Electromagnetic Waves in Optoelectronics, IPT 524000

Time: T5T6F2 *(1:10-3:00 PM, Tuesday; 09:00 AM, Friday), at Room* **105**, *EECS bldg.*

Ray-Kuang Lee[1]

[1] *R523, EECS Bldg., National Tsing-Hua University, Hsinchu, Taiwan.*
*Tel: 886-3-5715131-***2439***

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- **Course Description:**
  - Fundamental concepts for Electromagnetic Waves in Optoelectronics, including the mathematical methods, physical concepts, device ideas, and simulation techniques.
  - Extensions and applications of these basic concepts to update research fields will also be addressed.
  - Although this course is given primarily for the first year graduate students, those who are undergraduates or senior graduates are encouraged to take this course.
  - Background: Electromagnetism I, II.
- **Text Books and References:**

  [T1]: Hermann A. Haus, *"Waves and Fields in Optoelectronics,"* Prentice-Hall (1984).

  [T2]: Ammon Yariv, *"Optical Electronics,"* 4th Edition, John Wiley & Sons (1991).

  [R1]: S. Ramo, J. Whinnery and T. Van Duzer, *"Field and Waves in Communication Electronics,"* John Wiley & Sons (1994).

  [R2]: Ammon Yariv and Pochi Yeh, *"Optical waves in crystals,"* Wiley-Interscience (2003).

  [R3]: John D. Jackson, *"Classical Electrodynamics,"* 3rd edition, John Wiley & Sons (1984).

  [R4]: Ammon Yariv, *"Quantum electronics,"* 3rd edition, John Wiley & Sons (1989).
- **Teaching Method:**
  in-class lectures with some studies on journal papers
- **Syllabus:**
  1. (13/09-16/09) Introduction
  2. (20/09-23/09) Maxwell's equations, base on Chap. 1 [T1, T2], C3 [R1], and C6-7 [R3].
  3. (27/09-30/09) Plane waves propagation and reflection, base on C2 [T1] and C6 [R1].
  4. (04/10-07/10) Mirrors and interferometers, base on C3 [T1]
  5. (11/10-14/10) Fresnel diffraction and paraxial wave equation, base on C4 [T1].
  6. (18/10-21/10) Hermite-Gaussian beams, base on C4 [T1] and C2 [T2].
  7. (25/10-28/10) Midterm exam
  8. (01/11-04/11) Optical waveguides and fibers, base on C6 [T1], C3 [T2], and C8 [R1].
  9. (08/11-11/11) Coupled-mode theory for resonators and couplers, base on C7 [T1], C4 [T2], and C10 [R1].
  10. (15/11-18/11) Distributed feedback structures, base on C8 [T1].
  11. (22/11-25/11) Anisotropic media, base on C11 [T1].
  12. (29/11-02/12) Acoustic-, Electro-, and Magnetic-optic modulators, base on C9, 12 [T1], and C7-10 [R2].
  13. (06/12-09/12) Nonlinear systems, base on C10, 13 [T1].
  14. (13/12-16/12) Phase-conjugate optics, base on C17 [T2] and C19 [R4].
  15. (20/12-23/12) Optical detection, base on C14 [T1], and C10, 11 [T2].
  16. (27/12-30/12) Introduction to quantum optics.
  17. (03/12-06/01) Final Exam
- **Evaluation:**
  1. Four Homework, 40%;
  2. Two Exams: one midterm exam, 30%, and one final exam, 30%;
  3. Bonus: One paper study report with detailed model explanation and formula derivations, 20%.
- **Office hours:**
  10:30-12:00, Friday at Room 523, EECS bldg.

I. MAXWELL'S EQUATIONS

A. Maxwell's equations in real, time-dependent form

Maxwell's equations in *mks* units, the electric field E (V/m), and the magnetic field H (A/m),
Faraday's law:

$$\nabla \times E = -\frac{\partial}{\partial t}\mu_0 H - \frac{\partial}{\partial t}\mu_0 M. \tag{I.1}$$

Ampére's law:

$$\nabla \times H = \frac{\partial}{\partial t}\epsilon_0 E + \frac{\partial}{\partial t}P + J. \tag{I.2}$$

Gauss's law for the electric field:

$$\nabla \cdot \epsilon_0 E = -\nabla \cdot P + \rho. \tag{I.3}$$

Gauss's law for the magnetic field:

$$\nabla \cdot \mu_0 H = -\nabla \cdot \mu_0 M. \tag{I.4}$$

where M is the *magnetization density*; P is the *polarization density*; ρ is the *charge density*; and J is the *current density*. Two constants ϵ_0 and μ_0 are *dielectric constant* and *magnetic permeability* of free space.

