VIII. DISTRIBUTED FEEDDACK STRUCTURES

A. The equations of DFB structures

In the preceding chapter, we consider that two optical waveguides are coupled to each other via their fringing fields. To produce appreciable coupling, synchronism was necessary; that is, the two propagation constants could not differ by much more than the magnitude of the coupling coefficient,

$$|\frac{\beta_1-\beta_2}{2}|<|\kappa_{12}|. \tag{VIII.1}$$

Consider a structure of the *"distributed feedback"* (DFB) structure as shown below, for two waves of opposite group velocity, denoted by a for the "forward" wave with positive group velocity and b for the "backward" wave with negative group velocity. If there is no periodicity, then the differential equations obeyed by a and b are,

$$\frac{d\,a}{d\,z} = -j\beta a, \tag{VIII.2}$$

$$\frac{d\,b}{d\,z} = j\beta b. \tag{VIII.3}$$

Suppose that a periodic perturbation of the guiding structure is introduced. The *space harmonics* of the periodic structure are capable of coupling the forward wave, a, to another wave via

$$exp(-j\beta z)\cos(\frac{2\pi}{\Lambda}z)=\frac{1}{2}\{exp[-j(\beta-\frac{2\pi}{\Lambda})z]+exp[-j(\beta+\frac{2\pi}{\Lambda})z]\}. \tag{VIII.4}$$

When $\beta-\frac{2\pi}{\Lambda}$ is close to $-\beta$, the forward wave, a, is coupled to the backward wave, b. The other exponential with the argument $[\beta+(2\pi/\Lambda)]z$ does not produce backward wave because its spatial dependence differs greatly for that of $exp(+j\beta z)$. The effect of the coupling of a to b can be included by introducing a coupling term produced by the forward/backward waves from the space harmonics in the differential equations for a/b,

$$\frac{d\,a}{d\,z} = -j\beta a+\kappa_{ab}be^{-j(2\pi/\Lambda)z}, \tag{VIII.5}$$

$$\frac{d\,b}{d\,z} = j\beta b+\kappa_{ba}be^{+j(2\pi/\Lambda)z}. \tag{VIII.6}$$

The equations above can be reduced to coupling-of-modes equations with space-independent coefficients by introducing the new variables,

$$a = A(z)e^{-j(\pi/\Lambda)z}, \tag{VIII.7}$$

$$b = B(z)e^{+j(\pi/\Lambda)z}, \tag{VIII.8}$$

with the result

$$\frac{d\,A}{d\,z} = -j(\beta-\frac{\pi}{\Lambda})A+\kappa_{ab}B, \tag{VIII.9}$$

$$\frac{d\,B}{d\,z} = j(\beta-\frac{\pi}{\Lambda})B+\kappa_{ba}A. \tag{VIII.10}$$

These coupled mode equations can be simplified in appearance by the introduction of the detuning parameter,

$$\delta\equiv\beta-\frac{\pi}{\Lambda}, \tag{VIII.11}$$

and the relation of the coupling constants,

$$\kappa_{ab}=\kappa_{ba}^*\equiv\kappa, \tag{VIII.12}$$

$$\frac{d\,A}{d\,z} = -j\delta A+\kappa B, \tag{VIII.13}$$

$$\frac{d\,B}{d\,z} = j\delta B+\kappa^* A, \tag{VIII.14}$$

The propagation constant β for the coupled system is

$$\beta = \pm\sqrt{\delta^2 - |\kappa|^2}, \tag{VIII.15}$$

which is a real number when

$$|\delta| > |\kappa|. \tag{VIII.16}$$

B. Reflection filter

Consider the reflection from a DFB structure of length l, which acts as a filter, with one end matched, $B = 0$ at $z = 0$. For $|\delta| < |\kappa|$, the solutions of the coupled equations are of the form,

$$exp(\mp\gamma z), \tag{VIII.17}$$

where

$$\pm\gamma = \pm\sqrt{|\kappa|^2 - \delta^2}, \tag{VIII.18}$$

with $\kappa = \kappa_{ab} = \kappa_{ba}^*$ and $\delta = \beta - (\pi/\Lambda)$. The solutions are growing and decaying exponentials, whereas they are periodic functions in the range $|\delta| > |\kappa|$.
The general solutions with arbitrary constants are,

$$\begin{aligned} A &= A_+e^{-\gamma z} + A_-e^{+\gamma z}, & \text{(VIII.19)}\\ B &= B_+e^{-\gamma z} + B_-e^{+\gamma z}, & \text{(VIII.20)} \end{aligned}$$

where only two of the four constants are independent, i.e.

