2.5 \times 10^{-5}$.

monochromatic convergence rate of the average errors can be obtained, as shown in Fig. 3d. Fig. 4 shows the corresponding final synthesized index modulation profiles, the calculated transmission spectra, and the curve of the average errors for an initial uniform coupling coefficient profile. To achieve a smooth convergence rate for the average error, our calculations also indicate that the maximum value of the initial index modulation Δn_0 should not be too larger in the LMO algorithm.

In Fig. 5, we compare the synthesized results with different initial guess of the apodization functions (uniform, sinc, Gaussian, and sinusoidal) with a fixed $\Delta n_0 = 2.5 \times 10^{-4}$ for linear LPG filters. Again, the final apodization profile of the index modulation has very similar one with respect to the initial guess. To decrease the impact from the initial index modulation profiles, in Fig. 6 we use smaller value of the maximum modulation index Δn_0 (around 2×10^{-5}) for different initial apodization functions. In this situation, the impact from the initial guess functions becomes less dominant and almost the same results can be obtained for a suitable *ad hoc* parameter $\alpha = 1 \times 10^4$.

Based on these systematical calculations of the synthesized linear LPG, we show the typical designed complex index modulation profiles for a linear LPG with total grating length of 4 cm in Fig. 7, including the amplitude and phase of index modulation profiles. It can be clearly seen that the designed coupling coefficient profile for linear LPGs based on the LMO method is a smooth one and not too complicated in their amplitudes as well as phases. In addition, the total grating length is not too long to fabricate by current experimental setups.

Another example to demonstrate by LMO method is the well known EDFA gain flattening LPG (EDFA-GFLPG). In previous work, various design schemes of LPG devices have been proposed to equalize the gain spectrum of an EDFA [17–20]. The main approaches are based on the phase shift LPGs, using multiple LPGs scraped together or specially arranged LPGs to match the spectral shaping to flatten EDFA gain spectrum. However, in this work, we directly synthesize the EDFA-GFLPGs just by using a single specific designed LPG.

For the designed EDFA-GFLPGs, the center wavelength is set at the wavelength $\lambda_D = 1.531 \times 10^{-3}$ mm. First of all, results of the designed EDFA-GFLPGs with different *ad hoc* values are shown in Fig. 8, where a uniform initial guess index modulation profile is used with $\Delta n_0 = 1.5 \times 10^{-5}$. The comparisons of the final index modulation profiles, the calculated transmission spectra, and the tendency of the average error convergence are shown in Fig. 8b–d. From the calculations, best convergence of the average errors occurs with the α value about 5×10^3. Next we try to use different initial guess of the apodization functions to synthesize EDFA-GFLPGs, with fixed $\Delta n_0 = 1.5 \times 10^{-5}$ and $\alpha = 5 \times 10^3$. Fig. 9a and b show the initial and the final designed profiles of index modulations with uniform, sinc, Gaussian, and sinusoidal functions. In this case, again the final synthesized apodization functions are shaped into similar profiles as the initial guess functions. For a smaller value of the maximum value of $\Delta n_0 = 1.5 \times 10^{-5}$, the convergence rates of the average errors are all efficient no matter what kind of the initial guess index modulation profile is chosen, as shown in

