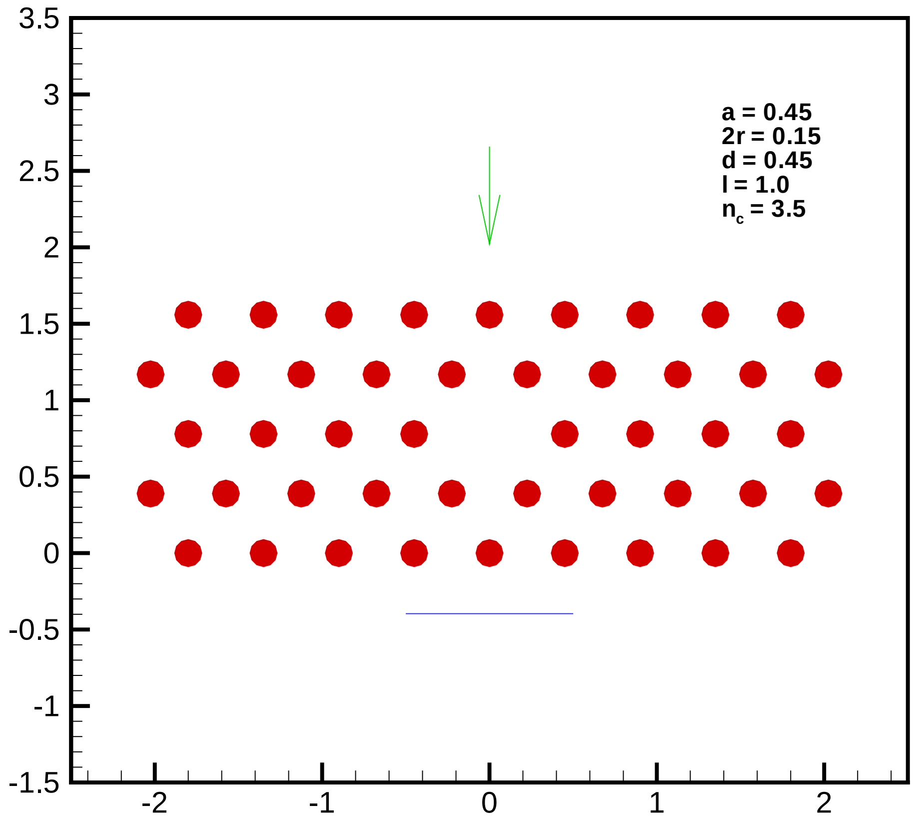
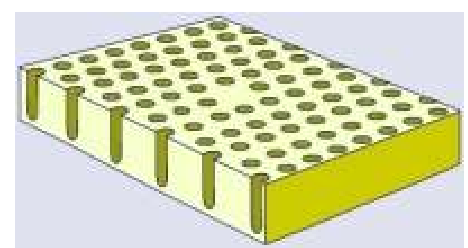
# Course Projects

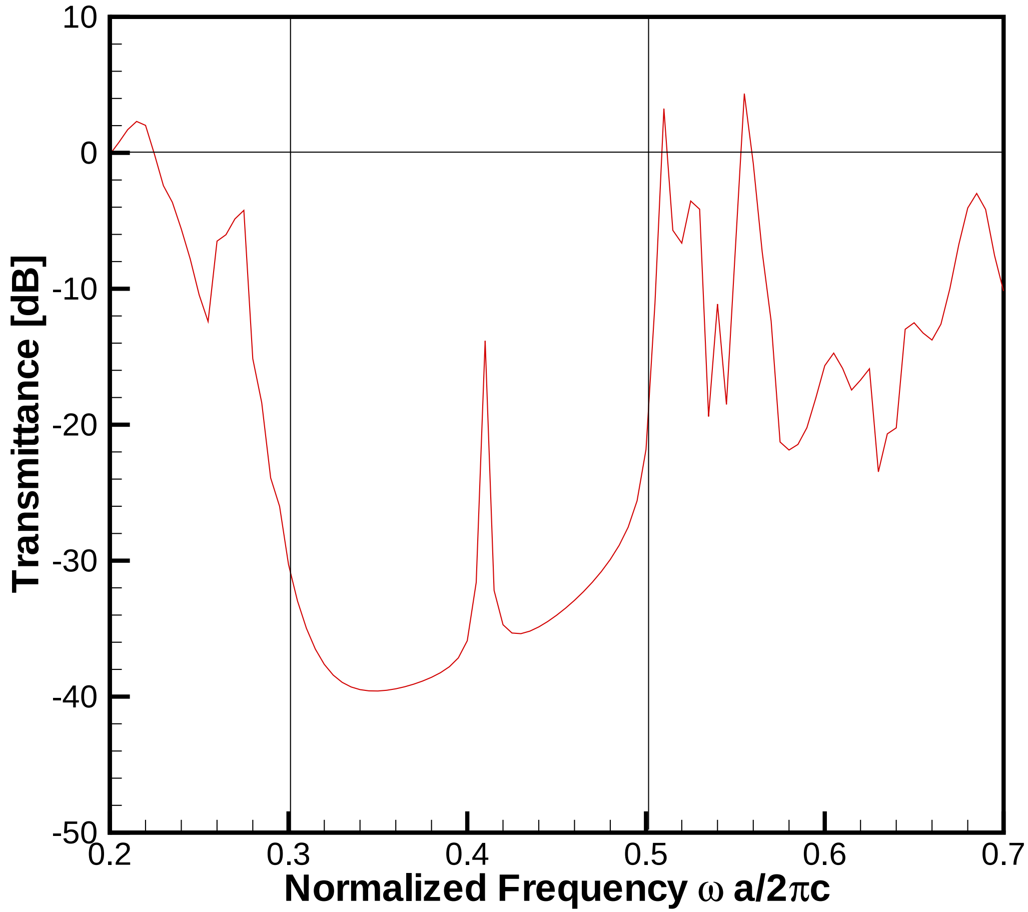
- 2D/3D Finite-Difference Time-Domain mehtod
- Band-spectrum for 1D nonlinear Schrödinger equation
- Band-spectrum for 2D/3D Maxwell equation
- Coupled nonlinear PDE in 2+1 dimensions
- Soliton solutions for nonlinear PDE
- Finite Element method
- Optimization problem
- Monte-Carlo simulation

