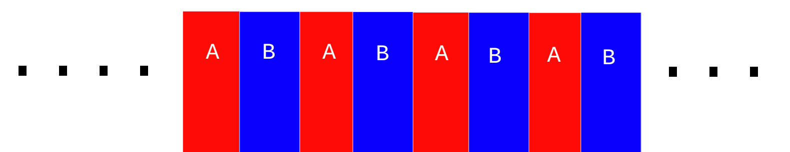
Boundary value problems: 1D Bragg reflector

coupled-mode equation:

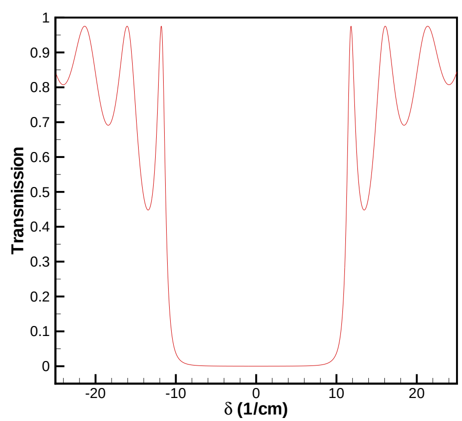
$$\begin{aligned}\frac{dE_+(z)}{dz} &= i\delta E_+(z) + i\kappa E_-(z)\\ \frac{dE_-(z)}{dz} &= -i\delta E_-(z) - i\kappa^* E_+(z)\end{aligned}$$

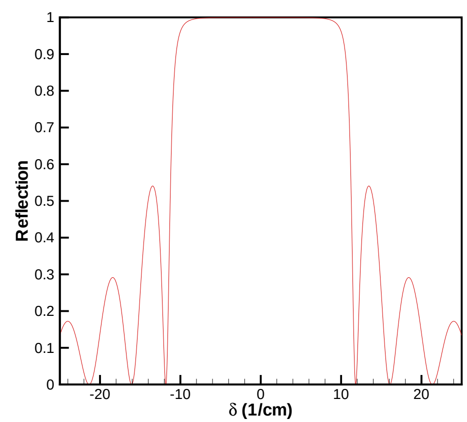
with the Boundary Condition:



