Interaction between Photons and Atoms in Photonic Crystals

李瑞光 Ray-Kuang Lee[†]

國立清華大學電機工程學系暨光電工程研究所

Department of Electrical Engineering
and Institute of Photonics Technologies

National Tsing-Hua University, Hsinchu, Taiwan

國立清華大學 National Tsing Hua University

[†] e-mail: rklee@ee.nthu.edu.tw

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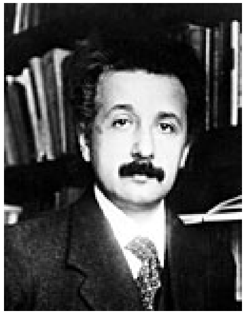
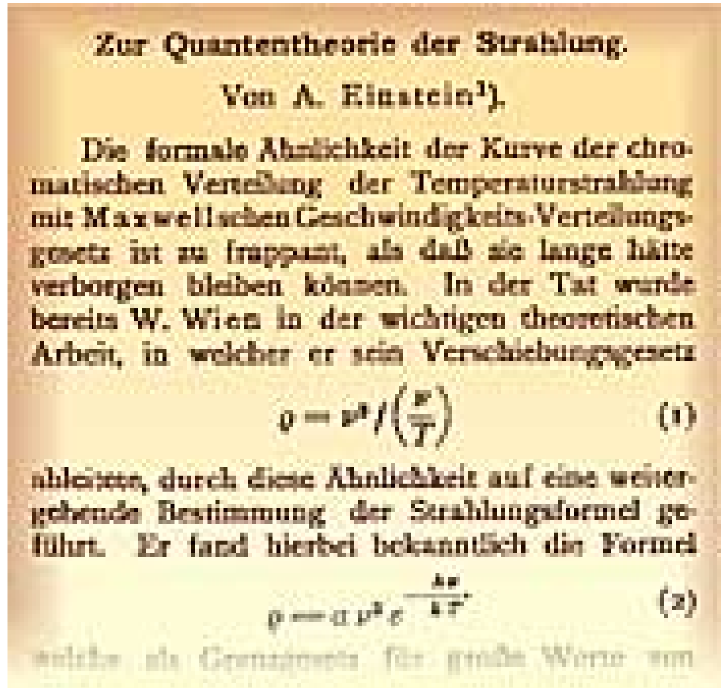
Ref: Ray-Kuang Lee and Yinchieh Lai, *J. Opt. B* **6**, S715 (special issue 2004).

Outline

1. On the Shoulders of Giants
2. Resonance Fluorescence Spectra in PhCs
3. Fluorescence Squeezing Spectra in PhCs
4. Conclusions

Einstein on Radiation





Zur Quantentheorie der Strahlung.

Von A. Einstein[1]).

Die formale Ähnlichkeit der Kurve der chromatischen Verteilung der Temperaturstrahlung mit Maxwellschen Geschwindigkeits-Verteilungsgesetz ist zu frappant, als daß sie lange hätte verborgen bleiben können. In der Tat wurde bereits W. Wien in der wichtigen theoretischen Arbeit, in welcher er sein Verschiebungsgesetz

$$\rho = \nu^3 f\left(\frac{\nu}{T}\right) \qquad (1)$$

ableitete, durch diese Ähnlichkeit auf eine weitergehende Bestimmung der Strahlungsformel geführt. Er fand hierbei bekanntlich die Formel

$$\rho = \alpha \nu^3 e^{-\frac{h\nu}{kT}} \qquad (2)$$

welche als Grenzgesetz für große Werte von

"On the Quantum Theory of Radiation"

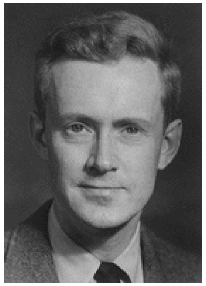
$$\rho(v_0) = \frac{A/B}{e^{hv_0/kT} - 1}$$

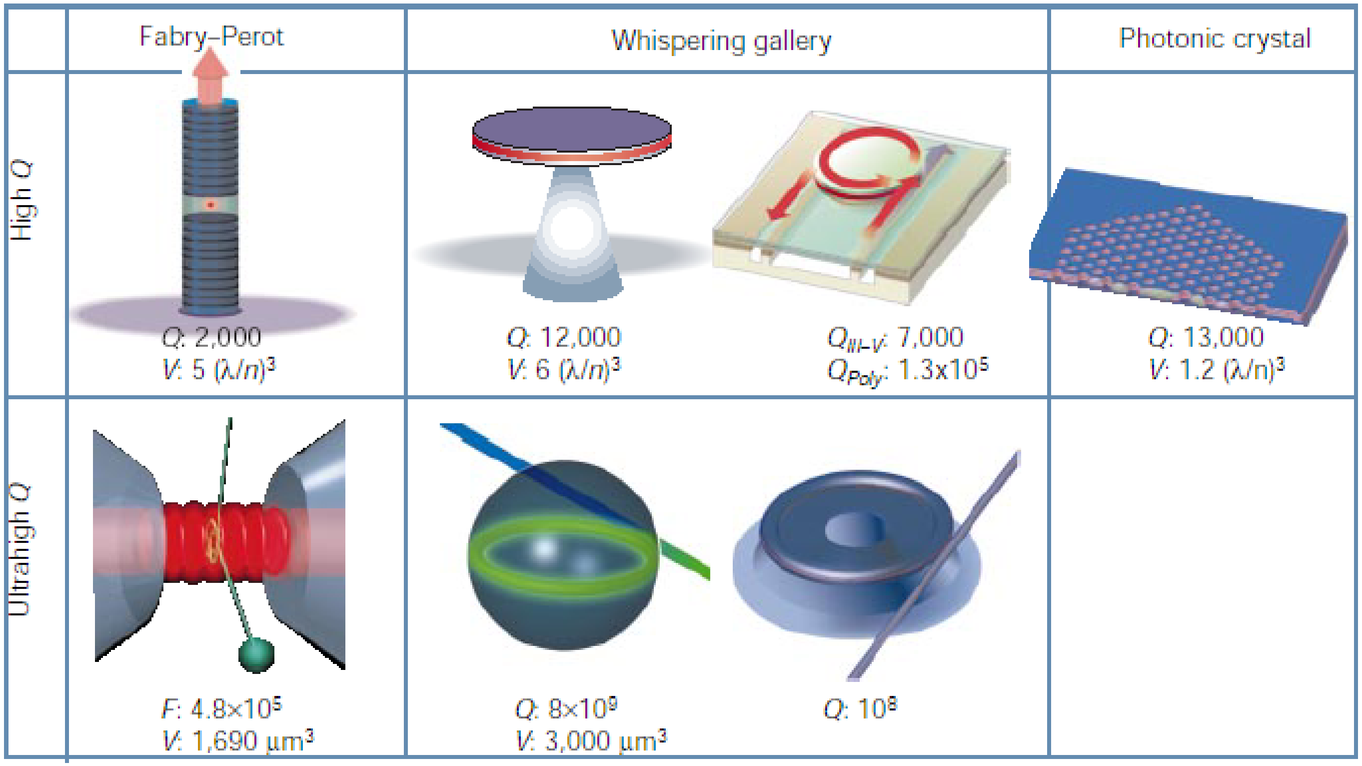
$$\frac{A}{B} = \frac{8\pi h v_0^3}{c^3}$$

A. Einstein, *Phys. Z.* **18**, 121 (1917).

D. Kleppner, "Rereading Einstein on Radiation," *Physics Today* **58**, 30 (Feb. 2005).

Purcell effect: Cavity-QED (Quantum ElectroDynamics)





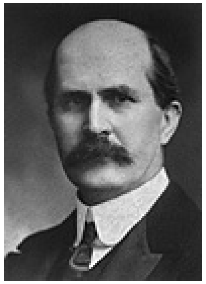
E. M. Purcell, *Phys. Rev.* **69** (1946).

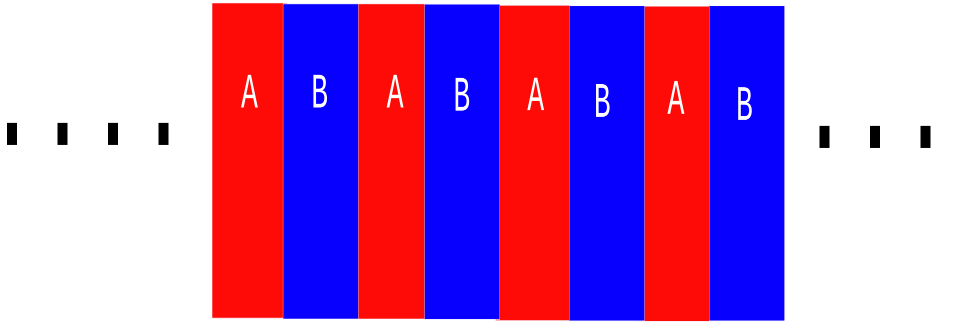
Nobel laureate Edward Mills Purcell (shared the prize with Felix Bloch) in 1952,

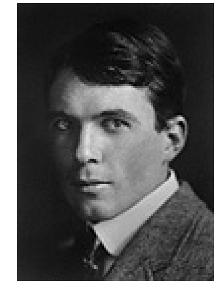
for their contribution to nuclear magnetic precision measurements.