# One-defect in photonic crystal

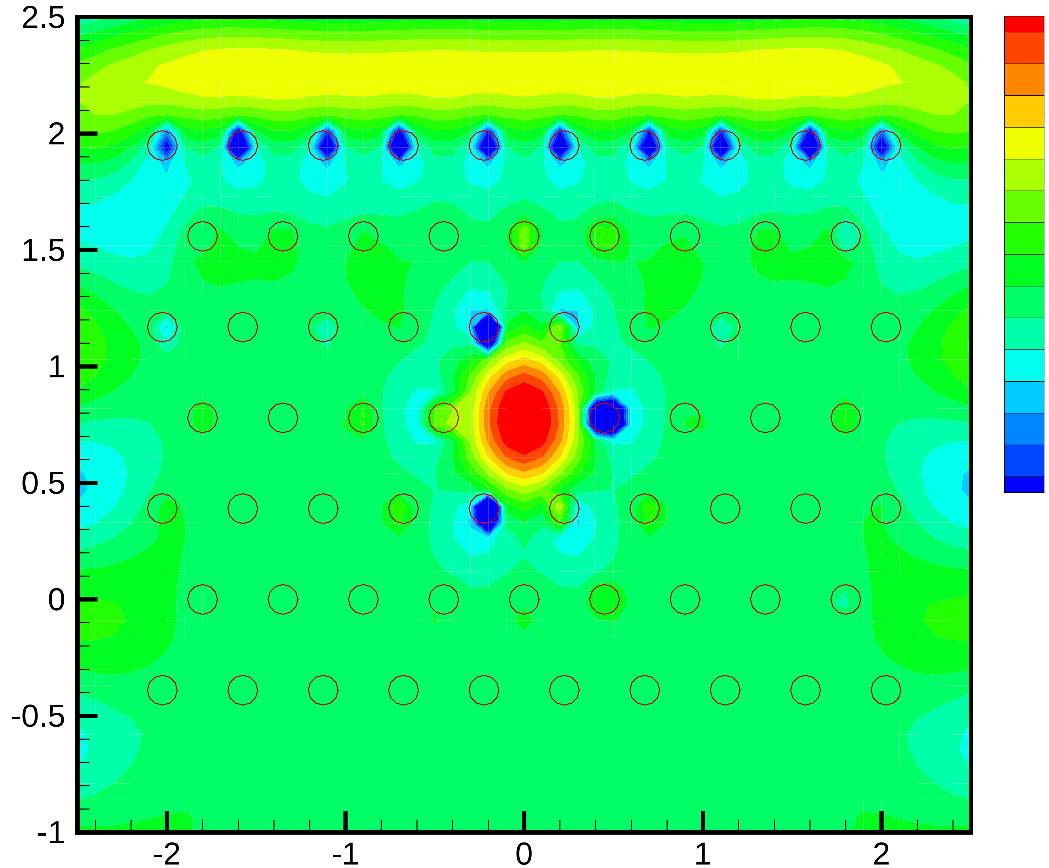


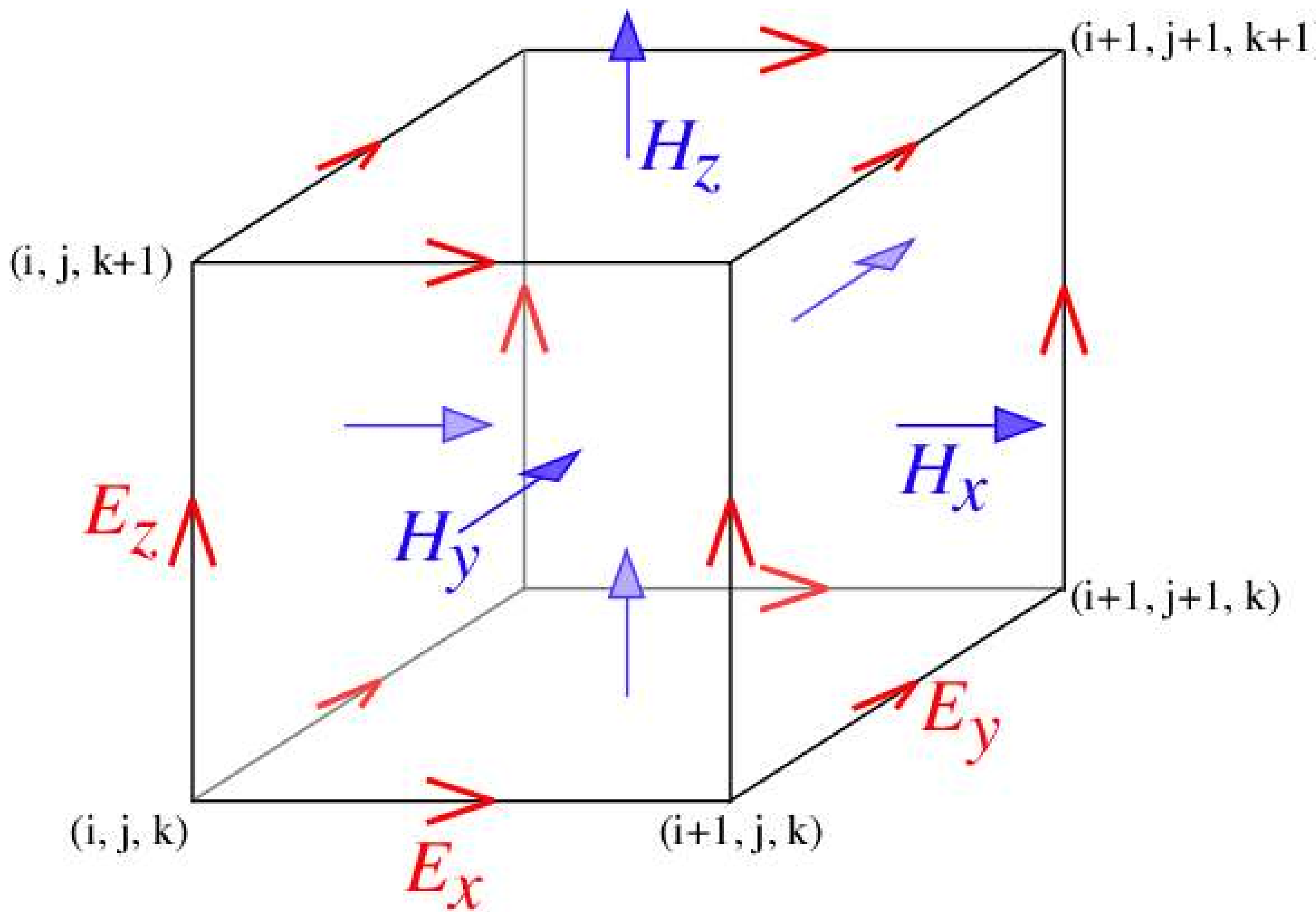
# Transmittance: one-defect



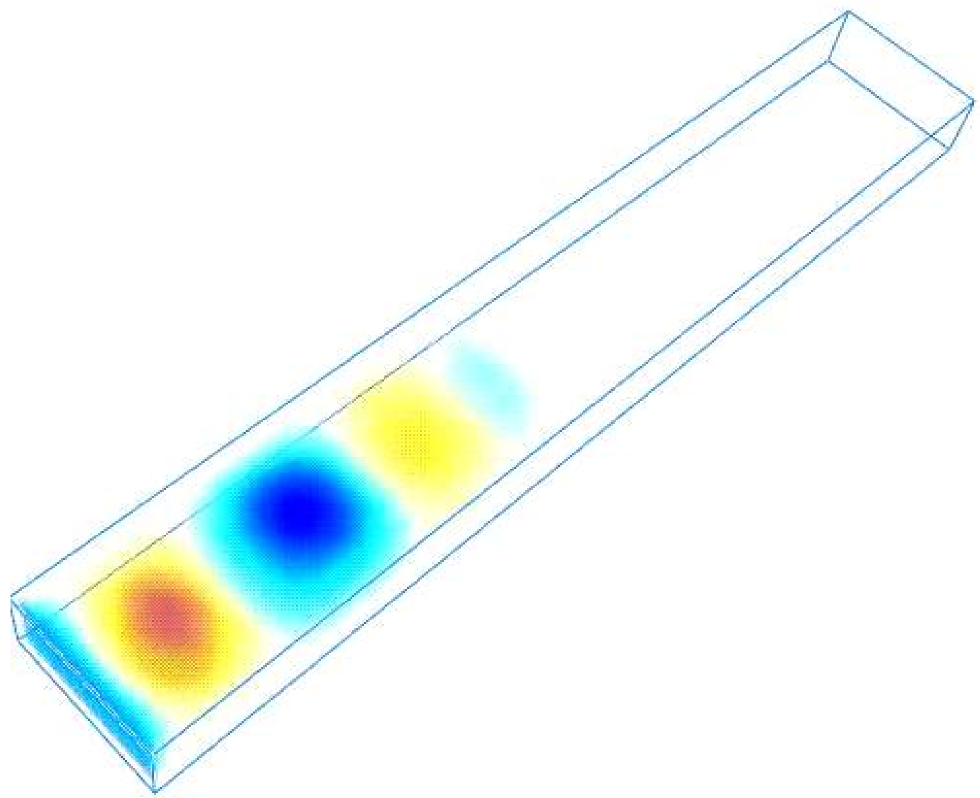
# Localized field: one-defect



# Yee's algorithm



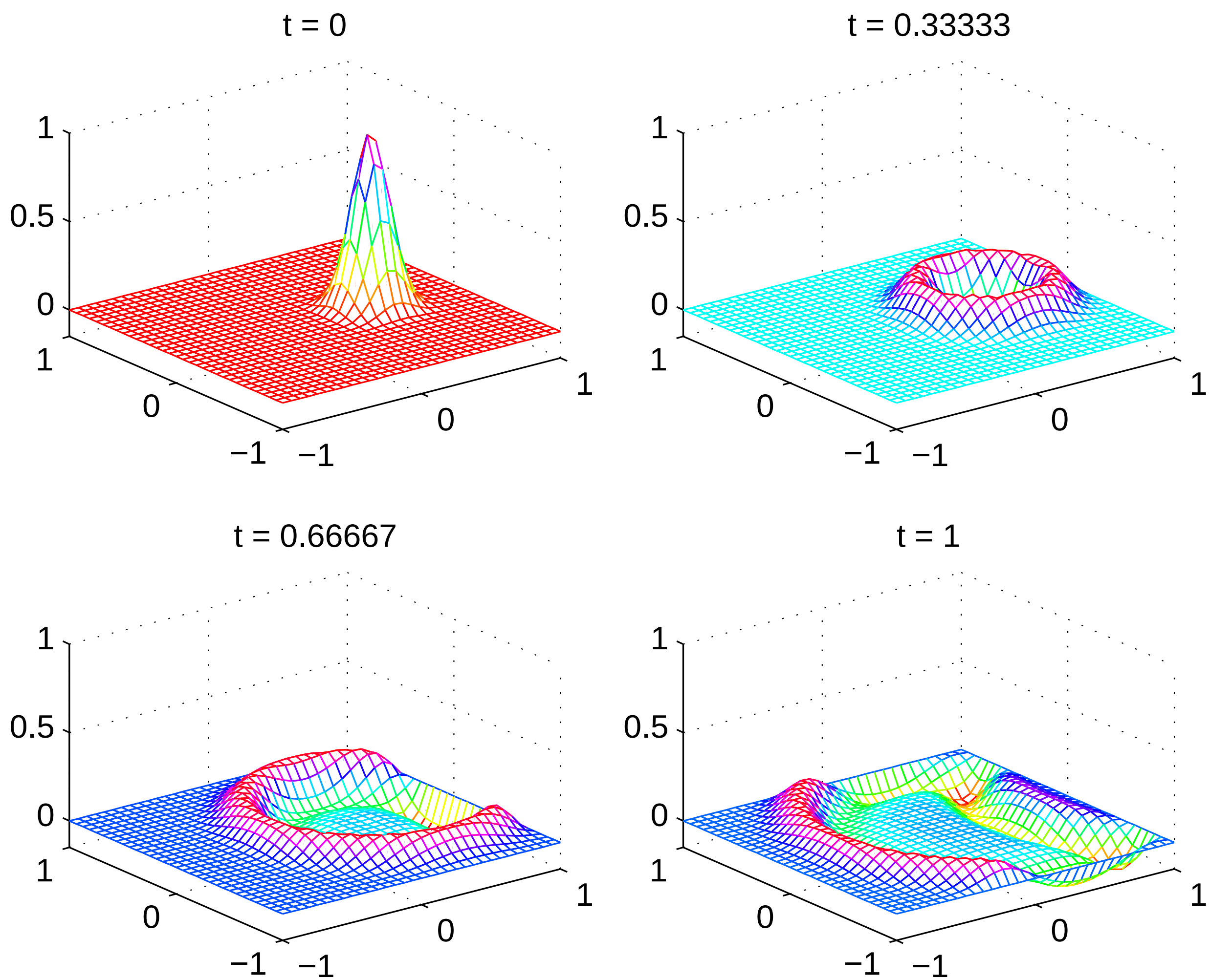
# Metallic Waveguide



by ToyFDTD

# FFT method for wave equation

$$u_{tt} = u_{xx} + u_{yy}, \qquad -1 < x, y < 1, \quad t > 0, \quad u = 0 \quad \text{on the boundary}$$