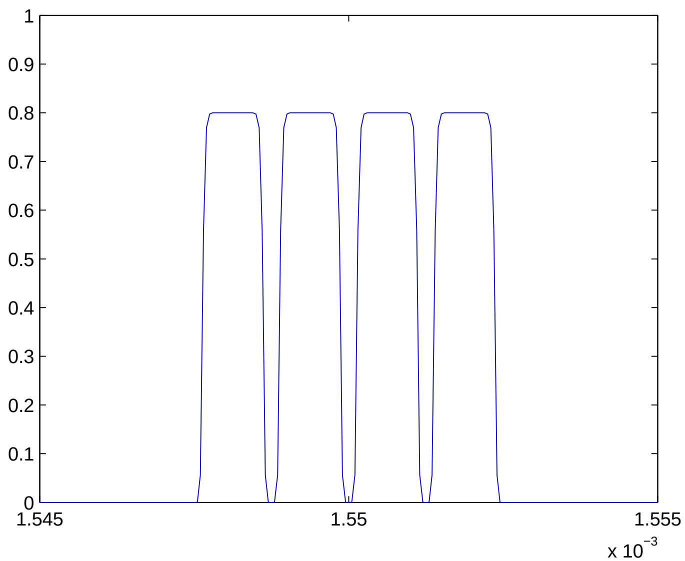
$$E_+(z=0) = 1$$

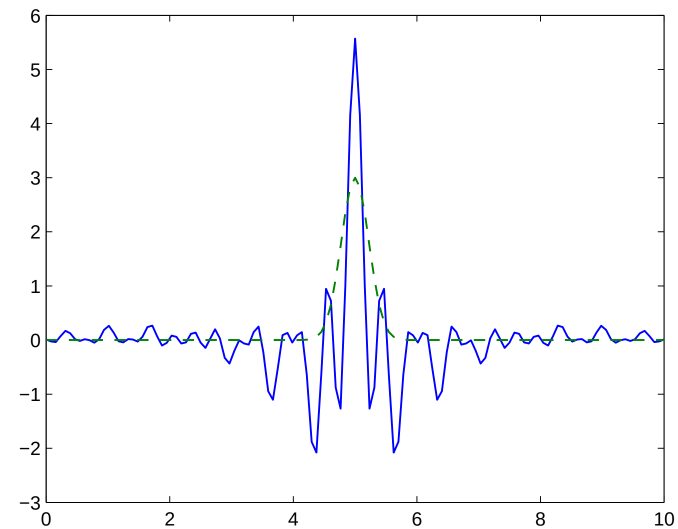
$$E_-(z=L) = 0$$





9, Optimization





- Simulated annealing
- Genetic algorithm
- Penalty function
- Optimal control method
- Matlab built-in routines

Unconstrained optimization

- Golden Search method
- Quadratic approximation method
- Nelder-Mead method
- Steepest Descent method
- Newton method
- Conjugate Gradient method
- Simulated annealing
- Genetic algorithm

Golden Search method

- Bisection method
- Golden Search method
  - Given (a, b, c), where b is a fraction w of the way between a and c,

