Ultraslow bright and dark solitons in semiconductor quantum wells

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We study the low-intensity light pulse propagation through an asymmetric double quantum well via Fano-type interference based on intersubband transitions. The propagation of the pulse across the quantum well is studied analytically and numerically with the coupled Maxwell-Schrödinger equations. We show the generation of ultraslow bright and dark optical solitons in this system. Whether the solitons are dark and bright can be controlled by the ratio of dipole moments of the intersubband transitions. Such investigation of ultraslow optical solitons in the present work may lead to important applications such as high-fidelity optical delay lines and optical buffers in semiconductor quantum wells structure.

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Solitons describe a class of fascinating shaping-preserving wave propagation phenomena in nonlinear media. Over the past few years, the subject of extensive theoretical and experimental investigations on solitons in optical fibers [1,2], cold-atom media [3–7], Bose-Einstein condensates (BEC) [8,9], and other nonlinear media [10], has received a great deal of attention mainly due to that these special types of wave packets are formed as the result of interplay between nonlinearity and dispersion properties of media under excitations, and can lead to undistorted propagation over an extended distance. Among the various solitons studied so far, optical solitons of the interacting system of atoms and electromagnetic field via electromagnetically induced transparency (EIT) have received much attention because of the potential applications in quantum information processing and transmission [1,2,11]. In fact, ever since ultraslow light propagation, large Kerr nonlinearities, and refractive-index enhancement without absorption have been investigated and observed [12,13], light storage with the technique of EIT has been an exciting research field. Recently, optical solitons including two-color solitons with very low group velocities, based on Raman excitation, have been systematically proposed by Wu and Deng [3–7]. Consequently, the dynamics of ultraslow optical solitons in cold atomic medium were studied [14].

It should be noted that similar phenomena involving EIT and ultraslow propagation of optical pulses in semiconductor quantum well (QW) systems have also attracted great attention due to the potentially important applications in optoelectronics and solid-state quantum information science [15–36]. In fact, the analogies between coherent nonlinear phenomena in atomic two-level systems and two-band semiconductor models have been successfully exploited over the past few years, various effects including the resonant solitons have been considered in the literature. More recently, several studies can be found in the literature focusing on exploiting the analogy between atomic three level system and semiconductor heterostructures with a band structure. For example, coherently controlled photocurrent generation [26], EIT [29], and gain without inversion [20–22] have been extensively investigated in semiconductor QW systems. In particular, quantum tunneling to a continuum from two resonant subband levels in asymmetric double QW may give rise to Fano-type interference [17,18]. In contrast, devices based on the intersubband transitions in the semiconductor QW have many inherent advantages in quantum information processing. One may naturally ask if such techniques can also be used to facilitate the formation of an optical soliton in semiconductor QW media.

In the present paper, we wish to extend the above analogy by examining the low-intensity light pulse propagation across an asymmetric double quantum well that exhibits Fano-type interference between adjacent intersubband transitions. We obtain the equation of space-time-dependent Rabi frequency for the pulsed laser field and demonstrate the formation of ultraslow bright and dark solitons in semiconductor QW structures. A few works have discussed coherent control of intersubband transitions in QW [27,28] focusing on the absorption spectra and relaxation dynamics in three-level (or four-level) models. Unlike those works, we will mainly discuss the propagation of coherent light pulse. In addition, a few authors have also considered the pulse propagation dynamics [32,37]. Our work is also different from those investigations as we will consider the space-time-dependent propagation of a single pulsed laser field.

Let us consider a semiconductor double QW structure consisting of two quantum wells that are separated by a narrow barrier as shown in Fig. 1 [16]. At a certain bias voltage, the first subband of the shallow well labeled $|a\rangle$ is resonant with the second subband of the deep well labeled $|b\rangle$ [see Fig. 1(a)], and because of the strong coherent coupling via the thin barrier, the levels split into a doublet, i.e., $|2\rangle=(|a\rangle-|b\rangle)/\sqrt{2}$, $|3\rangle=(|a\rangle+|b\rangle)/\sqrt{2}$ [see Fig. 1(b)]. The splitting ω_s between $|2\rangle$ and $|3\rangle$ is given by the coupling strength and can be controlled by adjusting the height and width of the tunneling barrier with applied bias voltage [15–18]. A low-intensity pulsed laser field with optical frequency ω_p and amplitude E_p is subjected to couple simultaneously the transitions $|1\rangle\leftrightarrow|2\rangle$ and $|1\rangle\leftrightarrow|3\rangle$ with the respective Rabi fre-