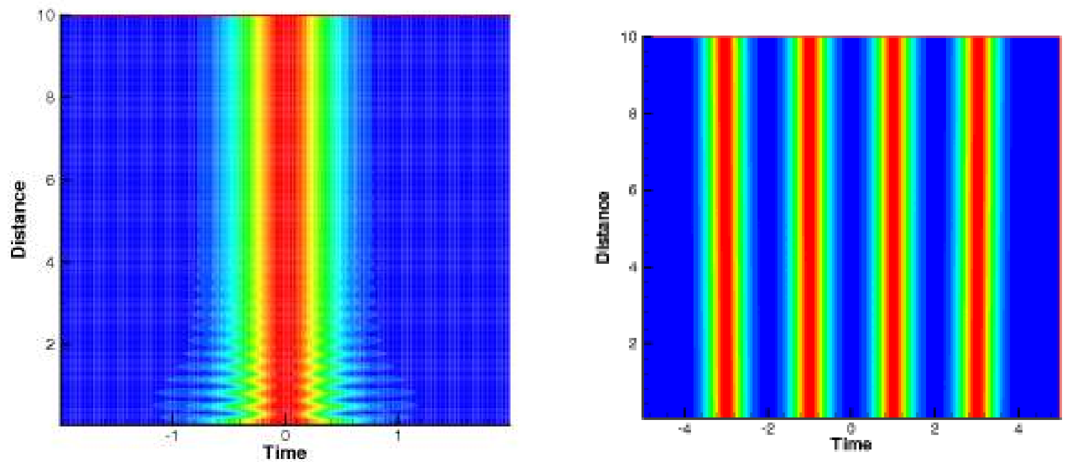
# Bound solitons in CGLE

Complex Ginzburg-Landau Equation:

$$
\begin{aligned}
iU_z + \frac{D}{2}U_{tt} + |U|^2U &= i\delta U + i\epsilon|U|^2U + i\beta U_{tt} \\
&+ i\mu|U|^4U - v|U|^4U,
\end{aligned}
$$

seek for bound-state solutions by propagation method.

$$
U(z,t) = \sum^{N} U_0(z, t+\rho_j)e^{i\theta_j}
$$

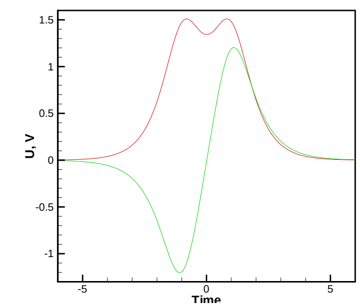



R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004).

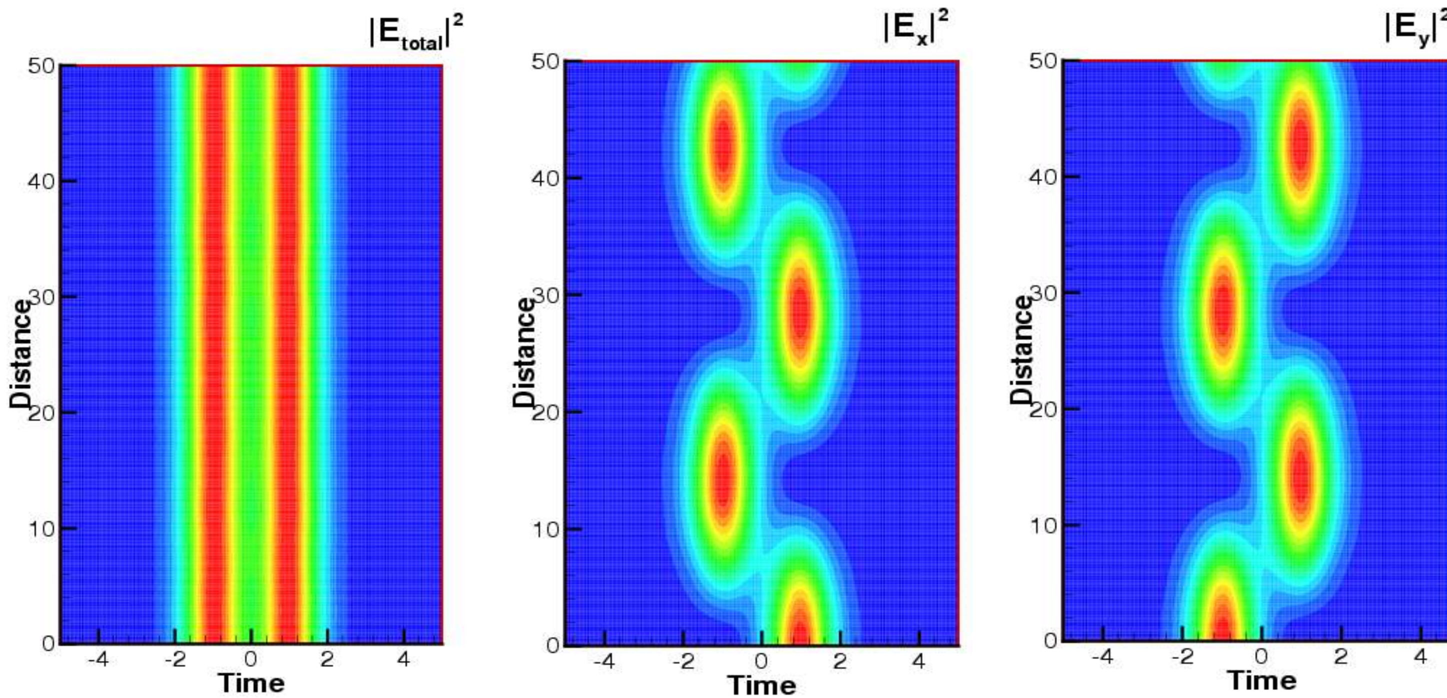
# Vector bound solitons

## Coupled Nonlinear Schrödinger Equations:

$$
\begin{aligned}
i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2U + B|V|^2U &= 0 \\
i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2V + B|U|^2V &= 0
\end{aligned}
$$




where $A = 1/3$, $B = 2/3$; and $U$, $V$ are circular polarization fields.



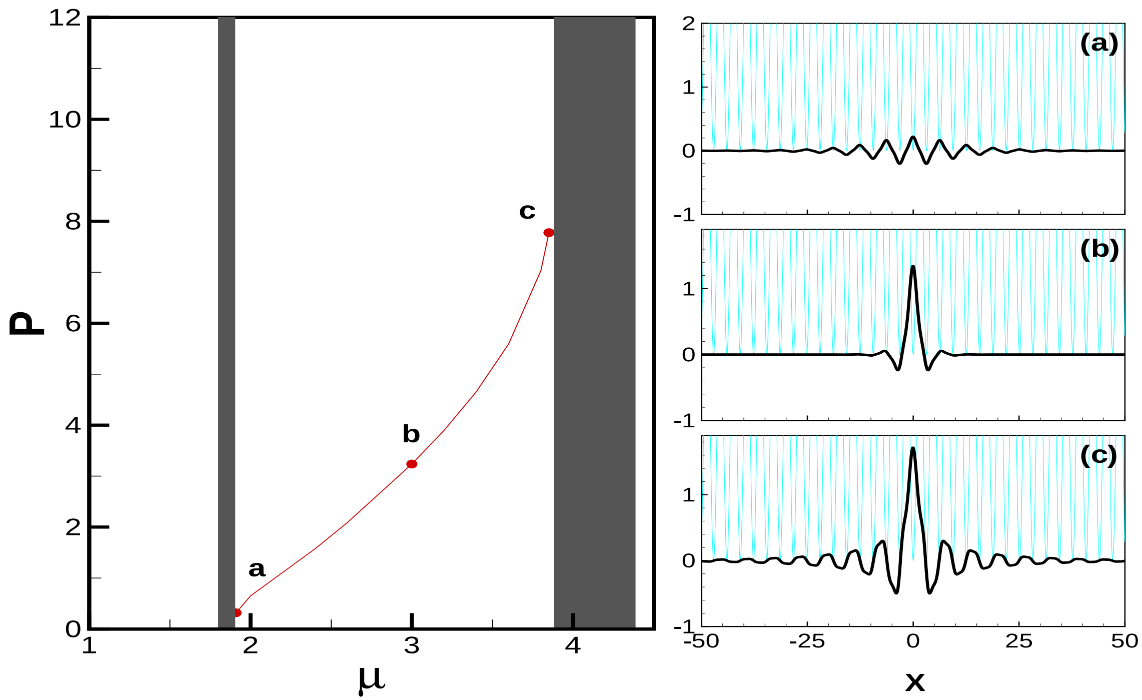

M. Haelterman, A. P. Sheppard, and A. W. Snyder, *Opt. Lett.* **18**, 1406 (1993).

# Gap solitons in optical lattices

1-D Gross-Pitaevskii equation with periodic potentials, $V(x) = V_0 \sin^2(k_0 x)$,

$$i\hbar \frac{\partial}{\partial t}\Phi_0(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi_0(t,x) + V(x)\Phi_0(t,x) + g_{1D}|\phi_0(t,x)|^2\phi_0(t,x)$$

which has gap soliton solutions.




by FS + NK methods, $N_x$ = 512, no. of iterations $< 10$; gpeol.m.