$$\frac{b-a}{c-a} = w, \qquad \frac{c-b}{c-a} = 1 - w$$

  - Our next trial point x is an additional fraction z beyond b,

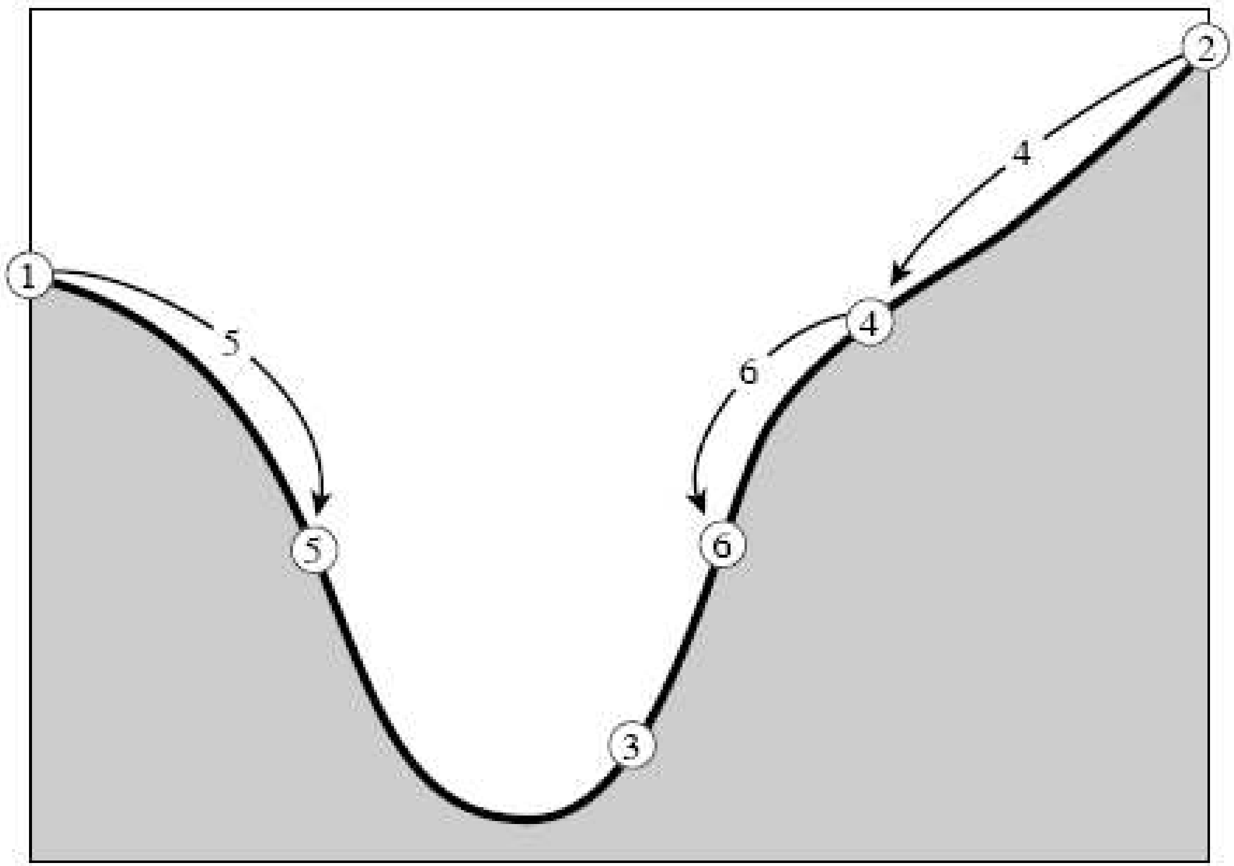
$$\frac{x-b}{c-a} = z,$$

    where $z = 1 - 2w$.
  - Apply the *scale similarity* for x, the same fraction of the way from b to c as b from a to c,

$$\frac{z}{1-w} = w,$$

    one has $w^2 - 3w + 1 = 0$, with the solution $w = \frac{3-\sqrt{5}}{2} \approx 0.38197$.

Golden Search method



Newton method

- By taking the Taylor series ofr a multi-variable objective function, say two-variables,

$$f(\mathbf{x}) \approx f(\mathbf{x}_k) + \nabla f(\mathbf{x})^T|_{\mathbf{x}_k}[\mathbf{x}-\mathbf{x}_k] + \frac{1}{2}[\mathbf{x}-\mathbf{x}_k]^T \nabla^2 f(\mathbf{x})^T|_{\mathbf{x}_k} + \ldots$$
$$\approx f(\mathbf{x}_k) + \mathbf{g}_k^T[\mathbf{x}-\mathbf{x}_k] + \frac{1}{2}[\mathbf{x}-\mathbf{x}_k]^T \mathbf{H}_k [\mathbf{x}-\mathbf{x}_k]$$

where

the gradient vector $\quad \mathbf{g}_k = \nabla f(\mathbf{x})|_{\mathbf{x}_k},$

the Hessian matrix $\quad \mathbf{H}_k = \nabla^2 f(\mathbf{x})|_{\mathbf{x}_k}.$

- The Newton method tries to go straight to the zero of the gradient of the approximate objective function,

$$\mathbf{g}_k + \mathbf{H}_k[\mathbf{x}-\mathbf{x}_k] = 0, \qquad \mathbf{x} = \mathbf{x}_k - \mathbf{H}_k^{-1}\mathbf{g}_k,$$

- By the updating rule

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{H}_k^{-1}\mathbf{g}_k.$$

Newton method: example

- For the objective function,

$$f(\mathbf{x}) = f(x_1, x_2) = x_1^2 - x_1x_2 - 4x_1 + x_2^2 - x_2,$$

- the gradient function

$$\mathbf{g}(\mathbf{x}) = \nabla f(\mathbf{x}) = [\frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2}]^T = [2x_1 - x_2 - 4 \quad 2x_2 - x_1 - 1]^T$$

- the Hessian matrix

$$\mathbf{H}_k = \nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix},$$

- by the iteration rule

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{H}_k^{-1}\mathbf{g}_k,$$

- with the initial guess $\mathbf{x}_0 = [0 \quad 0]^T$, one has the solution for the minimum point $\mathbf{x}_{k+1} = [3 \quad 2]^T$ within a few iterations.

Simulated Annealing method

- All of the optimization methods discussed so far only apply for local extreme, not for global extreme.
- Annealing is the physical process of heating up a solid metal aboe its melting point,
- and then cooling it down so slowly that the highly excited atoms can settle into a (global) minimum energy state, yielding a single crystal with a regular structure.
- Fast cooling by rapid quenching may result in widespread irregularities and defects in the crystal structure, analogous to being too hasty to find the global minimum.

Simulated Annealing method

- The simulated annealing process can be implemented using the Boltzmann probability distribution of an energy level $E \geq 0$ at temperature T described by

$$p(E) = \alpha \text{exp}(-E/KT)$$

with the Boltzmann constant K and $\alpha = 1/KT$.