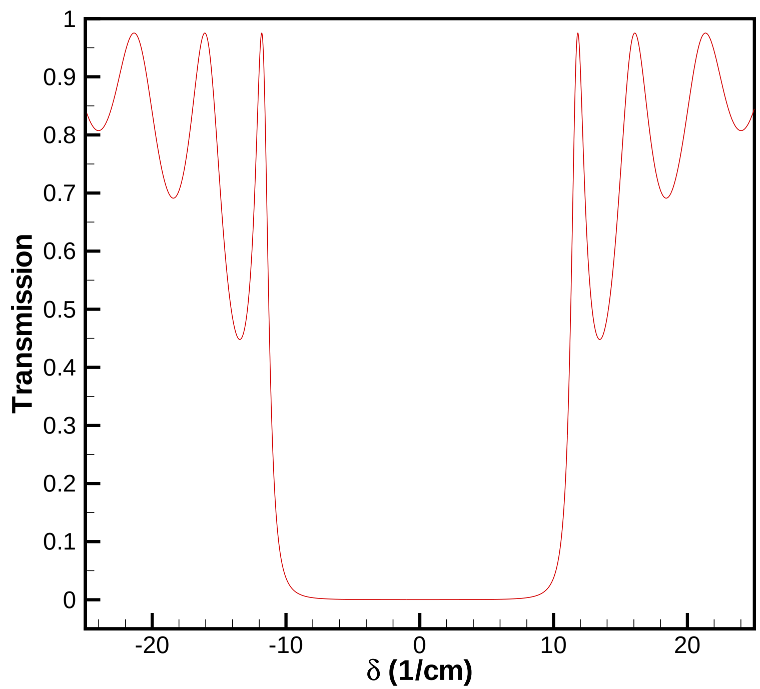
from: K. J. Vahala, *Nature* **424**, 839 (2003).

Bragg reflectors





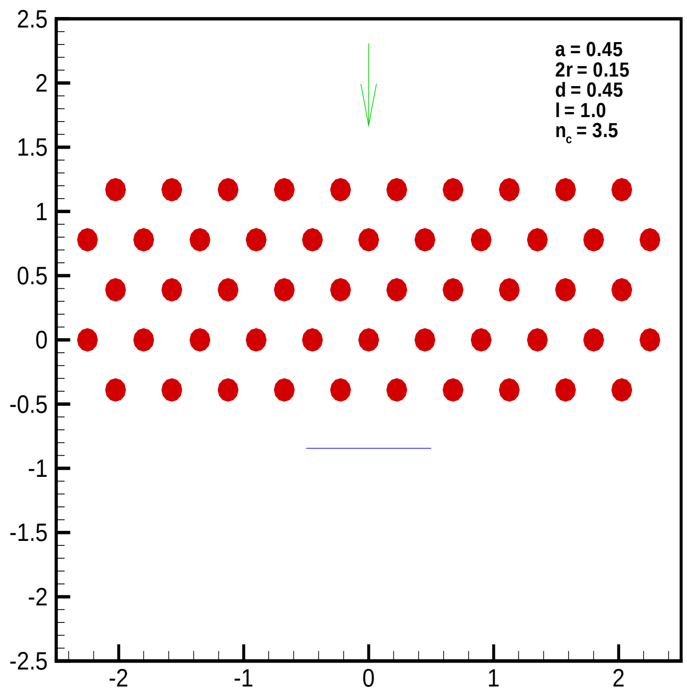


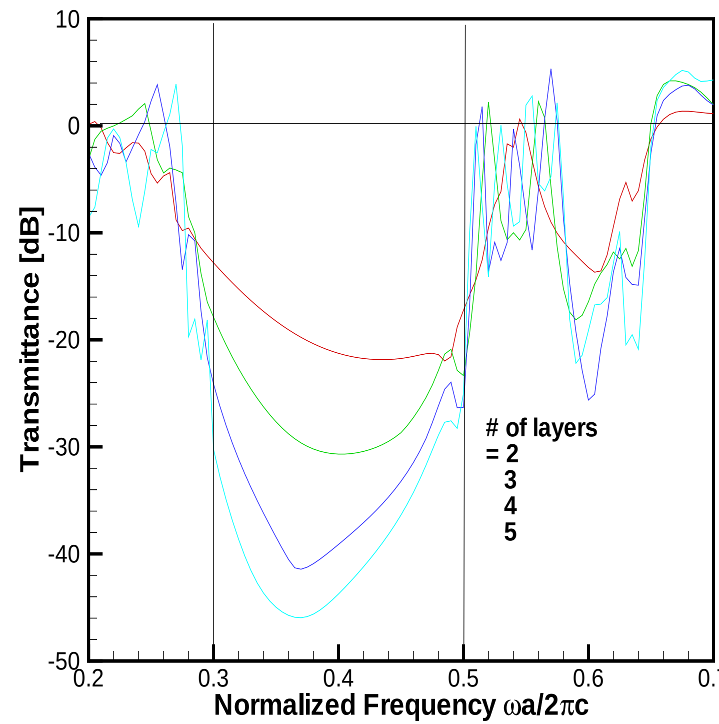


Nobel laureates William Henry Bragg and William Lawrence Bragg in 1915,

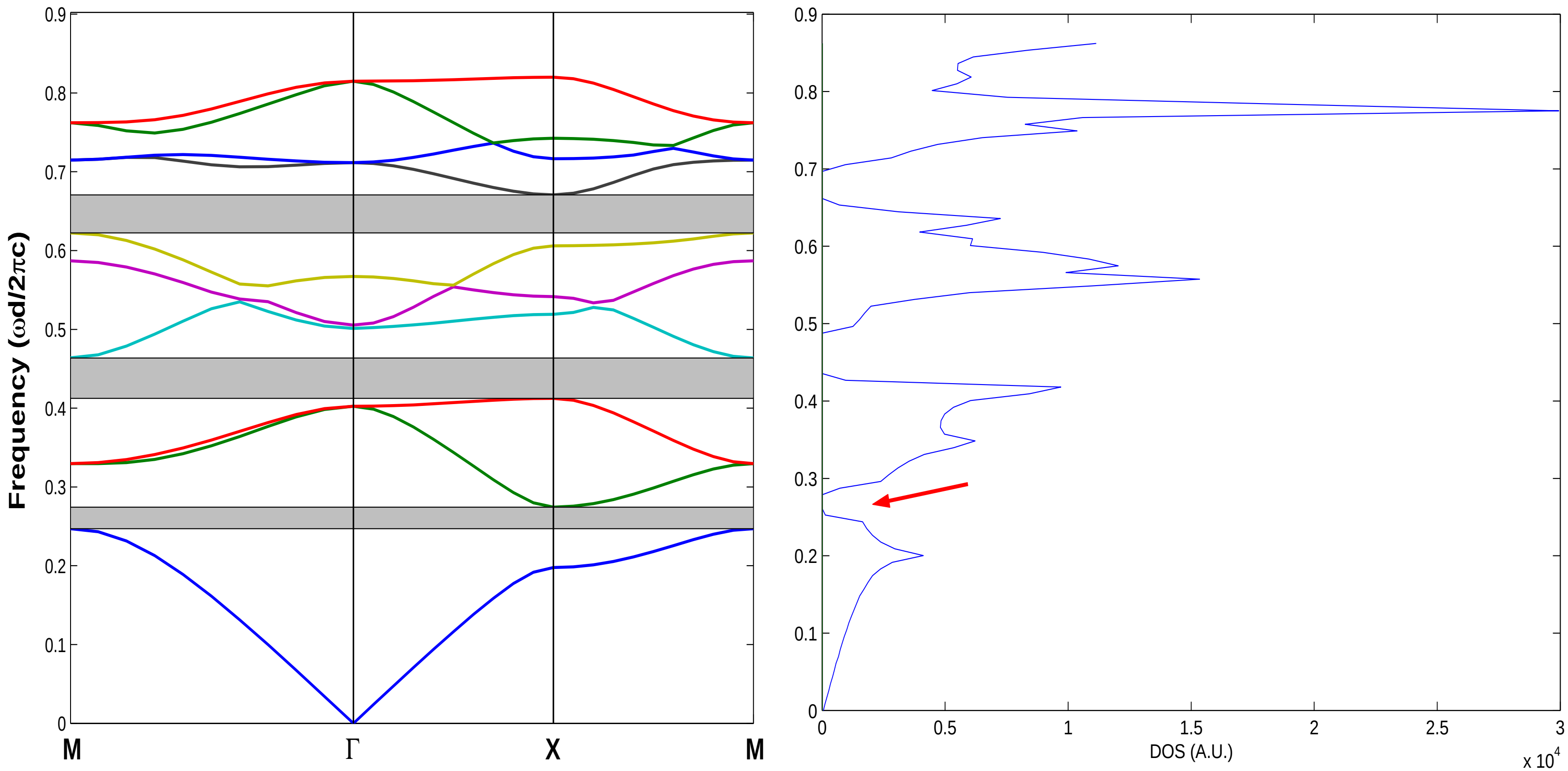
for their contribution to the X-ray diffraction analysis (Bragg diffraction).