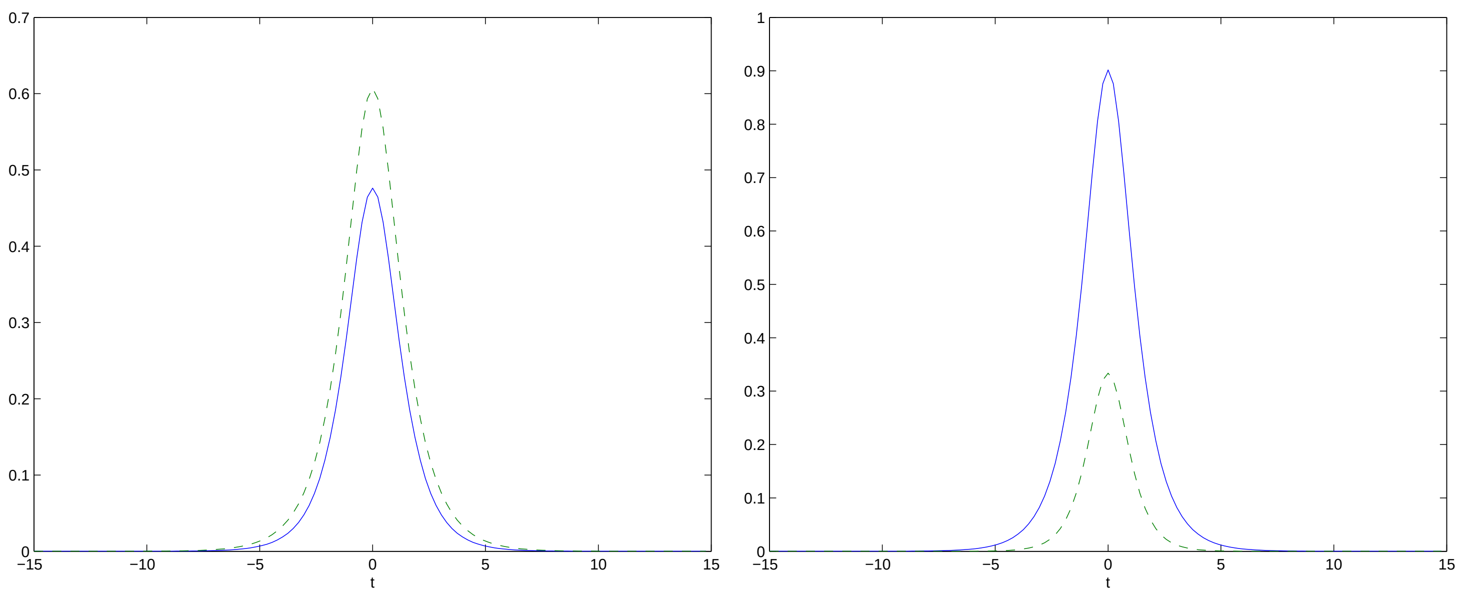
R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72**, 033607 (2005).

# Two-channel BPW solitons

For $N = 2$ ($u_0$ and $u_1$),

$$\frac{1}{2}\frac{d^2u_0}{dt^2} + (u_0^2 + 2\gamma_1 u_1^2)u_0 = \frac{1}{2}u_0,$$

$$\frac{\alpha_1}{2}\frac{d^2u_1}{dt^2} + (\gamma_1|u_1|^2 + 2\gamma_0|u_0|^2)u_1 = \lambda u_1.$$

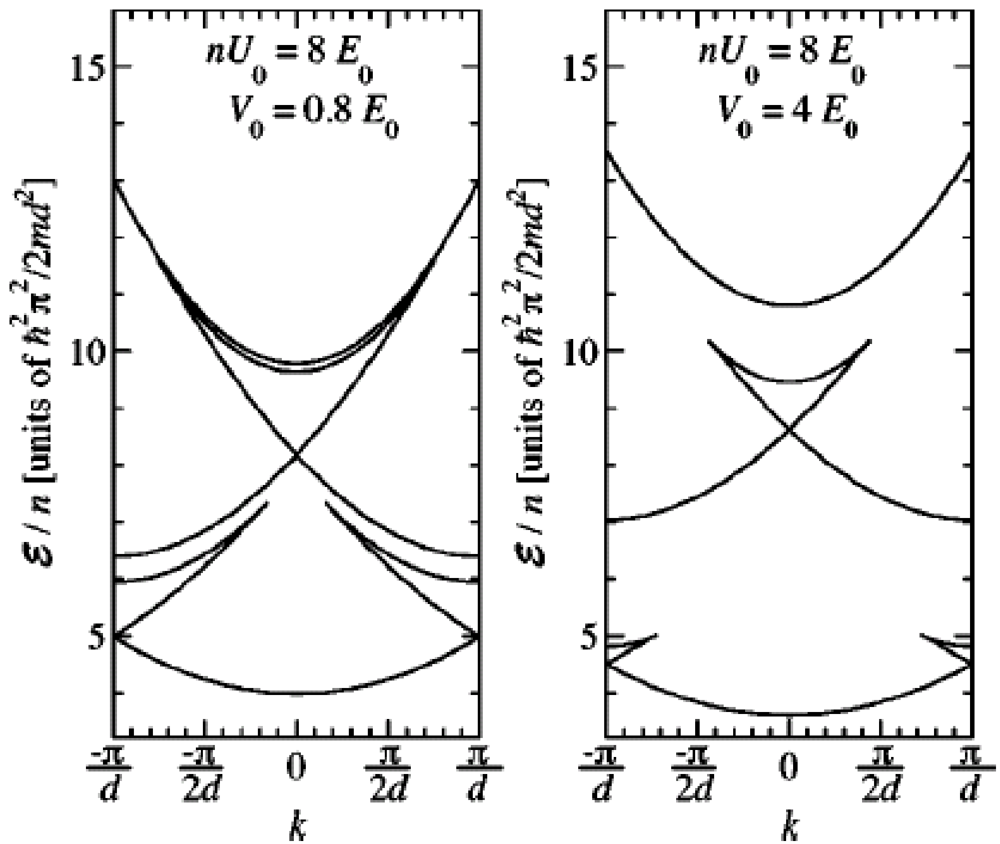



Left: $\lambda = 0.4$, Right $\lambda = 1.0$; solid-line: $u_0$, dashed-line: $u_1$;

by FS + NK methods, $N_x$ = 128, no. of iterations = $9$, $(16)$ for $\lambda = 0.4$, $(1.0)$; shepherd.m.

# Nonlinear band structure for BEC in OL

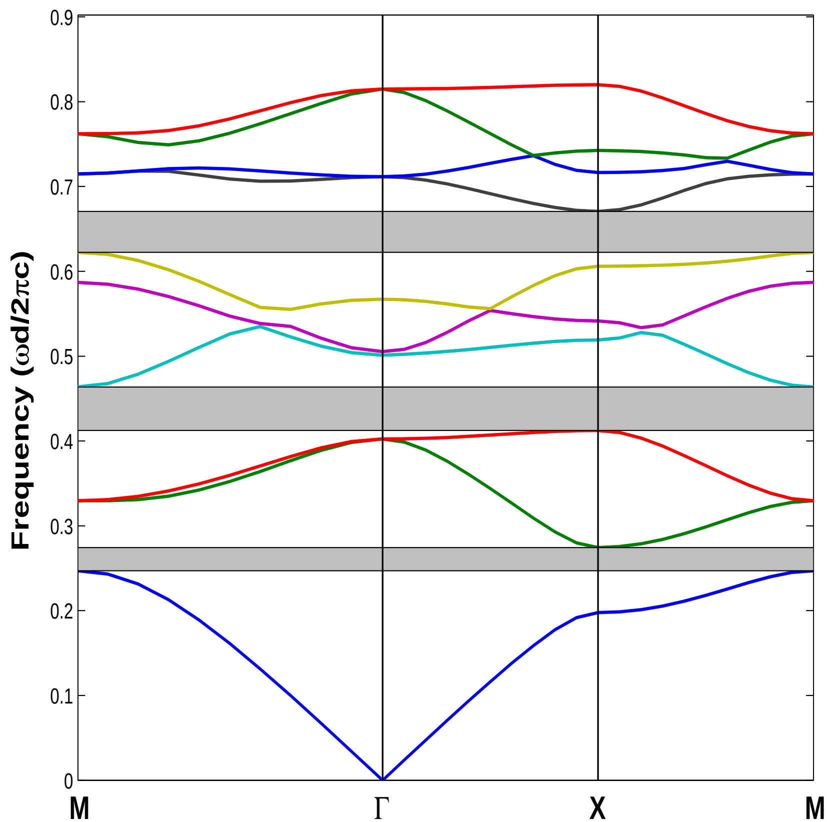
$$\frac{1}{2}\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} - [V_0 \sin^2(Kx)\phi - \mu\phi] - \sigma|\phi|^2\phi = 0.$$