- At high temperature the probability distribution curve is almost flat over a wide range of E, implying that the system can be in a high energy state as equally well as in a low energy state.
- At low temperature the probability distribution curve gets higher/lower for lower/higher E, implying that the system will most probably be in a low energy state,
- but still have a slim chance to be in a high energy state so that it can escape from the local minimum energy state.

Simulated Annealing method

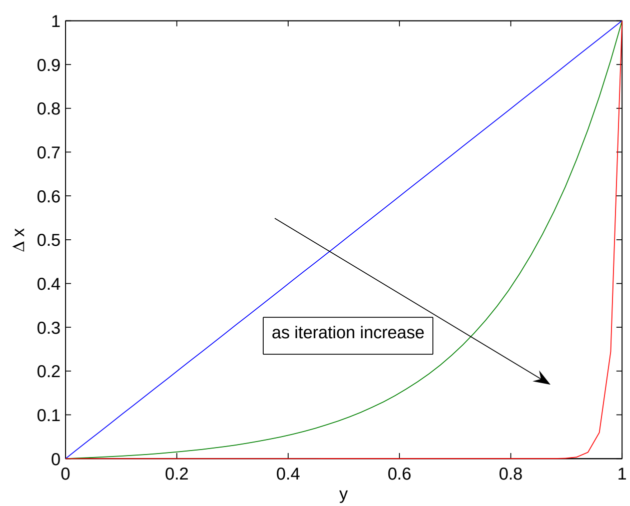
- Pick the initial guess $\mathbf{x}_0$,
- Generating a random vector $\mathbf{y}$ having uniform distribution $[-1,+1]$ and the same dimension as the variable $\mathbf{x}$, change the size of step $\Delta\mathbf{x}$ by,

$$\Delta\mathbf{x} = g_\mu^{-1}(\mathbf{y}), \qquad g_\mu^{-1}(\mathbf{y}) = \frac{(1+\mu)^{|y|}-1}{\mu},$$

where

$$\mu = 10^{100(k/k_{\max})^q}.$$

The quenching factor $q > 0$ is made small/large for slow/fast quenching.



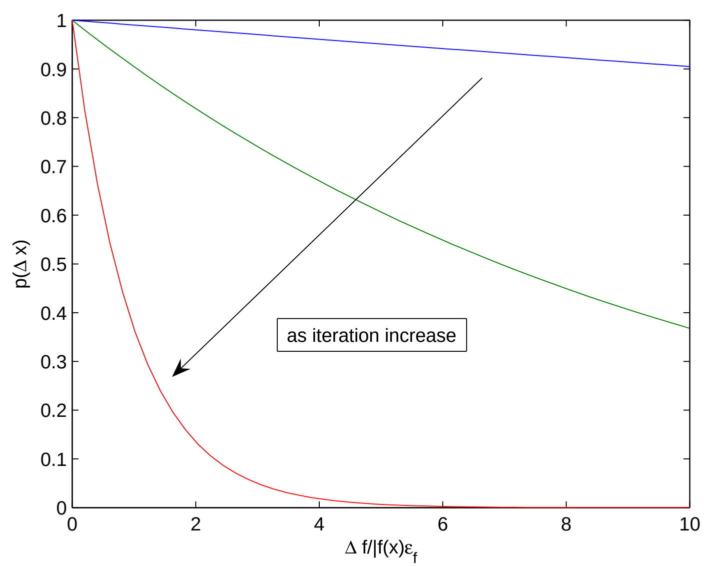
Simulated Annealing method

- As a selection $\Delta \mathbf{x}$ is analogue to the energy state.
- Like the Boltzmann distribution