1. *Linear, isotropic, and dispersion-free media*

Constitutive law:

$$P = \epsilon_0 \chi_e E, \tag{I.5}$$
$$M = \chi_m H, \tag{I.6}$$

where χ_e and χ_m are the *electric* and *magnetic susceptibilities*.
For linear, isotropic, and dispersion-free media, Maxwell's equations become

$$\nabla \times E = -\mu\frac{\partial}{\partial t}H, \tag{I.7}$$
$$\nabla \times H = \epsilon\frac{\partial}{\partial t}E + J, \tag{I.8}$$
$$\nabla \cdot \epsilon E = \rho, \tag{I.9}$$
$$\nabla \cdot \mu H = 0. \tag{I.10}$$

For a *source-free* medium, $\rho = J = 0$,

$$\nabla \times (\nabla \times E) = -\mu\epsilon\frac{\partial^2}{\partial t^2}E, \tag{I.11}$$
$$\Rightarrow \quad \nabla(\nabla \cdot E) - \nabla^2 E = -\mu\epsilon\frac{\partial^2}{\partial t^2}E. \tag{I.12}$$

When $\nabla \cdot E = 0$, one has *wave equation*

$$\nabla^2 E = \mu\epsilon\frac{\partial^2}{\partial t^2}E \tag{I.13}$$

which has following expression of the solutions, in 1D,

$$E = \hat{x}[f_+(z - vt) + f_-(z + vt)], \tag{I.14}$$
$$H = \sqrt{\frac{\epsilon}{\mu}}\hat{y}[f_+(z - vt) - f_-(z + vt)], \tag{I.15}$$

with the *characteristic admittance*, $\sqrt{\frac{\epsilon}{\mu}}$, of the medium.

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*Electronic address: rklee@ee.nctu.edu.tw

2. *Poynting's theorem*

Poynting's theorem is the law of power conservation for electromagnetic fields.

$$\nabla \cdot (E \times H) + \frac{\partial}{\partial t}(\frac{1}{2}\epsilon_0 E^2) + \frac{\partial}{\partial t}(\frac{1}{2}\mu_0 H^2) + E \cdot \frac{\partial P}{\partial t} + H \cdot \frac{\partial}{\partial t}(\mu_0 M) + E \cdot J = 0. \tag{I.16}$$

For the linear constitutive law,

$$E \cdot \frac{\partial P}{\partial t} = \frac{\partial}{\partial t}(\frac{1}{2}\epsilon_0 \chi_e E^2), \tag{I.17}$$

Then the Poynting's theorem for the *linear*, *isotropic* medium becomes,

$$\nabla \cdot (E \times H) + \frac{\partial}{\partial t}(w_e + w_m) + E \cdot J = 0, \tag{I.18}$$

where

$$w_e = \frac{1}{2}\epsilon E^2, \tag{I.19}$$

$$w_m = \frac{1}{2}\mu H^2. \tag{I.20}$$

Or write the Poynting's theorem in integral form,

$$\oint_S E \times H \cdot dA + \frac{\partial}{\partial t}\int_V (w_e + w_m)\, dV + \int_V E \cdot J\, dV = 0, \tag{I.21}$$

3. *Vector and scalar potentials*

For *nonmagnetic permeability*, $\mu = \mu_0$, $\mu_0 H$ is divergence-free, one can introduce a vector potential,

$$\mu_0 H = \nabla \times A. \tag{I.22}$$

Because unique specification of a vector field requires the specification of both its curl and its divergence, one must still specify the divergence of A,

$$E = -\frac{\partial A}{\partial t} - \nabla\Phi. \tag{I.23}$$

With vector potential, A, and the scalar potential, Φ, the Maxwell's equations become,

$$\nabla \times (\nabla \times A) = -\mu_0\epsilon\frac{\partial^2 A}{\partial t^2} - \mu_0\epsilon\frac{\partial}{\partial t}\nabla\Phi + \mu_0 J \tag{I.24}$$

$$\nabla \cdot (\epsilon\frac{\partial A}{\partial t} + \epsilon\nabla\Phi) = -\rho. \tag{I.25}$$

One can choose a *gauge*, for example, the *Lorentz gauge*,

$$\nabla \cdot A + \mu_0\epsilon\frac{\partial \Phi}{\partial t} = 0, \tag{I.26}$$

then the wave equations for A and Φ become,

$$\nabla^2 A - \mu_0\epsilon\frac{\partial^2 A}{\partial t^2} = -\mu_0 J - \mu_0(\nabla\epsilon)\frac{\partial}{\partial t}\Phi \tag{I.27}$$

$$\frac{1}{\epsilon}\nabla \cdot (\epsilon\nabla\Phi) - \mu_0\epsilon\frac{\partial^2 \Phi}{\partial t^2} = -\frac{\rho}{\epsilon} - \frac{1}{\epsilon}\nabla\epsilon \cdot \frac{\partial A}{\partial t}. \tag{I.28}$$