Photonic Bandgap Crystals: two(high)-dimension

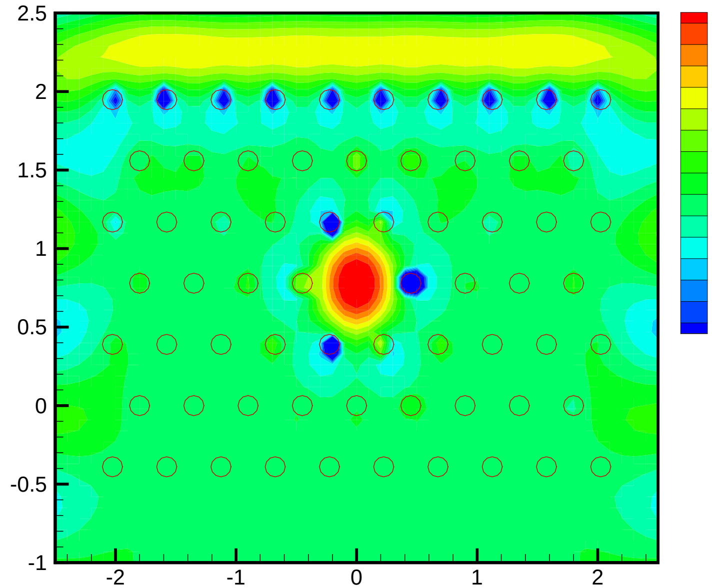


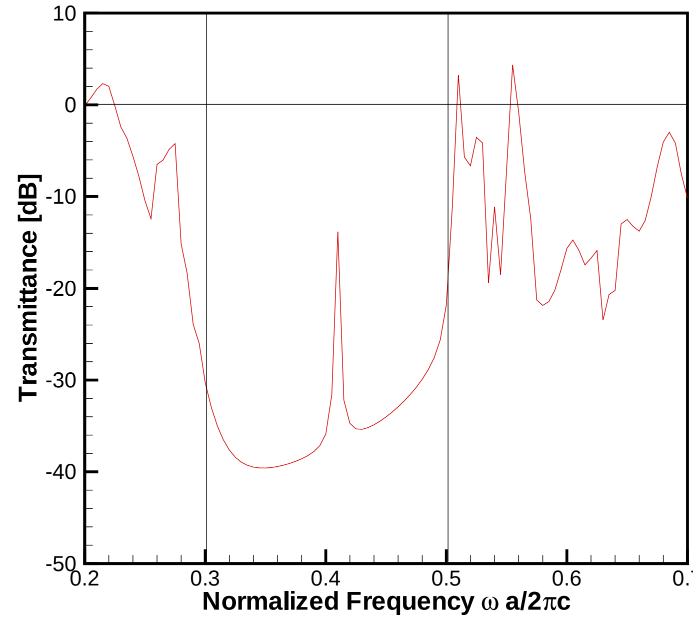


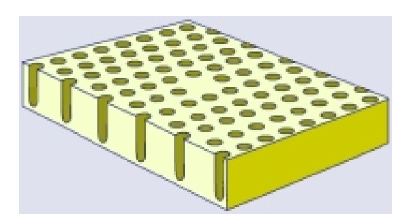
Band diagram and Density of States



Photonic Bandgap Crystals:point-defect (localized field)







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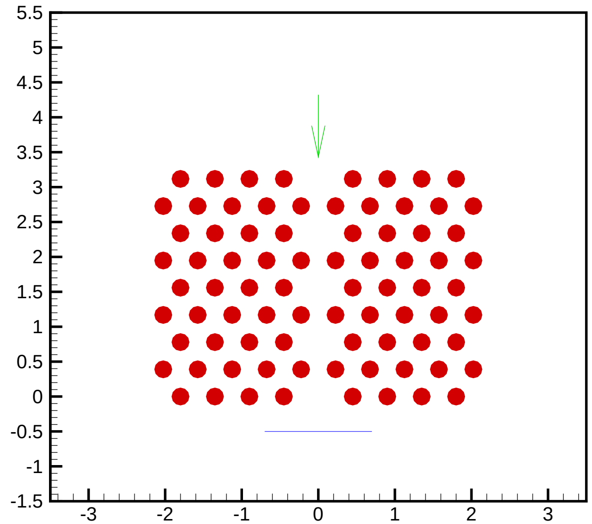
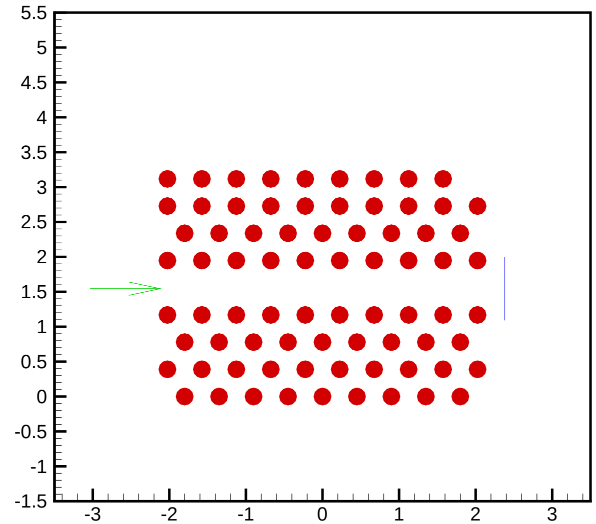
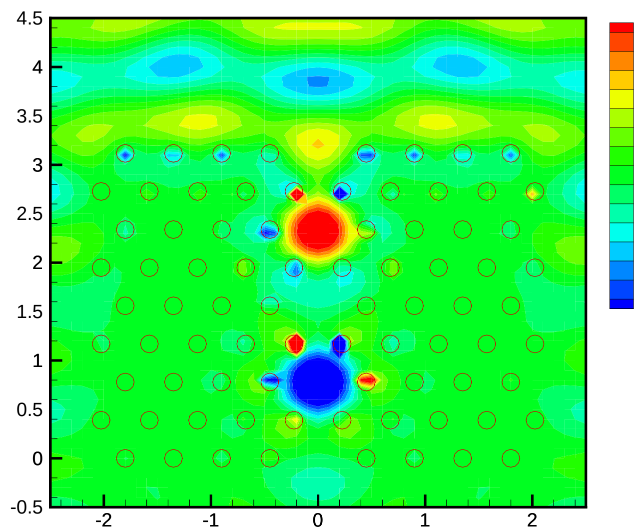
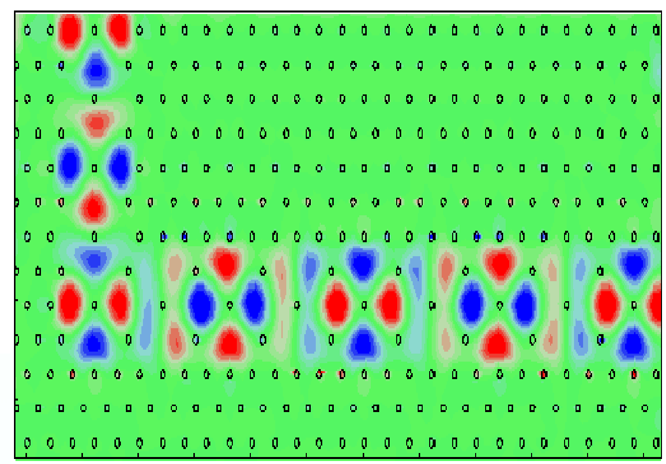
Photonic Bandgap Crystals:line-defects