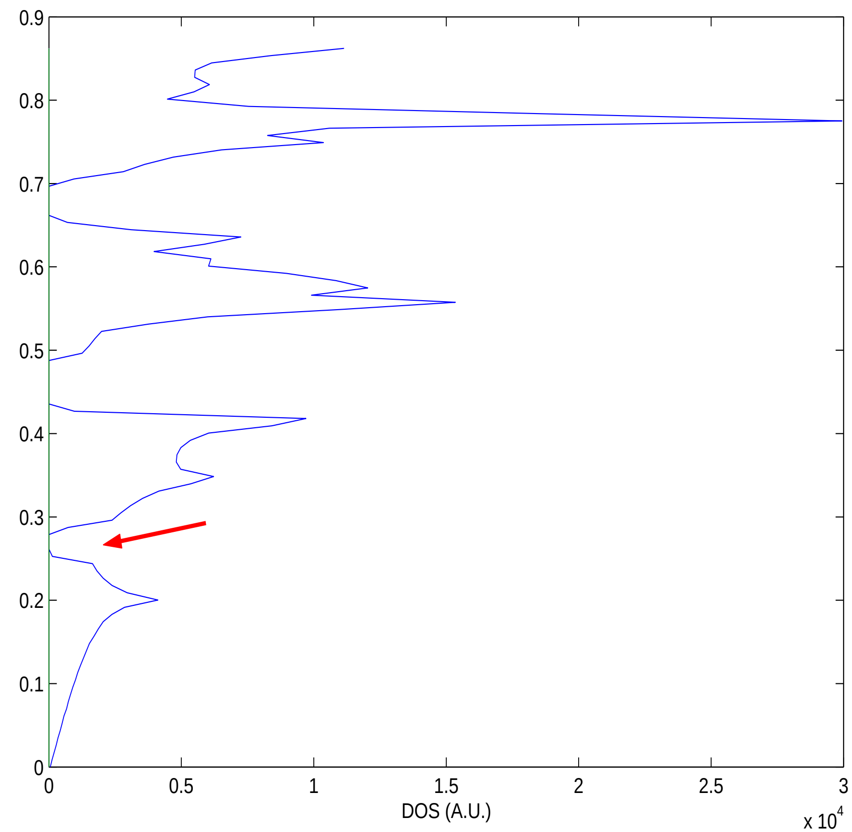



M. Machholm *et al.*, *Phys. Rev. A* **67**, 053613 (2003).

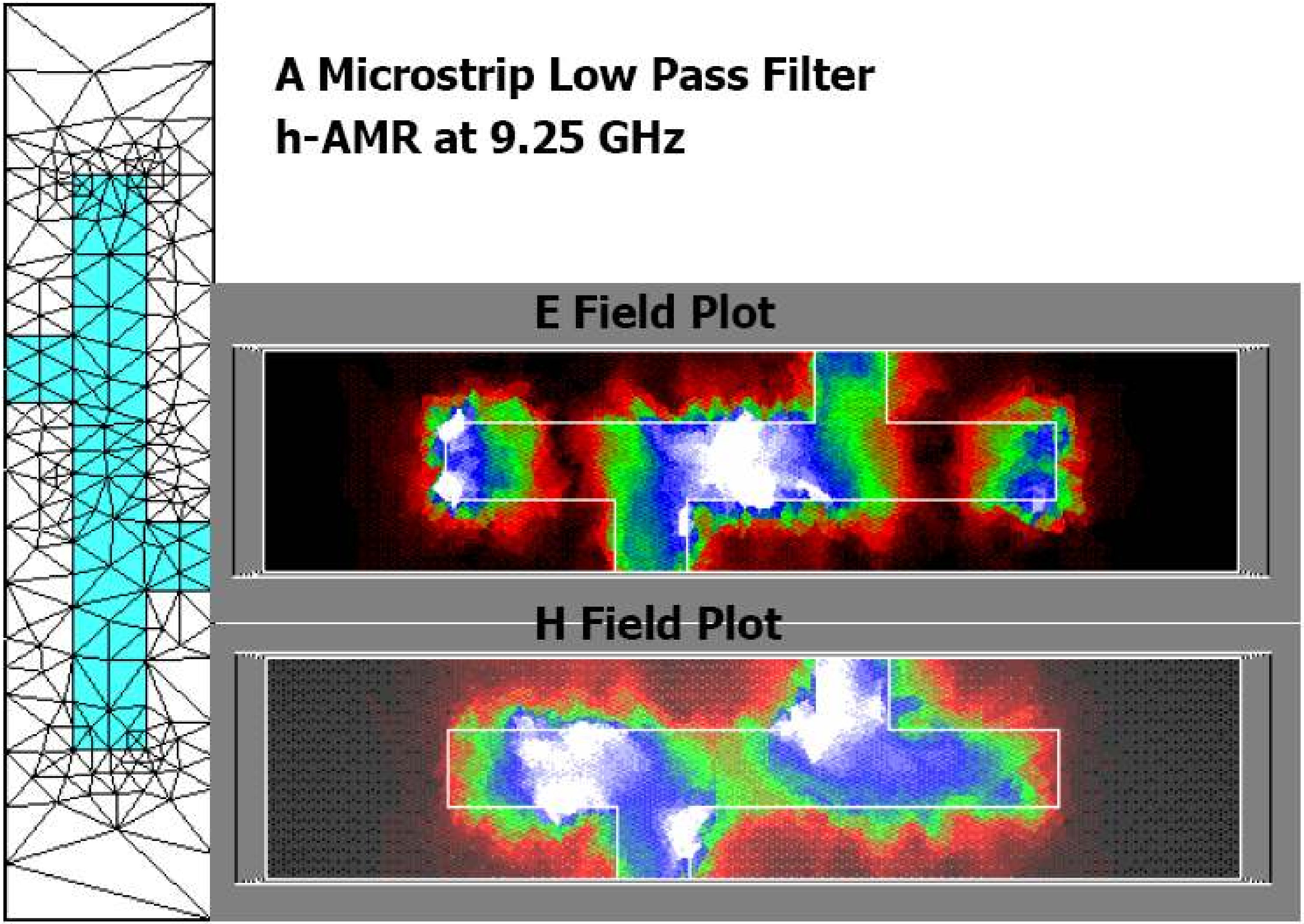
# Band diagram and Density of States

$$\frac{1}{\epsilon(\mathbf{r})}\nabla \times \{\nabla \times \mathbf{E}(\mathbf{r})\} = \frac{\omega^2}{c^2}\mathbf{E}(\mathbf{r}),$$

