XI. OPTICAL DETECTION

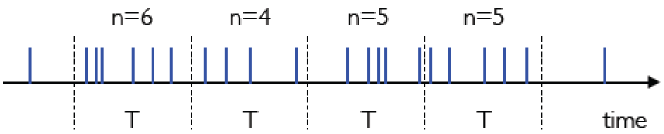
A. Quantum efficiency

Photodetectors produce an electric current proportional to the incident photon flux via generation of carriers in a semiconductor, or electron emission from the cathode in a vacuum diode. These detectors are characterized by a *quantum efficiency* η which expresses the ratio of the charged-particle current collected at the terminals to the rate of incident photons. The current due to an optical power P is,

$$i = \eta \frac{P}{\hbar\omega} e, \tag{XI.1}$$

where $P/\hbar\omega$ is equal to the rate of photons incident on the detector, ω is the radian frequency of the radiation and e is the electron charge.
The quantum efficiency is less than unity.

B. Photon number statistics



For photons are independent of each other, the probability of occurrence of n photons, or photoelectrons in a time interval T is random. Divide T into N intervals, the probability to find one photon per interval is,

$$p = \bar{n}/N, \tag{XI.2}$$

and the probability to find no photon per interval is,

$$1 - p. \tag{XI.3}$$

The probability to find n photons per interval is,

$$p(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}, \tag{XI.4}$$

which is a binomial distribution. When $N \to \infty$,

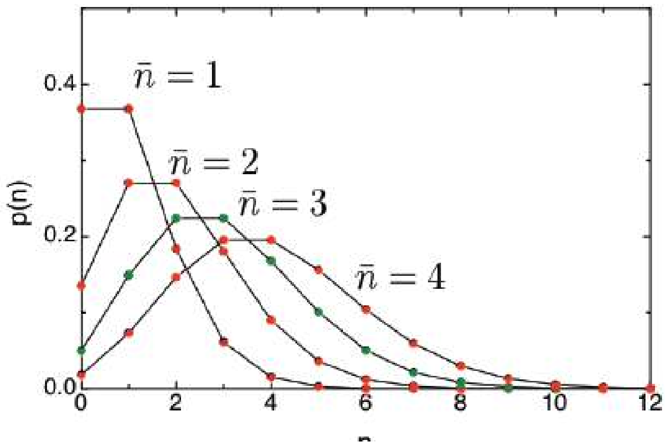
$$p(n) = \frac{\bar{n}^n \exp(-\bar{n})}{n!}, \tag{XI.5}$$

this is the *Poisson distribution* and the characteristics of coherent light.

C. Poisson statistics

The mean value of the Poisson statistics is,

$$\bar{n} = \sum_{n=0}^{\infty} n\, p(n). \tag{XI.6}$$



And the variance of the Poisson statistics is,

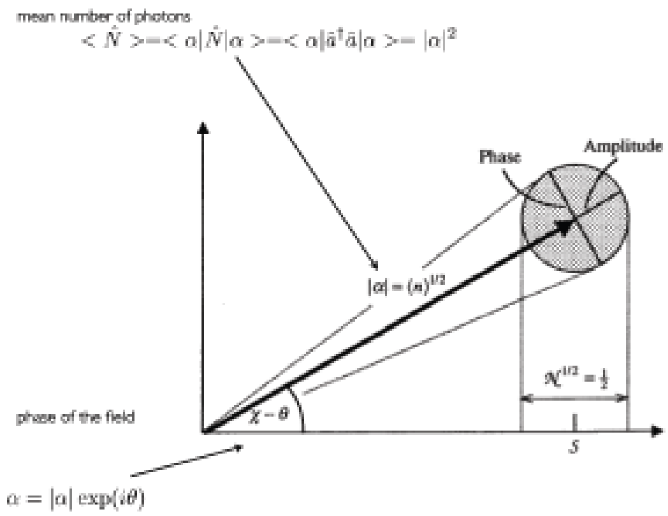
$$\sigma_n^2 = \sum_{n=0}^{\infty} (n - \bar{n})^2 \, p(n) = \bar{n}. \tag{XI.7}$$

The signal to noise ratio (SNR) for the Poisson statistics is,

$$\text{SNR} = \frac{(\text{mean})^2}{\text{variance}} = \frac{\bar{n}^2}{\sigma_n^2} = \bar{n}. \tag{XI.8}$$

There is always a probability to detect two photons even if $\bar{n} = 1$.

D. Coherent state



Coherent state (Glauber state), $|\alpha>$ is a superposition of the number state (Fock states), $|n>$, i.e.

$$|\alpha >= \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n >, \tag{XI.9}$$

with

$$\alpha = |\alpha|\text{ext}(i\theta). \tag{XI.10}$$

The mean number of photons in a coherent state is,

$$< \hat{N} >=< \alpha|\hat{N}|\alpha >=< \alpha|a^\dagger a|\alpha >= |\alpha|^2. \tag{XI.11}$$

The probability to find n photons in a coherent mode is,

$$p(n) = | < n|\alpha > |^2 = \exp(-|\alpha|^2)\frac{|\alpha|^{2n}}{n!} = \text{ext}(- < n >)\frac{< n >^n}{n!} \tag{XI.12}$$

This is the Poisson distribution. Coherent light is thus light with statistically independent photons.