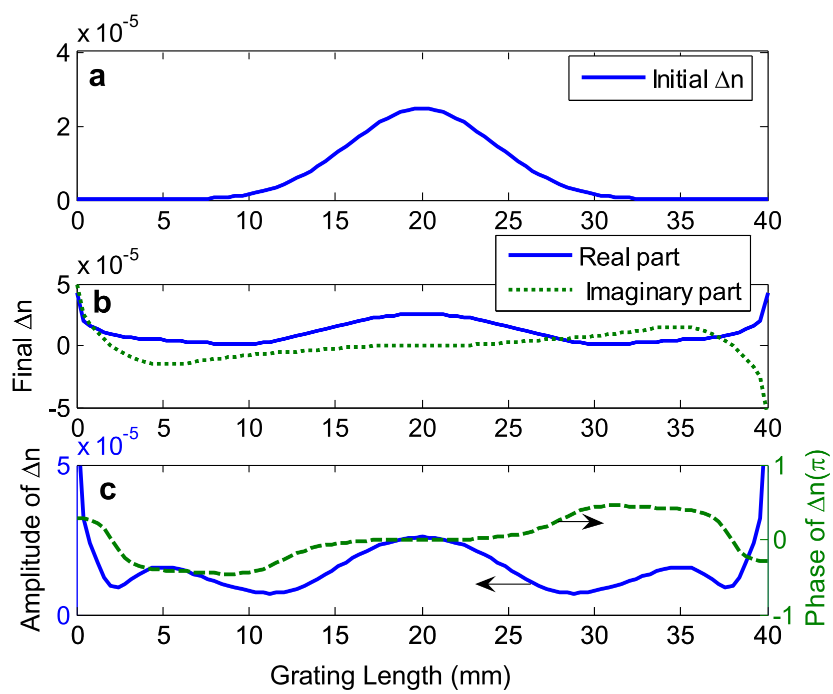

Fig. 7. Final designed complex index modulation profiles of the synthesizing linear LPG in Fig. 1 with $\alpha = 1 \times 10^4$: (a) the initial guess index modulation profile, (b) the real and imaginary parts, and (c) the amplitude and phase of the final index modulation profiles.

Fig. 9d. However, for a larger value of Δn_0, say about 2.5×10^{-4}, the LMO algorithm becomes divergent for the uniform initial guess profile. In the other hand, the LMO algorithm are more stable by using initial guess functions with sinc and Gaussian profiles, as shown in Fig. 10. From the simulation results for designing EDFA-GFLPGs, we find that the best results can be obtained with the conditions that the initial guess is a Gaussian function, the maximum value of the index modulation is smaller than 5×10^{-5}, and α is about 5×10^3. For a typical result of our synthesizing EDFA-GFLPG filters, its target and designed transmission spectra, the corresponding flattened gain-profiles of EDFA spectra, and the ripple deviations in the flattened wavelength region are shown in Fig. 11a–c in dB scales, respectively. It can be seen clearly that from Fig. 11a the designed reflection spectrum meets well with the target spectrum with only a little deviation for a range of wavelength about 5 nm. And from Fig. 11c the studied spectrum can be flattened to less than ±1.4 dB discrepancy within the entire C-band and the average ripple deviation in the flattened wavelength region is lower than 0.5 dB. In Fig. 12, the designed

