IX. ACOUSTO-OPTIC MODULATORS

A. Acousto-optic wave coupler

In an acousto-optic modulator an acoustic wave sets up a spatial modulation of the index of an acousto-optic medium; an optical wave diffracts from the index modulation. In a linear acousto-optic medium, the change of index n is proportional to the strain,

$$\nabla \times (\nabla \times E) = -\epsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2} - \mu_0 \frac{\partial^2 P}{\partial t^2}. \tag{IX.1}$$

The displacement density $\epsilon_0 E + P$ can be separated into a part due to the time-independent background index n, and a part due to the *space-time-dependent* index Δn produced by the acoustic wave,

$$\epsilon_0 E + P = \epsilon_0[n + \Delta(r,t)]^2 E \approx \epsilon_0 n^2 E + 2\epsilon_0 n \Delta(r,t) E, \tag{IX.2}$$

under the assumption $\Delta n \ll n$. Then the equation for the electric field becomes,

$$\nabla \times (\nabla \times E) + \epsilon_0\mu_0 n^2 \frac{\partial^2 E}{\partial t^2} = -\mu_0\epsilon_0 \frac{\partial^2}{\partial t^2}[2n\Delta n(r,t)E]. \tag{IX.3}$$

We consider a plane acoustic wave with the propagation vector k_s in the $x - z$ plane, and the index change produced by the acoustic wave of frequency ω_s has the same spatial dependence as the plane wave,

$$\begin{aligned} \Delta n(r,t) &= \Delta n \cos(\omega_s t - k_s \cdot r), && \text{(IX.4)} \\ &= \frac{\Delta n}{2}[e^{j(\omega_s t - k_s\cdot r)} + e^{-j(\omega_s t - k_s \cdot r)}], && \text{(IX.5)} \end{aligned}$$

where Δn is the peak amplitude of the index modulation. If E field is polarized along y direction, $E \cdot \nabla\epsilon = 0$ and the equation for the electric field becomes,

$$\nabla^2 E - \mu_0\epsilon_0 n^2 \frac{\partial^2 E}{\partial t^2} = \mu_0\epsilon_0 \frac{\partial^2}{\partial t^2}[2n\Delta n(r,t)E]. \tag{IX.6}$$

The time-dependent index multiplying the incident plane wave with the E field,

$$E \propto exp[-j(k_i \cdot r - \omega_i t)], \tag{IX.7}$$

produces a source for the diffracted wave with the frequency,

$$\omega_d = \pm\omega_s + \omega_i; \qquad \text{energy conservation}, \tag{IX.8}$$

and with the spatial dependence,

$$k_s + k_i = k_d; \qquad \text{momentum conservation}. \tag{IX.9}$$

Usually, acoustic waves propagate at a velocity that is *five* or *six* orders of magnitude smaller than the speed of light, acoustic wave have wavelengths comparable to optical waves if the acoustic frequency is five to six orders of magnitude smaller than the optical frequency, $\omega_s \ll \omega_i$, and thus $|k_d| \approx |k_i|$.
An approximation solution for Eq. (IX.6) is writing the incident and diffracted waves in,

$$\begin{aligned} E_i &= \vec{y} A_i(z) e^{-j(k_i\cdot r - \omega_i t)}, && \text{(IX.10)} \\ E_d &= \vec{y} A_d(z) e^{-j(k_d\cdot r - \omega_d t)}, && \text{(IX.11)} \end{aligned}$$

where the z dependence of $A_i(z)$ and $A_d(z)$ are the result of the coupling of the incident and diffracted waves by the index modulation. Disregarding second-order derivatives of $A_d(z)$, we obtain

$$-(k_d^2 - \omega_d^2\mu_0\epsilon_0 n^2)A_d - 2jk_d \cdot \nabla A_d \approx -\frac{\omega_d^2}{c^2} n\Delta n A_i. \tag{IX.12}$$

For

$$k_d = \omega_d\sqrt{\mu_0\epsilon_0} n, \tag{IX.13}$$

and

$$k_d \cdot \nabla A_d(z) = k_d \cos\theta \frac{d\,A_d}{d\,z}, \tag{IX.14}$$

we obtain

$$\frac{d\,A_d}{d\,z} = -j\frac{\omega_d}{2c}\frac{\Delta n}{\cos\theta}A_i. \tag{IX.15}$$

Analogically, one may write an equation for the incident wave as affected by the diffracted wave,

$$\frac{d\,A_i}{d\,z} = -j\frac{\omega_i}{2c}\frac{\Delta n}{\cos\theta}A_d. \tag{IX.16}$$