$$B_\pm = \frac{\mp\gamma + j\delta}{\kappa}A_\pm. \tag{VIII.21}$$

At $z = 0$ there is no reflected wave, $B_+ = -B_-$, and thus

$$B = -2B_+\sinh\gamma z. \tag{VIII.22}$$

Then the solutions for A become,

$$\begin{aligned} A &= \frac{1}{\kappa^*}(\frac{d}{d\,z}B - j\delta B), & \text{(VIII.23)}\\ &= -2B_+(\frac{\gamma}{\kappa^*}\cosh\gamma z - \frac{j\delta}{\kappa^*}\sinh\gamma z). & \text{(VIII.24)} \end{aligned}$$

The reflection coefficient $\Gamma = B/A$ at $z = -l$ is

$$\Gamma(-l) = -\frac{\sinh\gamma z}{(\gamma/\kappa^*)\cosh\gamma l + (j\delta/\kappa^*)\sinh\gamma l}. \tag{VIII.25}$$

Generalize to an arbitrary reflection at $z = 0$, $\Gamma(0)$, one find,

$$\Gamma(z) = \frac{(\kappa^*/\kappa) + \Gamma(0)[(\gamma/\kappa)\coth\gamma z + (j\delta/\kappa)]}{\Gamma(0) + [(\gamma/\kappa)\coth\gamma z - (j\delta/\kappa)]}. \tag{VIII.26}$$

The distributed feedback structure acts as a reflector in its stop band, $|\delta| < |\kappa|$, and has a set of transmission resonance in the passband, $|\delta| > |\kappa|$. One can also use an apodization function for the coupling constant, with another modulation function, to suppress the side bands of reflection. The design for a DFB structure with specific reflection/transmission is called the *synthesis*, which is an inverse problem.

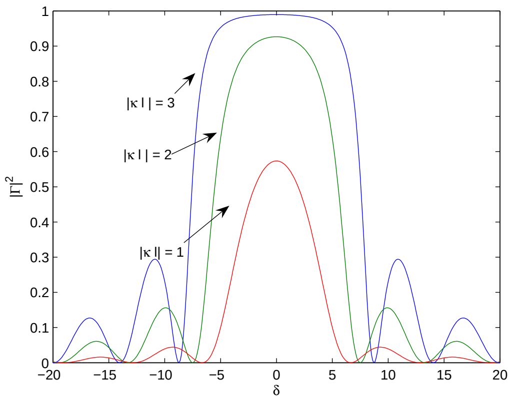

FIG. 2: Reflection $|\Gamma|^2$ as a function of the detuning δ for uniform distributed feed back structure.

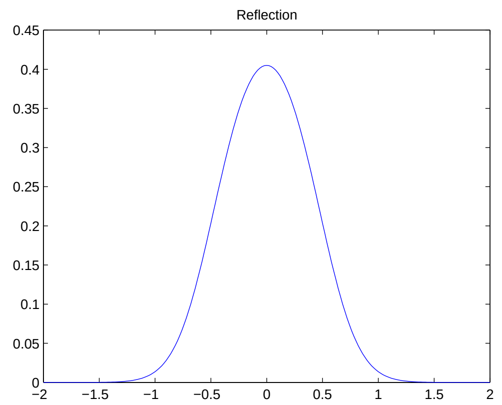

FIG. 3: Reflection $|\Gamma|^2$ as a function of the detuning δ for non-uniform distributed feed back structure, with a Gaussian apodization function.

C. High-Q transmission resonator

It is possible to achieve transmission within the stopband, if one spaces two periodic structures

1. by a phase shift between them,
2. by one (or an odd multiple of) quarter wave length(s).

The idea of making a high-Q transmission resonator can be constituted by two periodic structures, separated by a phase shift or by a space. Since the periodic structure offer an additional space harmonic to couple the forward wave, a, into the backward wave, b, two periodic structures have the same grating wavevector, $k_g = \frac{2\pi}{\Lambda}$, but with different directions. For a phase shift DFB structure with a phase shift $\phi = \pi$, two grating wavevector would cancel out with each other, the result makes the transmission at the wavelength $\delta = 0$ possible. For the center of the stop band is,

$$\frac{2\pi}{\lambda} = \frac{\pi}{\Lambda}, \tag{VIII.27}$$

DFB structures with a quarter-wave section, $\lambda/4$, in the space would also has the transmission window at the center of the stop band.