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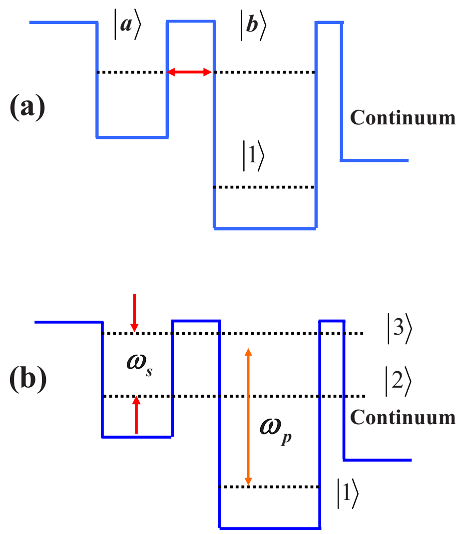

FIG. 1. (Color online) Conduction subband energy level diagram for an asymmetric double quantum wells separated by a thin tunneling barrier. (a) Subband $|a\rangle$ of the shallow well is resonant with the second subband $|b\rangle$ of the deep well. (b) Due to the strong coherent coupling via the thin barrier, the subbands split into a doublet $|2\rangle$ and $|3\rangle$, which are coupled to a continuum by a thin tunneling barrier adjacent to the deep well. ω_s is the energy splitting between the upper levels $|2\rangle$ and $|3\rangle$, ω_p is the frequency of the low-intensity pulsed laser field.

quencies $\mu_{31}E_p/(2\hbar)$ and $\mu_{21}E_p/(2\hbar)$ (here μ_{31} and μ_{21} is the intersubband dipole moments of the respective transitions). The low-intensity light pulse propagates in the z direction and likewise for the polarization. As in the experiments in Ref. [16], we consider a transverse magnetic polarized probe incident at an angle of 45° with respect to the growth axis so that all transition dipole moments include a factor $1/\sqrt{2}$ as intersubband transitions are polarized along the growth axis. What we are interested in is the propagation of the weak pulsed field across the QWs. We work in the interaction picture utilizing the rotating-wave approximation (RWA) and the electric-dipole approximation (EDA) following the standard processes which favor the physical insight into the nature of the probe propagation mechanism based on coupled Schrödinger-Maxwell equations. (There have been theoretical discussions concerning the equivalence between the Schrödinger-formalism adding phenomenal decay rates with the density matrix formalism in dealing with the dephasing processes in such circumstances in Ref. [38].)

$$\frac{\partial A_1}{\partial t} = i\Omega_p^* A_2 + i\left(\frac{\mu_{31}}{\mu_{21}}\right)^* \Omega_p^* A_3, \quad (1)$$

$$\frac{\partial A_2}{\partial t} = i\left(\frac{\omega_s}{2} + \delta + i\gamma_2\right)A_2 + i\Omega_p A_1 + \kappa A_3, \quad (2)$$

$$\frac{\partial A_3}{\partial t} = i\left(\delta - \frac{\omega_s}{2} + i\gamma_3\right)A_3 + i\frac{\mu_{31}}{\mu_{21}}\Omega_p A_1 + \kappa A_2, \quad (3)$$

together with $A_j (j=1,2,3)$, the amplitudes of subbands $|j\rangle$. Here $\Omega_p=\mu_{21}E_p/(2\hbar)$ (assumed real) denotes one-half Rabi frequencies for the transition $|1\rangle\leftrightarrow|2\rangle$, the coefficient μ_{31}/μ_{21} describes the ratio of a pair of dipole moments, and $\mu_{ij}=\mu_{ij}\cdot\tilde{e}_L$ with $\tilde{e}_L$ $(i,j=1,2,3)$ being the polarization unit vector of the laser field describing the intersubband dipole moments of the respective transitions. $\omega_s=E_3-E_2$ is the energy splitting between the upper levels, given by the coherent coupling strength of the tunneling. $\delta=\omega_p-\omega_0$ is the detuning between the frequency of the pulsed laser field and the average transition frequency $\omega_0=(E_3+E_2)/(2\hbar)$.