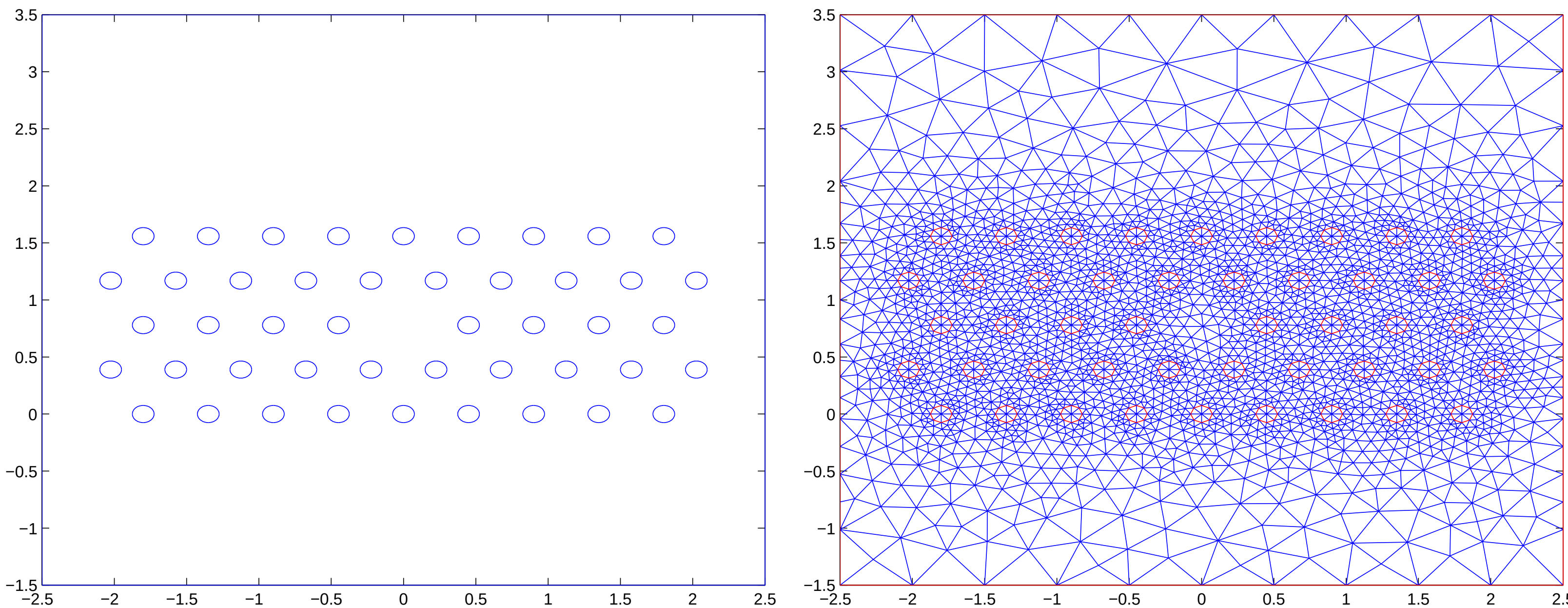


# Microwave circuit



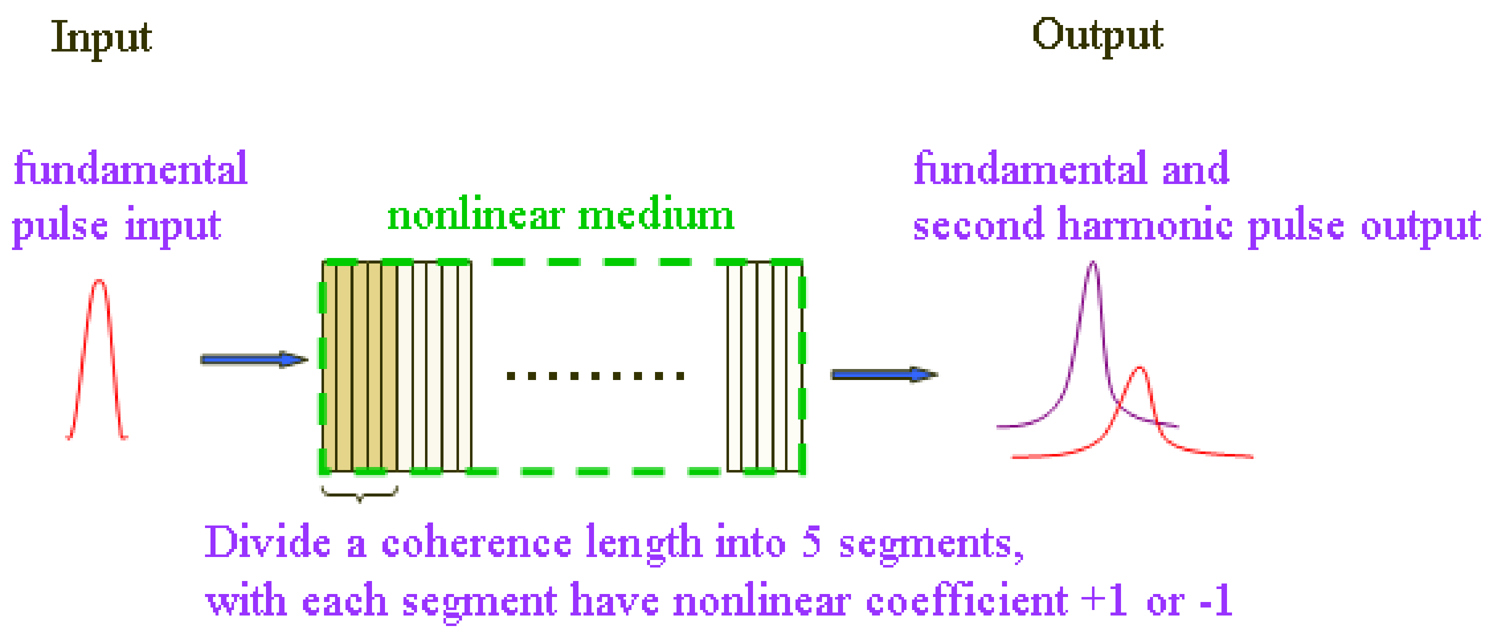
# Photonic Crystals



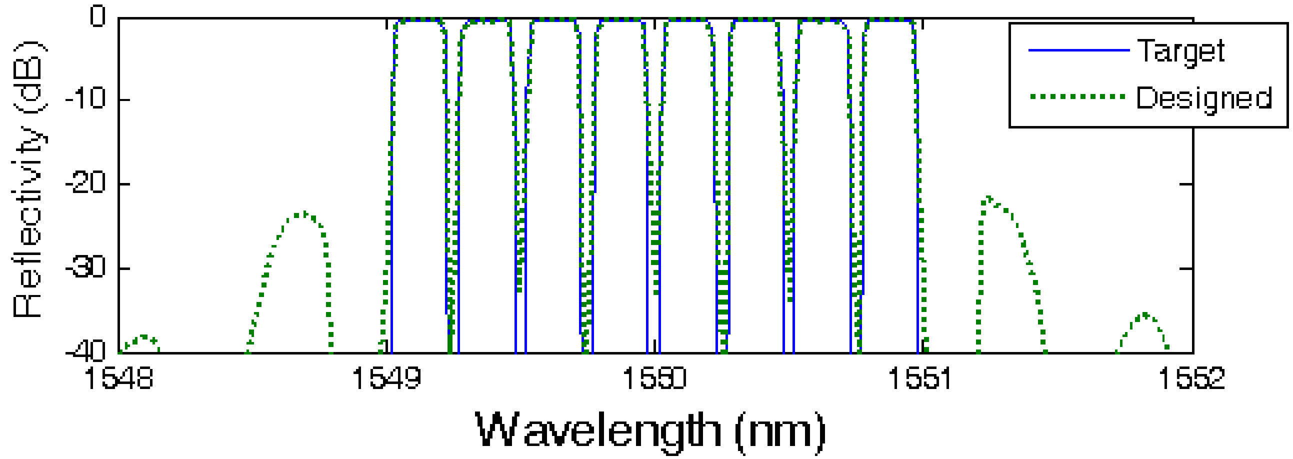
# Optimization of SHG pulse

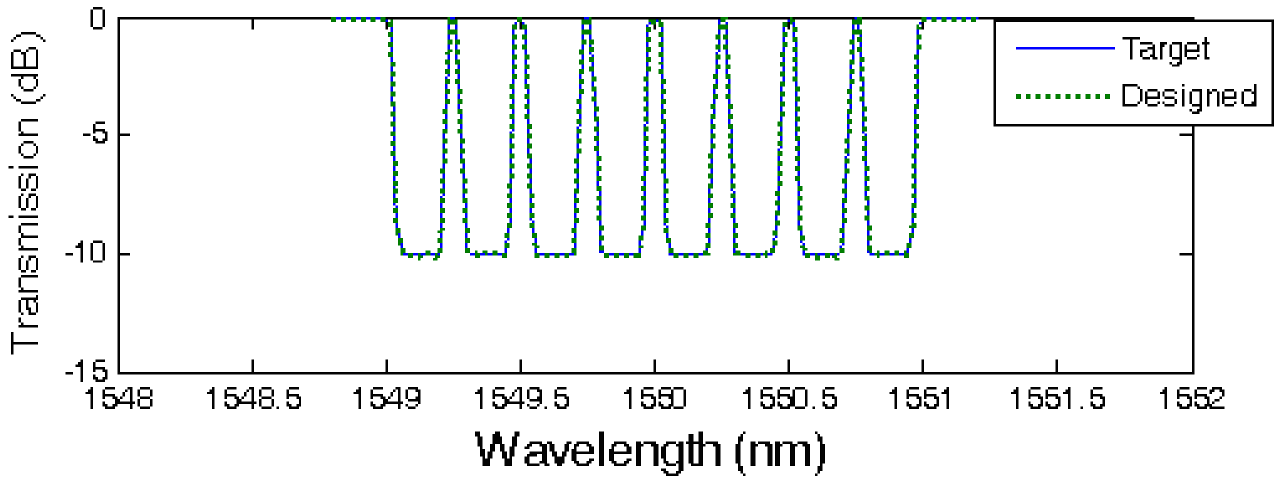
$$\frac{\partial A}{\partial z} = \frac{\eta}{2}\frac{\partial A}{\partial T} + i\xi_1\frac{\partial^2 A}{\partial T^2} - i\rho_1 A^* B,$$

$$\frac{\partial B}{\partial z} = -\frac{\eta}{2}\frac{\partial B}{\partial T} + i\xi_2\frac{\partial^2 A}{\partial T^2} - i\Delta k B - i\rho_1 A^2,$$