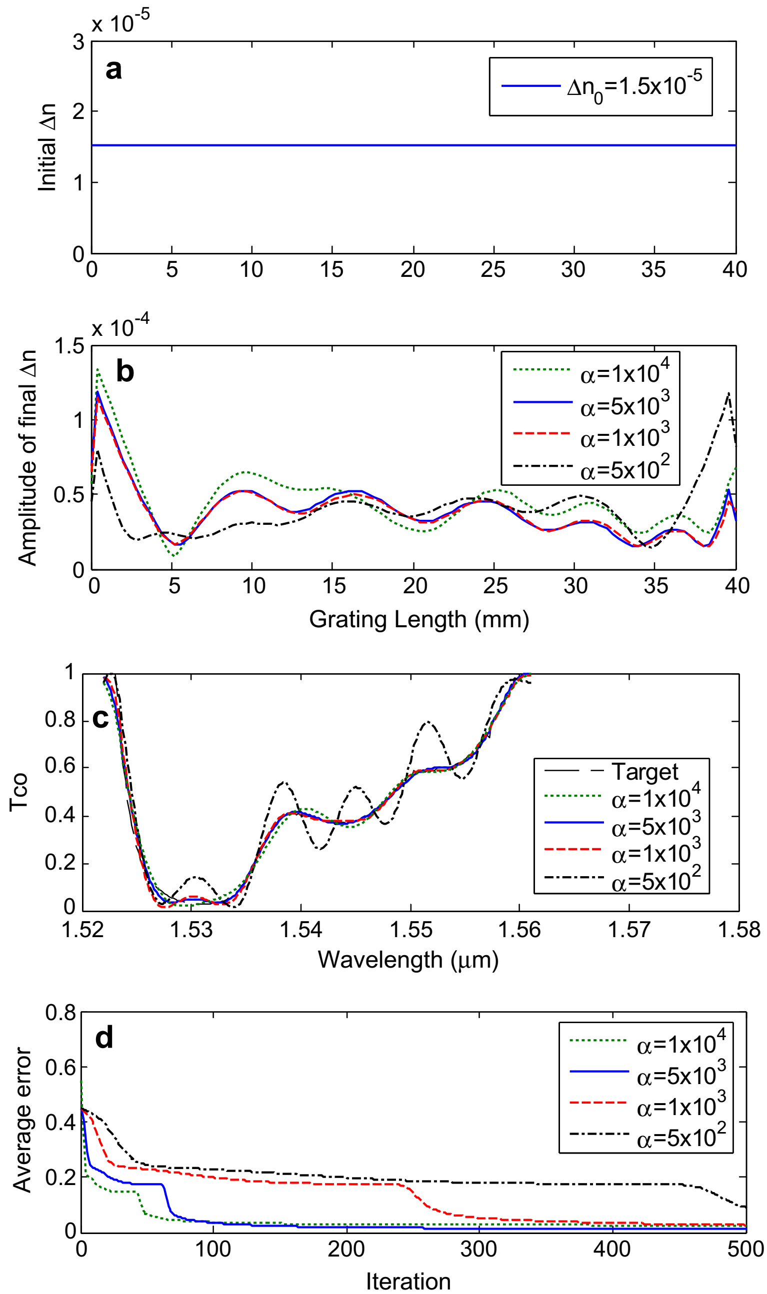

Fig. 8. Designing of a 4 cm long EDFA-gain flattering LPG with different *ad hoc* parameter α in LMO algorithm; (a) the initial guess index modulation profile, (b) final designed amplitude profiles, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of average error.

complex index modulation profiles of the EDFA gain flattering LPG synthesized in Fig. 11 are shown with the amplitude and the phase distributions of the index modulations. In general, LPGs are easier to fabricate than FBGs due to the periods of LPGs are several hundreds of μm. One can use the UV-beam-scanning point by point exposure techniques [21,22] to achieve a complex coupling coefficient $\kappa(z)$ of the gratings. By controlling and adjusting grating periods precisely and continuously in every scanning step for the continuous variation of the phase