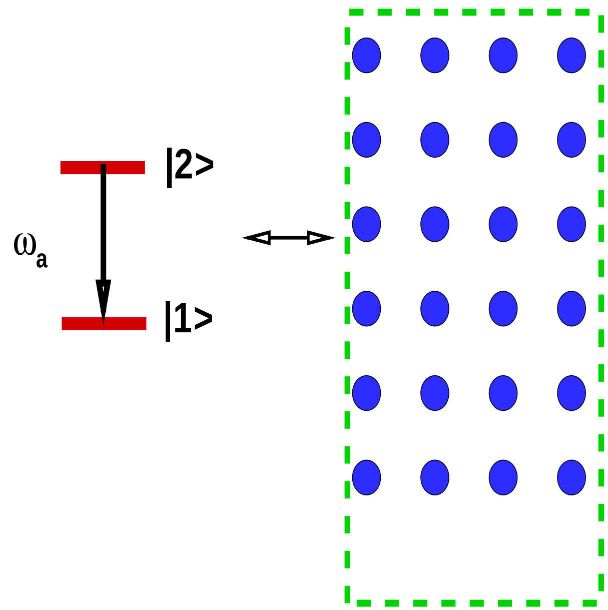


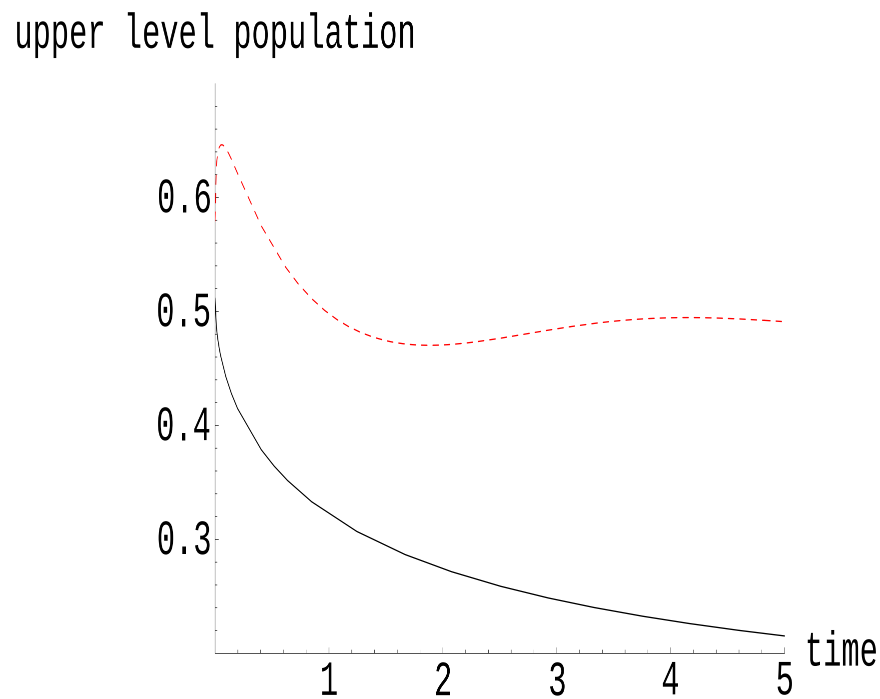




Courtesy: Bin-Shei Lin

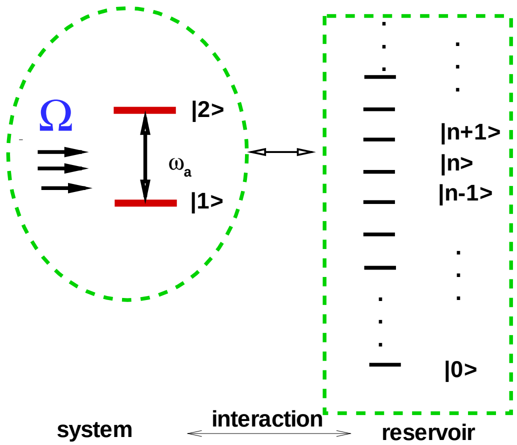
photon-atom bound state

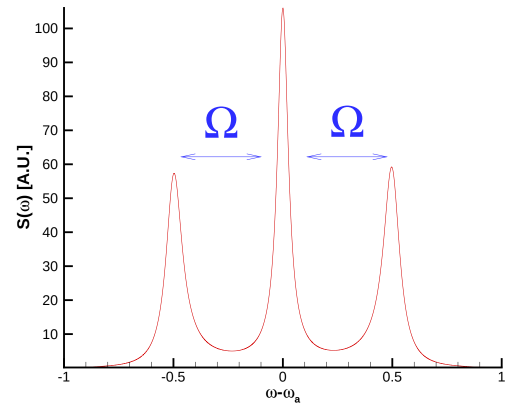




S. John and J. Wang, *Phys. Rev. Lett.* **64**, 2418 (1990).

Mollow's triplet: Resonance Fluorescence Spectrum

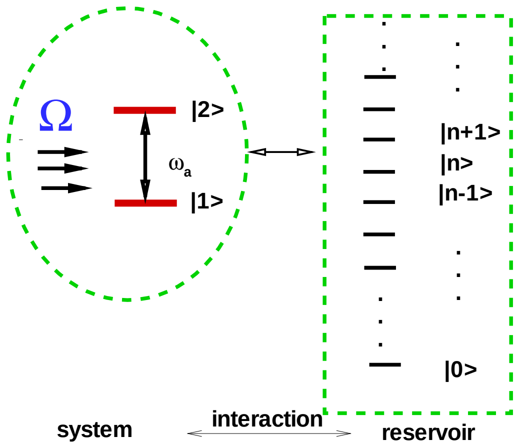


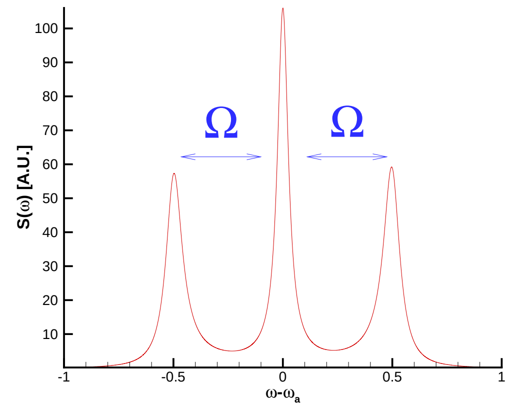


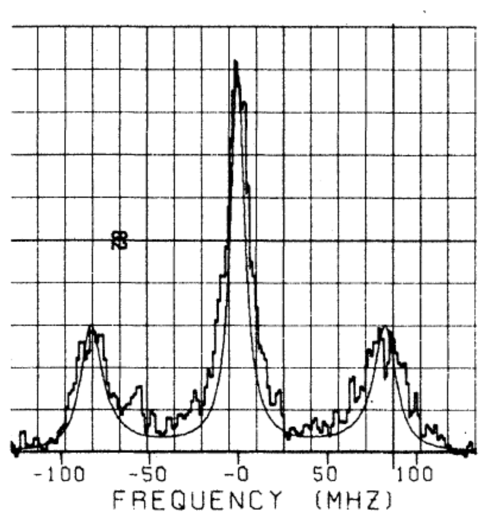
elastic Rayleigh scattering and inelastic Raman scattering

Theory: B. R. Mollow, *Phys. Rev.* **188**, 1969 (1969).

Mollow’s triplet: Resonance Fluorescence Spectrum



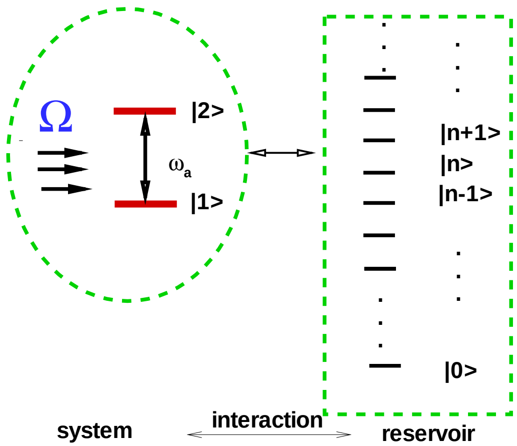


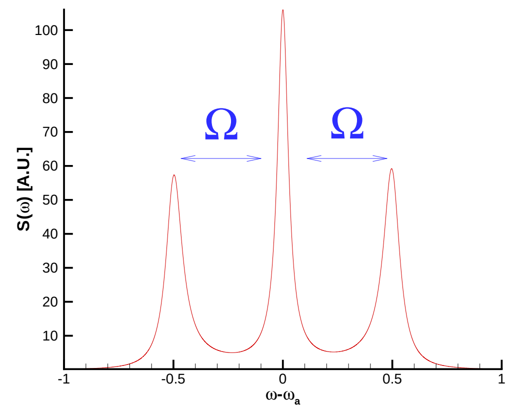


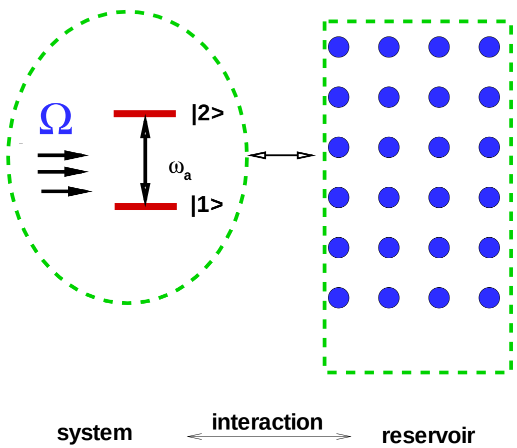
Theory: B. R. Mollow, *Phys. Rev.* **188**, 1969 (1969).

Exp: F. Y. Wu, R. E. Grove, and S. Ezekiel, *Phys. Rev. Lett.* **35**, 1426 (1975).