# Lagrange Multiplier method for FBG

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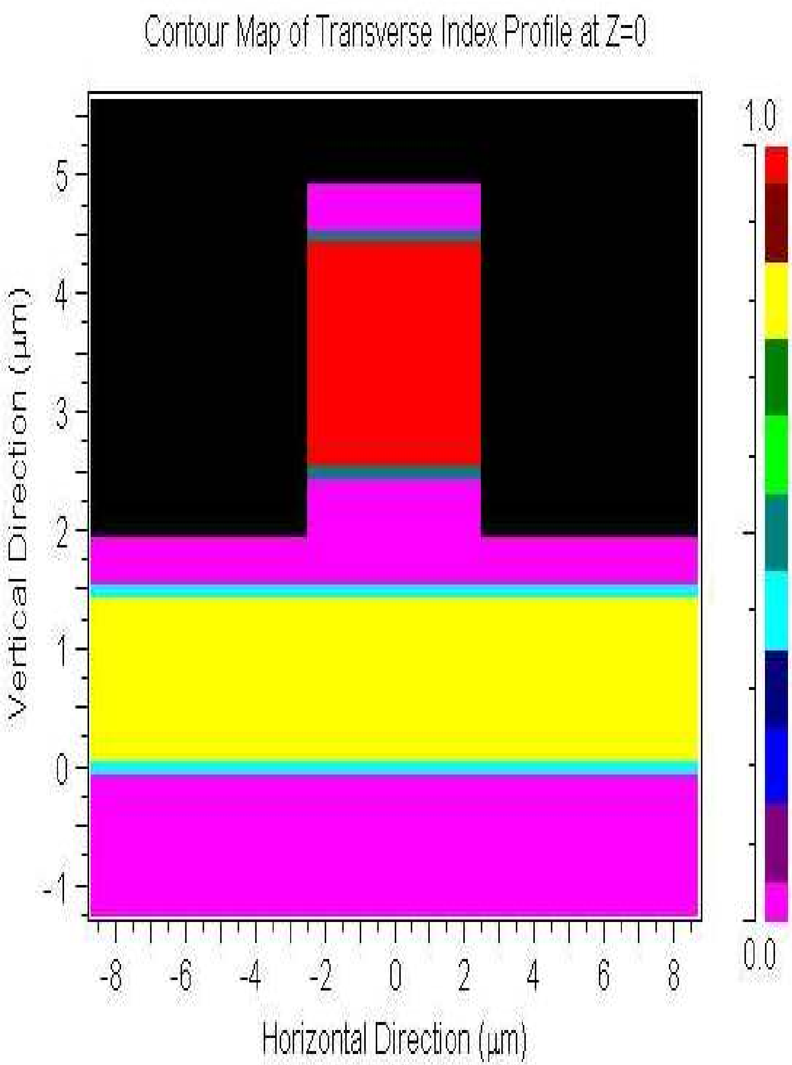
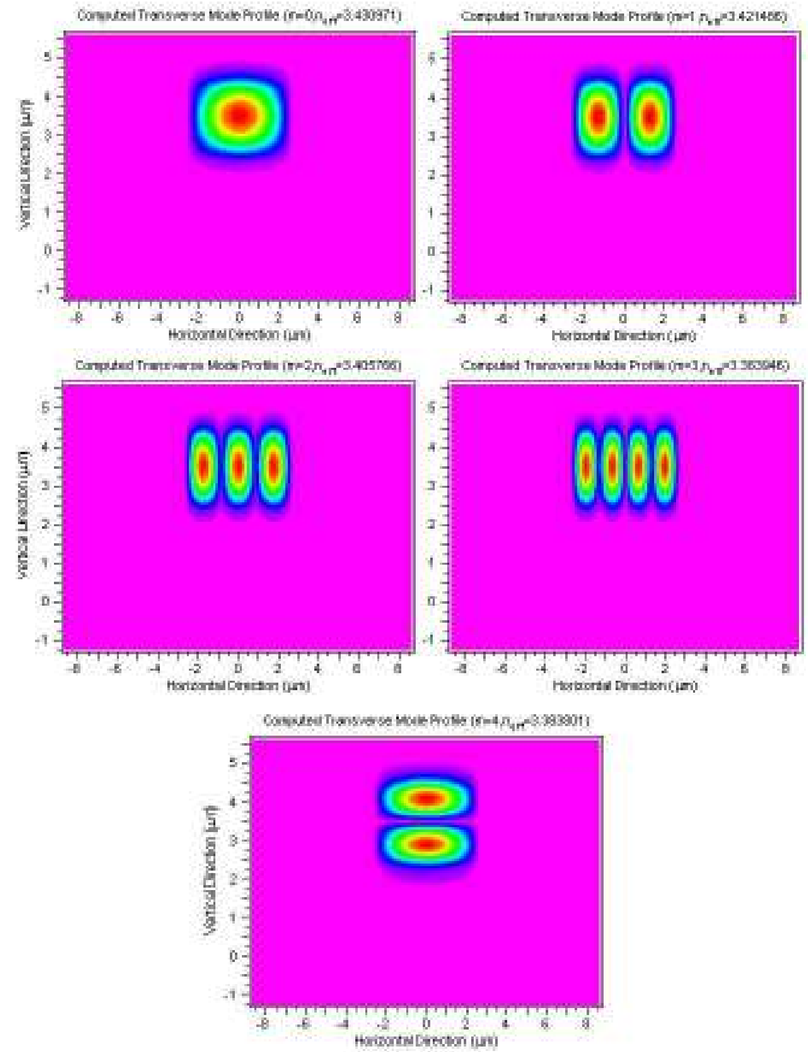
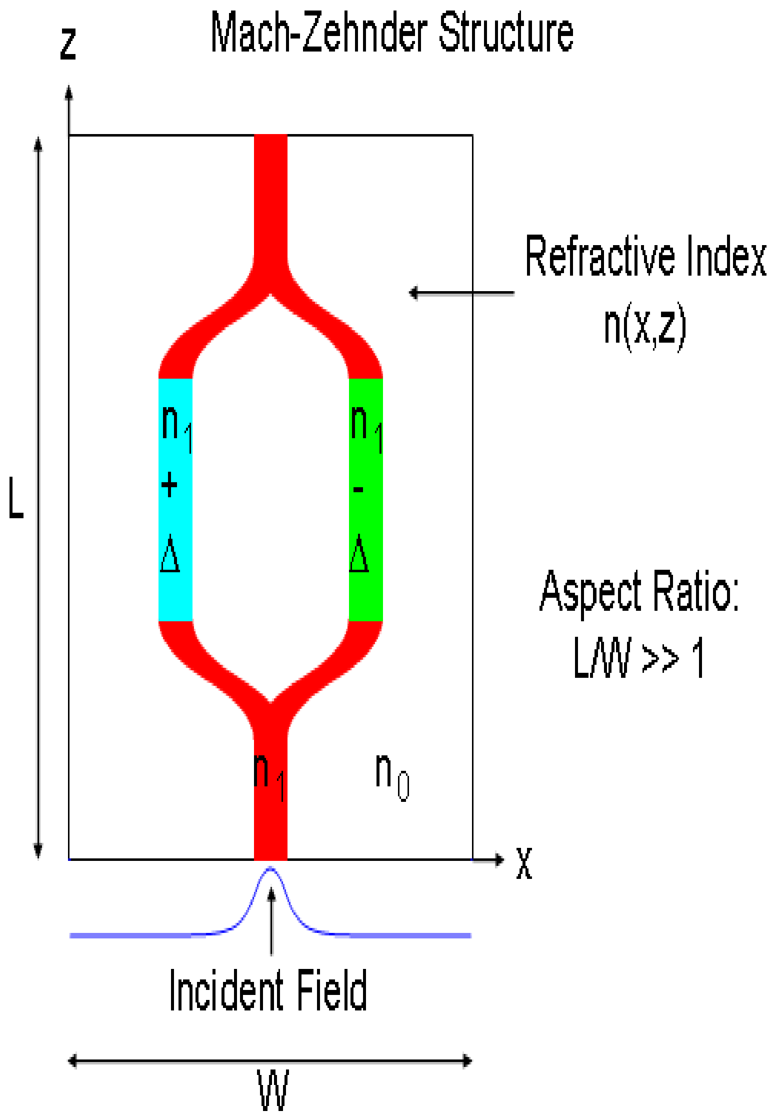
# Waveguide structures