The population decay rates and the dephasing rates are added phenomenologically in the above equations. The population decay rates for subband $|i\rangle$, denoted by γ_{il}, are due primarily to longitudinal optical (LO) phonon emission events at low temperature. The total decay rates γ_i are given by $\gamma_2=\gamma_{2l}+\gamma_{21}^{\rm dph}$, $\gamma_3=\gamma_{3l}+\gamma_{31}^{\rm dph}$, where $\gamma_{ij}^{\rm dph}$, determined by carrier-carrier scattering, interface roughness, and phonon scattering processes, is the dephasing decay rates of quantum coherence of the $|i\rangle\leftrightarrow|j\rangle$ transitions. The population decay rates can be calculated by solving the effective mass Schrödinger equation. And, as we know, the initially nonthermal carrier distribution is quickly broadened due to inelastic carrier-carrier scattering, with the broadening rate increasing as carrier density is increased. For the temperatures up to 10 K, the carrier density smaller than 10^{12} cm^{-2}, the dephasing decay rates $\gamma_{ij}^{\rm dph}$ can be estimated according to Refs. [18,31]. For our QWs considered, they turn out to be $\gamma_{21}^{\rm dph}=1.5$ meV, $\gamma_{31}^{\rm dph}=2.3$ meV. $\kappa=\sqrt{\gamma_{2l}\gamma_{3l}}$ represents the cross coupling of states $|2\rangle$ and $|3\rangle$ via the LO phonon decay; it describes the process in which a phonon is emitted by subband $|2\rangle$ and is recaptured by subband $|3\rangle$. These cross-coupling terms can be obtained if tunneling is present, e.g., through an additional barrier next to the deeper well. As mentioned above, $|2\rangle$ and $|3\rangle$ are both the superpostions of the resonant states $|a\rangle$ and $|b\rangle$. Because $|b\rangle$ is strongly coupled to a continuum via a thin barrier, the decay from state $|b\rangle$ to the continuum inevitably results in these two dependent decay pathways: from the excited doublet to the common continuum. That is to say, the two decay pathways are related: the decay from one of the excited doublets can strongly affect the neighboring transition, resulting in Fano-type interference characterized by those cross-coupling terms. The probe absorption can be canceled due to the Fano destructive interference between the two decay paths. Such destructive interference is similar to the decay-induced coherence in atomic systems with two closely lying energy states. If $\epsilon=\kappa/\sqrt{\gamma_2\gamma_3}$ is used to assess the strength of the cross-coupling, where the limit values $\epsilon=0$ and 1 correspond, respectively, to no interference and perfect interference.

In order to correctly describe the propagation of the generated optical solitons in the medium, equations of motion must be simultaneously solved with Maxwell's equation in a self-consistent manner. In the limit of plane waves and slowly varying amplitude approximations, the amplitude of the pulsed laser field $E_p=E_p(z,t)$ obeys Maxwell's equation. Making full use of the polarization amplitude $P(\omega_p)$ of the pulsed laser field $P(\omega_p)=N(\mu_{21}A_2A_1^*+\mu_{31}A_3A_1^*)$ with N being the electron density in the conduction band of the QW and Rabi frequency $\Omega_p=\mu_{21}E_p/(2\hbar)$, we can obtain the equation of motion for Ω_p

$$\frac{\partial\Omega_p}{\partial z}+\frac{1}{c}\frac{\partial\Omega_p}{\partial t}=iB\left[A_2+\left(\frac{\mu_{31}}{\mu_{21}}\right)^*A_3\right]A_1^*, \qquad (4)$$

where $B=2\pi N\omega_p|\mu_{21}|^2/\hbar c$ is related to the frequently used oscillator strength of the intersubband transition $|1\rangle\leftrightarrow|2\rangle$. It should be noted that the polarization amplitude $P(\omega_p)$ is the slow oscillating term of the induced polarization in both the intersubband transitions $|1\rangle\leftrightarrow|2\rangle$ and $|1\rangle\leftrightarrow|3\rangle$. Let us assume that $A_j=\Sigma_k A_j^{(k)}$ with $A_j^{(k)}$ is the kth-order part of A_j in terms Ω_p. Within an adiabatic frame work it can be shown that $A_j^{(0)}=\delta_{j0}$ and $A_1^{(1)}=0$. Considering the first order of the field Ω_p, we assume that the populations are initially in the ground state $|1\rangle$. Performing the Fourier transformations for Eqs. (2)–(4)