Reservoir Theory







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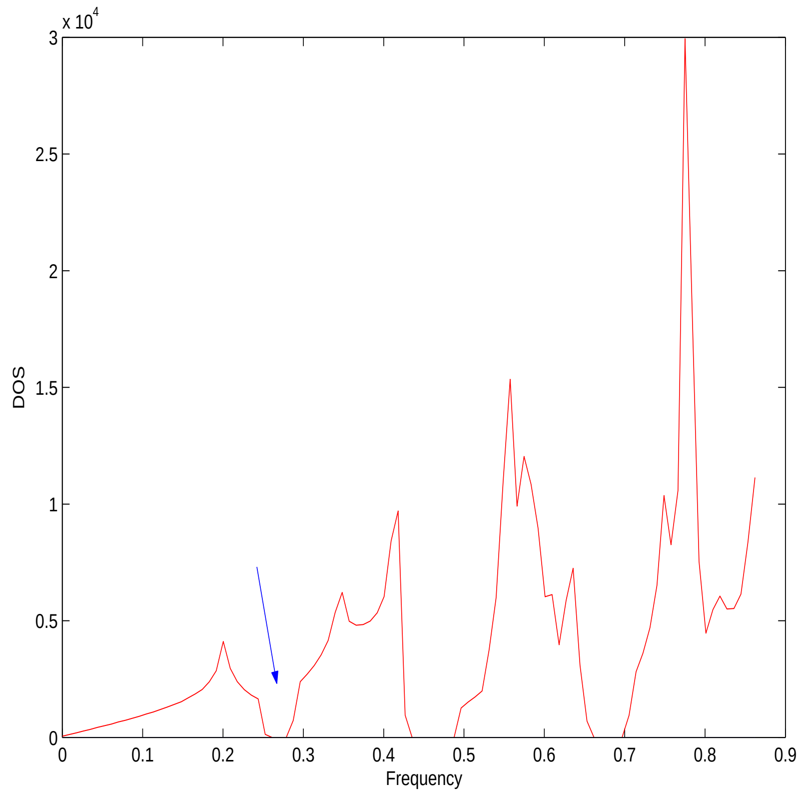
Density of States for Phcs



Hamiltonian of our system: Jaynes-Cummings model

$$
\begin{aligned}
H &= \frac{\hbar}{2}\omega_a\sigma_z + \hbar\sum_k \omega_k a_k^\dagger a_k + \frac{\Omega}{2}\hbar(\sigma_- e^{i\omega_L t} + \sigma_+ e^{-i\omega_L t}) \\
&+ \hbar\sum_k (g_k\sigma_+ a_k + g_k^* a_k^\dagger \sigma_-)
\end{aligned}
$$

And we want to solve the generalized Bloch equations:

$$
\begin{aligned}
\dot{\sigma}_-(t) &= i\frac{\Omega}{2}\sigma_z(t)e^{-i\Delta t} + \int_{-\infty}^{t} dt' G(t-t')\sigma_z(t)\sigma_-(t') + n_-(t) \\
\dot{\sigma}_+(t) &= -i\frac{\Omega}{2}\sigma_z(t)e^{i\Delta t} + \int_{-\infty}^{t} dt' G_c(t-t')\sigma_+(t')\sigma_z(t) + n_+(t) \\
\dot{\sigma}_z(t) &= i\Omega(\sigma_-(t)e^{i\Delta t} - \sigma_+(t)e^{-i\Delta t}) + n_z(t) \\
&- 2\int_{-\infty}^{t} dt' [G(t-t')\sigma_+(t)\sigma_-(t') + G_c(t-t')\sigma_+(t')\sigma_-(t)]
\end{aligned}
$$

Remarks:

1. coupling constant:

$$g_k \equiv g_k(\hat{\mathbf{d}}, \overrightarrow{r}_0) = |d|\omega_a \sqrt{\frac{1}{2\hbar\epsilon_0\omega_k V}}\hat{\mathbf{d}} \cdot \mathbf{E}_k^*(\overrightarrow{r_0})$$

2. memory functions:

$$G(\tau) \equiv \sum_k |g_k|^2 e^{i\Delta_k t}\Theta(\tau)$$

$$G_c(\tau) \equiv \sum_k |g_k|^2 e^{-i\Delta_k t}\Theta(\tau)$$

3. Markovian approximation:

$$G(t) = G_c(t) = \Gamma\delta(t)$$

Quantum noise operators

$$n_-(t) = i\sum_k g_k e^{i\Delta_k t}\sigma_z(t)a_k(-\infty)$$

$$n_+(t) = -i\sum_k g_k^* e^{-i\Delta_k t}a_k^+(-\infty)\sigma_z(t)$$

$$n_z(t) = 2i\sum_k [g_k^* e^{-i\Delta_k t}a_k^+(-\infty)\sigma_-(t) - g_k e^{i\Delta_k t}\sigma_+(t)a_k^+(-\infty)]$$

where the mean and the correlation functions of the reservoir before interaction,

$$< a_k(-\infty) >_R = < a_k^\dagger(-\infty) >_R = 0$$

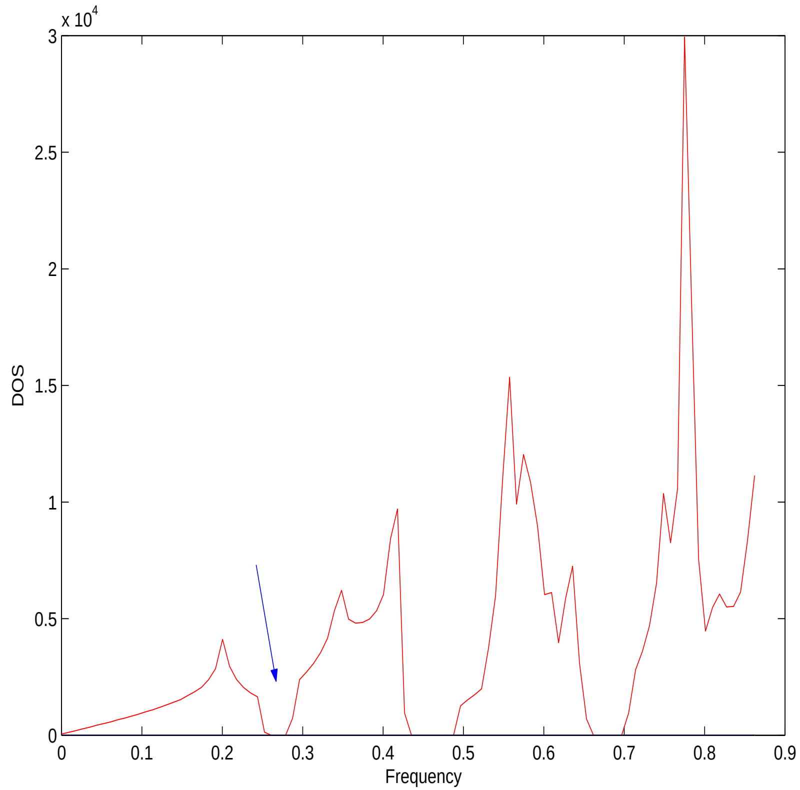
$$< a_k(-\infty)a_{k'}(-\infty) >_R = 0$$

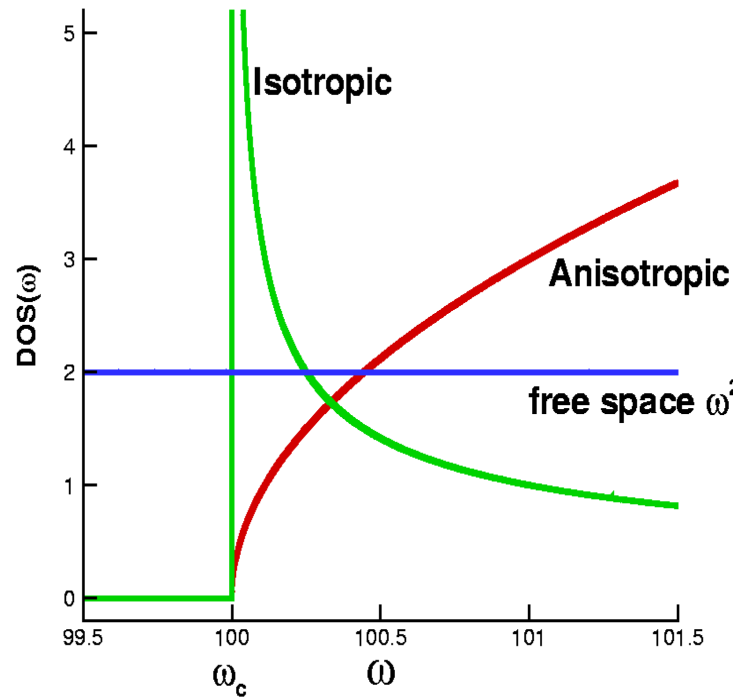
$$< a_k^\dagger(-\infty)a_{k'}^\dagger(-\infty) >_R = 0$$