E. Shot noise

When an electron of charge $-e$ is emitted from the photocathode of a vacuum photodiode at the time instant t_i, and travels to the anode, a current $i(t)$ flows in the external leads connecting the cathode and anode. During the time of traversal, a current flows in the leads to the photodiode from cathode to anode that integrates over time to $+e$, so that,

$$i(t) = eh(t - t_i), \tag{XI.13}$$

with

$$\int_{-\infty}^{\infty} h(t - t_i)dt = 1. \tag{XI.14}$$

The net current $i(t)$ produced by the flow of charges is,

$$i(t) = \sum_i h(t - t_i), \tag{XI.15}$$

where t_i is a random variable.
The discrete Fourier transform $I(n)$ is,

$$I(n) = e\frac{1}{T}\int_{-T/2}^{T/2} e^{-j\omega t}h(t - t_i)dt, \tag{XI.16}$$

$$= e\sum_i e^{-j\omega t_i}\frac{1}{T}\int_{-T/2-t_i}^{T/2-t_i} e^{-j\omega t'}h(t')dt'. \tag{XI.17}$$

If T is very large, the integrals can be replaced by the integral Fourier transform of $h(t)$, $H(\omega)$, where,

$$H(\omega) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt. \tag{XI.18}$$

Thus the Fourier component $I(n)$ of the current can be written,

$$I(n) = e\frac{2\pi}{T}\sum_i e^{-j\omega t_i}H(\omega). \tag{XI.19}$$

The spectrum Φ is given by

$$\Phi = \lim_{T\to\infty}\frac{T}{2\pi}\overline{|I(n)|^2} = e^2\lim_{T\to\infty}\{\frac{2\pi}{T}|H(\omega)|^2\overline{\sum_{i,k} e^{-j\omega(t_i - t_k)}}\}. \tag{XI.20}$$

If the photodetector is illuminated by light of constant intensity, the t_i's are random variables that are uncorrelated with each other. The statistical average gives zero for $t_i \neq t_k$ and $\omega \neq 0$, so that the double sum is replaced by the single sum for $i = k$. For $\omega = 0$, the dc component of the spectrum, the double sum must be carried out.

1. $\omega \neq 0$, if the rate of flow of the photoelectrons is I_0/e, where I_0 is the dc current produced by the illumination, the sum becomes,

$$\overline{\sum_{i,k} e^{-j\omega(t_i-t_k)}} = \overline{\sum_{i=k} e^{-j\omega(t_i-t_k)}} = T\frac{I_0}{e}, \tag{XI.21}$$

and the spectrum at $\omega \neq 0$ is,

$$\Phi(\omega) = e^2 \lim_{T\to\infty} 2\pi|H(\omega)|^2\frac{I_0}{e} = 2\pi e I_0|H(\omega)|^2. \tag{XI.22}$$

2. $\omega = 0$,

$$\overline{\sum_{i,k} e^{-j\omega(t_i-t_k)}} = (\frac{TI_0}{e})^2, \tag{XI.23}$$

and the spectrum at $\omega = 0$ is,

$$\Phi(\omega) = \lim_{T\to\infty} I_0^2\frac{T}{2\pi}. \tag{XI.24}$$

Consider that the idealized current response function is a rectangular of width τ_0, where τ_0 is the transit time for the electron through the photoconductor. The spectrum $H(\omega)$ is,

$$H(\omega) = \frac{1}{2\pi}\int_0^{\tau_0} e^{-j\omega t}\frac{1}{\tau_0}dt = \frac{1}{2\pi}\frac{\sin(\omega\tau_0/2)}{\omega\tau_0/2}e^{-j\omega\tau_0/2}. \tag{XI.25}$$

Then one can obtain the shot noise spectrum for the idealized case. It is of interest to define an effective bandwidth of the noise spectrum that is defined by the width $4\pi B$ of the rectangle of the same peak height and of equal area. The are of the function $\sin^2(x/2)/(x/2)^2$ is 2π so that,

$$\int |H(\omega)|^2 d\omega = \frac{1}{\tau_0}\frac{1}{2\pi} = 4\pi B(\frac{1}{2\pi})^2, \tag{XI.26}$$

or

$$B = \frac{1}{2\tau_0}. \tag{XI.27}$$

Suppose that we observe the mean-square current fluctuations $\bar{i_n^2}$ in the leads to the detector through a filter of bandwidth $B = 1/2\tau_0$. Within the timer interval τ_0, $\bar{N}$ charges pass on the average through the detector, where,

$$\bar{N} = \frac{eI_0}{e} = \frac{I_0}{2Be}. \tag{XI.28}$$

For a random arrival of particles, N is a fluctuating quantity ant the mean-square deviation is ,

$$\sigma_N^2 = \bar{N^2} - \bar{N}^2 = \bar{N}. \tag{XI.29}$$

The mean-square current fluctuation are,

$$\bar{i^2} - \bar{i}^2 = (\frac{e\bar{N}}{\tau_0})^2 - (\frac{e\bar{N}}{\tau_0})^2 = \frac{e^2\bar{N}}{\tau_0^2} = 2eI_0B. \tag{XI.30}$$

This is the shot-noise formula.

F. Extended studies

1. Quantum optics;
2. Statistics optics.