$$A_j(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\alpha_j(\omega)\exp(-i\omega t)d\omega,\quad j=2,3, \qquad (5)$$

$$\Omega_p(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Lambda_p(\omega)\exp(-i\omega t)d\omega, \qquad (6)$$

where ω is the Fourier-transform variable. We have

$$\alpha_2=-\frac{i\kappa\left(\frac{\mu_{31}}{\mu_{21}}\right)+\left(\omega+\delta-\frac{\omega_s}{2}+i\gamma_3\right)}{\left(\omega+\delta-\frac{\omega_s}{2}+i\gamma_3\right)\left(\omega+\delta+\frac{\omega_s}{2}+i\gamma_2\right)+\kappa^2}\Lambda_p, \qquad (7)$$

$$\alpha_3=-\frac{\left(\frac{\mu_{31}}{\mu_{21}}\right)\left(\omega+\delta+\frac{\omega_s}{2}+i\gamma_2\right)+i\kappa}{\left(\omega+\delta-\frac{\omega_s}{2}+i\gamma_3\right)\left(\omega+\delta+\frac{\omega_s}{2}+i\gamma_2\right)+\kappa^2}\Lambda_p, \qquad (8)$$

$$\frac{\partial\Lambda_p}{\partial z}-i\frac{\omega}{c}\Lambda_p=iB\left[\alpha_2+\left(\frac{\mu_{31}}{\mu_{21}}\right)^*\alpha_3\right]. \qquad (9)$$

Substituting Eqs. (7) and (8) into Eq. (9), we then obtain the solution for the pulsed laser field as follows:

$$\Lambda_p(z,\omega)=\Lambda_p(0,\omega)\exp[iK(\omega)z], \qquad (10)$$

where the propagation constant $K(\omega)$ is denoted by

$$K(\omega)=\frac{\omega}{c}-B\left[\frac{i\kappa\left(\frac{\mu_{31}}{\mu_{21}}\right)+\left(\omega+\delta-\frac{\omega_s}{2}+i\gamma_3\right)}{\left(\omega+\delta-\frac{\omega_s}{2}+i\gamma_3\right)\left(\omega+\delta+\frac{\omega_s}{2}+i\gamma_2\right)+\kappa^2}+\left(\frac{\mu_{31}}{\mu_{21}}\right)^*\frac{\left(\frac{\mu_{31}}{\mu_{21}}\right)\left(\omega+\delta+\frac{\omega_s}{2}+i\gamma_2\right)+i\kappa}{\left(\omega+\delta-\frac{\omega_s}{2}+i\gamma_3\right)\left(\omega+\delta+\frac{\omega_s}{2}+i\gamma_2\right)+\kappa^2}\right]$$
$$=K(0)+K'(0)\omega+\frac{1}{2}K''(0)\omega^2+\cdots\ . \qquad (11)$$

The expressions of $K(0)$, $K'(0)$, and $K''(0)$ are shown in Appendix A. The physical interpretation of Eq. (11) is rather clear. $K(0)=\phi+i\beta$ describes the phase shift ϕ per unit length and absorption coefficient β of the pulsed laser field, $K'(0)$ gives the group velocity $V_g=\mathrm{Re}[1/K'(0)]$, and $K''(0)$ represents the group-velocity dispersion that contributes to the laser pulse's shape change and the addition of the pulsed laser field intensity. It should be emphasized that optical solitons produced in this way generally travel with a group velocity given by $V_g=\mathrm{Re}[1/K'(0)]$.

Following the method developed by Refs. [3,4,7], we take a trial function $\Lambda_p(z,\omega)=\tilde{\Lambda}_p(z,\omega)\exp[iK(0)z]$ and substitute it into the wave equation

$$\frac{\partial\Lambda_p}{\partial z}=iK(\omega)\Lambda_p \qquad (12)$$

we can obtain

$$\exp[iK(0)z]\frac{\partial\tilde{\Lambda}_P(z,\omega)}{\partial z}=i\left[K'(0)\omega+\frac{1}{2}K''(0)\omega^2\right]\tilde{\Lambda}_P(z,\omega)\exp[iK(0)z]. \qquad (13)$$