$$< a_k^\dagger(-\infty)a_{k'}(-\infty) >_R = \bar{n}_k\delta_{kk'}$$

$$< a_k(-\infty)a_{k'}^\dagger(-\infty) >_R = (\bar{n}_k + 1)\delta_{kk'}$$

Modeling DOS of PBCs





anisotropic model: $\omega_k = \omega_c + A|\mathbf{k} - \mathbf{k}_0^i|^2$

$$D(\omega) = \sqrt{\frac{\omega - \omega_c}{A^3}}\Theta(\omega - \omega_c)$$

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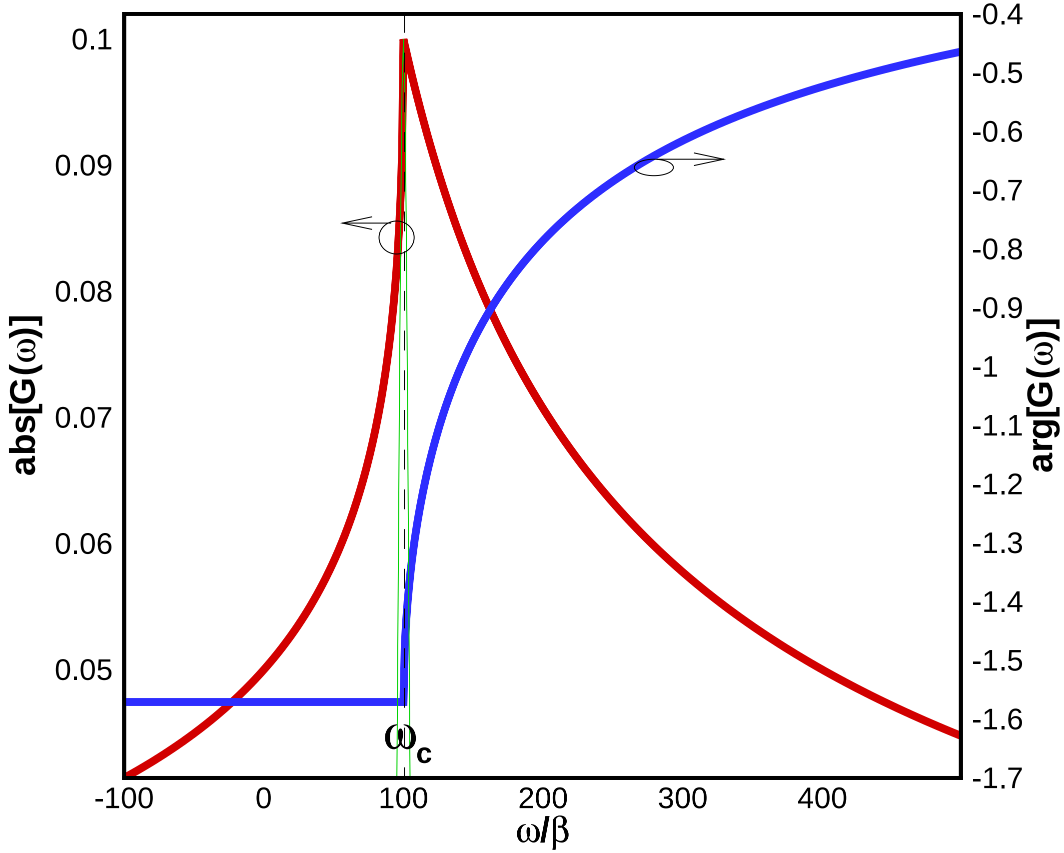
S. Y. Zhu, et al., *Phys. Rev. Lett.* **84**, 2136 (2000).

Memory functions of PBCs



Amplitude and phase spectrum of the memory function with $\omega_a = \omega_c = 100\beta$.

Liouville operator expansion:

$$\sigma_{ij}(t) = e^{-i\mathcal{L}(t-t')}\sigma_{ij}(t') = \sum_{n=0}^{\infty} \frac{[-i(t-t')]^n}{n!}\mathcal{L}^n \sigma_{ij}(t')$$

For zero-th order Liouville operator expansion, we get

$$\dot{\sigma}_-(t) = i\frac{\Omega}{2}\sigma_z(t)e^{-i\Delta t} - \int_{-\infty}^{t} dt' G(t-t')\sigma_-(t') + n_-(t)$$

$$\dot{\sigma}_+(t) = -i\frac{\Omega}{2}\sigma_z(t)e^{i\Delta t} - \int_{-\infty}^{t} dt' G_c(t-t')\sigma_+(t') + n_+(t)$$

$$\dot{\sigma}_z(t) = i\Omega(\sigma_-(t)e^{i\Delta t} - \sigma_+(t)e^{-i\Delta t}) - \int_{-\infty}^{t} dt'[G(t-t') + G_c(t-t')](1+\sigma_z(t')) + n_z(t)$$

valid for the case of

atom with longer lifetime and under weak pumping

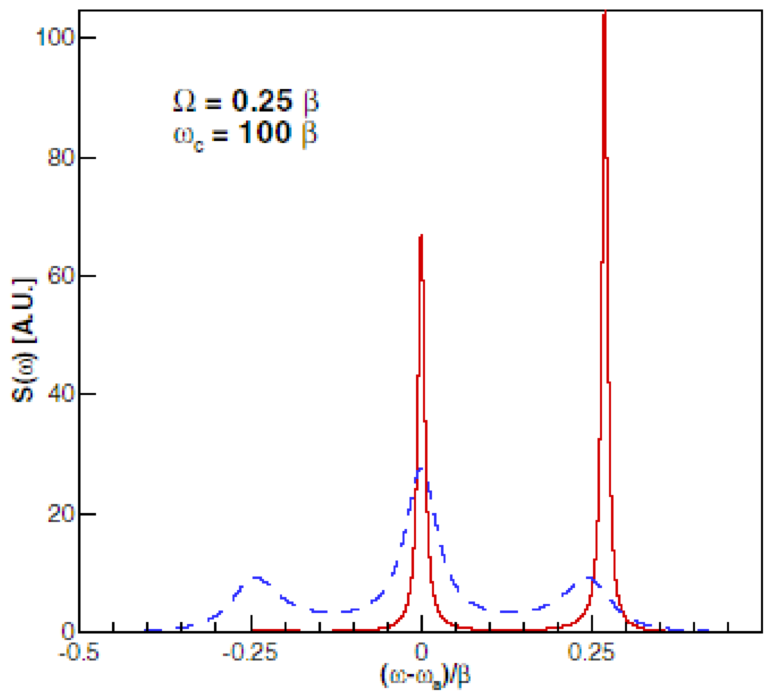
Correlations of noise operators at zero temperature

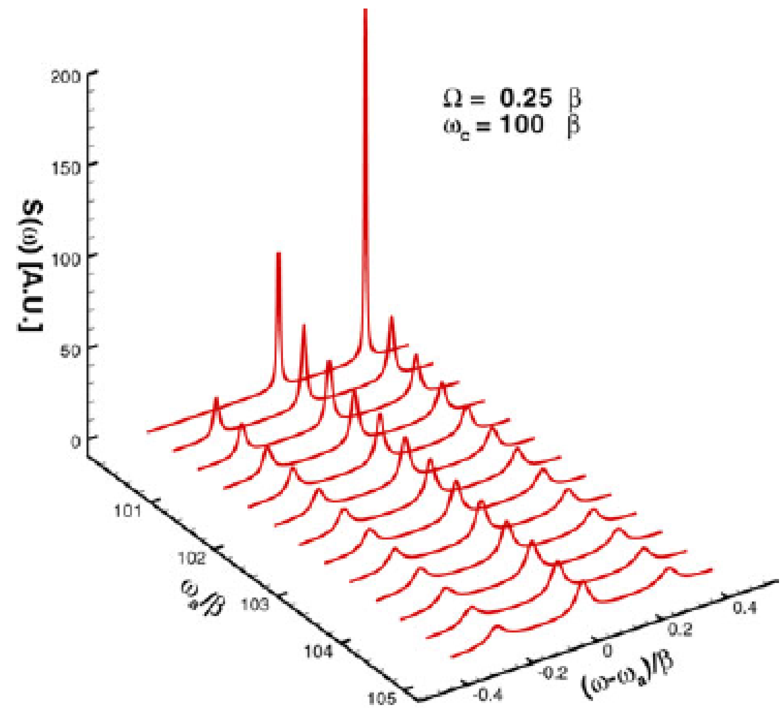
$$
\begin{aligned}
\langle \tilde{n}_-(\omega_1)\tilde{n}_+(-\omega_2)\rangle_R &= \pi N(\omega_1)\Theta(\omega_1+\omega_a-\omega_c)\delta(\omega_1-\omega_2) \\
\langle \tilde{n}_z(\omega_1)\tilde{n}_z(-\omega_2)\rangle_R &= N(\omega_1)[4\pi\delta(\omega_1-\omega_2)+\langle\tilde{\sigma}_z(\omega_1-\omega_2)\rangle_R] \\
&\quad \cdot\Theta(\omega_1+\omega_a-\omega_c) \\
\langle \tilde{n}_z(\omega_1)\tilde{n}_-(-\omega_2)\rangle_R &= 0 \\
\langle \tilde{n}_-(\omega_1)\tilde{n}_z(-\omega_2)\rangle_R &= N(\omega_1)\langle\tilde{\sigma}_-(\omega_1-\omega_2)\rangle_R\Theta(\omega_1+\omega_a-\omega_c) \\
\langle \tilde{n}_z(\omega_1)\tilde{n}_+(-\omega_2)\rangle_R &= N(\omega_1)\langle\tilde{\sigma}_+(\omega_1-\omega_2)\rangle_R\Theta(\omega_1+\omega_a-\omega_c) \\
\langle \tilde{n}_+(\omega_1)\tilde{n}_z(-\omega_2)\rangle_R &= 0
\end{aligned}
$$