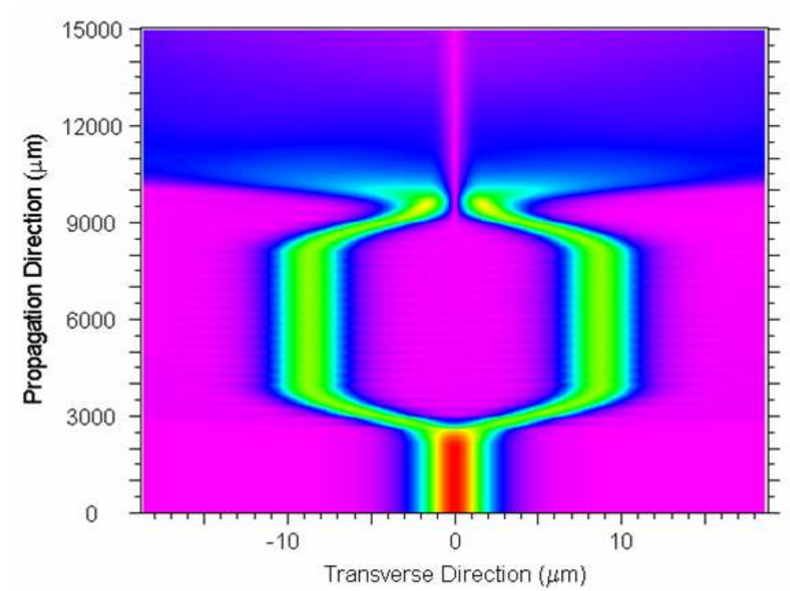


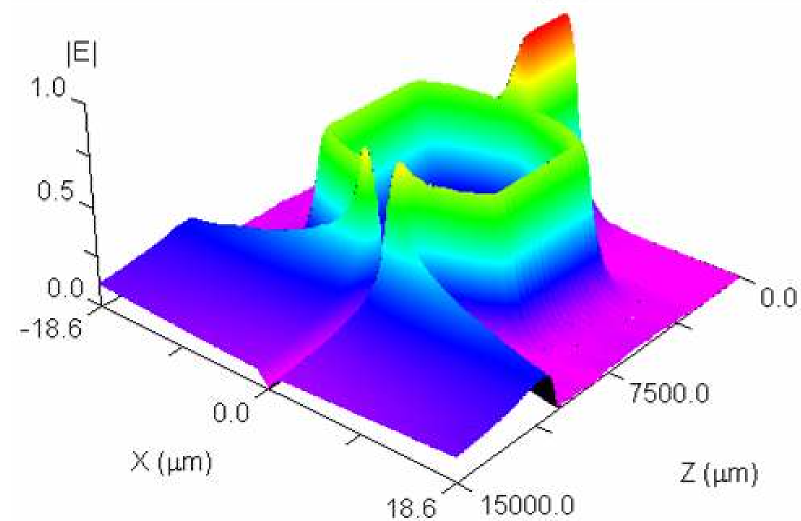



by BeamProp

# Mach-Zehnder structure



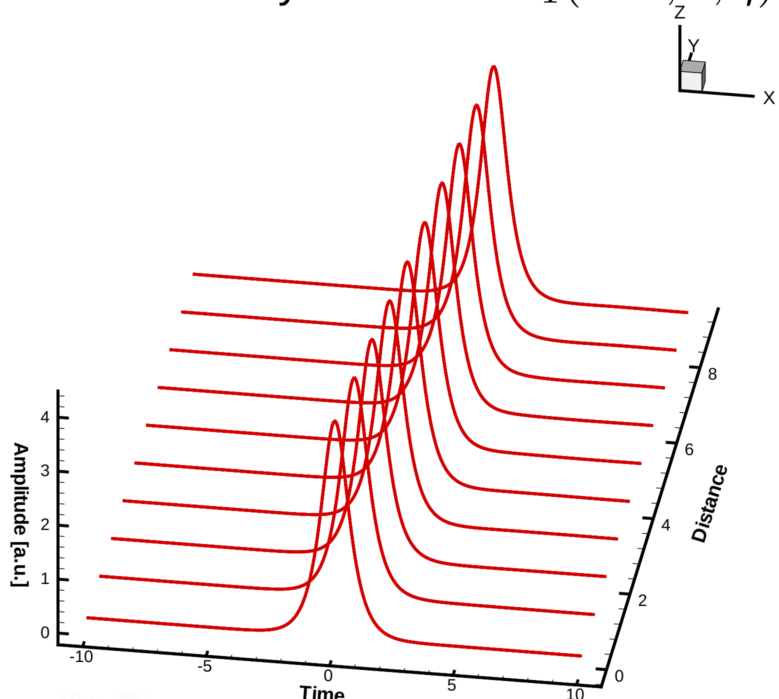



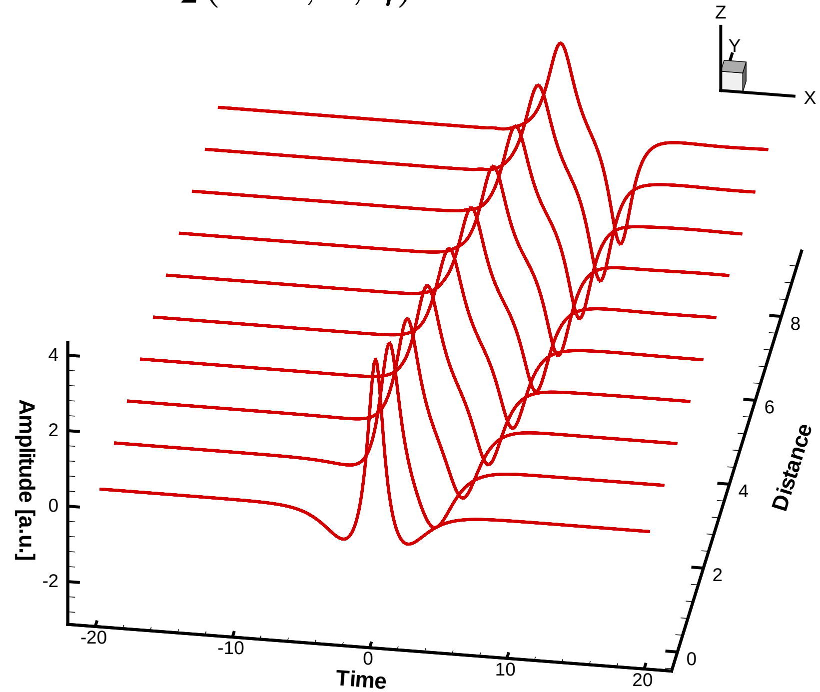

by BeamProp

# $2\pi$ and $0\pi$ solitons in Self-Induce Transparency media

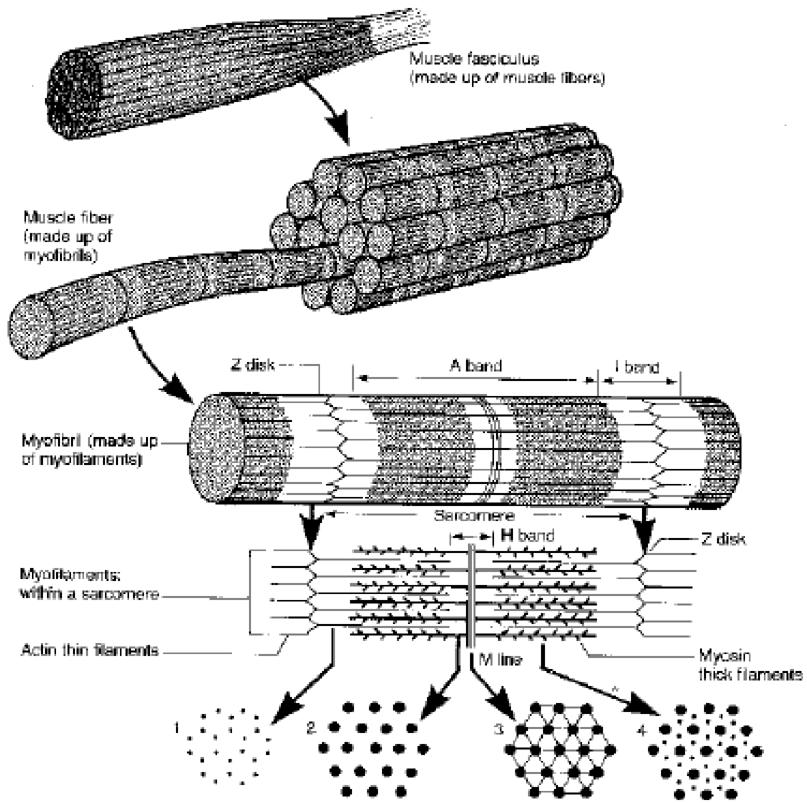
$$\frac{\partial}{\partial t}V_1(x,t,\eta) + i\eta V_1(x,t,\eta) = \frac{1}{2}U(x,t)V_2(x,t,\eta),$$
$$\frac{\partial}{\partial t}V_2(x,t,\eta) - i\eta V_2(x,t,\eta) = -\frac{1}{2}U^*(x,t)V_1(x,t,\eta),$$
$$\frac{\partial}{\partial x}U(x,t) = 2 < V_2^*(x,t,\eta)V_1(x,t,\eta) >,$$

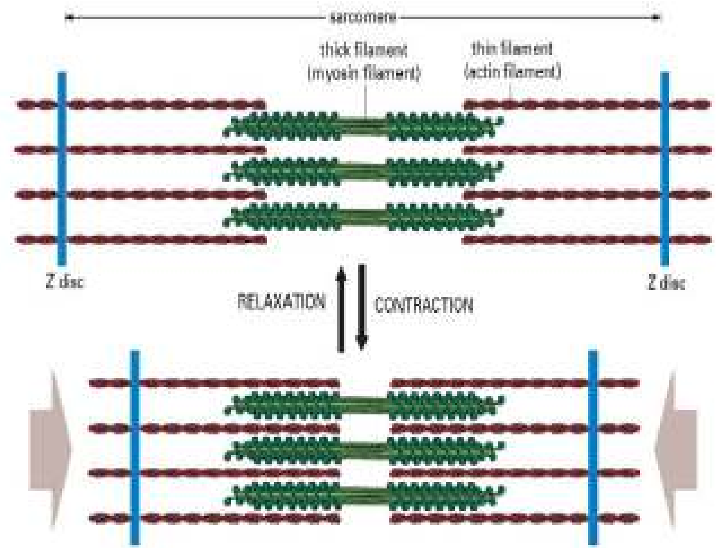
with the boundary condtions $V_1(-\infty, x, \eta) = 1$ and $V_2(-\infty, x, \eta) = 0$.

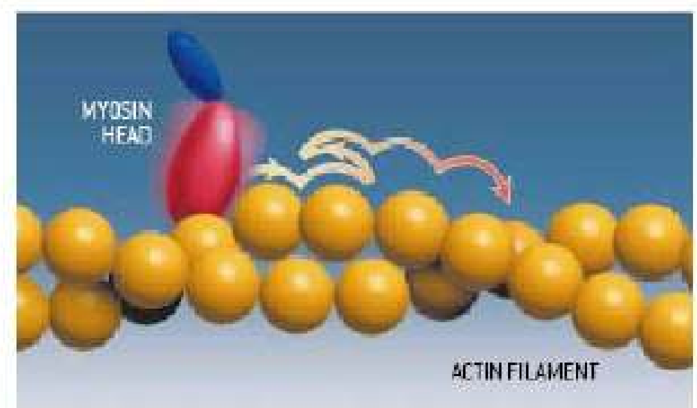


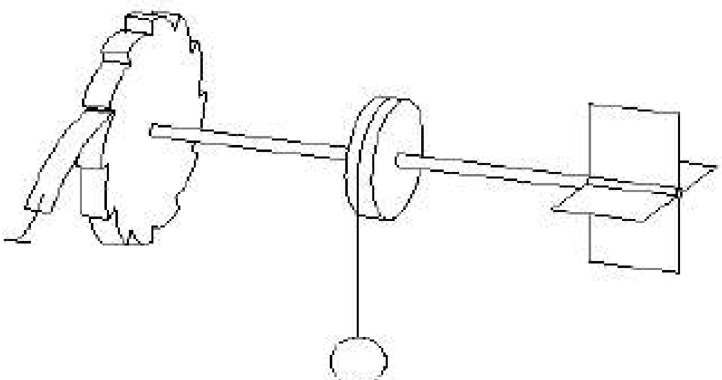
# Muscle Contraction: Myosin

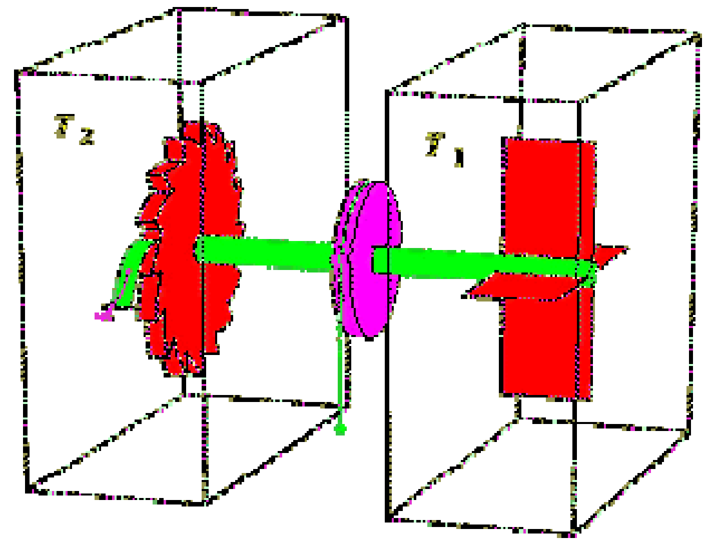


# Ratchets






**thought experiment** of a perpetual mobile against the 2nd Law of Thermodynamics

R. P. Feynman, *The Feynman Lectures on Physics*, Vol. 1, Chap. 46 (1963).

# Mathematical Model for Ratchets

**Ratchet ingredients:**

1. Brownian particle (mass $m$)
2. periodic asymmetric potential $V(x,t)$
3. zero mean driving forces $f(t)$, i.e. $\langle f(t)\rangle = 0$

**Ratchet model:** $m\ddot{x} + \gamma\dot{x} + \frac{d}{dx}V(x,t) = f(t)$ **Interesting**
**Behavior:** $\langle x(t)\rangle \neq 0$ even when $\langle f(t)\rangle = 0$