AMO Summer School @ 國立中正大學物理系

Gap solitons in optical lattices: their classical and quantum properties

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Ref: Ray-Kuang Lee and Yinchieh Lai, *Phys. Rev. A* **69**, 021801*(R)* (2004);
R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72**, (Sep. 2005).

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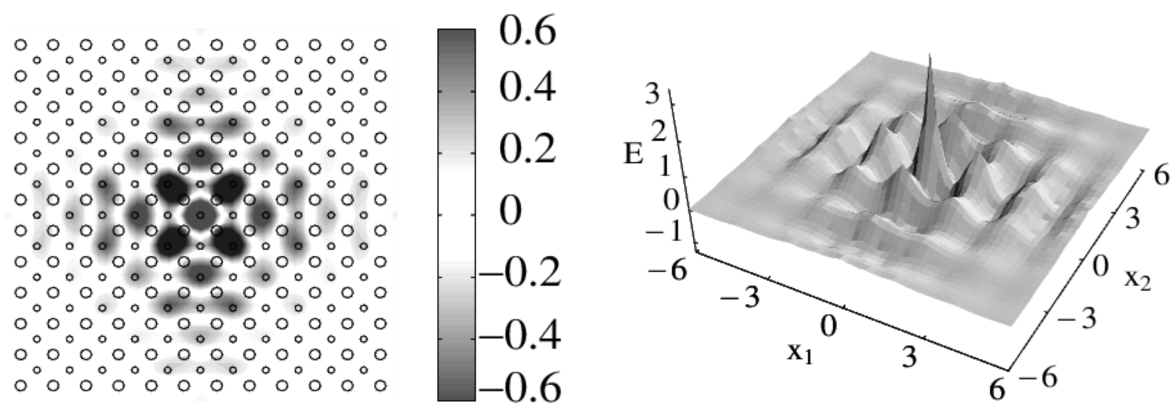
Outline

1. The great wave of translation
2. Bragg grating solitons
3. Gap solitons in optical lattices
4. Entangled solitons for quantum information
5. Conclusions

Nonlinear Photonic Crystals

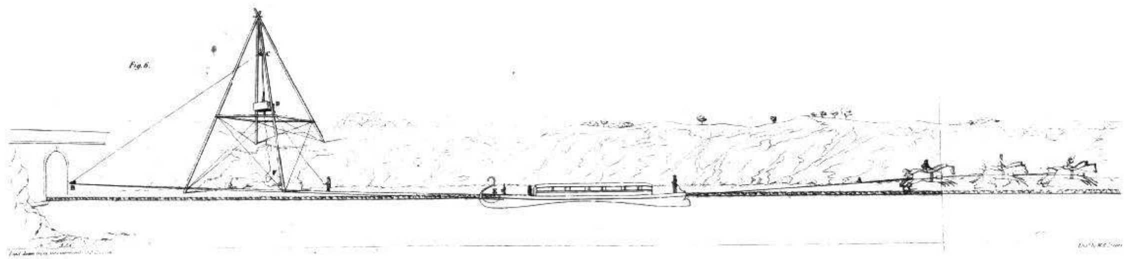
- Can pulse transmit the bandgap without using defect waveguides ?
- With *Nonlinear* photonic bandgap crystals, pulse can propagate without changing its shape.
- It's called Bragg/Gap solitons.

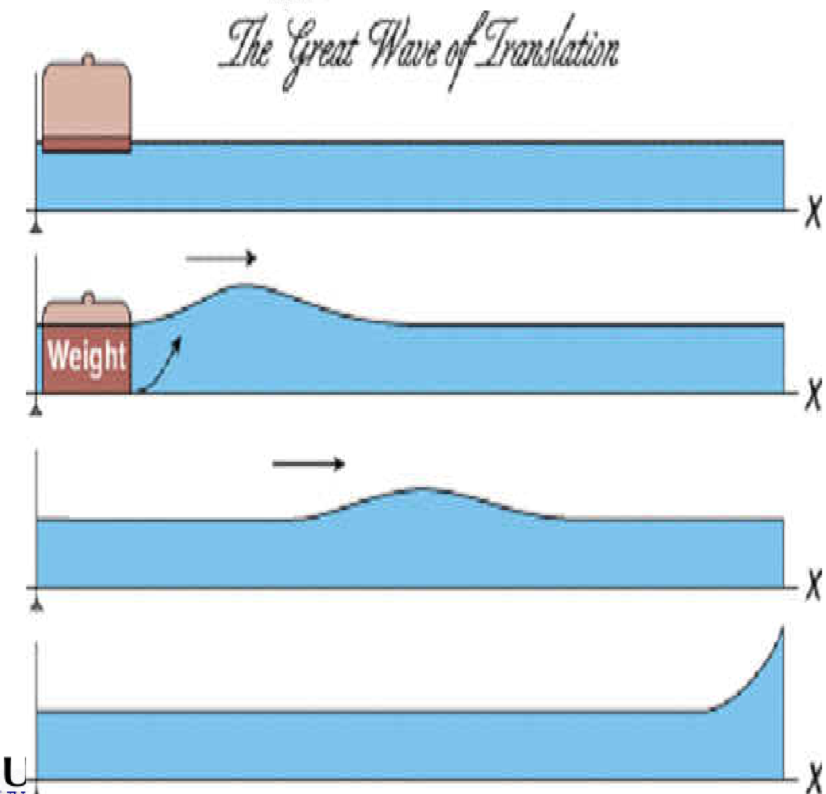


S. F. Mingaleev and Yu. S. Kivshar, *Phys. Rev. Lett.* **86**, 5474 (2001).

The Great Wave of Translation

Scottish engineer John Scott Russell (1808-1882), *fourteenth meeting of the British Association for the Advancement of Science*, York, September 1844 (London 1845).

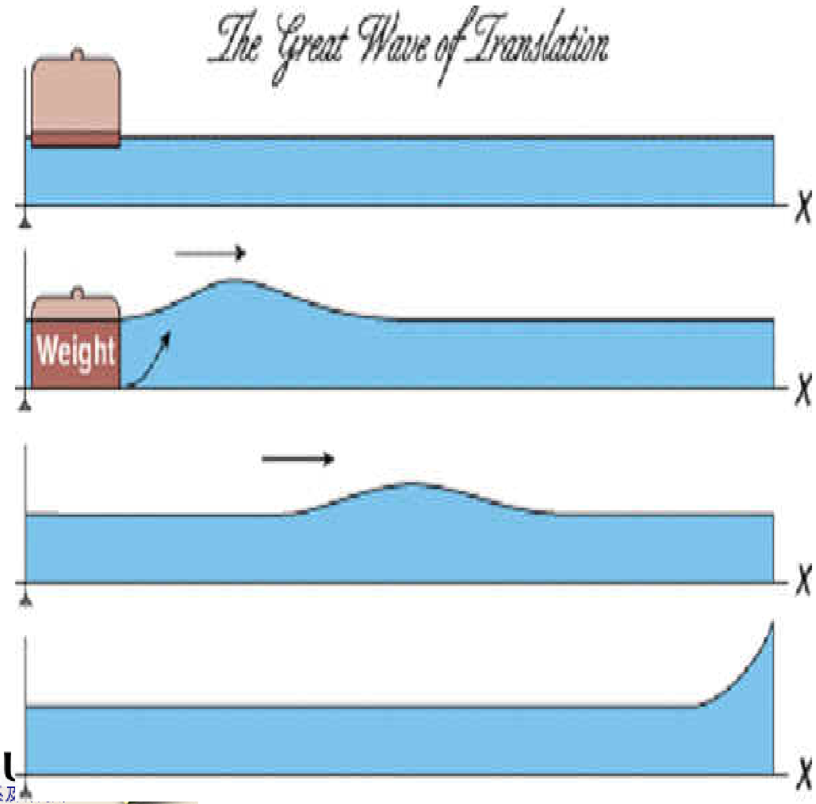




The Great Wave of Translation

Scottish engineer John Scott Russell (1808-1882), *fourteenth meeting of the British Association for the Advancement of Science*, York, September 1844 (London 1845).

Soliton on the Scott Russell Aqueduct on the Union Canal near Heriot-Watt University, 12 July 1995.



Tsunami

The Great Wave of Kanag'awa is an example of a soliton.

Hokusai, 1879, Japanese woodcut.

Solitons

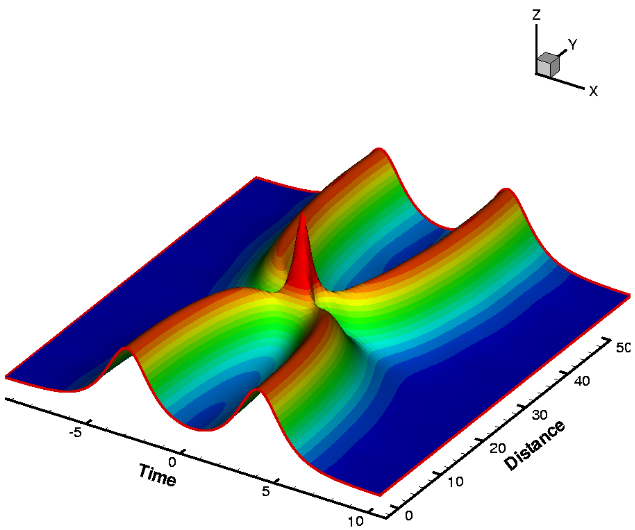
A Universal phenomenon of self-trapped wave packets.

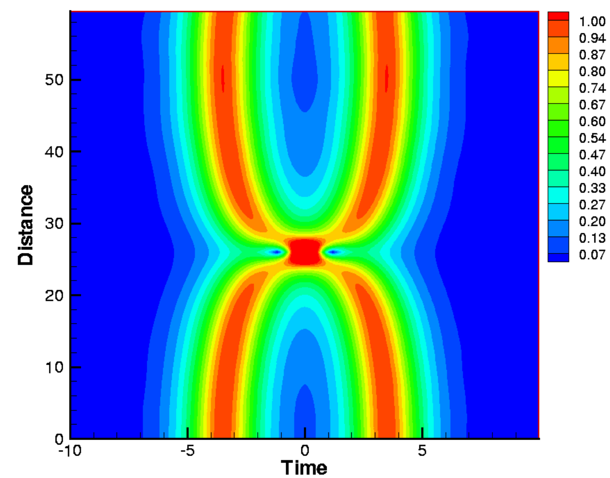
- EM waves in nonlinear optical materials;
- shallow- and deep-water waves;
- charge-density waves in plasmas;
- sound waves in liquid ^{3}He;
- matter waves in Bose-Einstein condensates;
- excitations on DNA chains;
- domain walls in supergravity, and
- "branes" at the end of open strings in superstring theory; to name only a few.

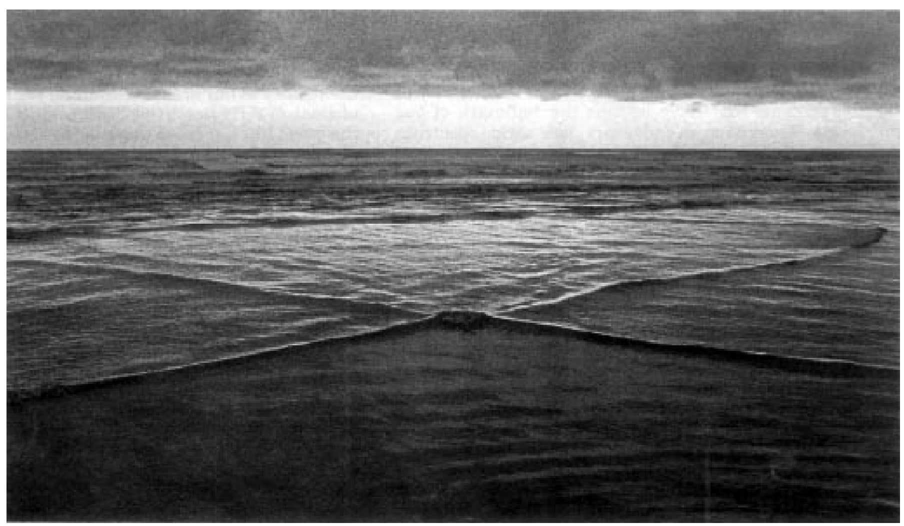
M. Segev, *Optics & Photonics News*, pp. 27 (Feb. 2002).

Wave-particle characteristics of solitons

Collision between solitons







Courtesy of T. Toedterneier

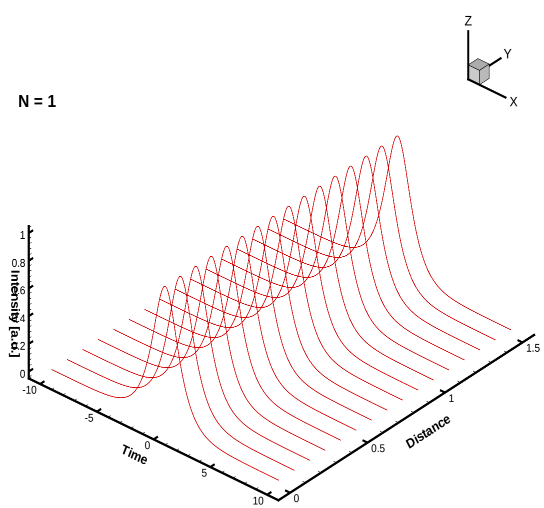
Solitons in optical fibers

Nonlinear Schrödinger Equation:

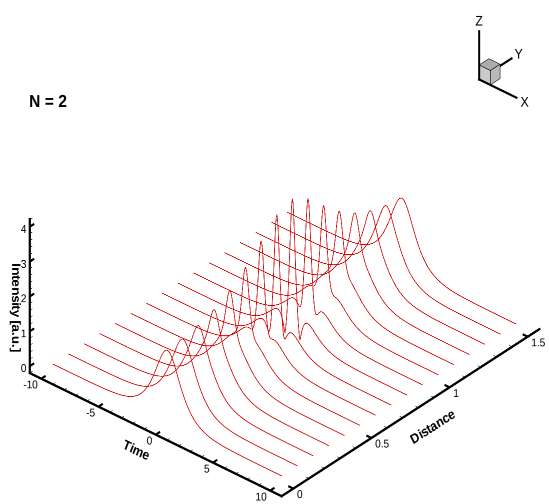
$$iU_z(z,t) \;=\; -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2 U(z,t)$$

Fundamental soliton:

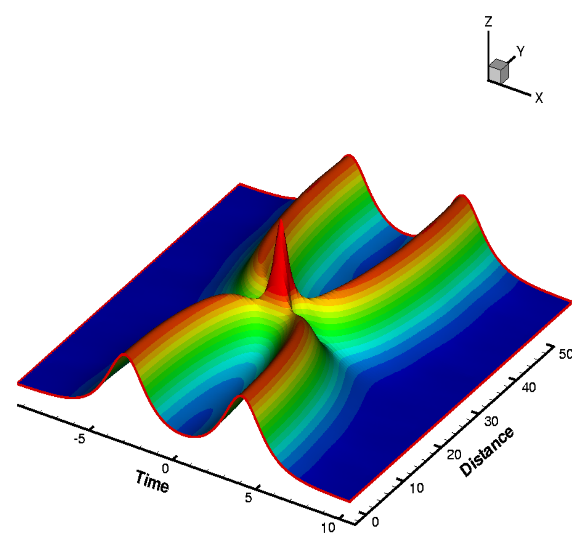
$$U(z,t) = \frac{n_0}{2} exp[i\frac{n_0^2}{8}z + i\theta_0] sech[\frac{n_0}{2}t]$$



Fundamental



Higher-order ($N = 2$)