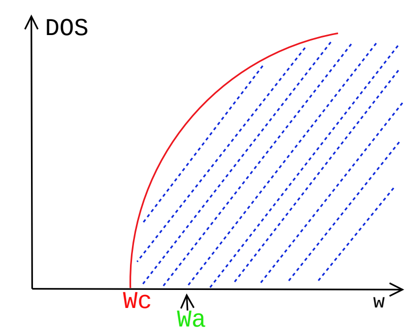
with $N(\omega) \equiv 4\beta^{3/2}\frac{\sqrt{\omega_a+\omega-\omega_c}}{\omega_a+\omega}$

Quantum noises of the photonic bandgap reservoir are not only color noises but also exhibit bandgap behaviour.

Resonance fluorescence spectra near the band-edge







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Quadrature spectra

Define quadrature field operator as:

$$\hat{E}_\theta(t) = e^{i\theta}\hat{E}^{(+)}(t) + e^{-i\theta}\hat{E}^{(-)}(t)$$

$\theta = 0 \ \left(\frac{\pi}{2}\right)$ are the *in-phase* (*out-of-phase*) quadrature fields.

Then the corresponding spectra with normally order variance is:

$$\begin{aligned} S_\theta(\omega) &\equiv < \tilde{E}_\theta(\omega), \tilde{E}_\theta(-\omega) > \\ &\propto \frac{1}{4}[< \tilde{\sigma}_-(\omega)\tilde{\sigma}_-(-\omega) > e^{-2i\theta} + < \tilde{\sigma}_+(\omega)\tilde{\sigma}_-(-\omega) > \\ &\quad + < \tilde{\sigma}_+(-\omega)\tilde{\sigma}_-(\omega) > + < \tilde{\sigma}_+(-\omega)\tilde{\sigma}_+(\omega) > e^{2i\theta}] \end{aligned}$$

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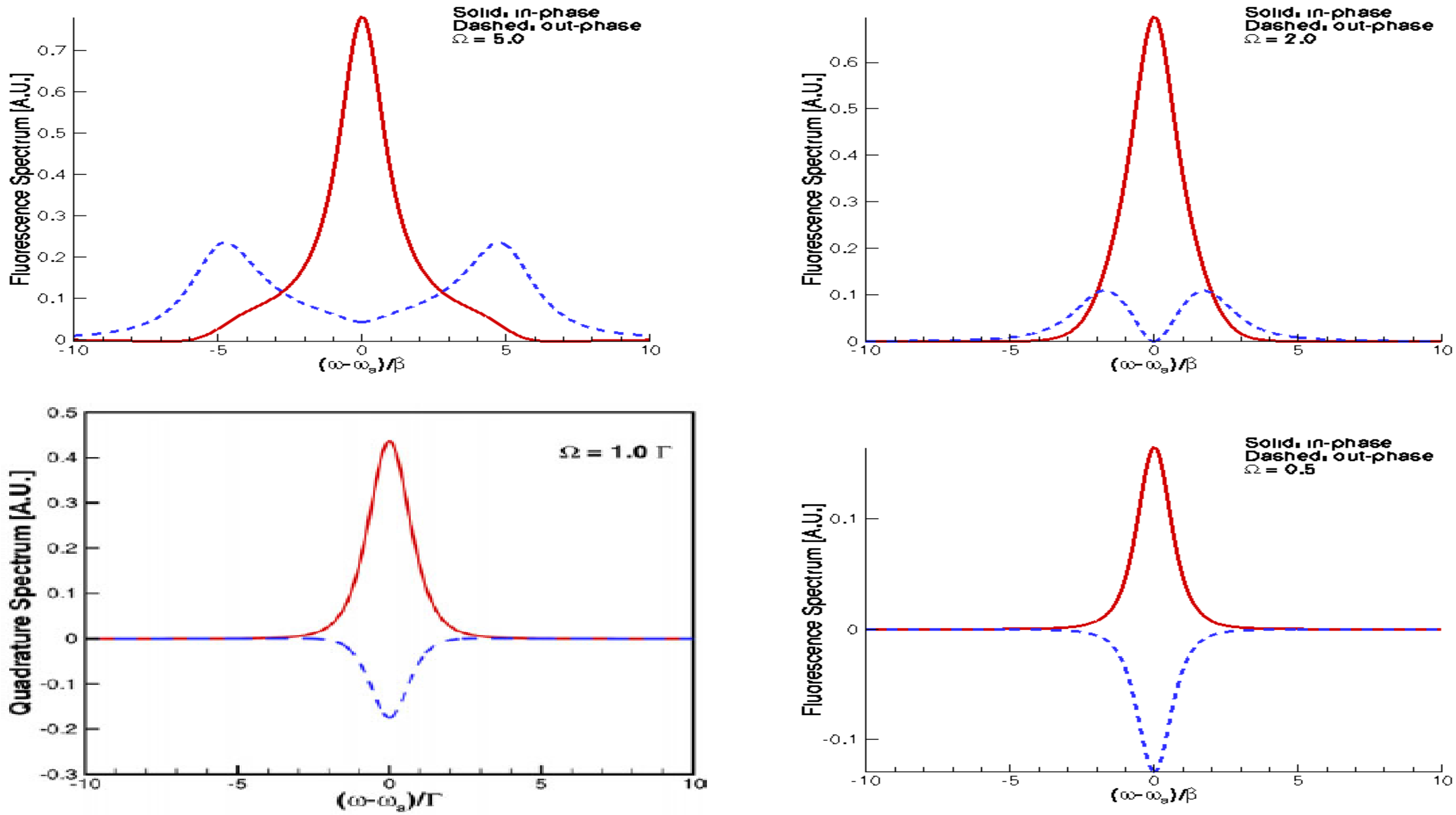
Quadrature spectra in free space

$\Omega = 5.0\Gamma$ $\Omega = 2.0\Gamma$



$\Omega = 1.0\Gamma$ $\Omega = 0.5\Gamma$

Theory: D. F. Walls and P. Zoller, *Phys. Rev. Lett.* **47**, 709 (1981).

Theory: L. Mandel, *Phys. Rev. Lett.* **49**, 136 (1982).