These are coupling of modes equations in space. The coupling coefficients are

$$-j\frac{\omega_d}{2c}\frac{\Delta n}{\cos\theta} \quad \text{and} - j\frac{\omega_i}{2c}\frac{\Delta n}{\cos\theta}. \tag{IX.17}$$

They are not exactly equal, in that $\omega_i \neq \omega_d$, for the present system does not obey power conservation - it is driven mechanically (acoustically). However, ω_d differs from ω_i by 1 part in 10^5 and the system behaves just like a lossless coupled-wave system. The solutions are, for $A_d(0)$ at $z = 0$,

$$\begin{aligned} Ai(z) &= A_i(0)\cos|\kappa|z, && \text{(IX.18)} \\ Ai(z) &= -jA_i(0)\sin|\kappa|z, && \text{(IX.19)} \end{aligned}$$

where

$$|\kappa| = \frac{\sqrt{\omega_i\omega_d}}{2c}\frac{\Delta n}{\cos\theta} \approx \frac{\omega_i}{2c}\frac{\Delta n}{\cos\theta}. \tag{IX.20}$$

1. The incident wave is depleted by the diffracted wave as it travels along the medium supporting the acoustic wave.
2. The diffracted wave is shifted in frequency, $\omega_d = \pm\omega_s + \omega_i$.
3. If the travel distance is long enough, $|\kappa|z > \pi/2$, the diffracted wave feeds back into the incident wave.
4. When $\omega_s = 0$, the acousto-optic modulator acts a time-independent "bulk grating", a cosinusoidal "frozen" index variation.

B. Acousto-optic amplitude modulator

The intensity of an incident wave is modulated if it interacts with a standing acoustic wave. In the acousto-optic modulator, a transducer is excited electrically and sets up acoustic standing waves in the crystal with the index variation,

$$\begin{aligned} n(r,t) &= \Delta n \sin\omega_s t \cos(k_s \cdot r), && \text{(IX.21)} \\ &= \frac{\Delta n}{4j}\{exp[j(\omega_s t - k_s \cdot r)] + exp[j(\omega_s t + k_s \cdot r)] + exp[-j(\omega_s t - k_s \cdot r)] + exp[-j(\omega_s t + k_s \cdot r)]\} && \text{(IX.22)} \end{aligned}$$

1. If we only have one acoustic wave, with the space-time dependence $exp[\pm j(\omega_s t - k_s \cdot r)]$, the incident wave at ω_i produced a diffracted wave at $\omega_d = \omega_i + \omega_s$ with the propagation vector $k_d = k_i + k_s$. And the diffracted wave reacted back by producing $\omega_d - \omega_s = \omega_i$ with propagation vector $k_d - k_s = k_i$.
2. Now for a standing acoustic wave, we have four dependences introduced by the index variation, $exp[\pm j(\omega_s t \pm k_s \cdot r)]$. The incident wave at frequency ω_i can produce a diffracted wave at frequency $\omega_i \pm \omega_s$ with the propagation vector $k_d = k_i + k_s$, which reacts back to produce $(\omega_i \pm \omega_s) \pm \omega_s$ with the propagation vector $k_i = k_d - ks$. Since the acoustic frequency ω_s is small compared with the optical frequency, wave produced at $\omega_i \pm m\omega_s$ remain *phase matched*. The sidebands *cascade*.

For the bulk grating, at $z = l$,

$$Ai(z) = A_i(0)\cos(\frac{\omega_i}{2c}\frac{\Delta n \sin\omega_s t}{\cos\theta}l), \tag{IX.23}$$

$$Ai(z) = -jA_i(0)\sin(\frac{\omega_i}{2c}\frac{\Delta n \sin\omega_s t}{\cos\theta}l), \tag{IX.24}$$

where Δn now varies with time. By using the Bessel-function identity,

$$exp(jx\sin\omega_s t) = \sum_{m=-\infty}^{\infty} J_m(x)e^{jm\omega_s t}, \tag{IX.25}$$

we have

$$\cos(x\sin\omega_s t) = \sum_{m=\text{even}} J_m(x)e^{jm\omega_s t}, \tag{IX.26}$$

$$\sin(x\sin\omega_s t) = -j\sum_{m=\text{odd}} J_m(x)e^{jm\omega_s t}, \tag{IX.27}$$

where we use the fact that $J_m(-x) = (-1)^m J_m(x)$. The incident wave amplitude and diffracted wave amplitude decomposed into Fourier components are,

$$A_i(l) = A_i(0)\sum_{m=\text{even}} J_m(\frac{\omega_i}{2c}\frac{\Delta n l}{\cos\theta})e^{jm\omega_s t}, \tag{IX.28}$$

$$A_i(l) = A_i(0) - \sum_{m=\text{odd}} J_m(\frac{\omega_i}{2c}\frac{\Delta n l}{\cos\theta})e^{jm\omega_s t}. \tag{IX.29}$$