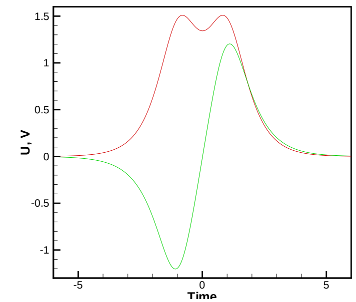


Soliton interaction

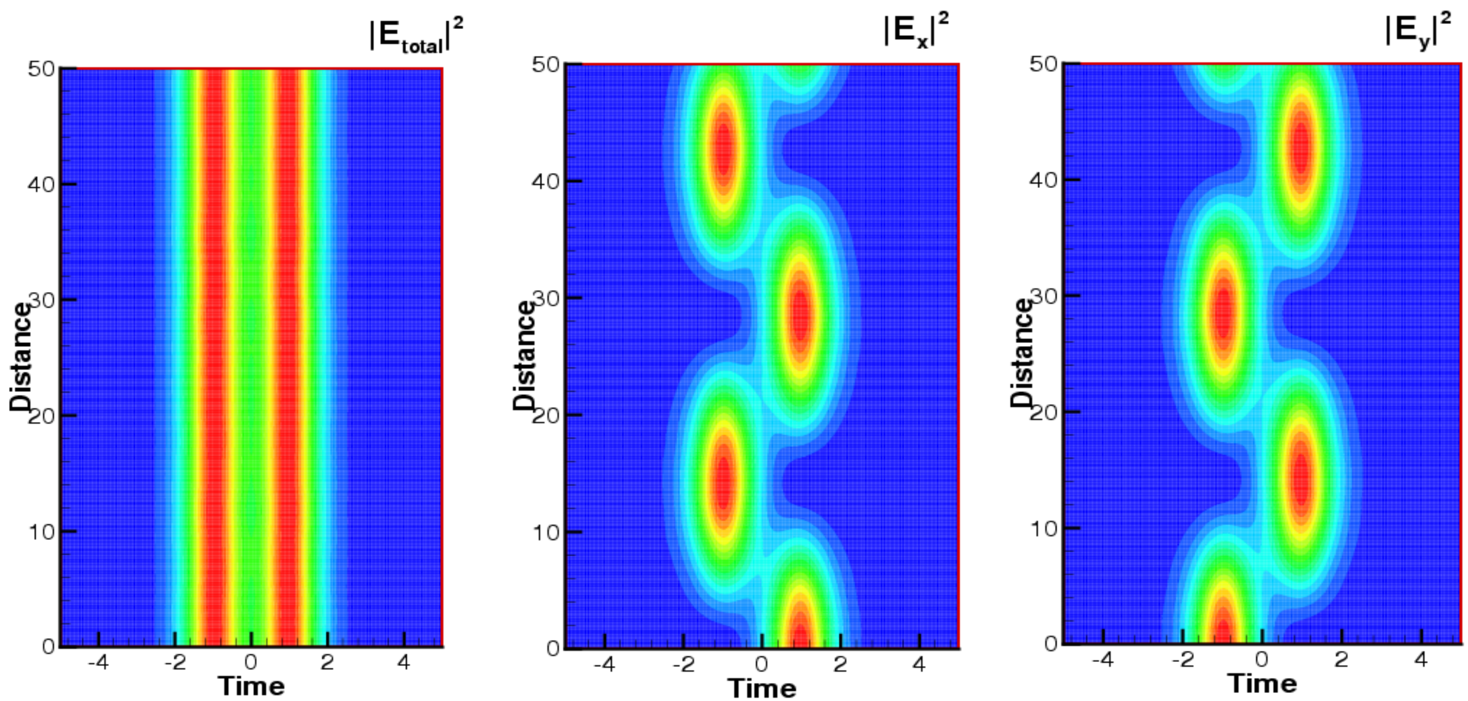
Vector bound solitons

Coupled Nonlinear Schrödinger Equations:

$$
\begin{aligned}
i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2 U + B|V|^2 U &= 0 \\
i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2 V + B|U|^2 V &= 0
\end{aligned}
$$



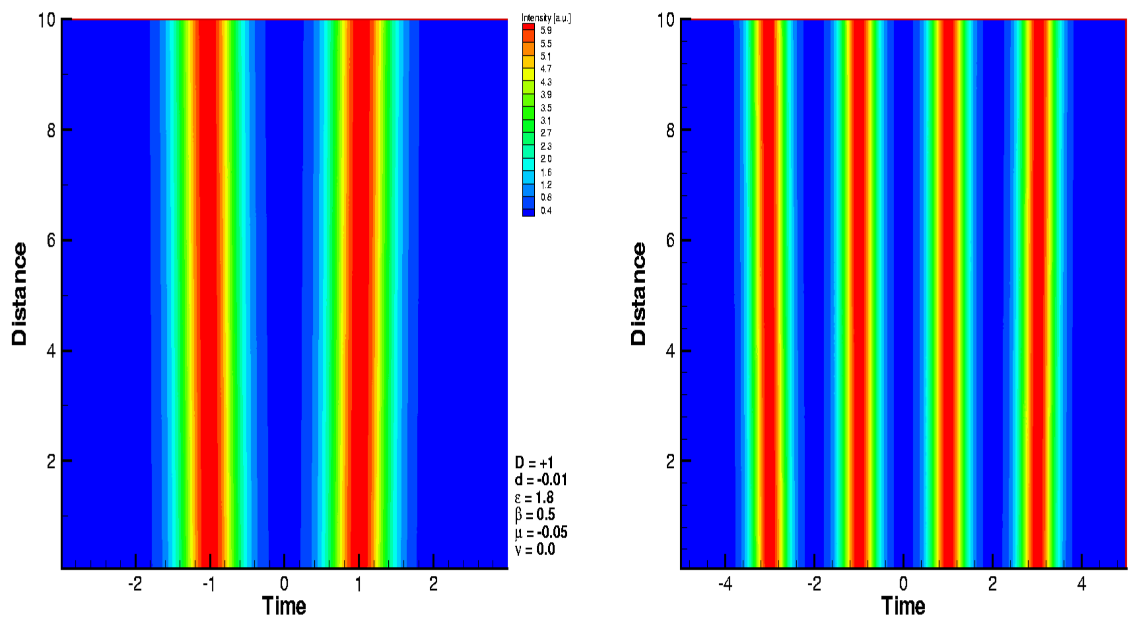
where $A = 1/3$, $B = 2/3$; and U, V are circular polarization fields.



M. Haelterman, A. P. Sheppard, and A. W. Snyder, *Opt. Lett.* **18**, 1406 (1993).

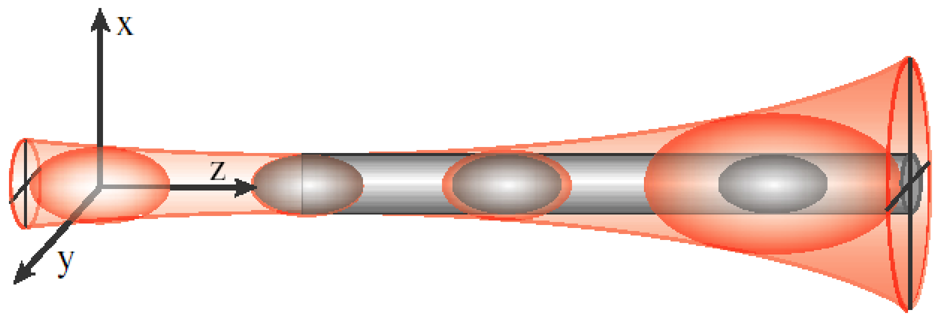
Complex Ginzburg-Lanau Equation:

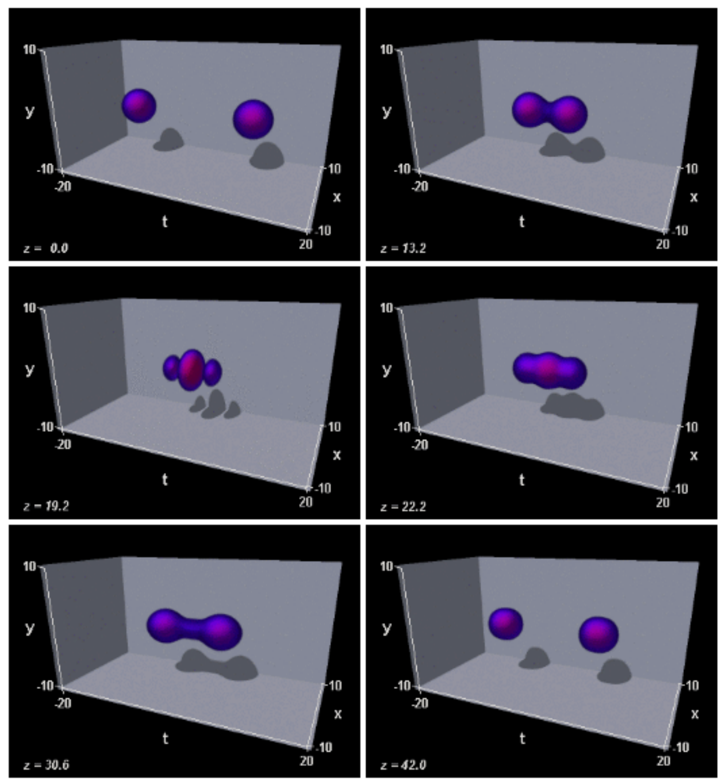
$$
\begin{aligned}
iU_z + \frac{D}{2}U_{tt} + |U|^2U &= i\delta U + i\epsilon|U|^2U + i\beta U_{tt} \\
&+ i\mu|U|^4U - v|U|^4U
\end{aligned}
$$



N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, *J. Opt. Soc. Am. B* **15**, 515 (1998).

Spatio-temporal solitons: light bullet





B. A. Malomed, D. Mihalache, F. Wise, and L. Torner, *J. Op. B* **7**, R53-R72 (2005).

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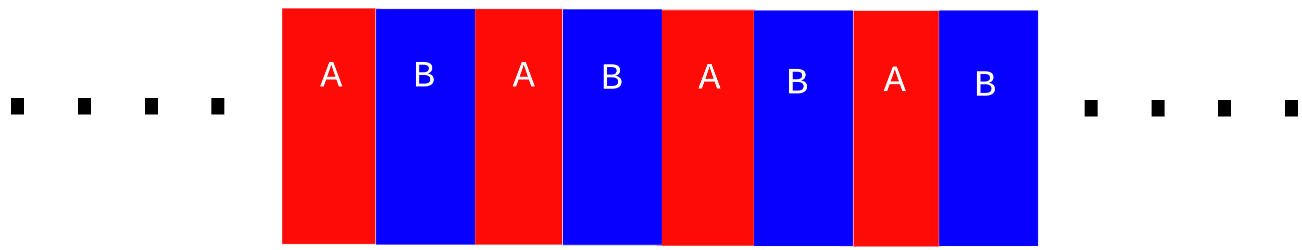
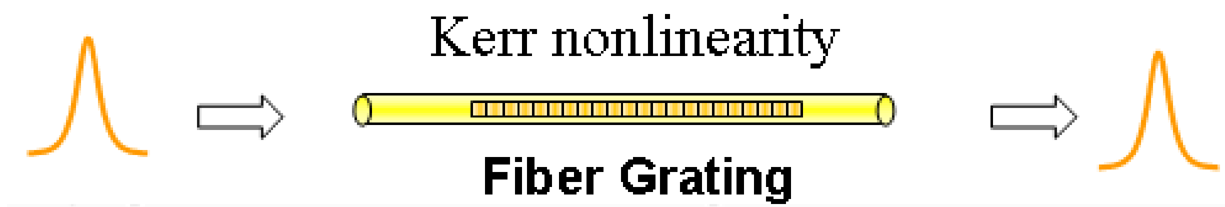
Fiber Bragg grating solitons





Coupled mode theory: linear case

Linear wave propagation in a 1-D periodic structure:

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2(z)}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

where $n^2(z) = \bar{n}^2 + \tilde{\epsilon}(z)$ is a periodic structure.

For FBGs, we expand $\tilde{\epsilon}(z)$ by the Fourier series and only keep the phase-matching ± 1 order terms. Then decomposes the light field into the forward (U_a) and backward (U_b) propagation pulses, $E(z,t) = U_a(z,t)e^{-i(\omega t - k_0 z)} + U_b(z,t)e^{-i(\omega t + k_0 z)} + c.c.$:

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_a(z,t) + \frac{\partial}{\partial z}U_a = i\delta U_a + i\kappa U_b$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_b(z,t) - \frac{\partial}{\partial z}U_b = i\delta U_b + i\kappa U_a$$

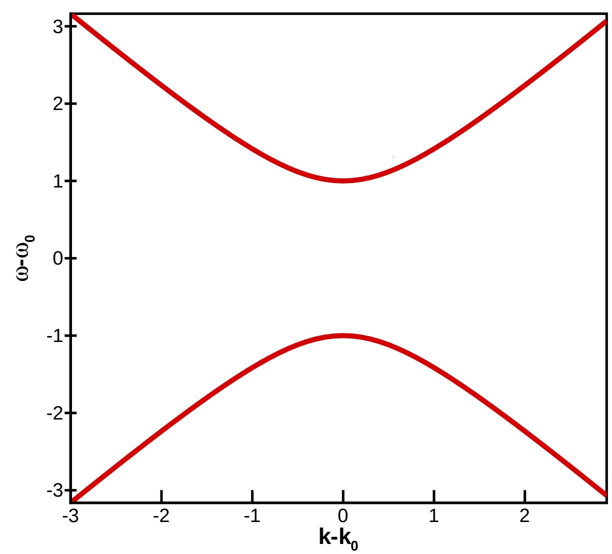
where $v_g = \bar{n}/c$ is the group velocity of the pulses, $\delta = \omega - \omega_0$ is the wavelength detuning parameter, and $\kappa = \omega_0\tilde{\epsilon}/2\bar{n}c$ is the coupling coefficient.

Dispersion relations for FBGs

Using the envelope functions, $E_{\pm}(z,t) = A_{\pm} e^{-i(\Omega t - Qz)}$, one can have

$$\begin{bmatrix} c\Omega/\bar{n} & \kappa \\ \kappa & c\Omega/\bar{n} + Q \end{bmatrix} \begin{bmatrix} A_+ \\ A_- \end{bmatrix} = 0,$$

with the dispersion relation, $c\Omega/\bar{n} = \pm\sqrt{\kappa^2 + Q^2}$.



Coupled mode theory: nonlinear case

Consider third-harmonic generation, χ_3 nonlinearity,

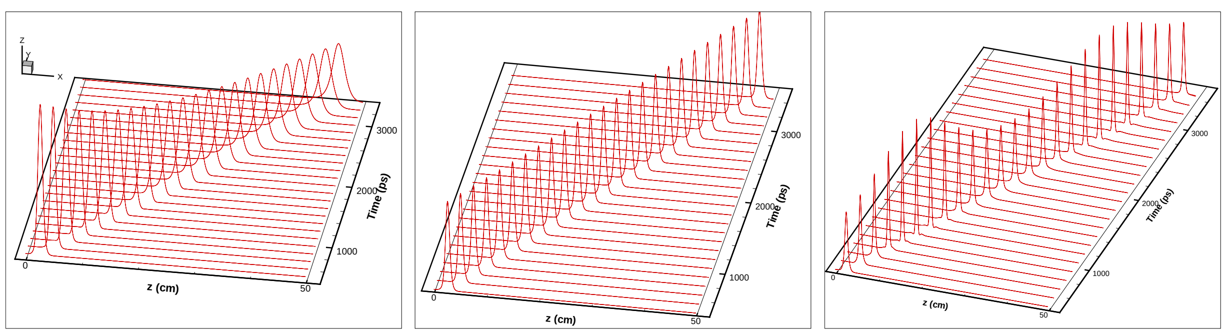
$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2(z)}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 \mathcal{P}_{NL}}{\partial t^2},$$

then we have nonlinear coupled-mode theory:

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_a(z,t) + \frac{\partial}{\partial z}U_a = i\delta U_a + i\kappa U_b + i\Gamma|U_a|^2 U_a + 2i\Gamma|U_b|^2 U_a$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_b(z,t) - \frac{\partial}{\partial z}U_b = i\delta U_b + i\kappa U_a + i\Gamma|U_b|^2 U_b + 2i\Gamma|U_a|^2 U_b$$

decay stationary oscillate



Theory: A. Aceves and S. Wabnitz, *Phys. Lett. A* **141**, 37 (1986).

Exp: B. J. Eggleton *et al.*, *Phys. Rev. Lett.* **76**, 1627 (1996).