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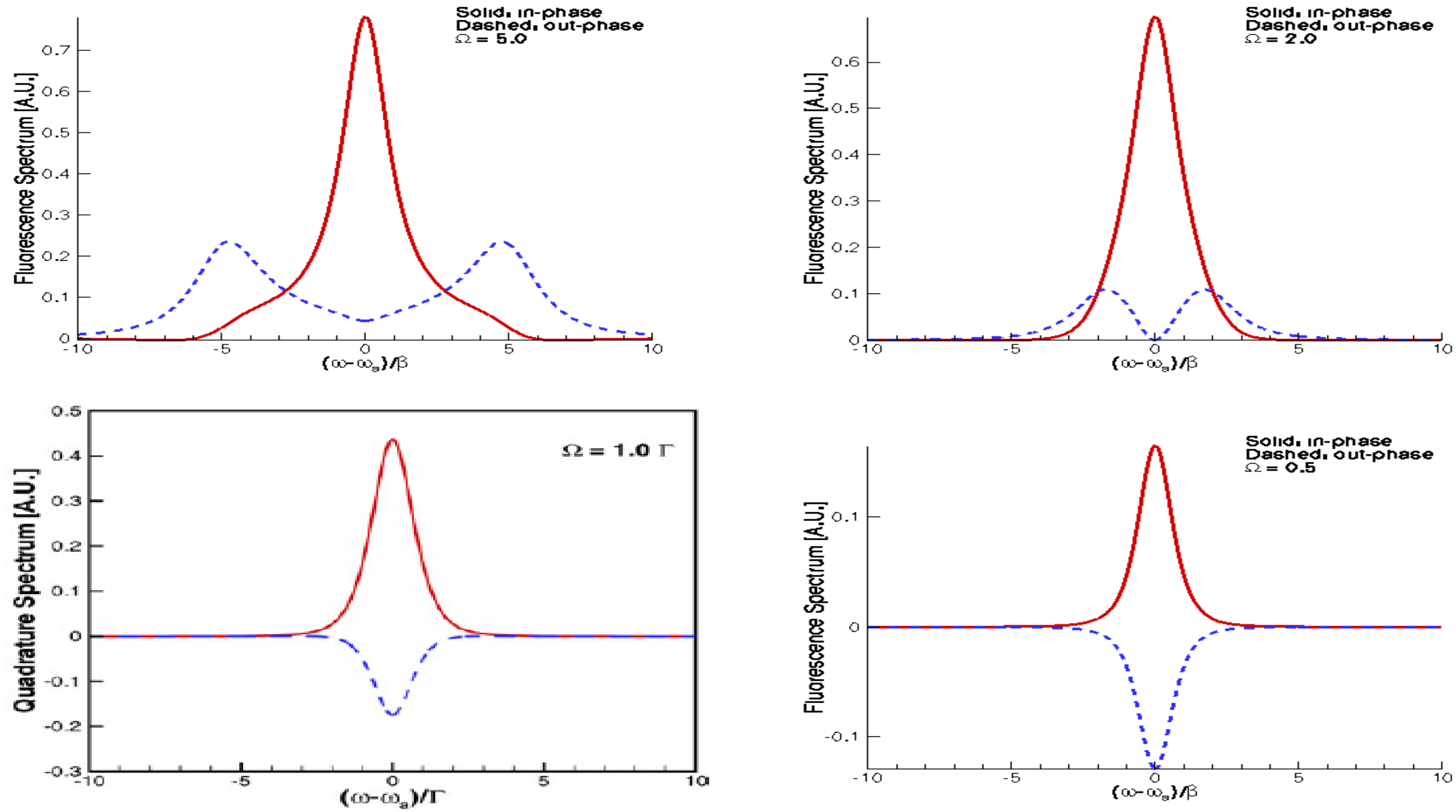
Quadrature spectra in free space

$\Omega = 5.0\Gamma$ $\Omega = 2.0\Gamma$



$\Omega = 1.0\Gamma$ $\Omega = 0.5\Gamma$

Squeezing occurs when $\Omega^2 < 4\Gamma^2$.

Observation of squeezing fluorescence spectra

VOLUME 81, NUMBER 17 PHYSICAL REVIEW LETTERS 26 OCTOBER 1998

Observation of Squeezing in the Phase-Dependent Fluorescence Spectra of Two-Level Atoms

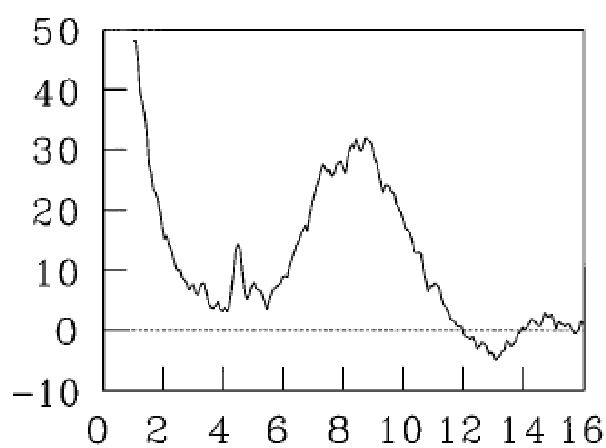
Z. H. Lu, S. Bali, and J. E. Thomas

Physics Department, Duke University, Durham, North Carolina 27708-0305

(Received 18 June 1998)

We observe squeezing in the phase-dependent fluorescence spectra of two-level atoms that are coherently driven by a near-resonant laser field in *free space*. In contrast to previous predictions that emphasized the in- and out-of-phase quadratures, we find that maximum squeezing occurs for homodyne detection at a phase near $\pm 45°$ relative to the exciting field. A new physical picture of phase-dependent noise is developed that incorporates quantum collapses into a Bloch vector model and yields a very simple form for the complete squeezing spectrum. [S0031-9007(98)07454-7]

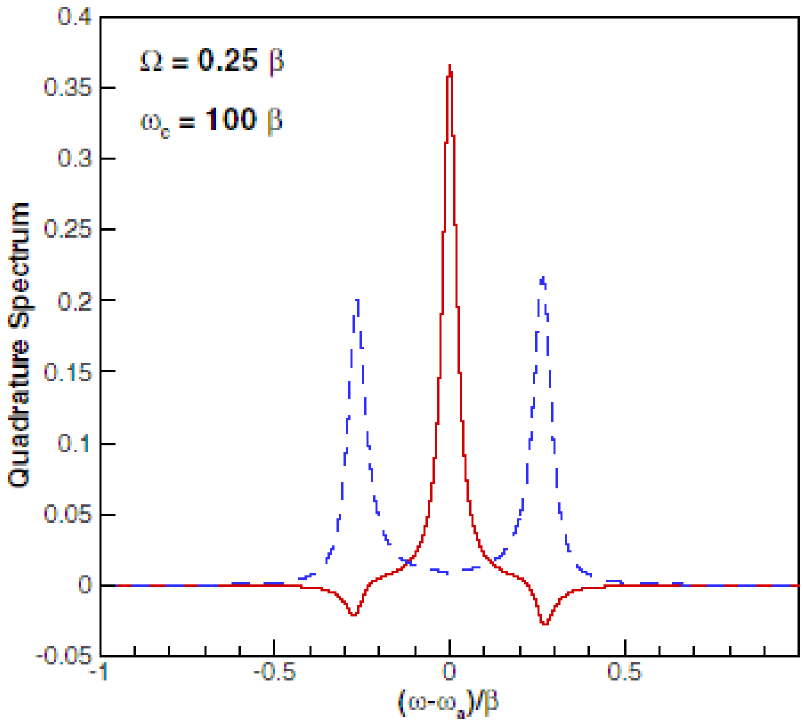
PACS numbers: 42.50.Lc, 32.80.–t

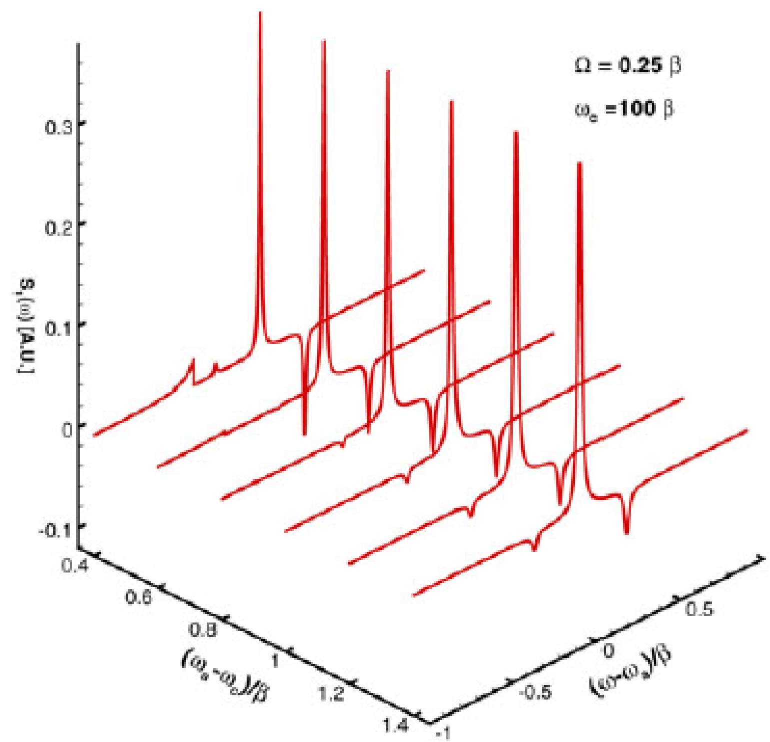


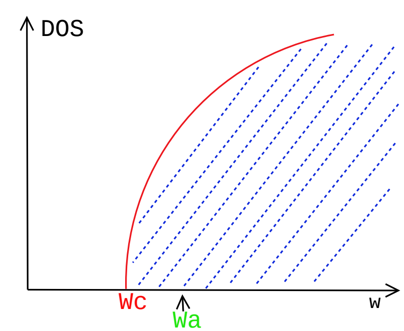
with ^{147}Yb atoms

Exp: Z. H. Lu, S. Bali, and J. E. Thomas, *Phys. Rev. Lett.* **81**, 3635 (1998).

Fluorescence quadrature spectra near the band-edge







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Conclusions

1. Suppression and enhancement of the relative fluorescence peak amplitudes varied at different wavelength offsets.
2. Squeezing occurs in the out-of-phase quadrature for free space when $\Omega^2 < 4\Gamma^2$.
3. Squeezing occurs in the in-phase quadrature for PhCs when $\Omega^2 > 4\Gamma^2$.
4. Resonance fluorescence squeezing spectra come from the interference between two sidebands of Mollow's triplet.

R.-K. Lee and Y. Lai, *J. Opt. B*, **6**, S715 (2004).