Here we only kept terms up to order ω^2 in expanding the propagation constant $K(\omega)$. In order to balance the interplay between group velocity dispersion and nonlinear Kerr-effect due to self-phase modulation [39], it is necessary for us to consider the terms on the right-hand side of Eq. (4) and to analyze the nonlinear polarization of the pulsed laser field, i.e.,

$$iB\left[\tilde{A}_2^{(1)}+\left(\frac{\mu_{31}}{\mu_{21}}\right)^*\tilde{A}_3^{(1)}\right][A_1^{(0)}]^*=iB\left[A_2^{(1)}+\left(\frac{\mu_{31}}{\mu_{21}}\right)^*A_3^{(1)}\right]+\mathcal{N}, \qquad (14)$$

where $\mathcal{N}$ means the nonlinear terms given by $\mathcal{N}=-iB[A_2^{(1)}+(\mu_{31}/\mu_{21})^*A_3^{(1)}][|A_2^{(1)}|^2+|A_3^{(1)}|^2]$. For the explicit derivation of Eq. (14), see Appendix B.

Below we will derive the nonlinear evolution equation for Ω_p. Performing the inverse Fourier transformation for the above evolution Eq. (13)

$$\tilde{\Omega}_p(z,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(-i\omega t)\tilde{\Lambda}_p(z,\omega)d\omega, \qquad (15)$$

associating with the nonlinear polarization terms, we can straightforwardly obtain the following nonlinear evolution equation for the slowly varying envelope $\tilde{\Omega}_p(z,t)$:

$$-i\frac{\partial\tilde{\Omega}_p(z,t)}{\partial z}-iK'(0)\frac{\partial\tilde{\Omega}_p(z,t)}{\partial z}+\frac{1}{2}K''(0)\frac{\partial^2\tilde{\Omega}_p(z,t)}{\partial t^2}=W\exp(-2\beta z)|\tilde{\Omega}_p(z,t)|^2\tilde{\Omega}_p(z,t), \qquad (16)$$

where absorption coefficient $\beta=\mathrm{Im}[K(0)]$ and the nonlinear coefficient W is given by

$$W=\frac{B(\delta+\omega_s/2)}{[(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2]^2}-i\frac{B\left(\gamma_2-\kappa\frac{\mu_{31}}{\mu_{21}}\right)}{[(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2]^2}+\left|\frac{\mu_{31}}{\mu_{21}}\right|^2\frac{B\left[(\delta-\omega_s/2)-i\gamma_3+2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2][(\delta-\omega_s/2)^2+\gamma_3^2+\kappa^2]}+\left|\frac{\mu_{31}}{\mu_{21}}\right|^2\frac{B\left[(\delta+\omega_s/2)-i\gamma_2-2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2][(\delta-\omega_s/2)^2+\gamma_3^2+\kappa^2]}$$

$$+\left|\frac{\mu_{31}}{\mu_{21}}\right|^4 \frac{B\left[(\delta-\omega_s/2)-i\gamma_3+2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta-\omega_s/2)^2+\gamma_3^2+\kappa^2]^2}. \quad (17)$$

We define $\xi = z$, and $\eta = t - K'(0)z$, according to $\partial/\partial z \sim \partial/\partial\xi - K'(0)\partial/\partial\eta$ and $\partial/\partial t \sim \partial/\partial\eta$, the nonlinear evolution equation of Eq. (16) can be simplified as

$$i\frac{\partial\tilde{\Omega}_p}{\partial\xi} - \frac{1}{2}K''(0)\frac{\partial^2\tilde{\Omega}_p}{\partial\eta^2} = -W\exp(-2\beta\xi)|\tilde{\Omega}_p|^2\tilde{\Omega}_p. \quad (18)$$