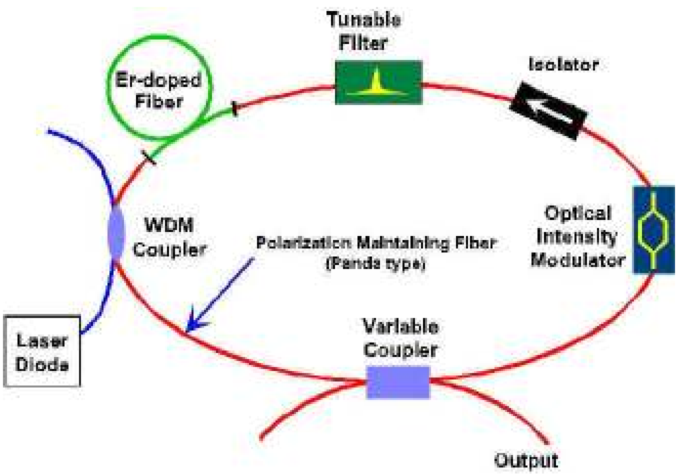
1. The sidebands of the incident wave are at *even* harmonics, the modulation of the diffracted wave at *odd* harmonics.

2. The fundamental is completely canceled if the modulation obeys the relation,

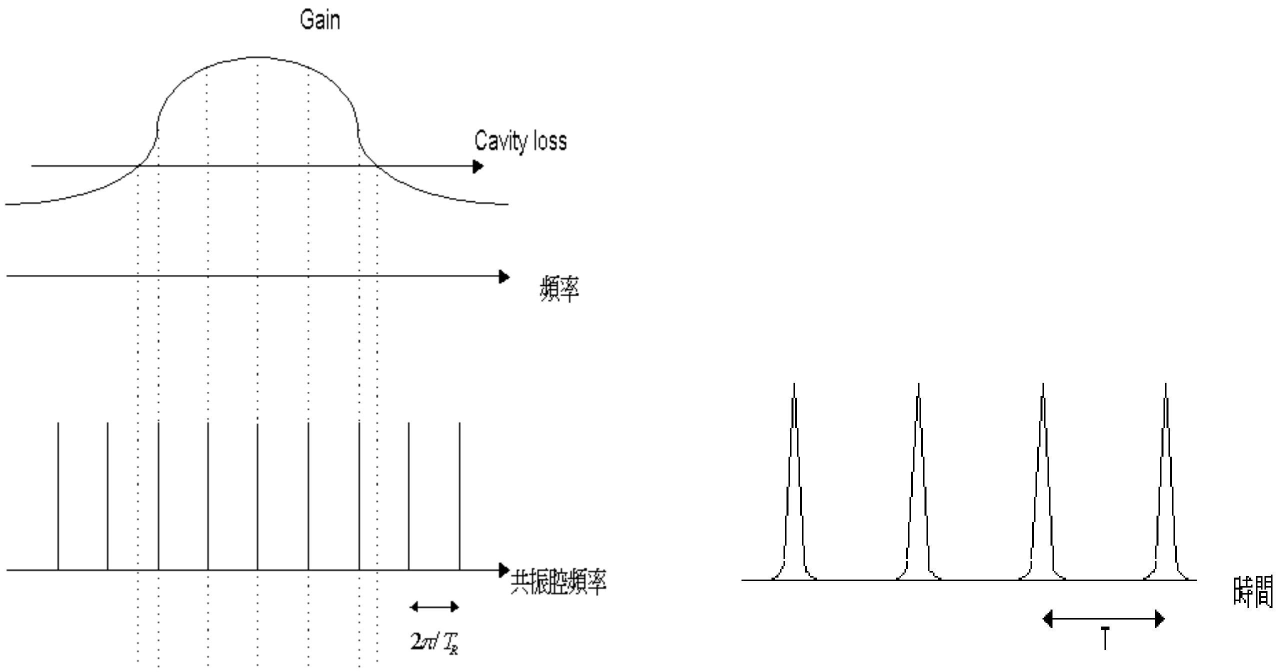
$$\frac{\omega_i}{2c}\frac{\Delta n l}{\cos\theta} = 2.405, \tag{IX.30}$$

   the first zero of the Bessel function J_0.