Coherent and Squeezed States

Laser beam can be represented by

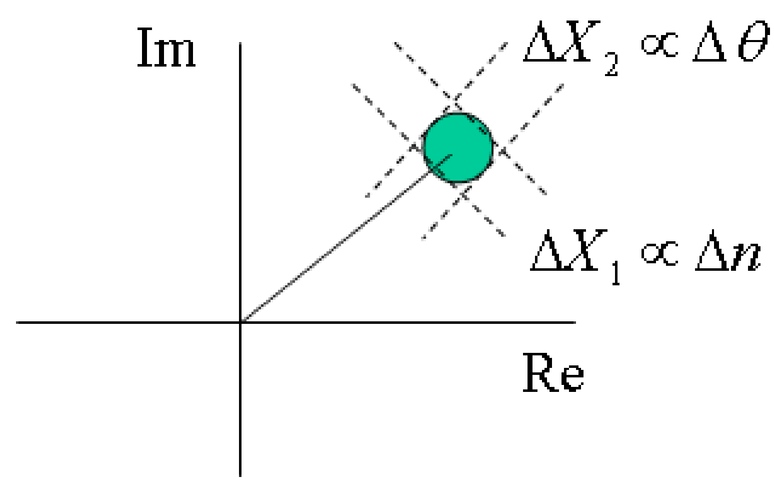
$$\hat{E}(t) = E_0[\hat{X}_1 \sin(\omega t) - \hat{X}_2 \cos(\omega t)]$$

where

$$\begin{aligned} \hat{X}_1 &= \text{amplitude quadrature} \\ \hat{X}_2 &= \text{phase quadrature} \end{aligned}$$

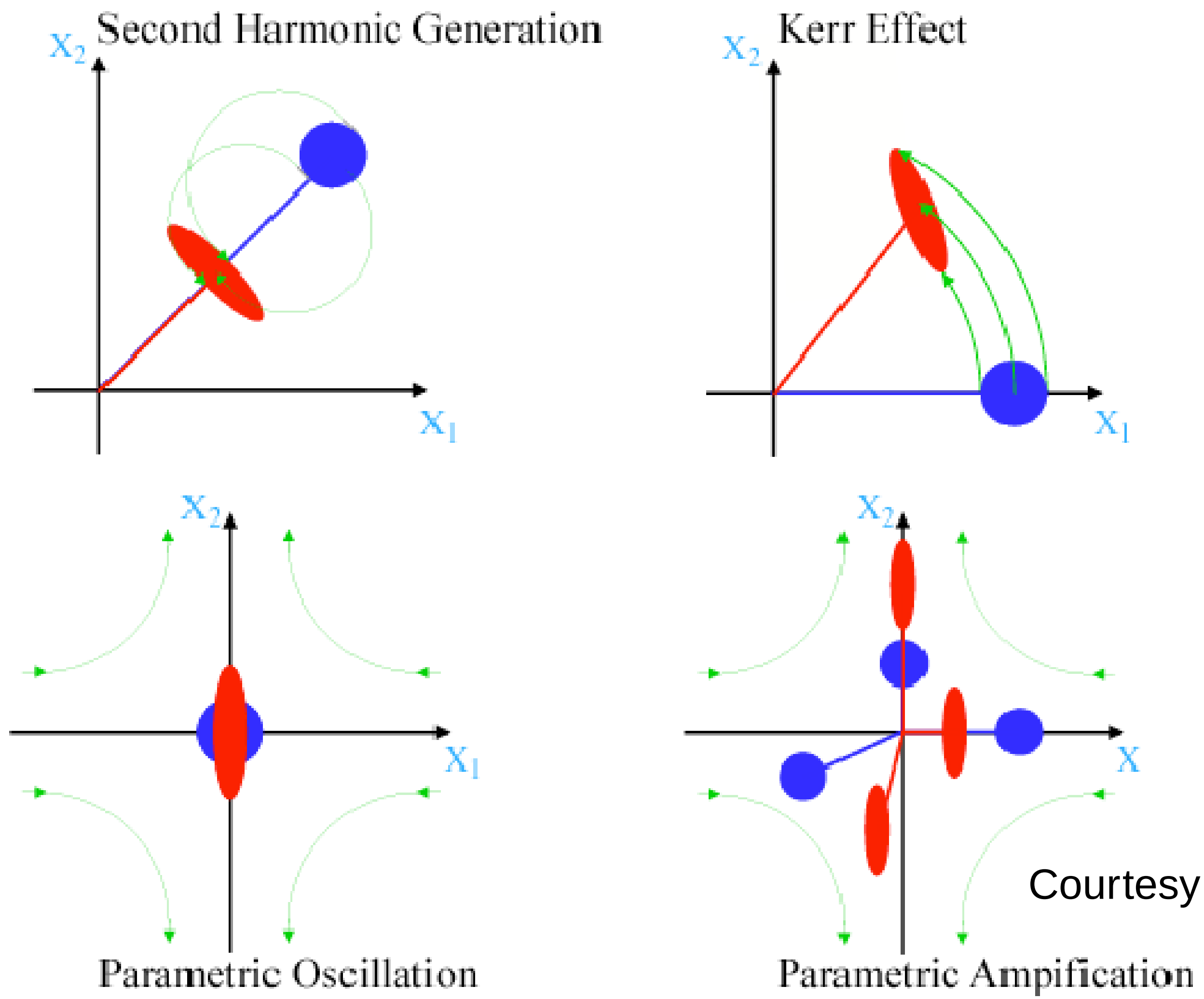
Uncertainty Principle: $\Delta\hat{X}_1 \Delta\hat{X}_2 \geq 1$.

1. Coherent states: $\Delta\hat{X}_1 = \Delta\hat{X}_2 = 1$,
2. Amplitude squeezed states: $\Delta\hat{X}_1 < 1$,
3. Phase squeezed states: $\Delta\hat{X}_2 < 1$,
4. Quadrature squeezed states.



Generations of Squeezed States

Nonlinear optics:



Courtesy of P. K. Lam

Definition of Squeezing and Correlation

Squeezing Ratio

$$\hat{M} = M + \Delta\hat{M}$$

$$\text{SR} = \frac{\langle \Delta\hat{M}^2 \rangle}{\langle \Delta\hat{M}^2 \rangle_{\text{c.s.}}}$$

SR $< 1 : Squeezing$

SR $> 1 : Anti-Squeezing$

Correlation

$$C = \frac{\langle : \Delta\hat{A}\Delta\hat{B} : \rangle}{\sqrt{\langle \Delta\hat{A}^2 \rangle \langle \Delta\hat{B}^2 \rangle}}$$

$0 \leq C \leq 1$: *Positive Correlation*

$C = 0$: *No Correlation*

$-1 \leq C \leq 0$: *Negative Correlatior*

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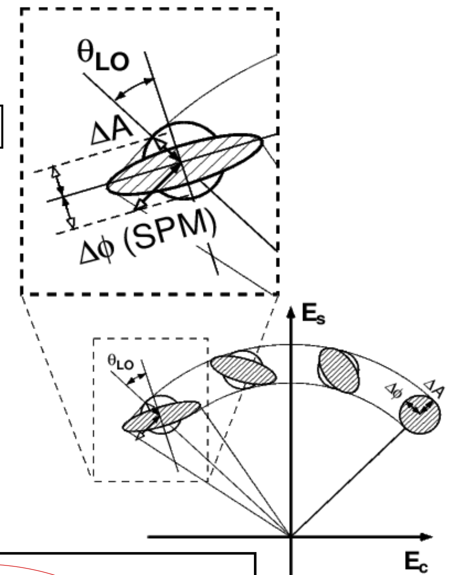
Quadrature Squeezing of Solitons

For $N = 1$ soliton:

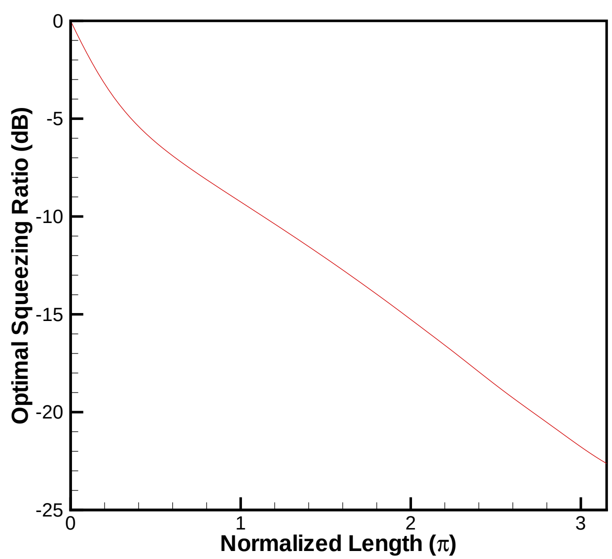
$$U(z,t) = \frac{n_0}{2} exp[i\frac{n_0^2}{8}z + i\theta_0] sech[\frac{n_0}{2}t]$$

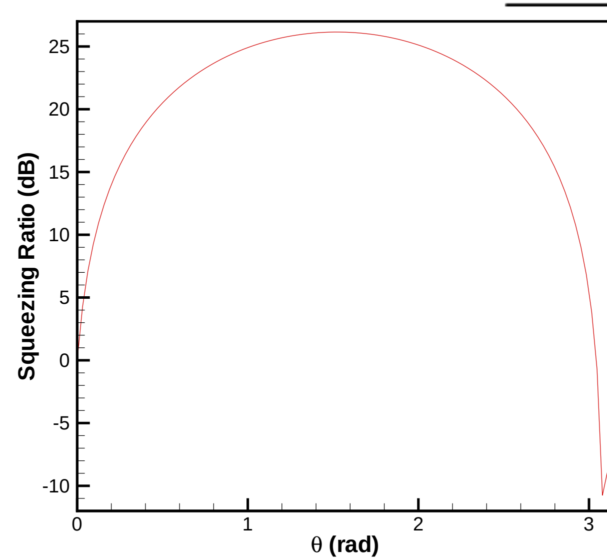


$$\Delta\hat{n}(z) = \Delta\hat{n}(0)$$

$$\Delta\hat{\theta}(z) = \Delta\hat{\theta}(0) + \frac{n_0}{4} z \Delta\hat{n}(0)$$

$$\Delta\hat{X}_\theta(z) = \alpha_1 \Delta\hat{n}(z) + \alpha_2 \Delta\hat{\theta}(z)$$





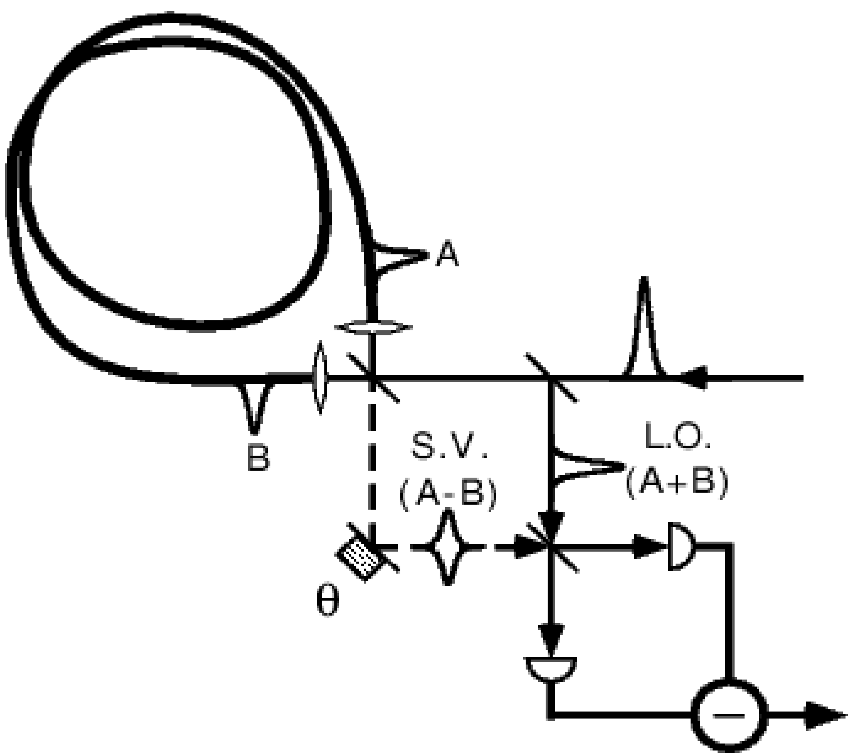
Optimal Squeezing Ratio $\equiv$ min $\frac{\text{var}[\Delta\hat{X}_\theta(z)]}{\text{var}[\Delta\hat{X}_\theta(0)]}$

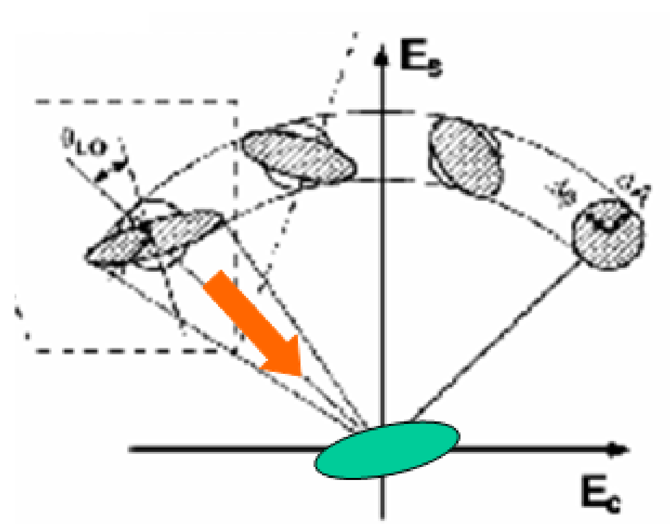
Y. Lai and H. A. Haus, *Phys. Rev. A* **40**, 844 (1989); *ibid* **40**, 854 (1989).

Generation and Detection of Squeezed Vacuum

1. Balanced Sagnac Loop (to cancel the mean field),

2. Homodyne Detection.

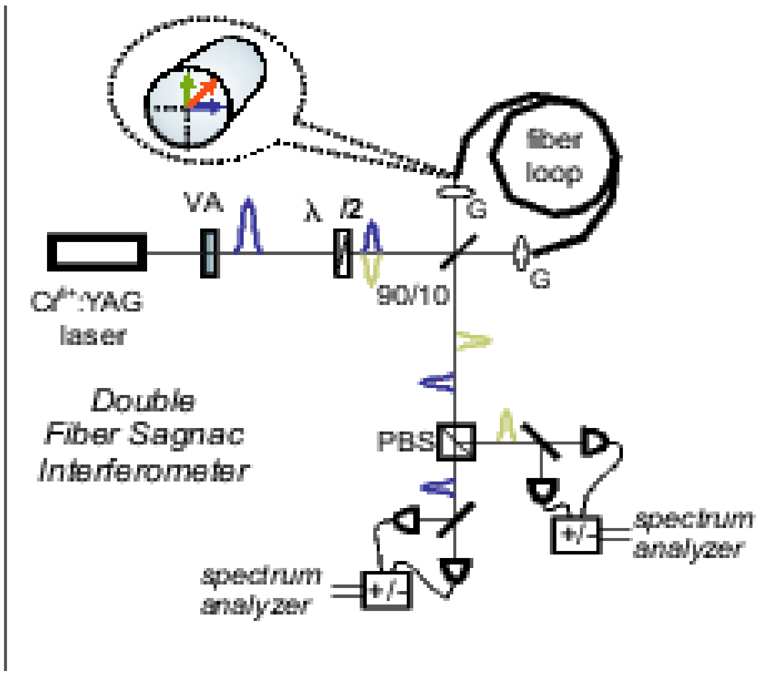


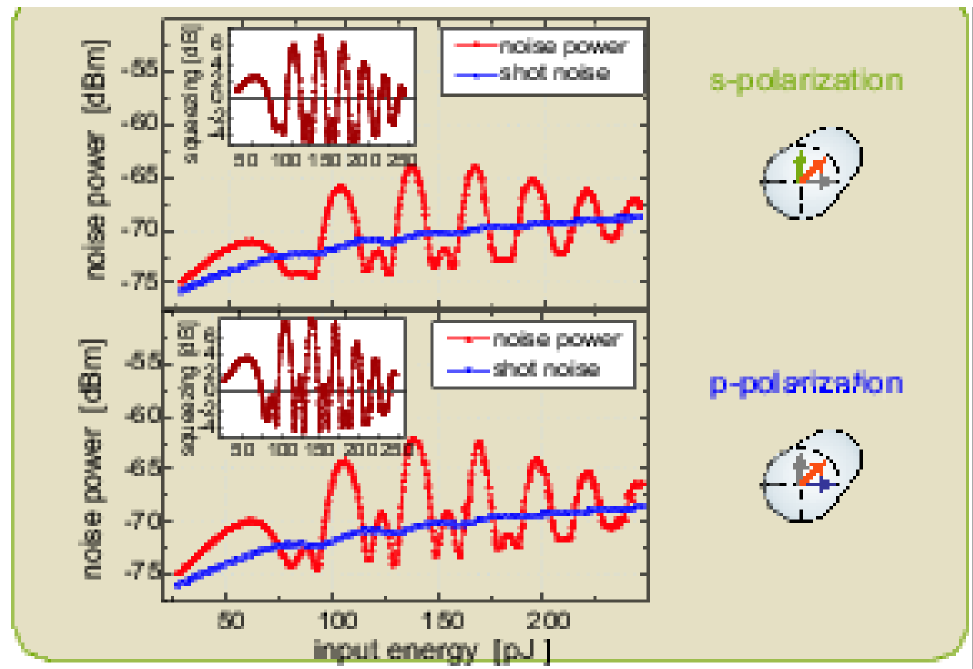


M. Rosenbluh and R. M. Shelby, *Phys. Rev. Lett.* **66**, 153(1991).

Generation and Detection of Amplitude Squeezed States

By asymmetric Sagnac Loop

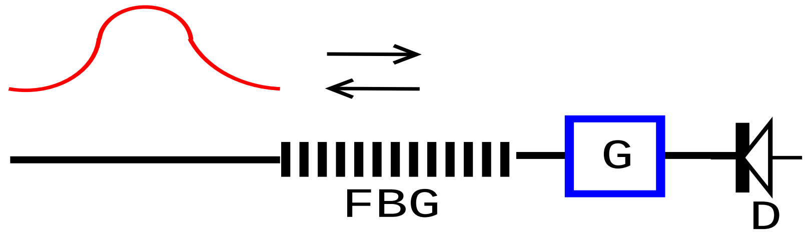




Ch. Silberhorn, P. K. Lam, O. Weis, F. Konig, N. Korolkova, and G. Leuchs,

Phys. Rev. Lett. **86**, 4267 (2001).