If the splitting between $|2\rangle$ and $|3\rangle$ can be controlled by adjusting the height and width of the tunneling barrier so that the absorption of the pulsed laser field was largely suppressed and thus we can neglect the collapse of the pulsed laser field, i.e., the power transmission $\exp(-2\beta\xi)=1$. We can choose the reasonable and realistic set of parameters to satisfy $\beta \simeq 0$, $K''(0) = \mathrm{Re}[K''(0)] + \mathrm{Im}[K''(0)] \simeq \mathrm{Re}[K''(0)]$, and $W = \mathrm{Re}(W) + \mathrm{Im}(W) \simeq \mathrm{Re}(W)$. Based on Eqs. (13) and (14), we can obtain the standard nonlinear Schrödinger equation governing the pulsed laser field evolution

$$i\frac{\partial\tilde{\Omega}_p}{\partial\xi} - \frac{1}{2}K''(0)\frac{\partial^2\tilde{\Omega}_p}{\partial\eta^2} = -\mathrm{Re}(W)|\tilde{\Omega}_p|^2\tilde{\Omega}_p, \quad (19)$$

which admits solutions describing bright and dark solitons. It is well known that whether the solutions to Eq. (19) are bright solitons or dark solitons depends on the sign of product $\mathrm{Re}[K''(0)]\mathrm{Re}(W)$, i.e., $\mathrm{Re}[K''(0)]\mathrm{Re}(W) < 0$ for bright solitons and $\mathrm{Re}[K''(0)]\mathrm{Re}(W) > 0$ for dark solitons. If we can adjust the tunneling barrier of QW so that the pulsed laser field is resonant with the average frequency ω_0 ($\delta = 0$), and the energy splitting between the levels $|2\rangle$ and $|3\rangle$ due to the coherent coupling of the tunneling is much larger than the population decay rates for subbands $[\omega_s \gg \max(\gamma_2, \gamma_3)]$, it is straightforward to show that $\mathrm{Re}[K''(0)] \simeq -32B[|\mu_{21}|^2 + |\mu_{31}|^2]/\omega_s^4|\mu_{21}|^2 < 0$, $\mathrm{Re}[W] \simeq -8B[|\mu_{21}|^4 - |\mu_{31}|^4]/\omega_s^3|\mu_{21}|^2$, and $V_g \simeq \omega_s^2|\mu_{21}|^2/4Bc[|\mu_{21}|^2 + |\mu_{31}|^2]$. As a result, the solutions of the Eq. (19) are closely associated with the value $|\mu_{31}/\mu_{21}|^2$, which corresponds to the ratio of the intersubband dipole moments μ_{31} and μ_{31} of the respect transitions. In the case of $|\mu_{31}/\mu_{21}| < 1$, bright solitons are produced; in contrast, dark solitons occur. The form of a fundamental bright soliton is given by

$$\Omega_p = \Omega_{p0}\,\mathrm{sech}(\eta/\tau)\exp[i\xi\,\mathrm{Re}(W)|\Omega_{p0}|^2/2]\exp[iK(0)\xi], \quad (20)$$

where $\mathrm{sech}(\eta/\tau)$ is the hyperbolic secant function. Amplitude Ω_{p0} and width τ are arbitrary constants subjected only to the constraint $|\Omega_{p0}\tau| = -\mathrm{Re}[K''(0)]/\mathrm{Re}(W)$.

We now present numerical examples to demonstrate the existence of ultraslow bright and dark solitons in the system studied through simulating Eq. (18). We consider a system where the population decay rates and the dephasing rates of the subbands $|2\rangle$ and $|3\rangle$ are $\gamma_{2l} = 5.6$, $\gamma_{3l} = 7.0$, $\gamma_{21}^{\mathrm{dph}} = 1.5$, and $\gamma_{31}^{\mathrm{dph}} = 2.3$ meV, respectively. From the above estimates, we obtain $\epsilon = 0.77$, which is close to the ideal value $\epsilon = 1$ and corresponds to a large tunneling efficiency leading a strong Fano-type interference effect. We first consider the case of dark solitons. Taking $B = 6\ \mathrm{cm}^{-1}$ meV, $|\mu_{31}/\mu_{21}| = 1.2$, $\omega_s = 50$ meV, and $\delta = 0$, we can obtain $V_g/c \sim 10^{-4}$, and the standard nonlinear Schrödinger Eq. (19) with $\mathrm{Re}[K''(0)]\mathrm{Re}(W) > 0$ is well characterized. Thus we have demonstrated the existence of dark solitons that travel with usltraslow group velocities in a semiconductor double quantum well structure. For bright solitons, we take $|\mu_{31}/\mu_{21}| = 0.9$ with all other parameters given above unchanged. In this case we obtain $V_g/c \sim 10^{-4}$. As shown in Fig. 2, these parameters and results again show that standard nonlinear Schrödinger Eq. (19) with $\mathrm{Re}[K''(0)]\mathrm{Re}(W) < 0$, which is well characterized and that the formation of bright solitons occurs. In Fig. 2, the numerical simulation of Eq. (18) for the bright soliton shows an excellent agreement with Eq. (20). In our calculations above, we have set the splitting on resonance (coupling strength) ω_s as 50 meV. The parameters chosen here can be realized in typical QW structures. For example, one can consider the QW structure consisting of two quantum wells: a 6.8-nm-thick $Al_{0.15}Ga_{0.85}As$ shallow well and a 7.0-nm-thick GaAs deep well separated by a 2.0-nm-thick $Al_{0.3}Ga_{0.7}As$ tunnel barrier, in which the barrier will couple the excited state of the deep well with the ground state of the shallow well to create a doublet states and splitting [15–18]. In fact, the coupling strength ω_s can vary in a wide range [17] which in experiments can be controlled by adjusting the height and width of the tunneling barrier experimentally through the bias voltage.