$$p(E) = \alpha \text{exp}(-E/KT),$$

- one has,

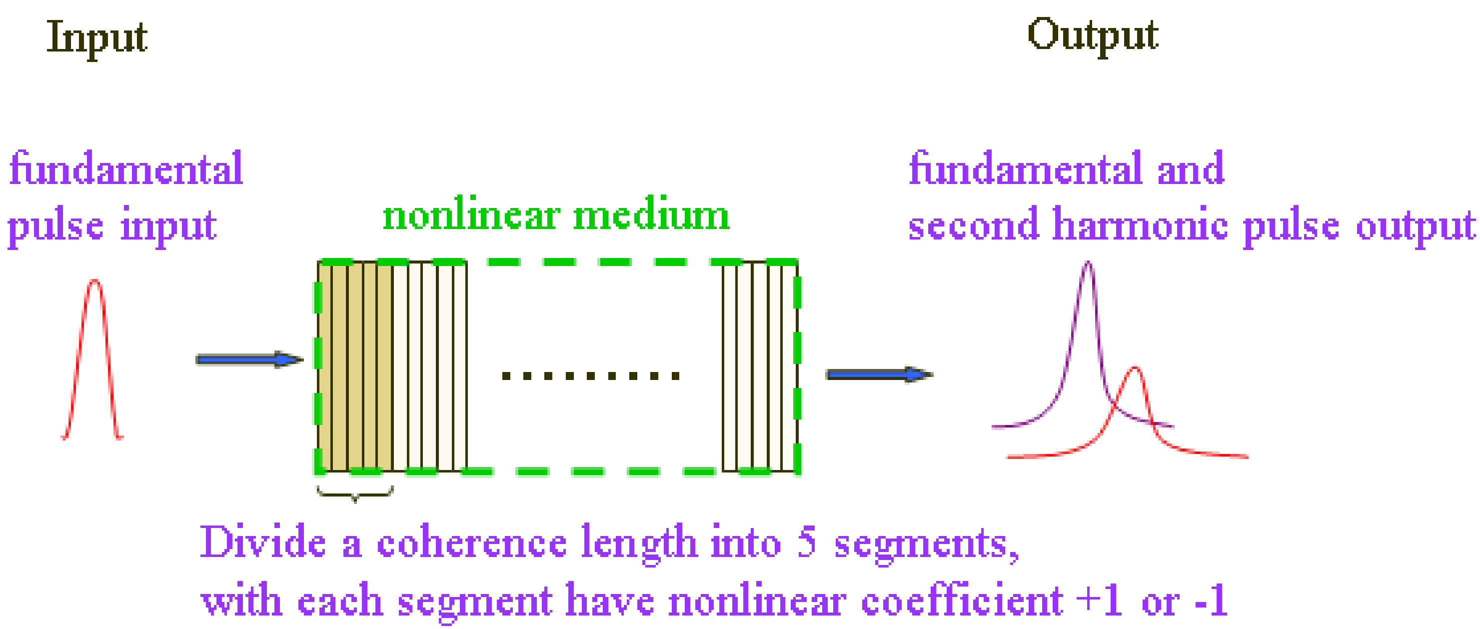
$$p(\Delta \mathbf{x}) = \text{exp}[-(\frac{k}{k_{\text{max}}})^q \frac{\Delta f}{|f(\mathbf{x})|\epsilon_f}].$$



Optimization of SHG pulse

$$\frac{\partial A}{\partial z} = \frac{\eta}{2}\frac{\partial A}{\partial T} + i\xi_1\frac{\partial^2 A}{\partial T^2} - i\rho_1 A^* B,$$

$$\frac{\partial B}{\partial z} = -\frac{\eta}{2}\frac{\partial B}{\partial T} + i\xi_2\frac{\partial^2 A}{\partial T^2} - i\Delta k B - i\rho_1 A^2,$$



Optimization of SHG pulse

At initial temperature T_0,
Given N sets of initial random structure (N binaries),
Calculate error function $\eta_0 = \eta = \int_{-\infty}^{+\infty} [B(\tau) - \text{target}(\tau)]^2 d\tau$

- Given random perturbation from N sets of recent structure,
  # of modified binaries = $(T/T_0) \times N$
- Calculate error function η and

  $probability = e^{-(\frac{(\eta-\eta_0)/\eta_0}{T})}$
- Create a random number "k".