Quantum Nonlinear Coupled Mode Equations



$$\frac{1}{v_g}\frac{\partial}{\partial t}\hat{U}_a(z,t) + \frac{\partial}{\partial z}\hat{U}_a = i\delta\hat{U}_a + i\kappa\hat{U}_b + i\Gamma|\hat{U}_a|^2\hat{U}_a + 2i\Gamma|\hat{U}_b|^2\hat{U}_a$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}\hat{U}_b(z,t) - \frac{\partial}{\partial z}\hat{U}_b = i\delta\hat{U}_b + i\kappa\hat{U}_a + i\Gamma|\hat{U}_b|^2\hat{U}_b + 2i\Gamma|\hat{U}_a|^2\hat{U}_b$$

where $\hat{U}_a$, $\hat{U}_b$ represent forward/backward fields, satisfying Bosonic commutation relations:

$$[\hat{U}_a(z_1,t), \hat{U}_a^\dagger(z_2,t)] = \delta(z_1 - z_2), \quad [\hat{U}_b(z_1,t), \hat{U}_b^\dagger(z_2,t)] = \delta(z_1 - z_2),$$

$$[\hat{U}_a(z_1,t), \hat{U}_a(z_2,t)] = [\hat{U}_a^\dagger(z_1,t), \hat{U}_a^\dagger(z_2,t)] = [\hat{U}_b(z_1,t), \hat{U}_b(z_2,t)] = 0$$

$$[\hat{U}_b^\dagger(z_1,t), \hat{U}_b^\dagger(z_2,t)] = [\hat{U}_a(z_1,t), \hat{U}_b(z_2,t)] = [\hat{U}_a(z_1,t), \hat{U}_b^\dagger(z_2,t)] = 0$$

Linearization Approach

By setting $\hat{U}(x,t) = U_0(z,t) + \hat{u}(z,t)$, we can linearize the QNLCME as follows:

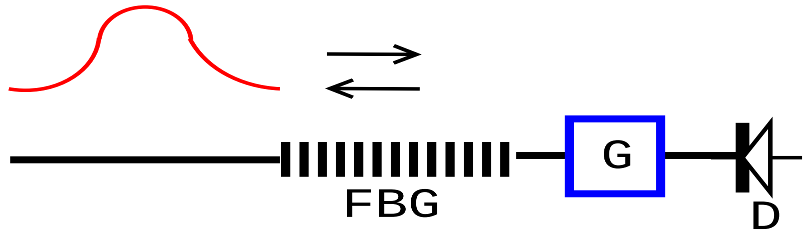
$$\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix} \hat{u}_a \\ \hat{u}_b \end{pmatrix} = \begin{pmatrix} i\Gamma U_{a0}^2 & 2i\Gamma U_{a0}U_{b0} \\ 2i\Gamma U_{a0}U_{b0} & +i\Gamma U_{b0}^2 \end{pmatrix}\begin{pmatrix} \hat{u}_a^\dagger \\ \hat{u}_b^\dagger \end{pmatrix} +$$

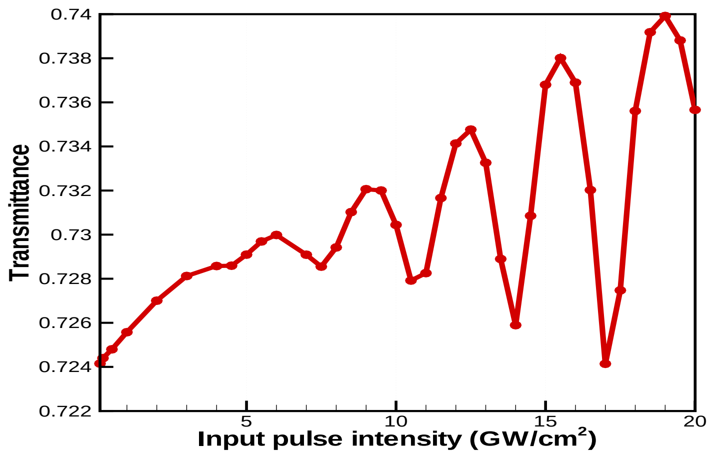
$$\begin{pmatrix} -\frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 & i\kappa + 2i\Gamma U_{a0}U_{b0}^* \\ +i\kappa + 2i\Gamma U_{a0}^* U_{b0} & \frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 \end{pmatrix} \cdot \begin{pmatrix} \hat{u}_a \\ \hat{u}_b \end{pmatrix}$$

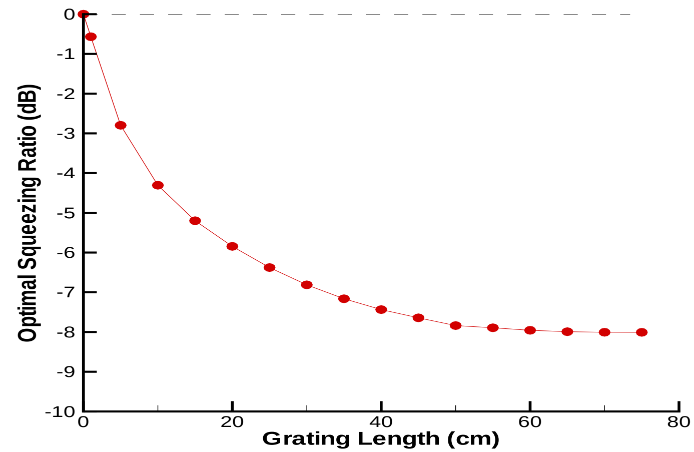
where the perturbation fields $\hat{u}_a(z,t)$ and $\hat{u}_b(z,t)$ also have to satisfy the same Bosonic commutation relations.

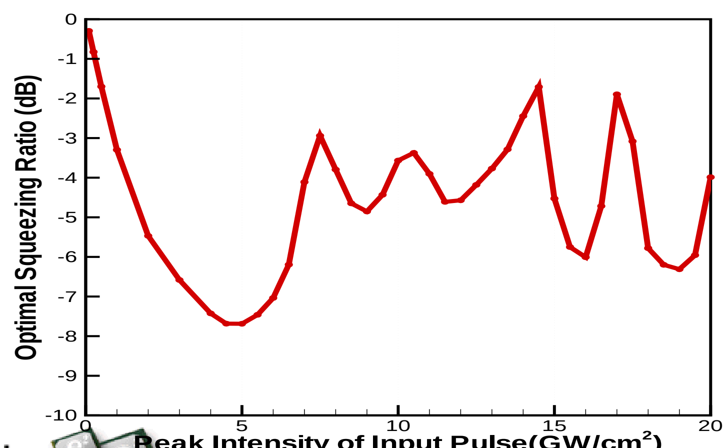
R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

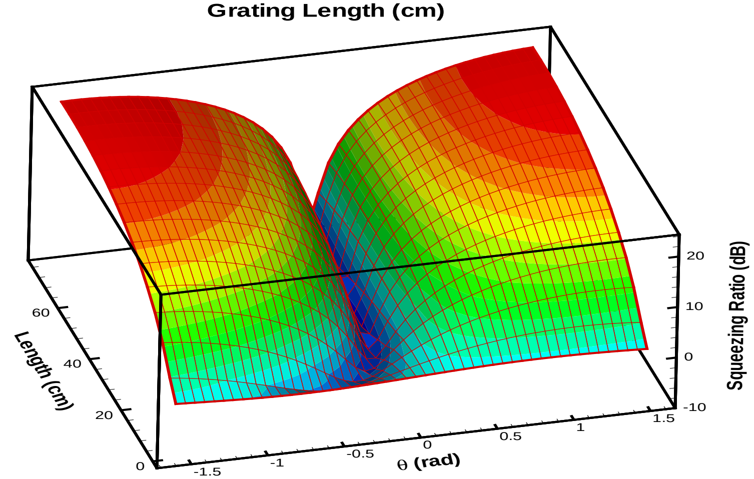
Amp. Squeezing of FBG solitons











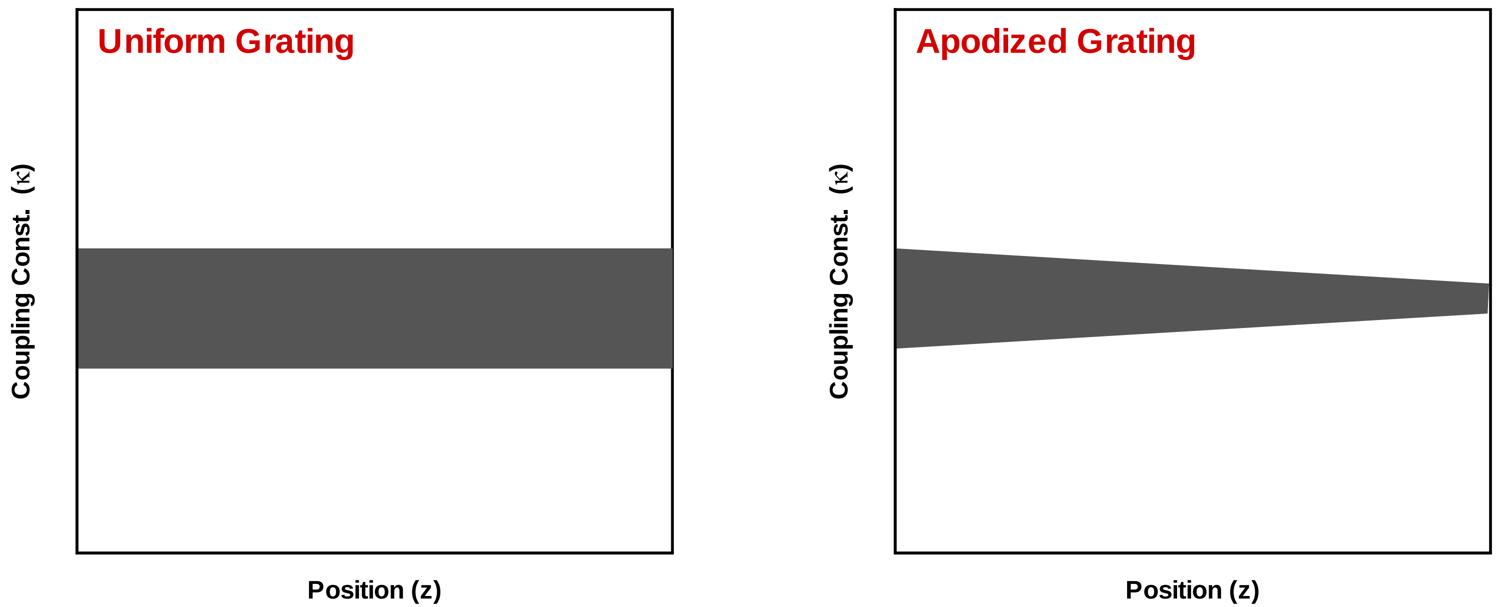
R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

Apodized Fiber Bragg Gratings

We consider a nonuniform FBG which has a position dependent coupling coefficient described by

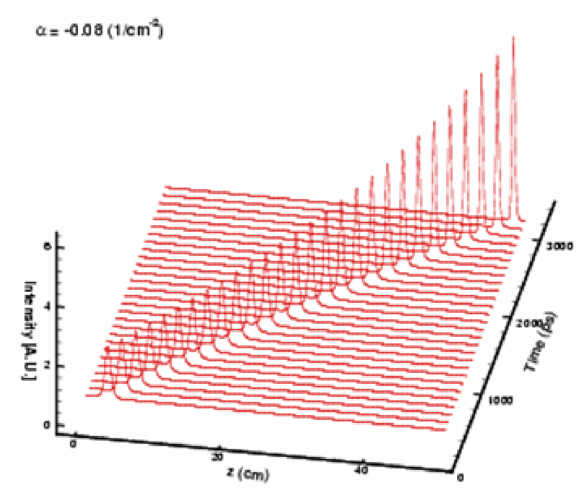
$$\kappa(z) = \kappa_0 + \alpha z$$

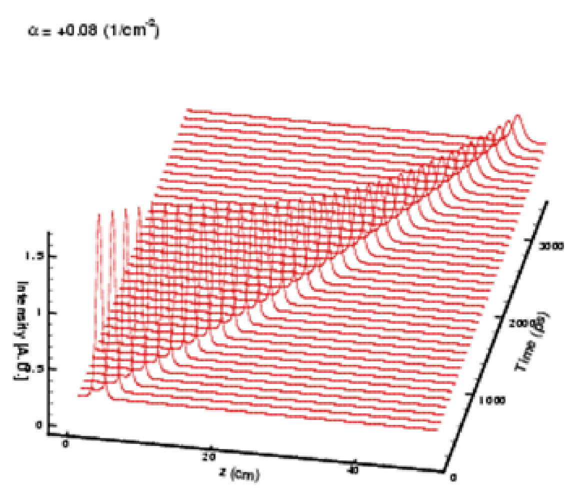
where κ_0 is the initial coupling coefficient and α is the slope of the coupling coefficient.

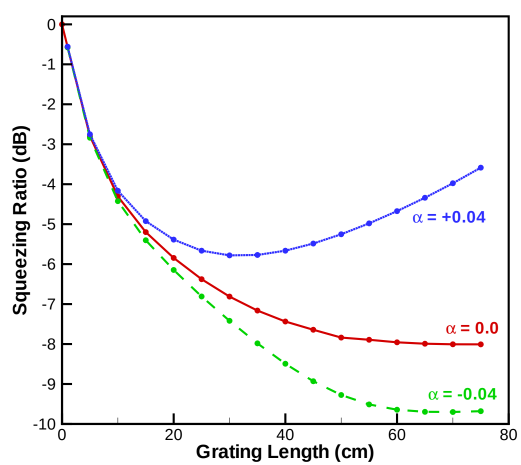


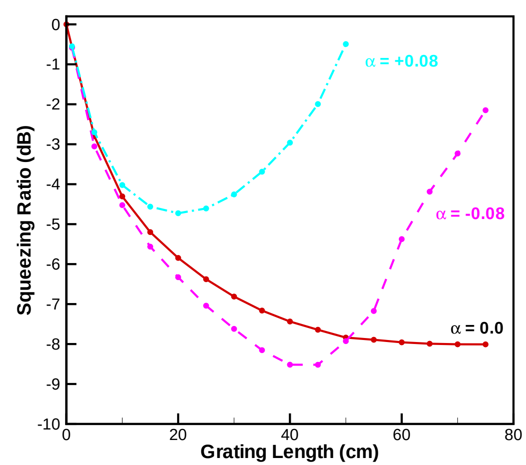
Tailor the Noise by Apodized Fiber Bragg Gratings

$\alpha < 0$ | $\alpha > 0$









$$\alpha = \pm 0.04 (1/cm^2) \qquad \alpha = \pm 0.08 (1/cm^2)$$

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R.-K. Lee and Y. Lai, *J. Opt. B* **6**, S638 (2004).