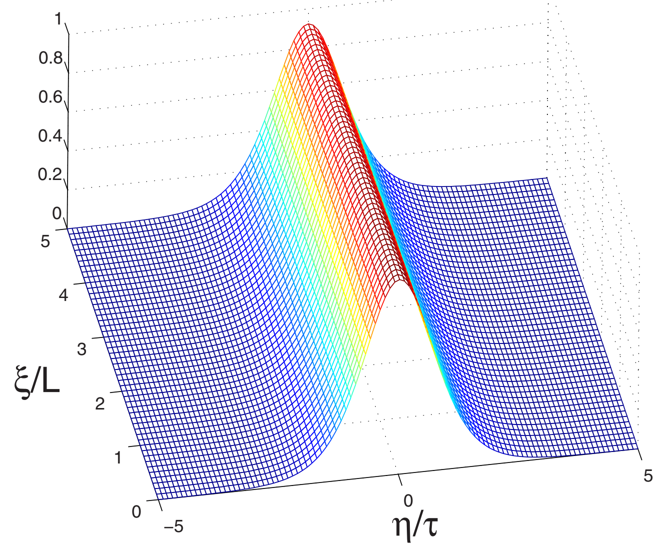

FIG. 2. (Color online) Surface plot of the amplitude for the generated fundamental bright soliton $|\Omega_p/\Omega_{p0}|^2\exp(-2\beta\xi)$ versus dimensionless time η/τ and distance ξ/L under the boundary condition $\Omega_p(\xi=0,\eta) = \Omega_{p0}\,\mathrm{sech}(\eta/\tau)$ by the numerical simulations. Here, we have chosen the relative parameter $\gamma_{2l} = 5.6$ meV, $\gamma_{3l} = 7.0$ meV, $\gamma_{21}^{\mathrm{dph}} = 1.5$ meV, $\gamma_{31}^{\mathrm{dph}} = 2.3$ meV, $B = 6\ \mathrm{cm}^{-1}$ meV, $\omega_s = 50$ meV, $|\mu_{31}/\mu_{21}| = 0.9$, $\delta = 0$, $L = 1.0$ cm, and $\tau = 1.0 \times 10^{-6}$ s.

It is worth noting that the above-described parameter sets also lead to negligible loss of the probe field for both the bright and the dark solitons described, as can be seen in Fig. 2. In addition, we have used a one-dimensional model in the calculation where the momentum-dependency of subband energies has been ignored. According to the Ref. [32], there is no large discrepancy between the reduced one-dimensional calculation and the full two-dimensional calculation. For details about two-dimensional calculations can be found in Refs. [28,29]. In the present paper, we have set the parameters γ_{ij}^{dph} and γ_{il} to satisfy $\gamma_{ij}^{\text{dph}} < \gamma_{il}$, a resonant probe can propagate with little absorption. If the dephasing decay rates γ_{ij}^{dph} is too large, the effect tunneling induced interference will become less pronounced according to the factor $\epsilon = \kappa/\sqrt{\gamma_2\gamma_3}$, and the probe will be more and more absorbed.