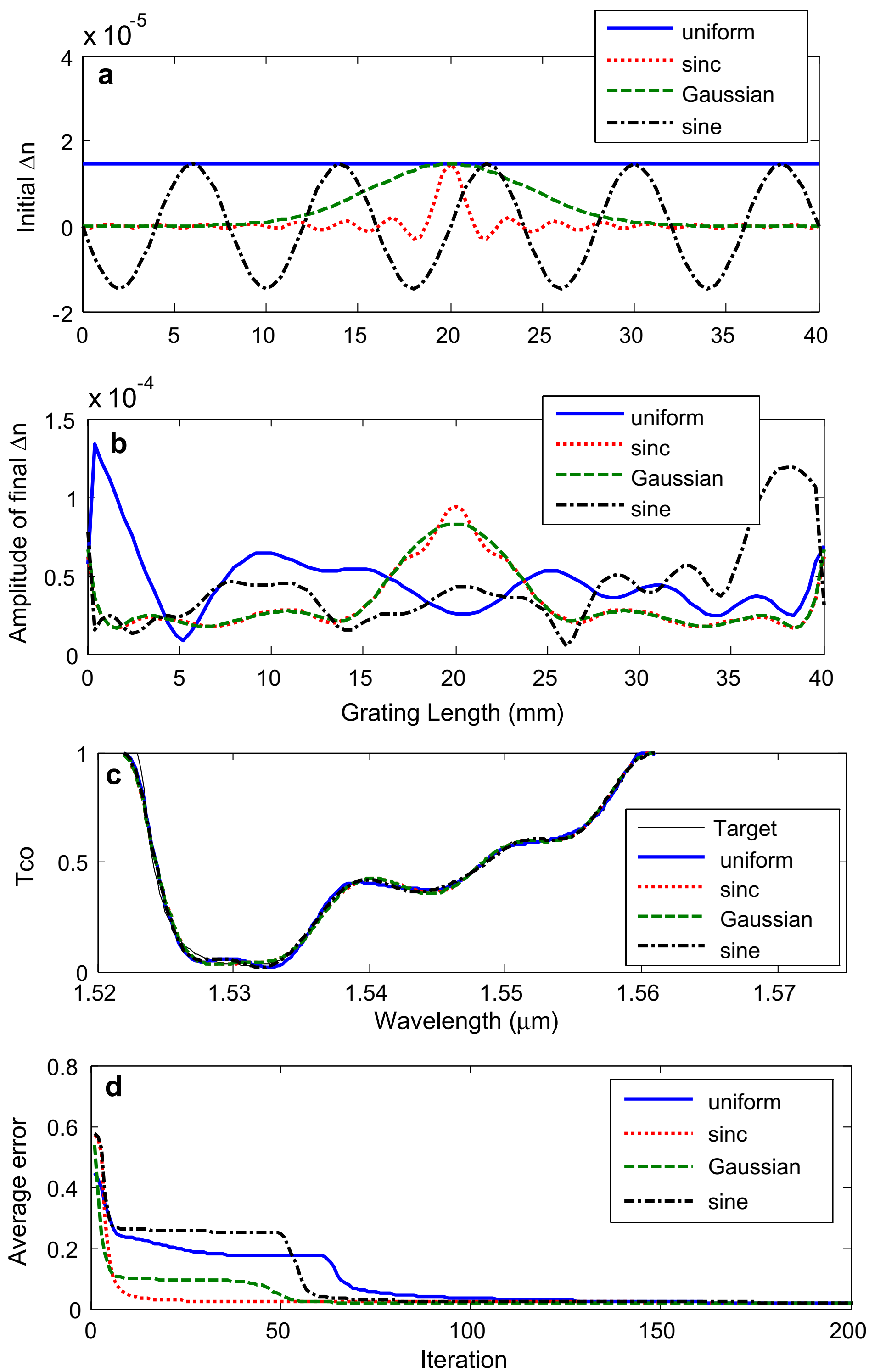

Fig. 9. (a) Different initial guesses of the apodization function (with fixed $\Delta n_0 = 1.5 \times 10^{-5}$ and $\alpha = 5 \times 10^3$), (b) final designed amplitude of the index modulation, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for EDFA-GFLPGs synthesis by LMO method.

of index modulation as required, as well as amplitude of the index modulation, the designed LPGs with a complex index modulation cab be achieved.

Finally, the last but not least thing, we want to emphasize is that all of the synthesized LPG filters demonstrated here are calculated with the constrained parameter β shown in Eq. (2) set to be zero (no constrain condition). The parameter β acts like an alternate and constrained parameter in the objective function to constrain the profile of the coupling coefficient: $\kappa(z)$. It can be set zero for the unconstrained coupling coefficient of the design condition and only the discrepancy between the output and target transmission spectra of LPGs is considered (the first term of Eq. (2)), moreover, based on our previous publication for reflection-type FBGs, non-zero β for further decrease the maximum value of the index modulation by sacrificing the reflectivity of the FBGs (i.e. increasing the reflectivity error) is the designed case that the $\kappa(z)$ always kept real number in FBGs. However, in this present work of optimal LPGs by the LMO method, the phase of the $\kappa(z)$ (complex) was introduced to tailor the target spectrum of LPGs and from the simulation results, the designed amplitude of