Outline

1. The great wave of translation
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3. Gap solitons in optical lattices
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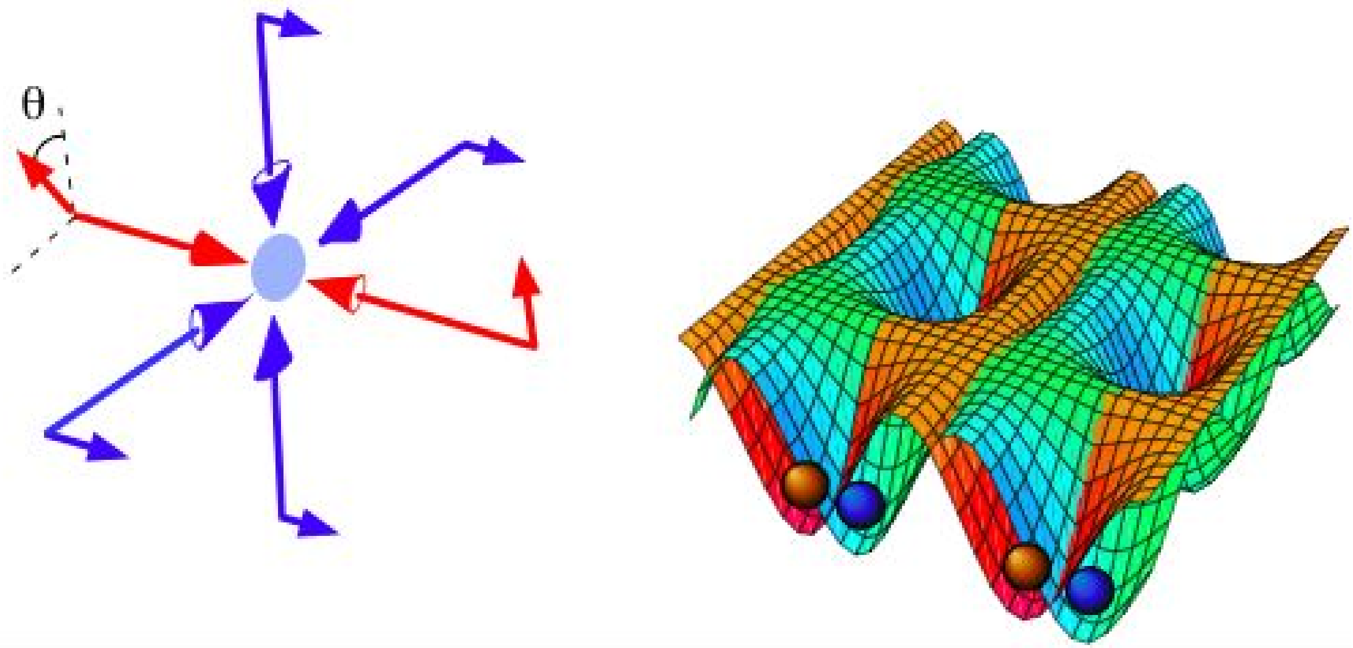
Optical lattices



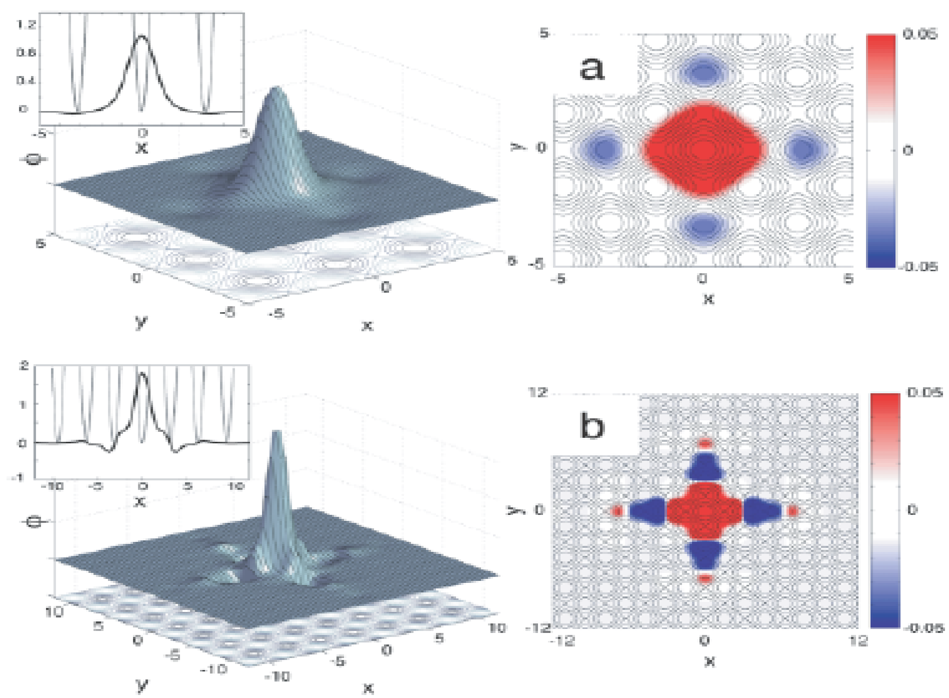
from: http://panda.unm.edu/

BEC in optical lattices

Gross-Pitaevskii equation with periodic potentials,

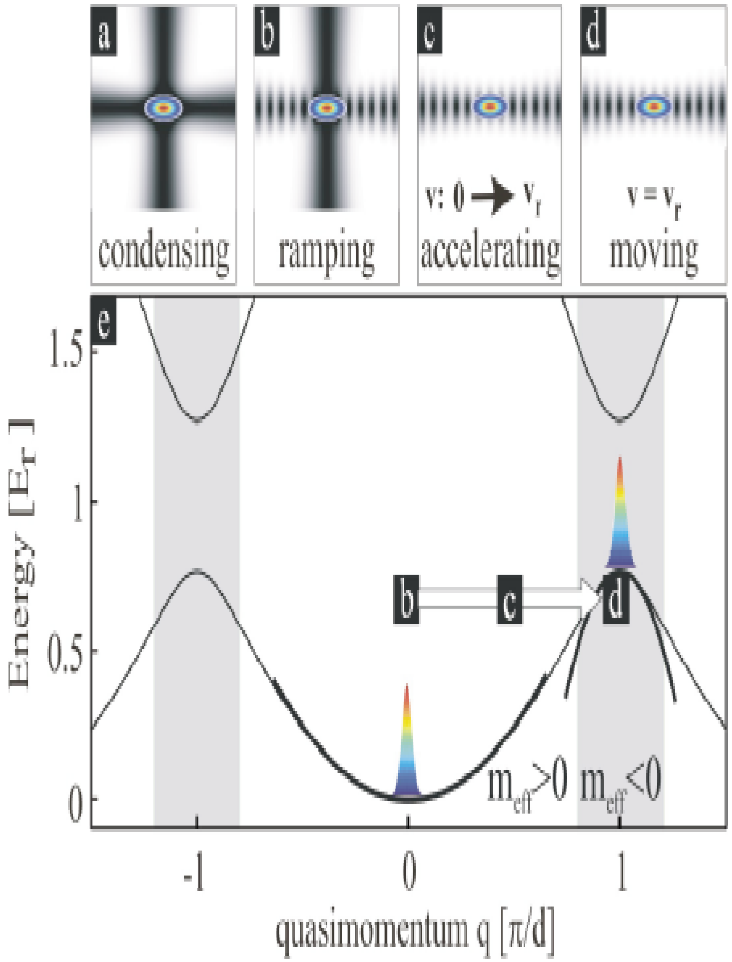
$$i\hbar\frac{\partial}{\partial t}\Phi = -\frac{1}{2}\nabla^2\Phi + V(t)\Phi + g|\phi|^2\phi$$

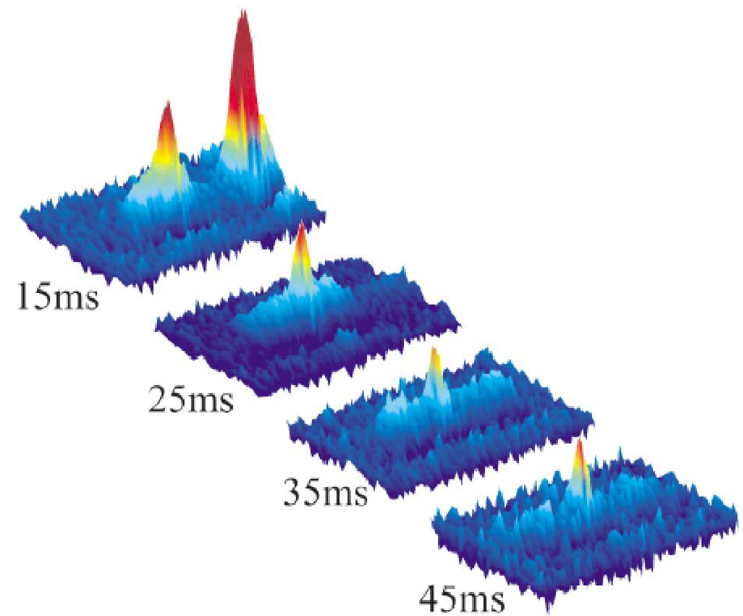
which has gap soliton solutions in 1D, 2D, and 3D.



E. A. Ostrovskaya and Yu. S. Kivshar, *Phys. Rev. Lett.* **90**, 160407 (2003).

Mater-wave gap soliton in optical lattices





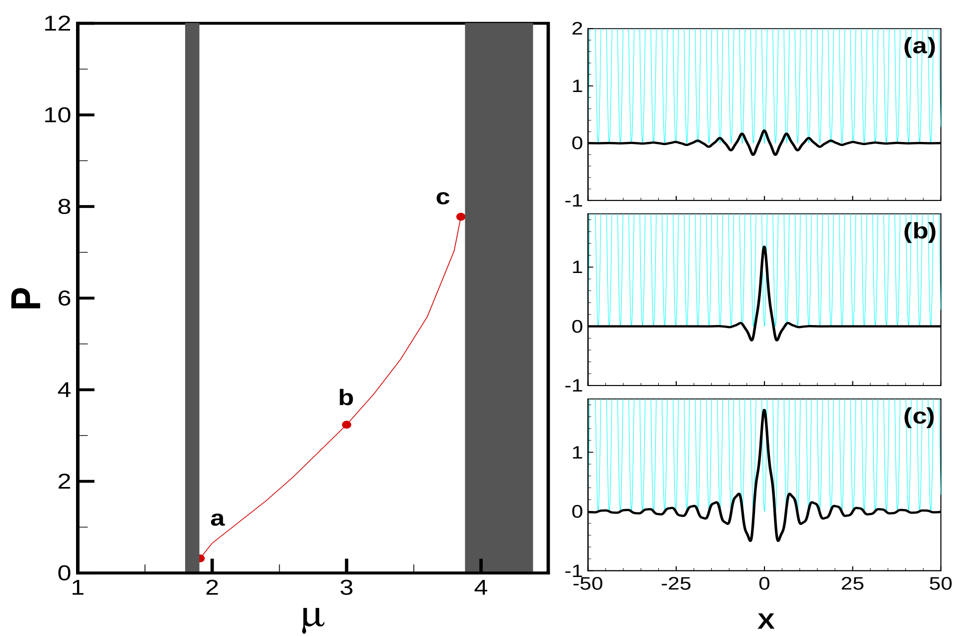
Exp: B. Eiermann, Th. Anker, M. Albiez, M. Taglieber, P. Treutlein, K.-P. Marzlin, and M. K. Oberthaler, *Phys. Rev. Lett.* **92**, 230401 (2004).

Gap solitons in 1D

1-D Gross-Pitaevskii equation,

$$i\hbar\frac{\partial}{\partial t}\Phi_0(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi_0(t,x)+V(x)\Phi_0(t,x)+g_{1D}|\phi_0(t,x)|^2\phi_0(t,x$$

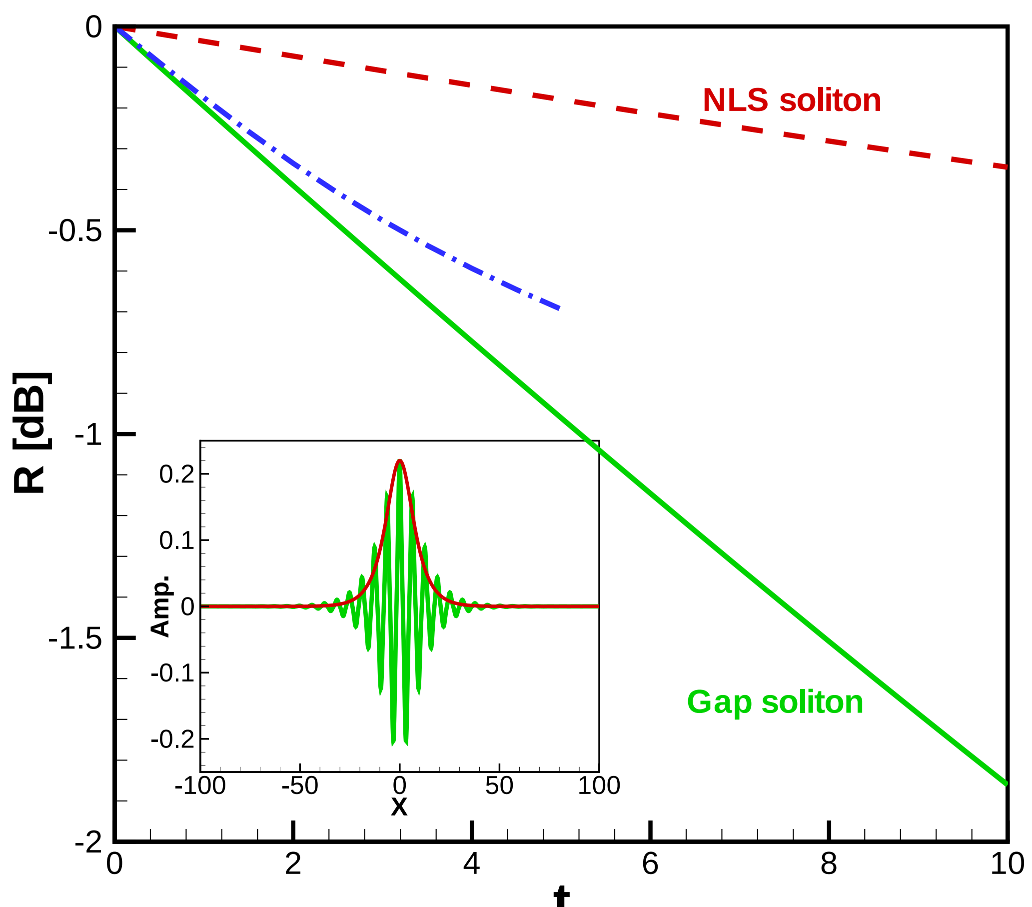
which has gap soliton solutions.