In summary, we have investigated the propagation of a single pulsed laser field in a specific asymmetric double QW structure via Fano-type interference from the Maxwell-Schrödinger equations of the pulsed laser field across the quantum wells, we have obtained a NLS Schrödinger equation governing the evolution of pulsed laser field. As a result, we achieve the ultraslow optical bright and dark solitons in the system, which is a scheme to achieve the generation of solitons in semiconductor QW. The present investigation is much more practical than its atomic counterpart due to its flexible design and the controllable interference strength. Such ultraslow optical solitons may provide a new possibility for designing high-fidelity optical delay lines and optical buffers in semiconductor quantum wells structure.

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APPENDIX A

In Eq. (11), the expressions of $K(0)$, $K'(0)$, and $K''(0)$ are written as follows:

$$K(0) = -\frac{B\left[(\delta+\omega_s/2)+i\gamma_2-i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2} - \frac{B\left|\frac{\mu_{31}}{\mu_{21}}\right|^2\left[(\delta-\omega_s/2)+i\gamma_3+i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{(\delta-\omega_s/2)^2+\gamma_3^2+\kappa^2}, \tag{A1}$$

$$K'(0) = \frac{1}{c} + \frac{B\left[(\delta+\omega_s/2)^2-\gamma_2^2-2i\gamma_2(\delta+\omega_s/2)-2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2]^2} + \frac{B|\mu_{31}/\mu_{21}|^2\left[(\delta-\omega_s/2)^2-\gamma_3^2-2i\gamma_3(\delta-\omega_s/2)+2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta-\omega_s/2)^2+\gamma_3^2+\kappa^2]^2}, \tag{A2}$$

$$K''(0) = -\frac{2B\left[(\delta+\omega_s/2)^2-\gamma_2^2-2i\gamma_2(\delta+\omega_s/2)-2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta+\omega_s/2)^2+\gamma_2^2+\kappa^2]^3} - \frac{2B\left|\frac{\mu_{31}}{\mu_{21}}\right|^2\left[(\delta-\omega_s/2)^2-\gamma_3^2-2i\gamma_2(\omega_s/2-\delta)+2i\kappa\frac{\mu_{31}}{\mu_{21}}\right]}{[(\delta-\omega_s/2)^2+\gamma_3^2+\kappa^2]^3}. \tag{A3}$$

APPENDIX B

Considering the right side of Eq. (4) and analyzing the nonlinear polarization of pulsed laser field, we can obtain

$$iB\left[\widetilde{A}_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* \widetilde{A}_3^{(1)}\right][A_1^{(0)}]^* = iB\left[A_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* A_3^{(1)}\right]|A_1^{(0)}|^2. \tag{B1}$$

By using the relations:

$$|A_1^{(0)}|^2 + |A_2^{(1)}|^2 + |A_3^{(1)}|^2 = 1, \tag{B2}$$

$$A_2^{(1)} = -\frac{i\kappa\frac{\mu_{31}}{\mu_{21}} + \left(\delta - \frac{\omega_s}{2} + i\gamma_3\right)}{\left(\delta-\frac{\omega_s}{2}+i\gamma_3\right)\left(\delta+\frac{\omega_s}{2}+i\gamma_2\right)+\kappa^2}\Omega_p, \tag{B3}$$

$$A_3^{(1)} = -\frac{\frac{\mu_{31}}{\mu_{21}}\left(\delta+\frac{\omega_s}{2}+i\gamma_2\right)+i\kappa}{\left(\delta-\frac{\omega_s}{2}+i\gamma_3\right)\left(\delta+\frac{\omega_s}{2}+i\gamma_2\right)+\kappa^2}\Omega_p, \tag{B4}$$

we have

$$\begin{aligned} &iB\left[\widetilde{A}_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* \widetilde{A}_3^{(1)}\right][A_1^{(0)}]^* \\ &= iB\left[A_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* A_3^{(1)}\right][1 - |A_2^{(1)}|^2 - |A_3^{(1)}|^2] \\ &= iB\left[A_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* A_3^{(1)}\right] - iB\left[A_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* A_3^{(1)}\right] \\ &\quad \times[|A_2^{(1)}|^2 + |A_3^{(1)}|^2]. \end{aligned} \tag{B5}$$

Thus $\mathcal{N}$ can be expressed as

$$\mathcal{N} = -iB\left[A_2^{(1)} + \left(\frac{\mu_{31}}{\mu_{21}}\right)^* A_3^{(1)}\right][|A_2^{(1)}|^2 + |A_3^{(1)}|^2]. \tag{B6}$$

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