C. Active modelocking



An acousto-optic medium under acoustic standing wave excitation may be used to produce short pulses from a laser oscillator - by the process of *active modelocking*. Consider the system of a ring fiber laser, which contains a laser gain medium (Er-doped fiber), a lossy medium, and a modulator. The modulator modulates the loss of the resonator. The operation of modelocking is that the radiation field in the resonator bouncing back and forth "see" a time dependent net gain. The part of the radiation that passes the modulator at an instant in time when it has minimum loss experiences maximum net gain. The radiation passing at other times experiences less gain, and even net loss. A pulse is formed which grows and narrows. In the steady state, the gain adjusts to equal the loss, and pulse narrowing in one pass is compensated by pulse broadening due to dispersion of the gain medium.



1. *Effect of gain, loss, and modulator*

1. Gain: The gain medium of length l_g is described by a frequency-dependent gain coefficient,

$$exp\{\frac{\alpha_g}{1+[(\omega-\omega_0)/\omega_g]^2}\}a(\omega) \approx [1+\alpha_g l_g(1-(\frac{\omega-\omega_0}{\omega_g}^2)]a(\omega), \tag{IX.31}$$

   where ω_0 is the center frequency of the gain medium, ω_g is its width, and $\alpha_g l_g$ is the integrated gain at line center.

2. Loss: All of the losses due to unavoidable absorption, and power escape from the laser resonator are describe by,

$$exp(-\alpha_l l_l)a(\omega) \approx (1-\alpha_l l_l)a(\omega). \tag{IX.32}$$

3. Modulator: The wave traversing the modulation medium with the time-dependent modulation loss is described in the *time domain* by,

$$exp[-\alpha_m l_m(1-\cos\omega_M t)]A(t), \tag{IX.33}$$

   where ω_M is the modulation frequency.

The wave passes the three elements *twice* in one round trip in the linear resonator, and *once* in the ring resonator. Expanded to first order in net gain and loss, the combined effect on $A(t)$ is,

$$[1 + \alpha_g l_g (1 + \frac{1}{\omega_g^2}\frac{d^2}{dt^2}) - \alpha_l l_l - \alpha_m l_m (1 - \cos\omega_M t)]A(t) = A(t + T_R), \quad \text{(IX.34)}$$

$$= A(t) - \delta T_R \frac{dA(t)}{dt}, \quad \text{(IX.35)}$$

where T_R is the round-trip time and δT_R is the change to the transit time wrought by pulse reshaping. This is the equation of active modelocking. For strong modelocking - well separated pulses occurring near the time instant of minimum loss, $t = n\frac{2\pi}{\omega_M}$, with n is an integer. Then, expanding $\cos\omega_M t$ around the minimum at $t = 0$,

$$[\alpha_g l_g (1 + \frac{1}{\omega_g^2}\frac{d^2}{dt^2}) - \alpha_l l_l - \frac{\alpha_m l_m}{2}\omega_M^2 t^2 + \delta T_R \frac{d}{dt}]A(t) = 0. \quad \text{(IX.36)}$$

In the case of *synchronism*, $\delta T_R = 0$, achieved by proper adjustment of the modulation frequency, the equation of modelocking is the Schrödinger equation for a particle in a parabolic potential with the solutions,

$$A(t) = H_\nu(\omega_p t) exp(-\frac{\omega_p^2 t^2}{2}), \quad \text{(IX.37)}$$

with

$$\omega_p^2 = \sqrt{\frac{\alpha_m l_m}{2\alpha_g l_g}}\omega_M \omega_g, \quad \text{(IX.38)}$$

$$l - \frac{\alpha_l l_l}{\alpha_g l_g} = \frac{\omega_p^2}{\omega_g^2}(2\nu + 1), \quad \text{(IX.39)}$$

where H_ν is the Hermite polynomial of order ν.

1. Generally, $\omega^2 \ll |\omega_g|^2$, the gain will not exceed the loss by much, $\alpha_g l_g \approx \alpha_l l_l$.
2. Saturation of the gain coefficient,

$$\alpha_g = \frac{\alpha_g^0}{1 + (I/I_s)}, \quad \text{(IX.40)}$$

   leads to stability of the fundamental Gaussian solution and instability of the higher-order Hermite-Gaussians.
3. The pulse width is inversely proportional to the forth root of the modulation depth, $\alpha_m l_m$, and the square root of the gain bandwidth, $\sqrt{\omega_g}$.
4. The full width at half maximum intensity of the pulse is,

$$\text{FWHM} = \frac{2\sqrt{\ln 2}}{\omega_p}, \quad \text{(IX.41)}$$

   which is *inversely* proportional to the square of gain bandwidth, ω_g, and in the order of *ps* for a typical laser.

D. Extended studies

1. Acoustic drive power for given modulation
2. Acousto-optic frequency modulator
3. Electro-optic modulators
4. Magnetic-optic modulators