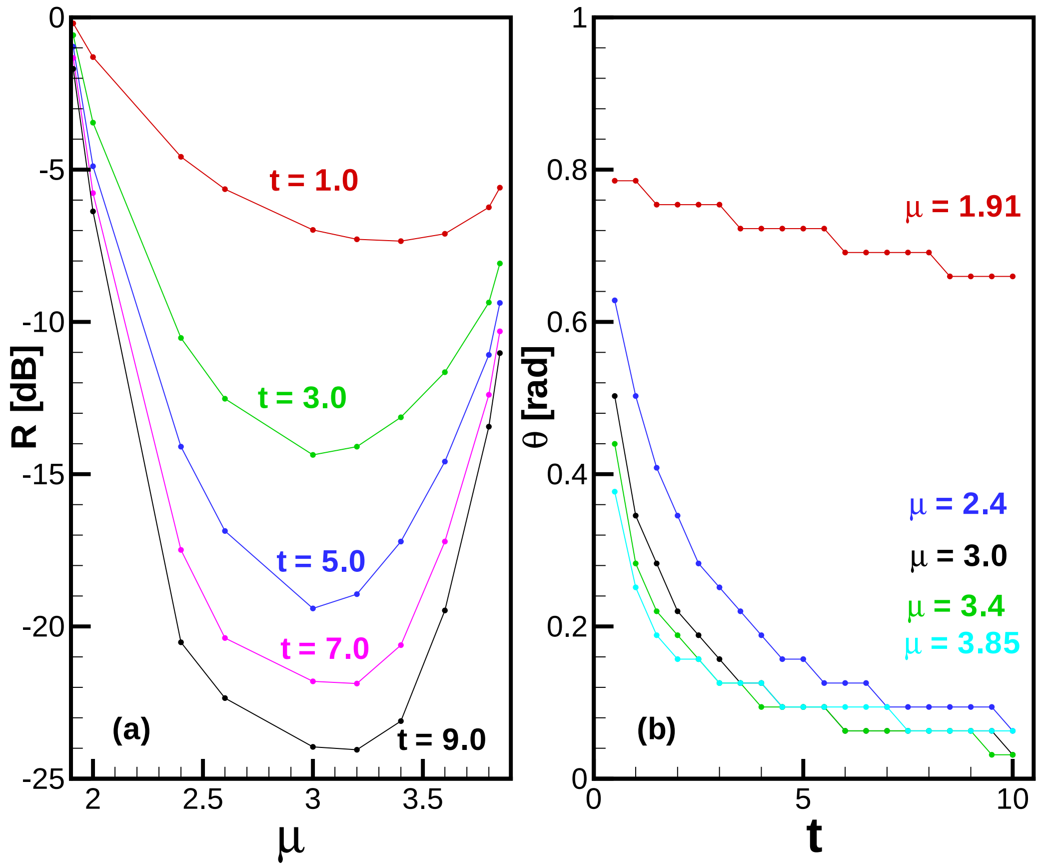
Comparison of envelope function near the bandedge

Near the bottom edge of the gap, one can use envelope approximation for the gap solitons

$$\psi(t,x) = AF(x)\phi(t,x).$$



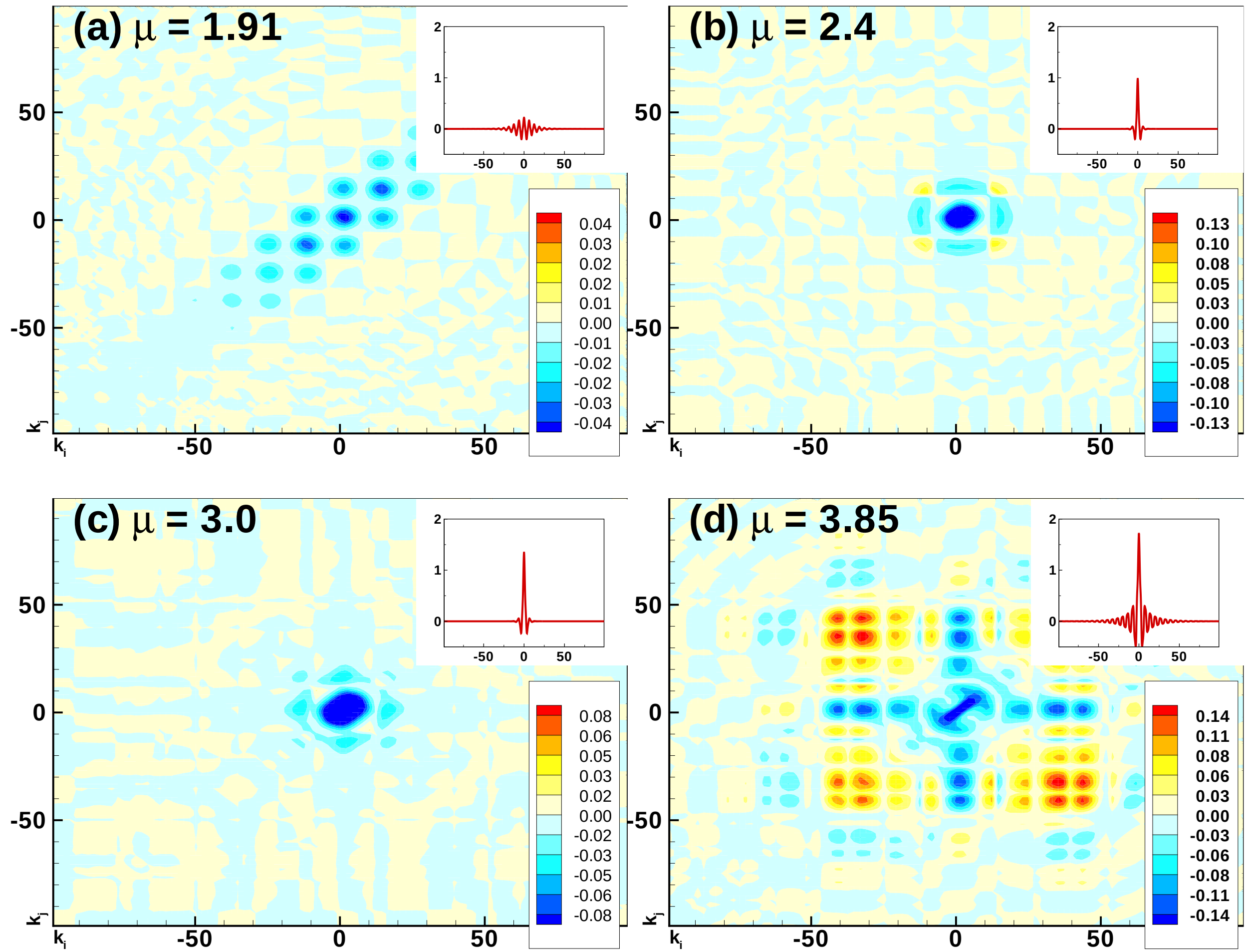
Squeezing ratio v.s. chemical potential



Squeezing effect is most profound in the depth of the gap and reduced near the band edges.

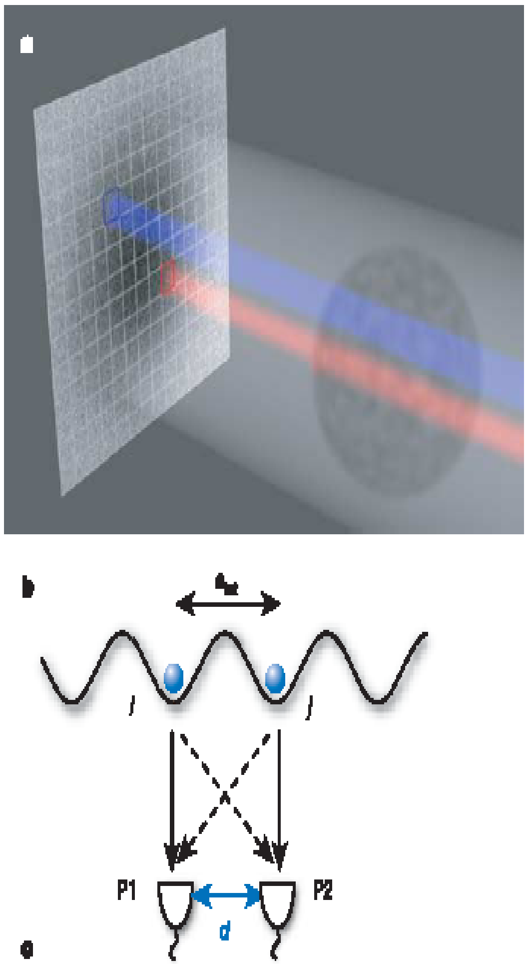
Quantum correlation patterns v.s. chemical potential

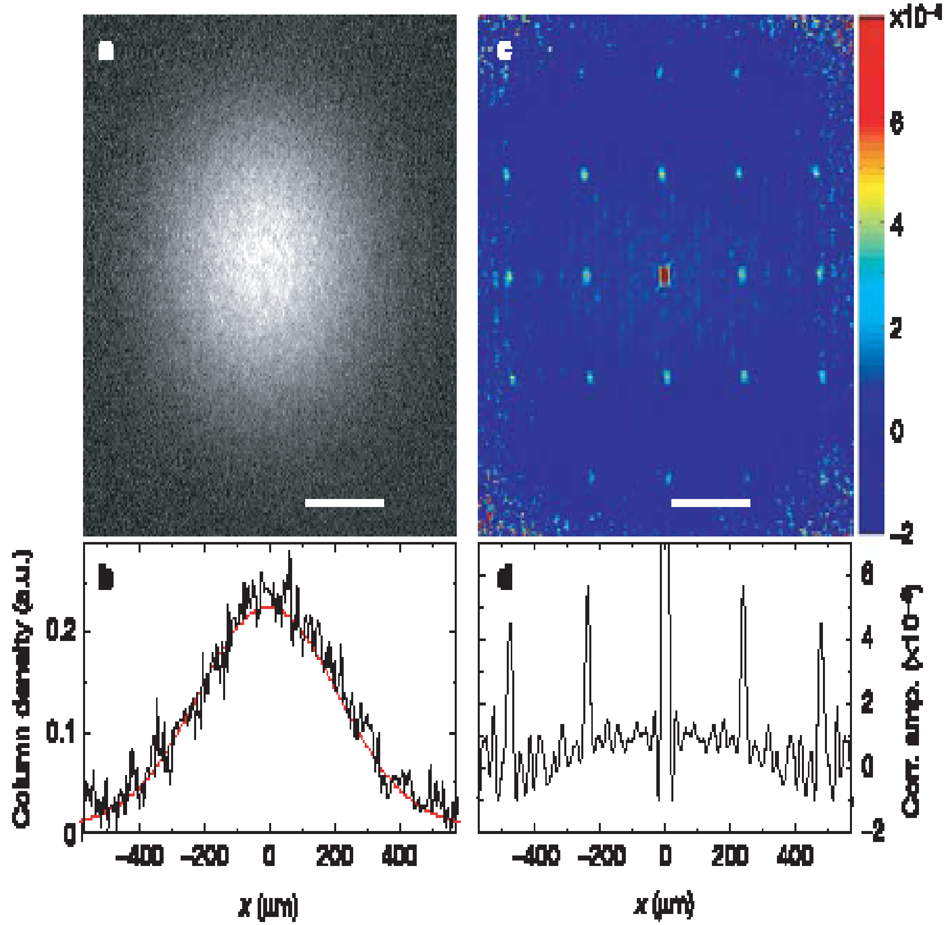
x-domain



R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72** (Sep. 2005).

Spatial quantum noise interferometry with cold atom





Exp: Simon Fölling *et al.*, *Nature* **434**, 481 (2005).

Summaries of: Squeezed Bragg solitons and gap solitons

1. The Bragg grating acts like a spectral filter and cause Bragg solitons amplitude squeezed.
2. Periodic potential makes the properties of gap solitons squeezing differ from the envelope solitons by NLS equation.
3. Quantum correlation spectra of gap solitons in the spatial domain show the intra-soliton structure induced by the Bragg scattering in the periodic potential.

R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).
R.-K. Lee and Y. Lai, *J. Opt. B*, **6**, S638 (2004).
R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72** (Sep. 2005).

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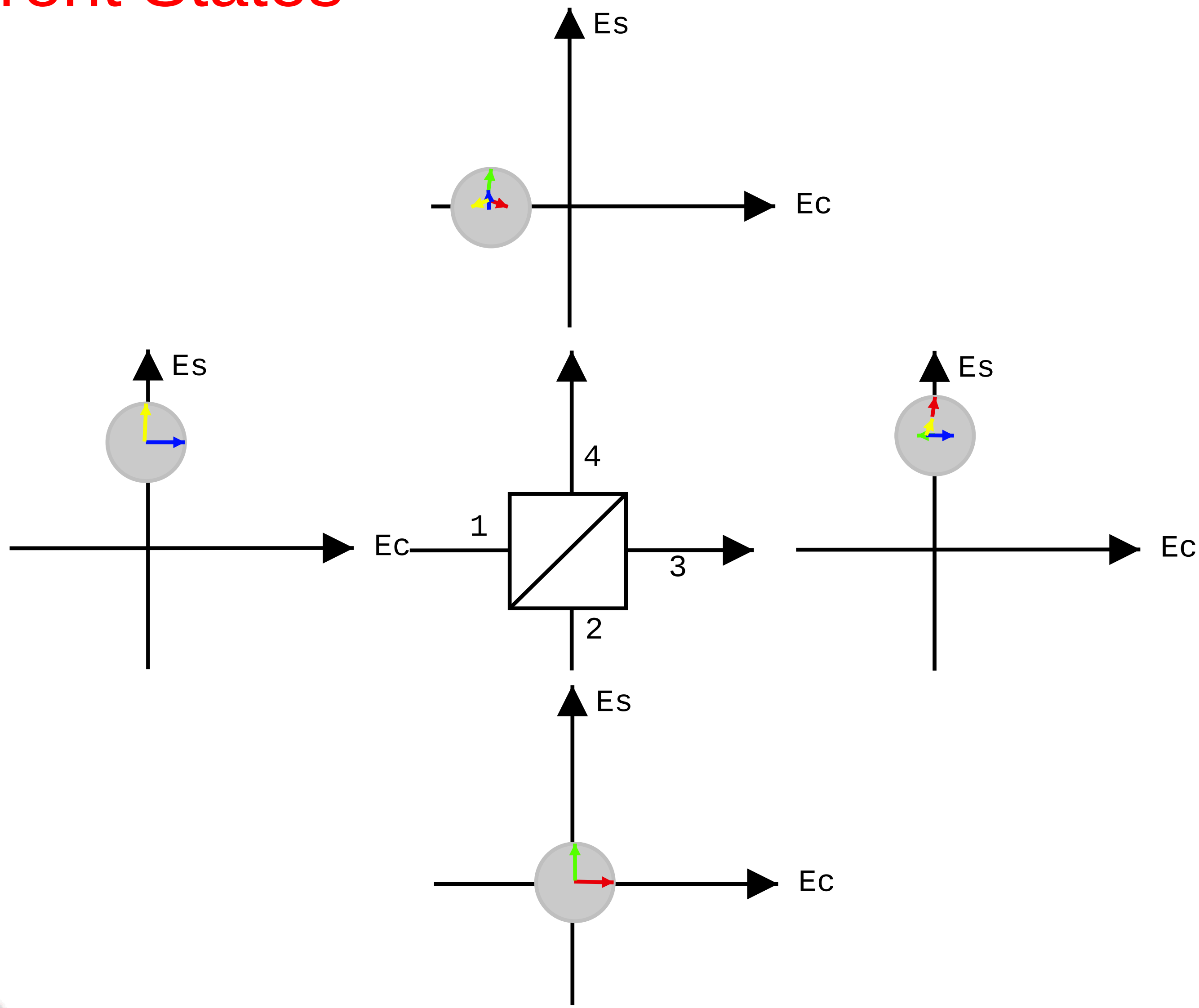
Applications of Squeezed Light

- Gravitational Waves Detection
- Quantum Non-Demolition Measurement (QND)
- Super-Resolved Images (Quantum Images)
- Generation of EPR Pairs