In a *uniform* charge-free medium, the wave equations for A and Φ are uncoupled,

$$\nabla^2 A - \mu_0\epsilon\frac{\partial^2 A}{\partial t^2} = 0 \tag{I.29}$$

$$\nabla^2 \Phi - \mu_0\epsilon\frac{\partial^2 \Phi}{\partial t^2} = 0. \tag{I.30}$$

B. Maxwell equations in complex form

For Maxwell's equations in linear media are linear, one can introduce the complex fields,

$$\mathbf{E}(r)e^{j\omega t}, \mathbf{H}(r)e^{j\omega t}, \mathbf{J}(r)e^{j\omega t} \tag{I.31}$$

to factor out the time dependence. Then Maxwell's equations are reduced to differential equations in the spatial variables only.

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}, \tag{I.32}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} + \mathbf{J}, \tag{I.33}$$
$$\nabla \cdot \epsilon\mathbf{E} = \rho, \tag{I.34}$$
$$\nabla \cdot \mu\mathbf{H} = 0. \tag{I.35}$$

In a *linear dispersive* medium,

$$\mathbf{P} = \epsilon\chi_e(\omega)\mathbf{E}. \tag{I.36}$$

A complex Poynting theorem has the form,

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) + j\omega(\mu\mathbf{H} \cdot \mathbf{H}^* - \epsilon^*\mathbf{E} \cdot \mathbf{E}^*) + \mathbf{E} \cdot \mathbf{J}^* = 0 \tag{I.37}$$

For lossless media, ϵ and μ must be real,

$$Im[\epsilon] = 0, \tag{I.38}$$
$$Im[\mu] = 0. \tag{I.39}$$

For a perfect conduct,

$$\oint_S \mathbf{E} \times \mathbf{H}^* + j\omega \int_V (\mu\mathbf{H} \cdot \mathbf{H}^* - \epsilon^*\mathbf{E} \cdot \mathbf{E}^*) = 0, \tag{I.40}$$

therefore, the field inside the volume must satisfies,

$$\int_V \epsilon^*\mathbf{E} \cdot \mathbf{E}^* = \int_V \mu\mathbf{H} \cdot \mathbf{H}^*. \tag{I.41}$$

In a dispersion-free medium, the time-averaged electric and magnetic energies are equal.

The complex form for the Helmholtz equation,

$$\nabla^2\mathbf{E} + \omega^2\mu\epsilon\mathbf{E} = 0. \tag{I.42}$$

1. *Propagation vector*

Assume a spatial dependence for **E** and **H** of the form $exp(-j\vec{k} \cdot \vec{r})$,

$$\mathbf{E} = \mathbf{E}_+ e^{-j\vec{k}\cdot\vec{r}}, \tag{I.43}$$
$$\mathbf{H} = \mathbf{H}_+ e^{-j\vec{k}\cdot\vec{r}}, \tag{I.44}$$

then, with $\nabla \leftrightarrow -j\vec{k}$, the Maxwell's equations become,

$$-j\vec{k} \times \mathbf{E}_+ = -j\omega\mu\mathbf{H}_+, \tag{I.45}$$
$$-j\vec{k} \times \mathbf{H}_+ = j\omega\epsilon\mathbf{E}_+, \tag{I.46}$$
$$j\vec{k} \cdot \mathbf{E} = 0 \tag{I.47}$$

One can find the *dispersion relation*:

$$k^2 = \omega^2\mu\epsilon. \tag{I.48}$$

The solution for **H** follows,

$$\mathbf{H}_+ = \frac{1}{\omega\mu}\vec{k} \times \mathbf{E} = \sqrt{\frac{\epsilon}{\mu}}\frac{\vec{k}}{k} \times \mathbf{E}. \tag{I.49}$$

The complex Poynting's theorem to the plane-wave solution,

$$\frac{1}{2}\mathbf{E} \times \mathbf{H}^* = \frac{\vec{k}}{k}\frac{1}{\sqrt{\mu\epsilon}} < w_e + w_m > . \tag{I.50}$$