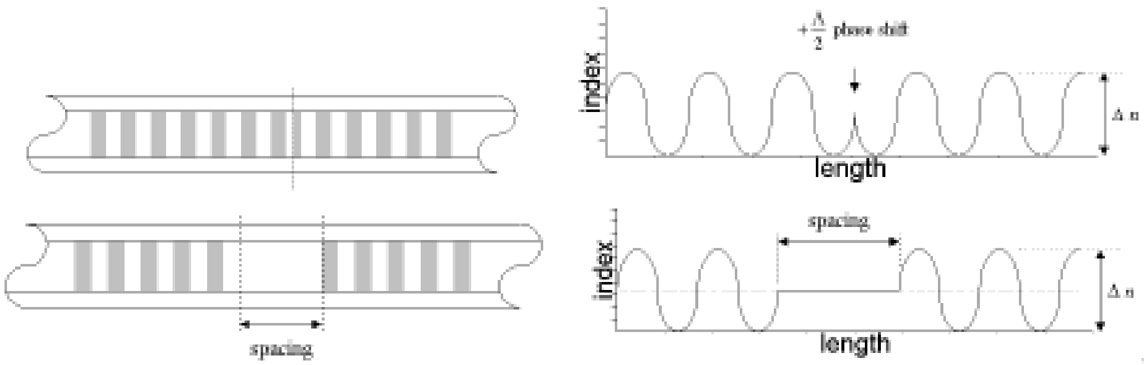

FIG. 4: A high-Q transmission resonators can be constructed by two DFB structures, with a discontinuation in their phase or with a space between them.

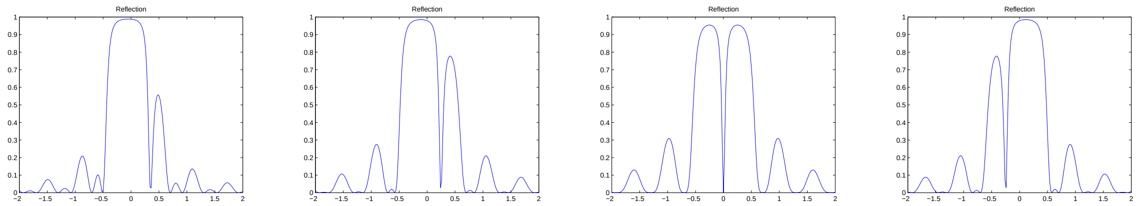

FIG. 5: Reflection $|\Gamma|^2$ as a function of the detuning δ for *phase-shift* distributed feed back structure, from left to right: $\phi = \pi/4, \pi/2, \pi, 3\pi/2$.

D. Extended studies

1. Coupling coefficient

2. Distributed FeedBack lasers

3. DBR lasers

4. Vertical Cavity Surface-Emitting laser

5. Photonic bandgap crystals

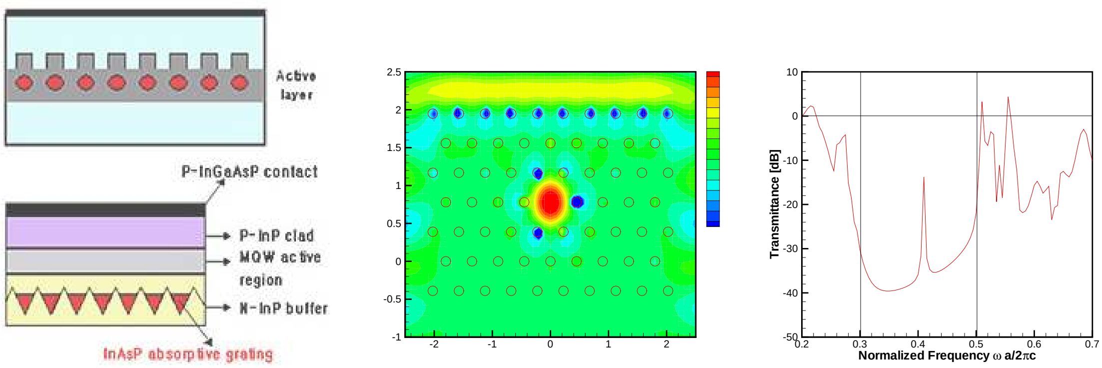

FIG. 6: Left: Typical structure for a DFB laser; Right: point-defect in photonic crystals