Interference of Coherent States

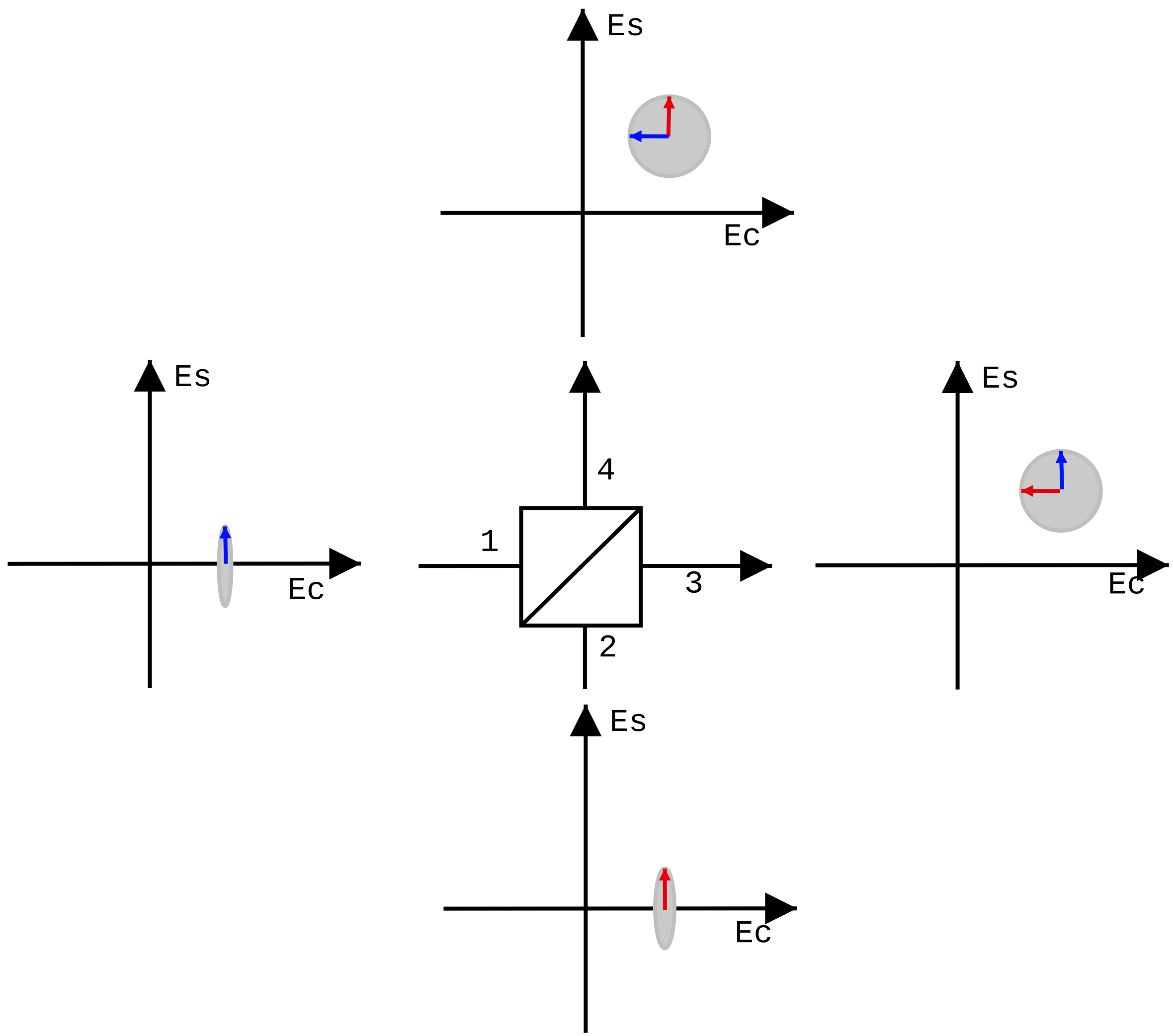
Coherent States



G. Leuchs, T. C. Ralph, C. Silberhorn, and N. Korolkova, *J. Mod. Opt.* **46**, 1927(1999).

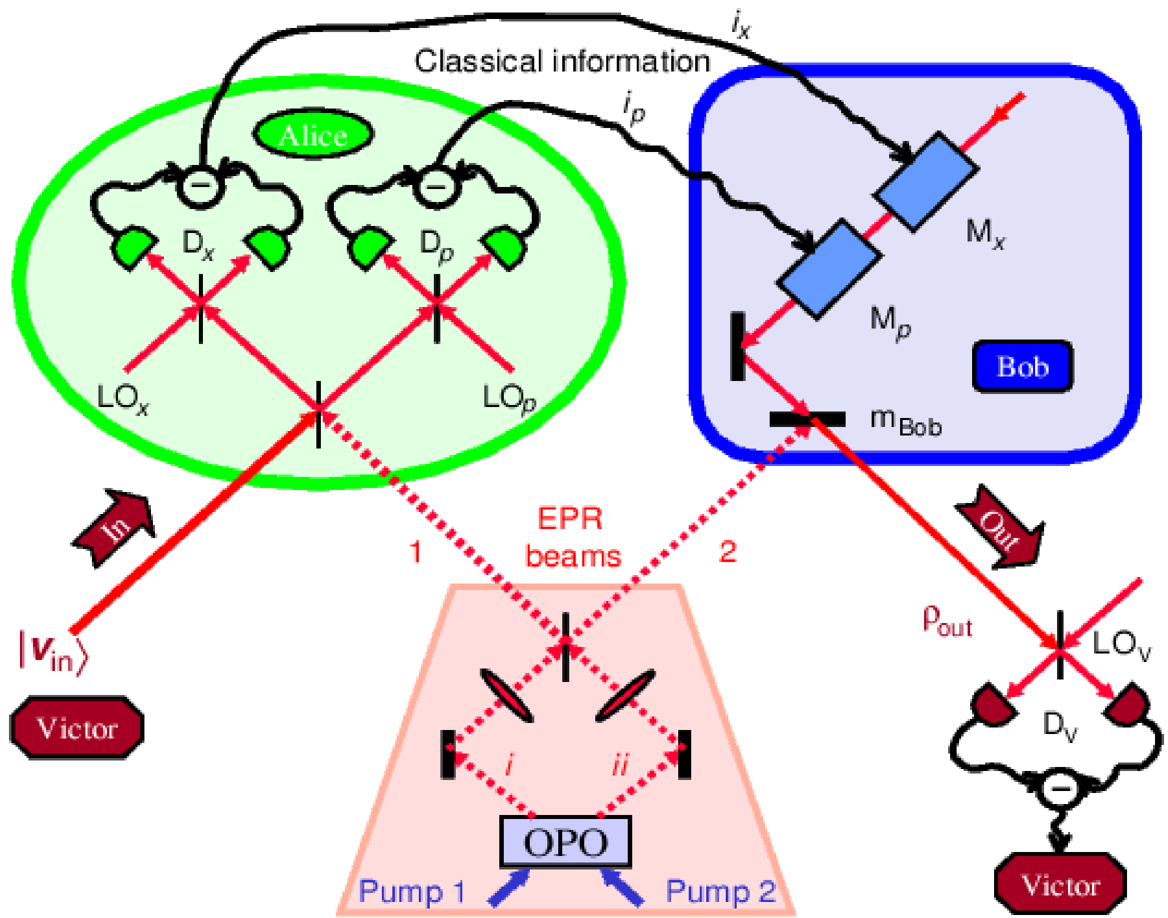
Generation of Continuous Variables Entanglement

Preparation EPR pairs by Squeezed Sates



$$\delta\hat{n}_3 = -\delta\hat{n}_4, \delta\hat{\theta}_3 = \delta\hat{\theta}_4.$$

Experiment of CV Teleportation

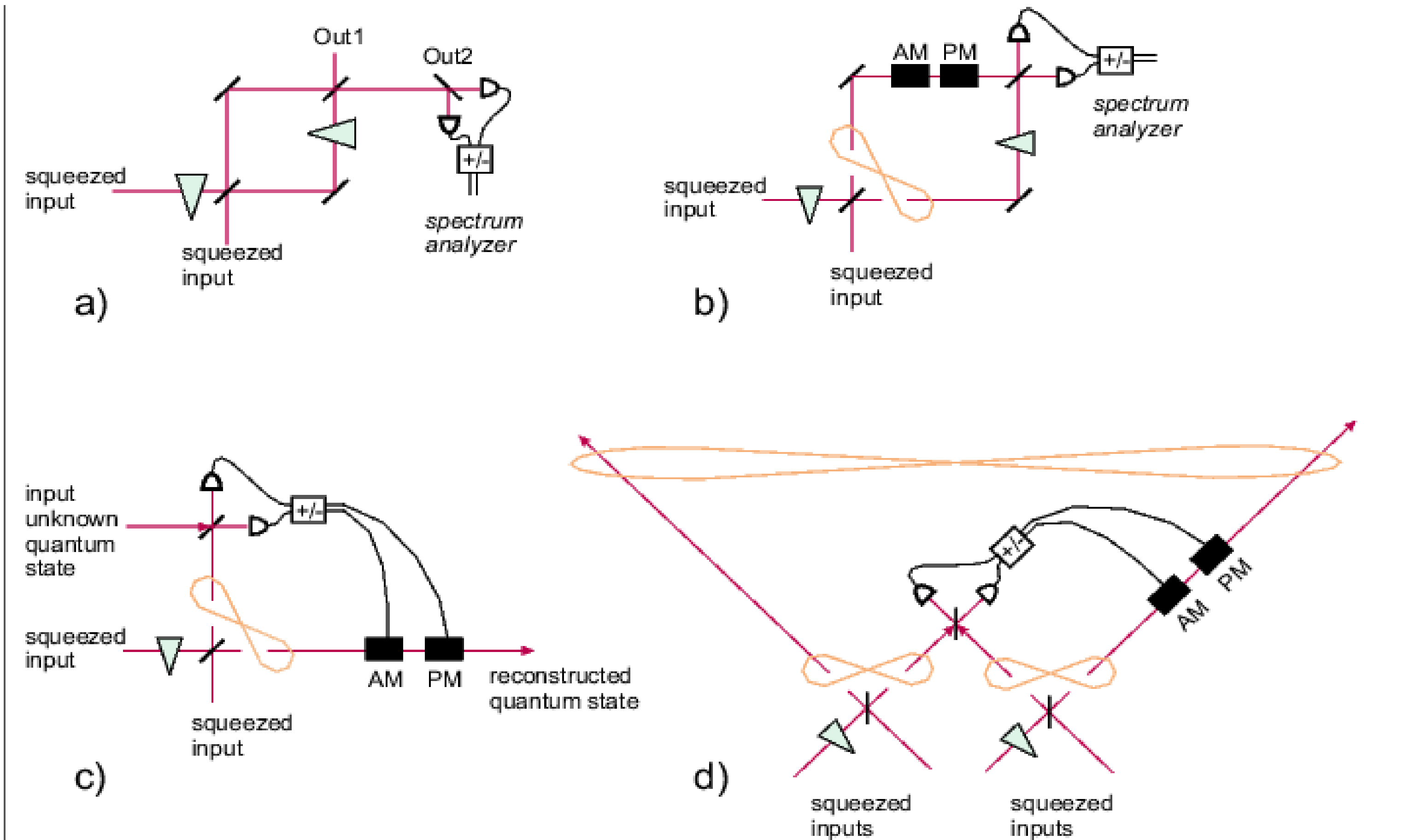


A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble,

and E. S. Polzik, *Science* **282**, 706 (1998).

Applications of EPR Pairs by Using Squeezed States

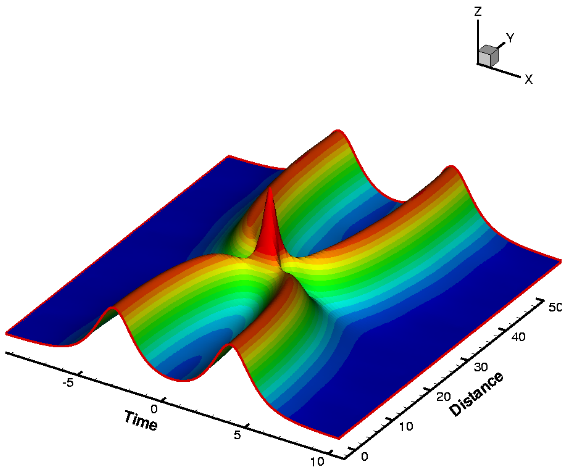
(a)entanglement; (b)quantum dense coding;
(c) teleportation; (d) entangle swapping.



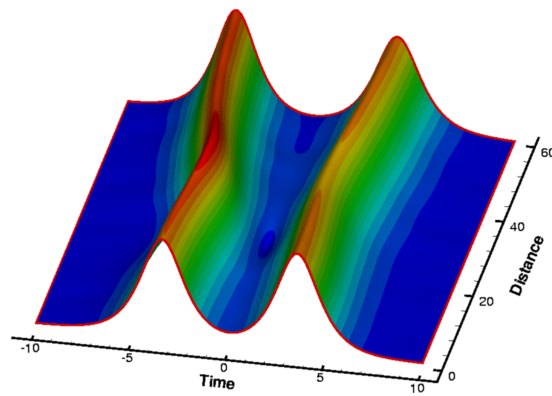
G. Leuchs and N. Korolkova, *Opt. & Photon. News* **Feb.**, 64 (2002).

Photon Number Correlation of 2-Solitons Interaction

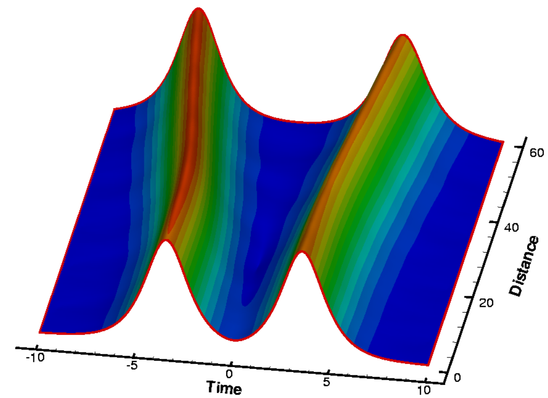
$$U(z,t) = sech(z, t+\rho) + r\, sech(z, t_\rho)e^{i\theta}$$



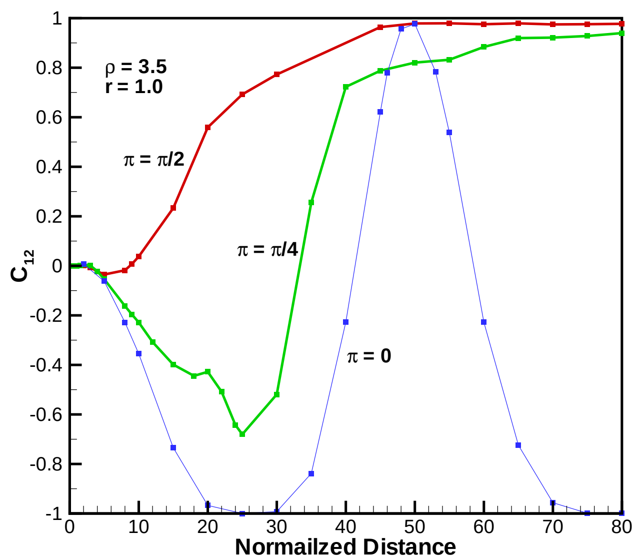
$\theta = 0$



$\theta = \pi/4$



$\theta = \pi/2$



$$C_{1,2} = \frac{\langle : \Delta\hat{n}_1 \Delta\hat{n}_2 : \rangle}{\sqrt{\Delta\hat{n}_1^2 \Delta\hat{n}_2^2}}$$

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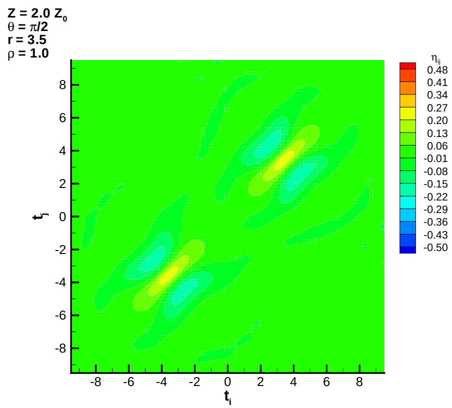
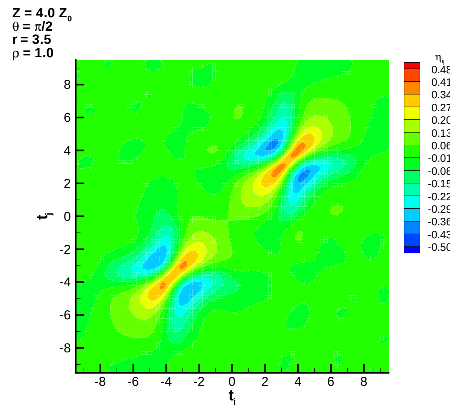
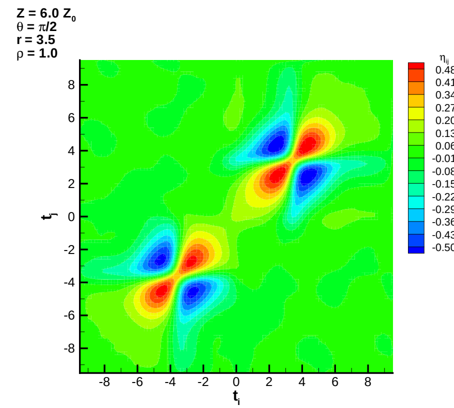
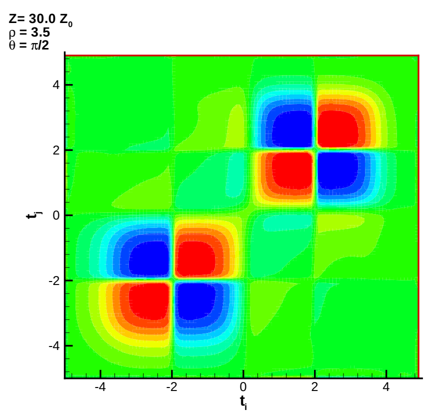
Evolutions of Photon Number Correlation Spectra

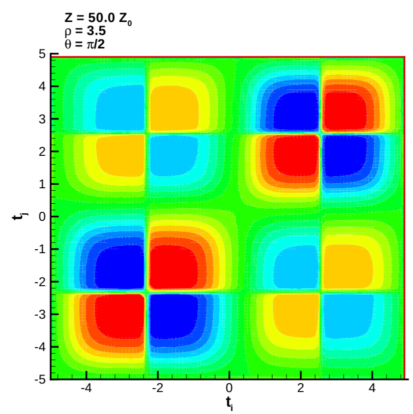
$Z = 2.0 Z_0$, $Z = 4.0 Z_0$, $Z = 6.0 Z_0$.