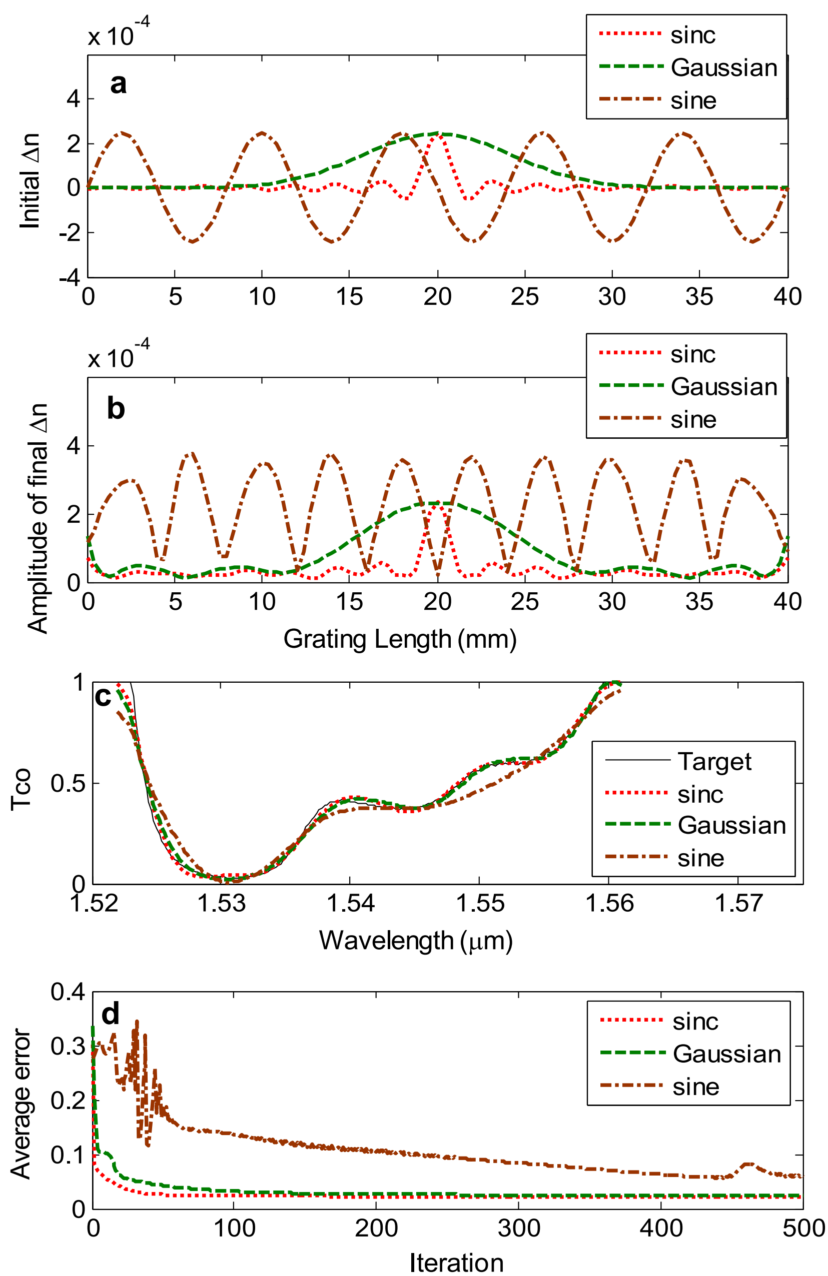

Fig. 10. (a) Different initial guesses of the apodization function (with fixed $\Delta n_0 = 2\times10^{-4}$ and $\alpha = 5\times10^{3}$), (b) final designed amplitude of index modulation, (c) synthesized core mode transmission spectra, and (d) typical evolution curves of the average error for EDFA-GFLPGs synthesis by LMO method.