C. Fourier transforms

A periodic function of time, f(t), of period T can be represented by a Fourier transform,

$$\mathbf{F}(n) = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}\, dt, \tag{I.51}$$

$$f(t) = \sum_{-\infty}^{\infty} \mathbf{F}(t)e^{+jn\omega_0 t}, \tag{I.52}$$

(I.53)

where $\mathbf{F}(n)$ is the corresponding Fourier series.
For an *aperiodic function*,

$$\mathbf{F}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\, dt, \tag{I.54}$$

$$f(t) = \int_{-\infty}^{\infty} \mathbf{F}(\omega)e^{+j\omega t}\, d\omega. \tag{I.55}$$

(I.56)

1. Properties of Fourier transform

- if $f(t)$ is real, then $\mathbf{F}(-\omega) = \mathbf{F}^*(\omega)$.
- if $f(t)$ is even, then $\mathbf{F}(-\omega) = \mathbf{F}(\omega)$, i.e., $\mathbf{F}(\omega)$ is even.
- if $f(t)$ is odd, then $\mathbf{F}(-\omega) = -\mathbf{F}(\omega)$, i.e., $\mathbf{F}(\omega)$ is odd.
- Time scaling, $f(a\,t) \leftrightarrow \frac{1}{|a|}\mathbf{F}(\frac{\omega}{a})$.
- Frequency scaling, $\frac{1}{|b|}f(\frac{t}{b}) \leftrightarrow \mathbf{F}(b\,\omega)$.
- Time shifting, $f(t - t_0) \leftrightarrow \mathbf{F}(\omega)e^{-j\omega t_0}$.
- Frequency shifting, $f(t)e^{j\omega_0 t} \leftrightarrow \mathbf{F}(\omega - \omega_0)$.

2. Convolution theorem

A convolution of two time functions, $f(t)$ and $g(t)$, is defined,

$$g \otimes f = \int_{-\infty}^{\infty} g(t - t')f(t')\, dt', \tag{I.57}$$

the Fourier transform of the convolution is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} g \otimes f\, e^{-j\omega t}\, dt = 2\pi\mathbf{G}(\omega)\mathbf{F}(\omega). \tag{I.58}$$

D. Phase velocity and group velocity

Suppose that ϵ is a function of ω (dispersive media), we may expand k in the neighborhood of ω_0,

$$k(\omega) = k(\omega_0) + \frac{d\,k}{d\,\omega}|_{\omega_0}\Delta\omega. \tag{I.59}$$

The bandwidth of the Fourier transform $\mathbf{E}(\omega)$ must occupy a narrow range of the values around ω_0,

$$E(t,z) = e^{j[\omega_0 t - k(\omega_0)z]}\int_{band}\mathbf{E}_+(\Delta\omega)e^{j\Delta\omega[t-(d\,k/d\,\omega)z]}d\,\Delta\omega. \tag{I.60}$$

$E(t,z)$ consists of two factors:

1. A rapidly varying term, the carrier, that propagates with the *phase velocity*,

$$v_p = \omega_0/k(\omega_0). \tag{I.61}$$

2. A slowly varying envelope, that proceeds with the *group velocity*,

$$v_g = 1/(d\,k/d\,\omega). \tag{I.62}$$

A general relation between group and phase velocities is,

$$\frac{d\,k}{d\,\omega} = \frac{d}{d\,\omega}(\frac{\omega}{v_p}) = \frac{1}{v_p} - \frac{\omega}{v_p^2}\frac{d\,v_p}{d\,\omega}, \tag{I.63}$$

then,

$$v_g = \frac{v_p}{1 - \frac{\omega}{v_p}\frac{d\,v_p}{d\,\omega}}. \tag{I.64}$$

Three possible cases,

- No dispersion: $v_g = v_p$;
- Normal dispersion: $v_g < v_p$;
- Anomalous dispersion: $v_g > v_p$.

In free space,

$$\omega = k\,v = \frac{k\,c}{n}, \tag{I.65}$$

hence,

$$v_g = \frac{d\,\omega}{d\,k} = v(1 - \frac{k}{n}\frac{d\,n}{d\,k}). \tag{I.66}$$

E. Power spectra and autocorrelation functions

Power spectra is the Fourier transformation of autocorrelation functions,

$$\lim_{T->\infty}[T\frac{|\bar{F(n)}|^2}{2\pi}] = \Phi(\omega), \tag{I.67}$$

$$\int_{\infty}^{\infty}\Phi(\omega)\,e^{j\omega t}\,d\,\omega = f(t)\bar{f(}t-\tau). \tag{I.68}$$

F. Extended studies

1. Stokes parameters and Poincaré sphere;
2. Fast Fourier Transform, FFT;
3. Slow light and fast light;
4. Atto-second pulse;
5. Time-Bandwidth product.
6. Auto-correlators: SPIDE, FROG, etc;
7. Kramers-Kronig relation.