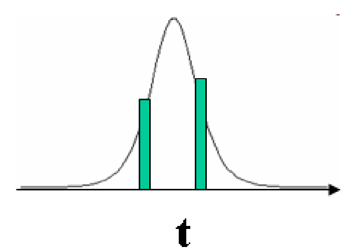








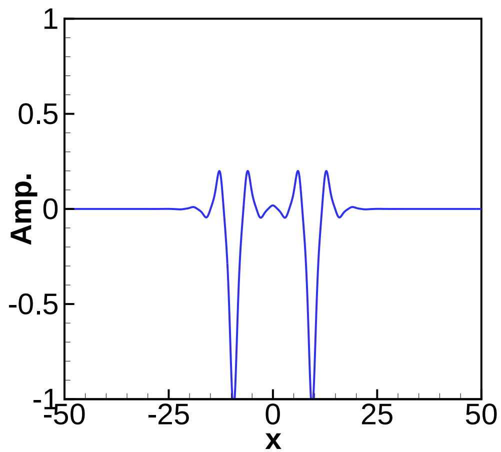


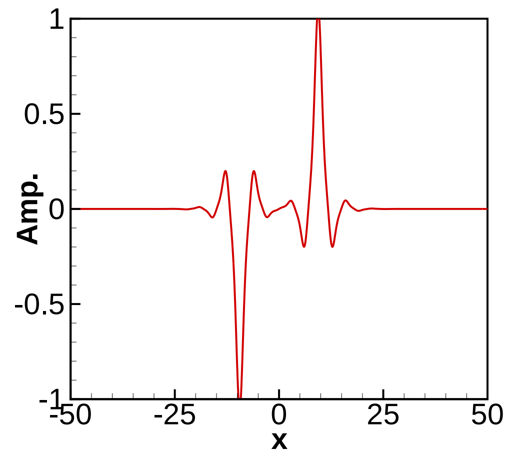


$Z = 30.0 Z_0$, $Z = 50.0 Z_0$

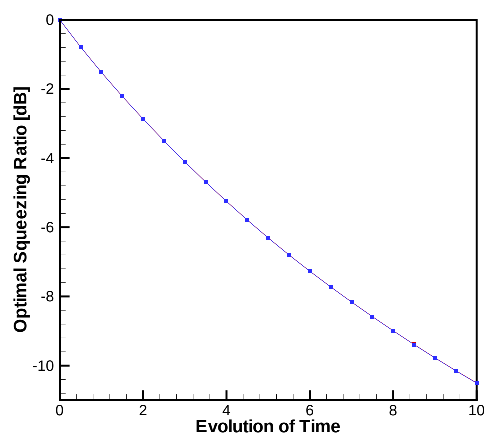
R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **71**, 013816 (2005).

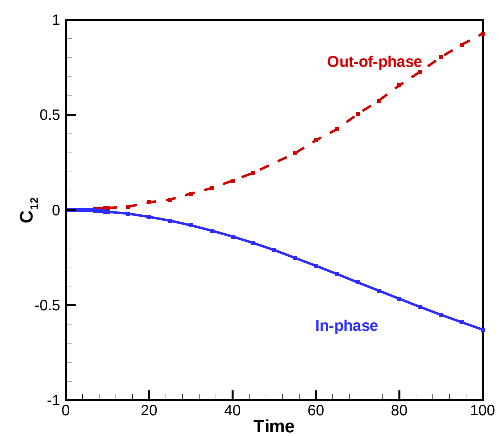
Bound gap solitons and high correlated EPR pairs





The noise fluctuations of bound gap soliton pairs are the same, but with different photon-number correlation parameter.

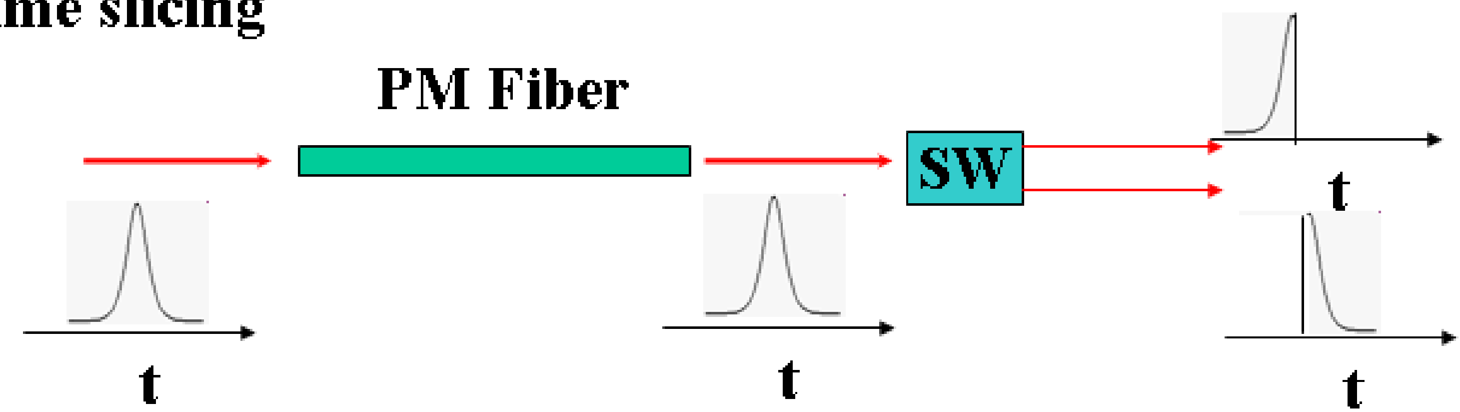




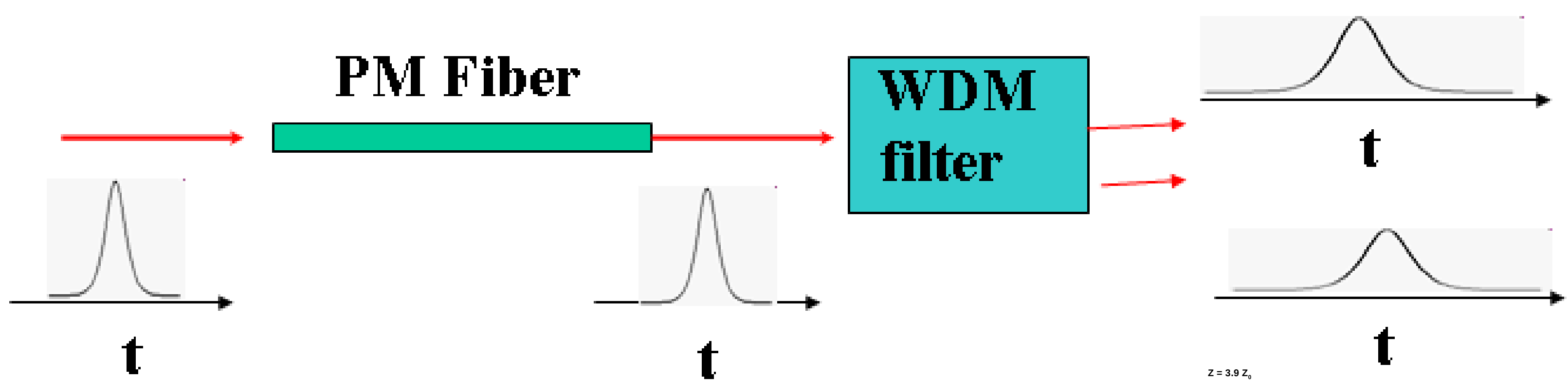
R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72** (Sep. 2005).

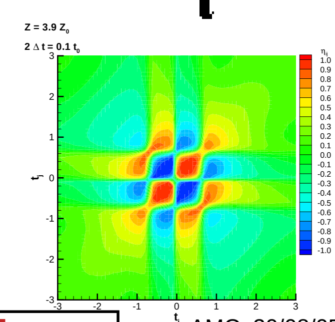
Entangled States by Time or Wavelength Slicing

(1) time slicing



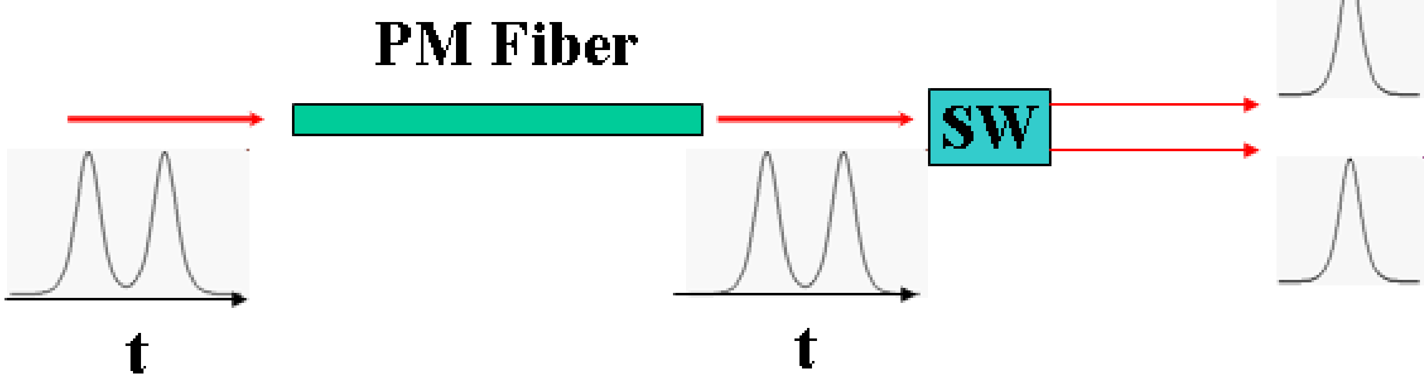
(2) Wavelength slicing



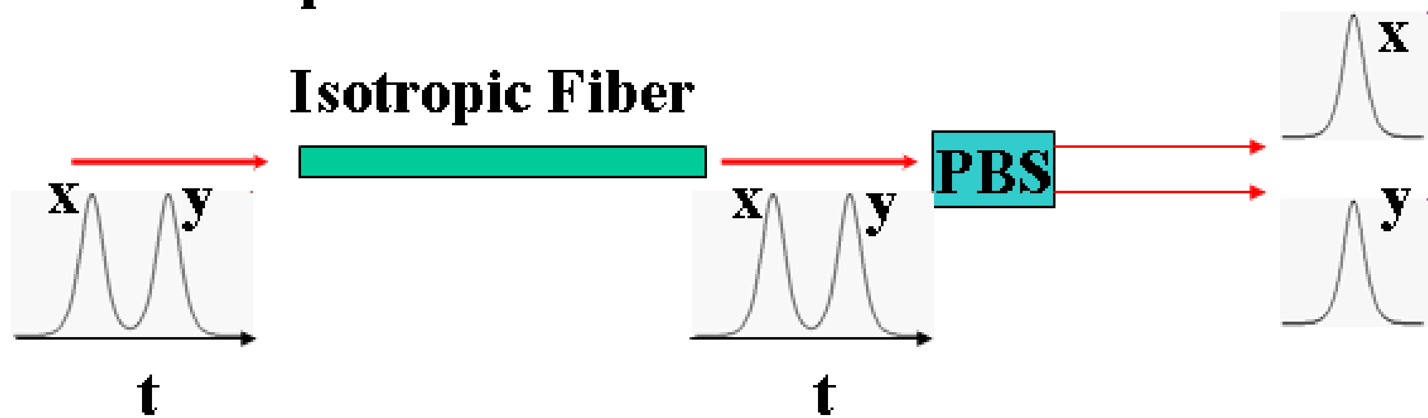


Entangled Soliton Pairs

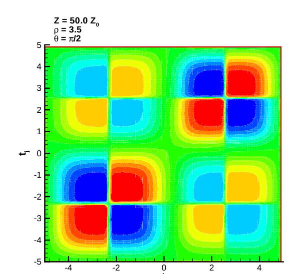
(1) TDM soliton pair



(2) PDM soliton pair



If necessary, the Sagnac loop configuration also can be used.



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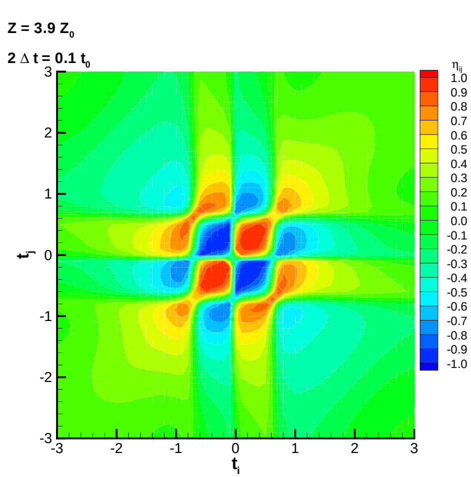
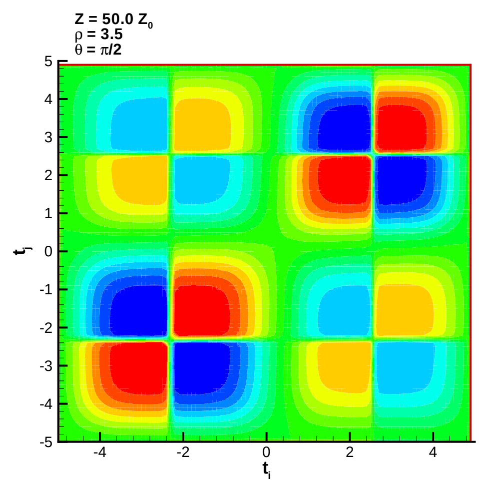
Conclusions

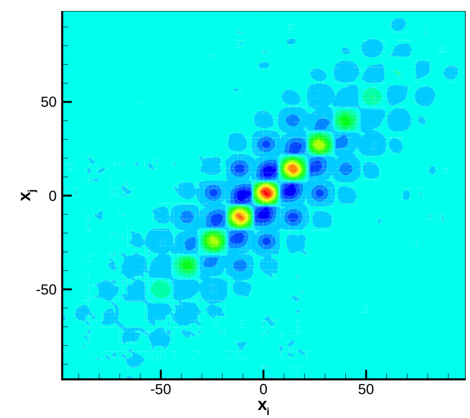
1. Periodic potential offers a new way to stabilize optical/matter-wave solitons in high dimensions.
2. Quantum properties and theories of gap solitons are reviewed.
3. Possible applications of quantum optical solitons in quantum information are needed to be explored more.



$N = 2$ soliton



2-solitons interaction



gap soliton