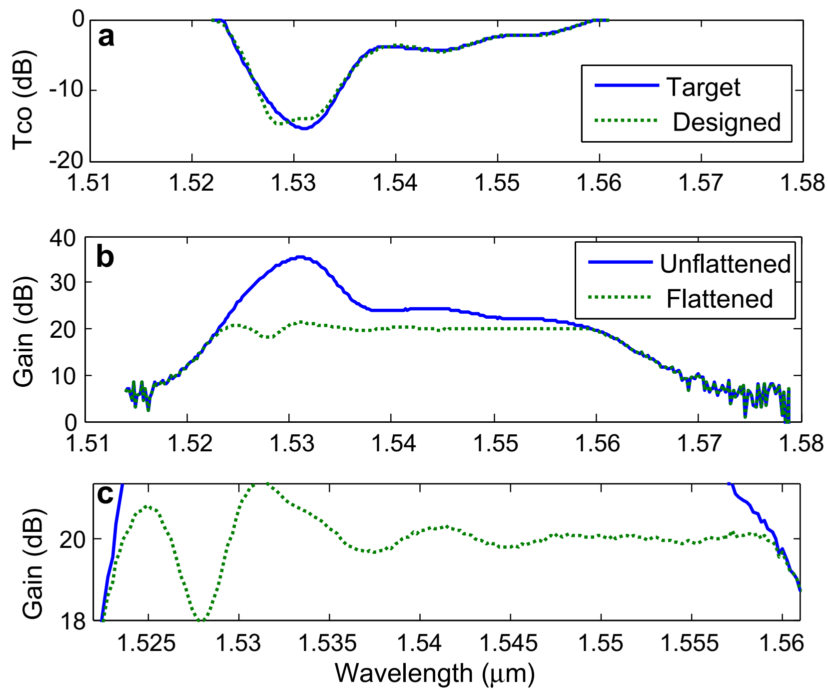

Fig. 11. (a) The comparisons between the target and synthesized core mode transmission spectra (in dB scale) of the EDFA-GFLPG filter, (b) unflattened and flattened gain-profiles of EDFA gain spectra by the designed gain flattering LPG, and (c) ripple deviations in the flattened wavelength region.

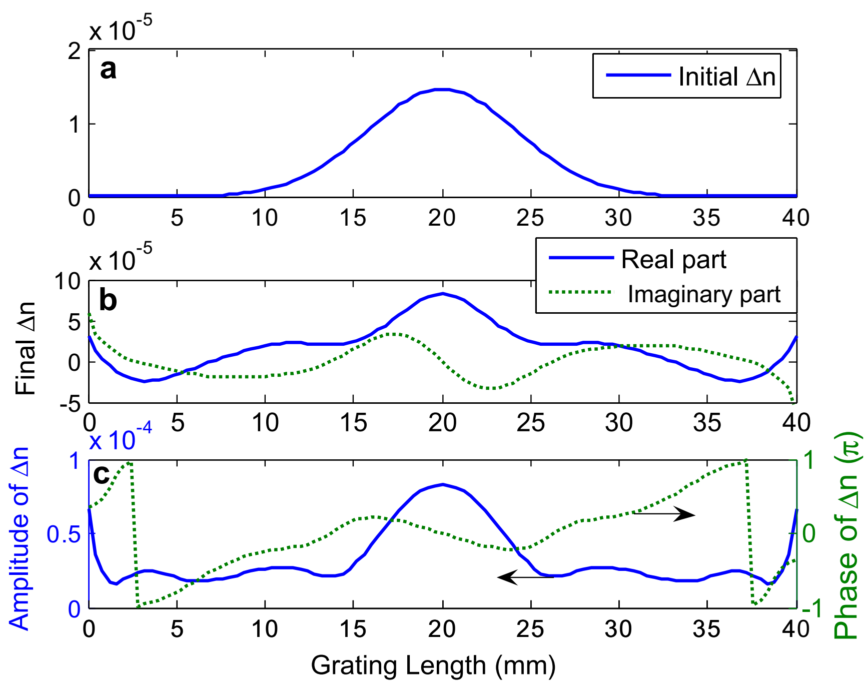

Fig. 12. Final designed complex index modulation profiles of the synthesized EDFA gain flattering LPG in Fig. 11: (a) the initial guess index modulation profile, (b) the real and imaginary parts, and (c) the amplitude and phase of the final index modulation profiles.

index modulations are under 2×10^{-4}, so that the designed LPGs could be implemented with the available commercial photosensitive fibers. Therefore, in the present paper for the designing optimal LPGs with complex coupling coefficient, only unconstrained condition $\beta = 0$ is needed to consider.

4. Conclusion

In this paper, for the first time, we have investigated and demonstrated a simple and fast LPG synthesis method-based on the LMO optimization algorithm. Transmission-type fiber gratings, such as linear LPG and EDFA gain flattening LPG filters with different parameters are successfully synthesized by using the proposed method. Based on the simulation results with different *ad hoc* parameter, maximum values, and profiles of initial guess index modulation, the effects of these parameters in the LMO algorithm for designing LPGs and the corresponding convergence of average errors are analyzed systematically. It can be seen clearly that the maximum value of initial guess for the index modulation should not be too larger and the best value of the evolutionary parameter α falls into the range of 5×10^3–1×10^4. Moreover, the profiles of the final coupling coefficients evolve into a similar one as the initial guess index modulation function. Therefore, an optimal solution might be obtained by adding a little perturbation on the partial profile of the index modulation that will be discussed and demonstrated in a future paper. From this first study, it appears that the proposed approach is an effective method which converges quickly on designing not only reflection-type FBG but also transmission-type LPG filters.

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