  (1) Case 1: $\eta <= \eta_0$ or k < probability
  $\eta_0 = \eta$, Keep the recent structure as best structure
  (2) Case 2: $\eta > \eta$ 0
  ignore the recent structure

$T > T_f$

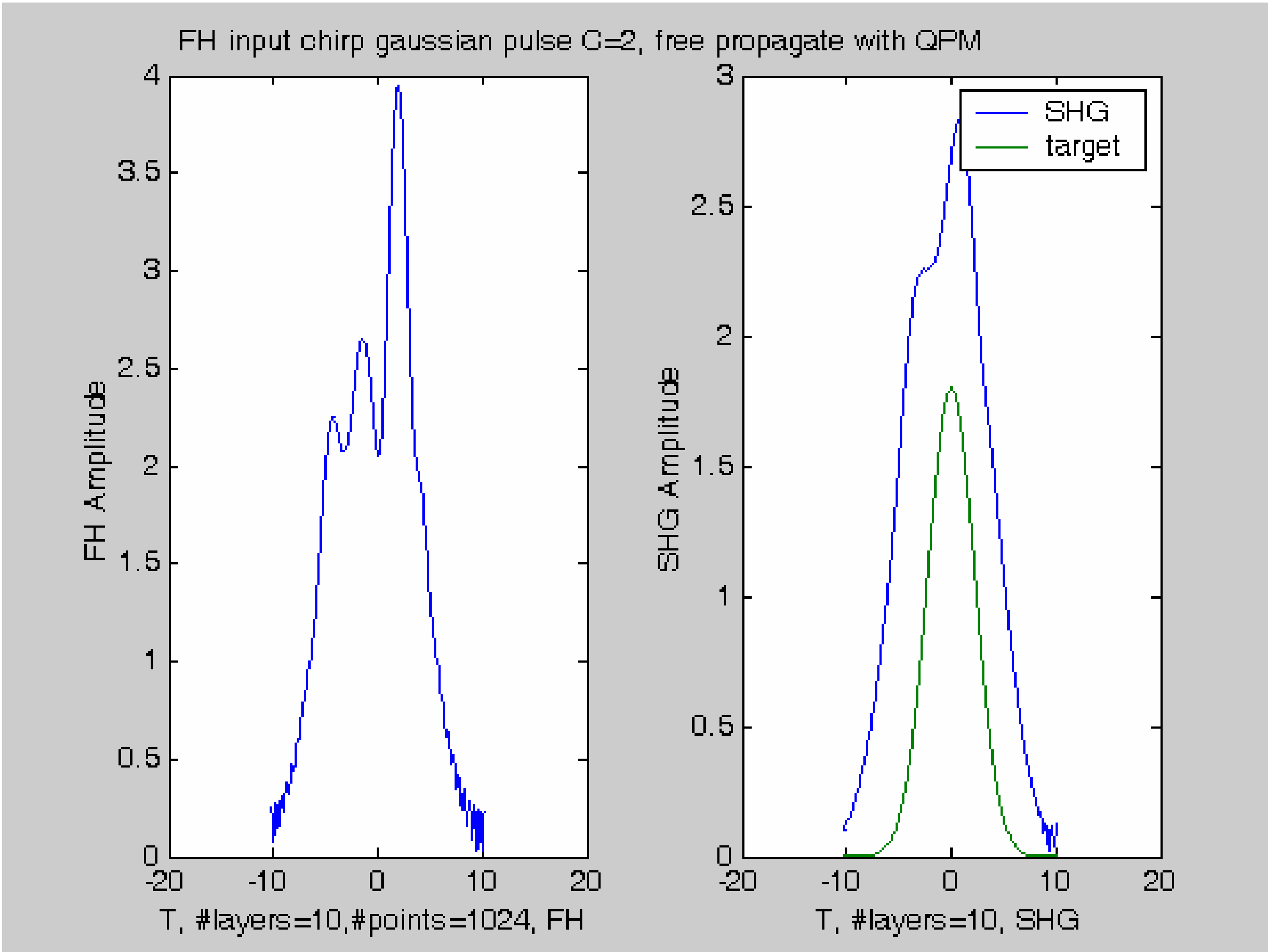
Case 1=5 or # Case 2 =20

Temperature cooling, $T = 0.95 \times T$

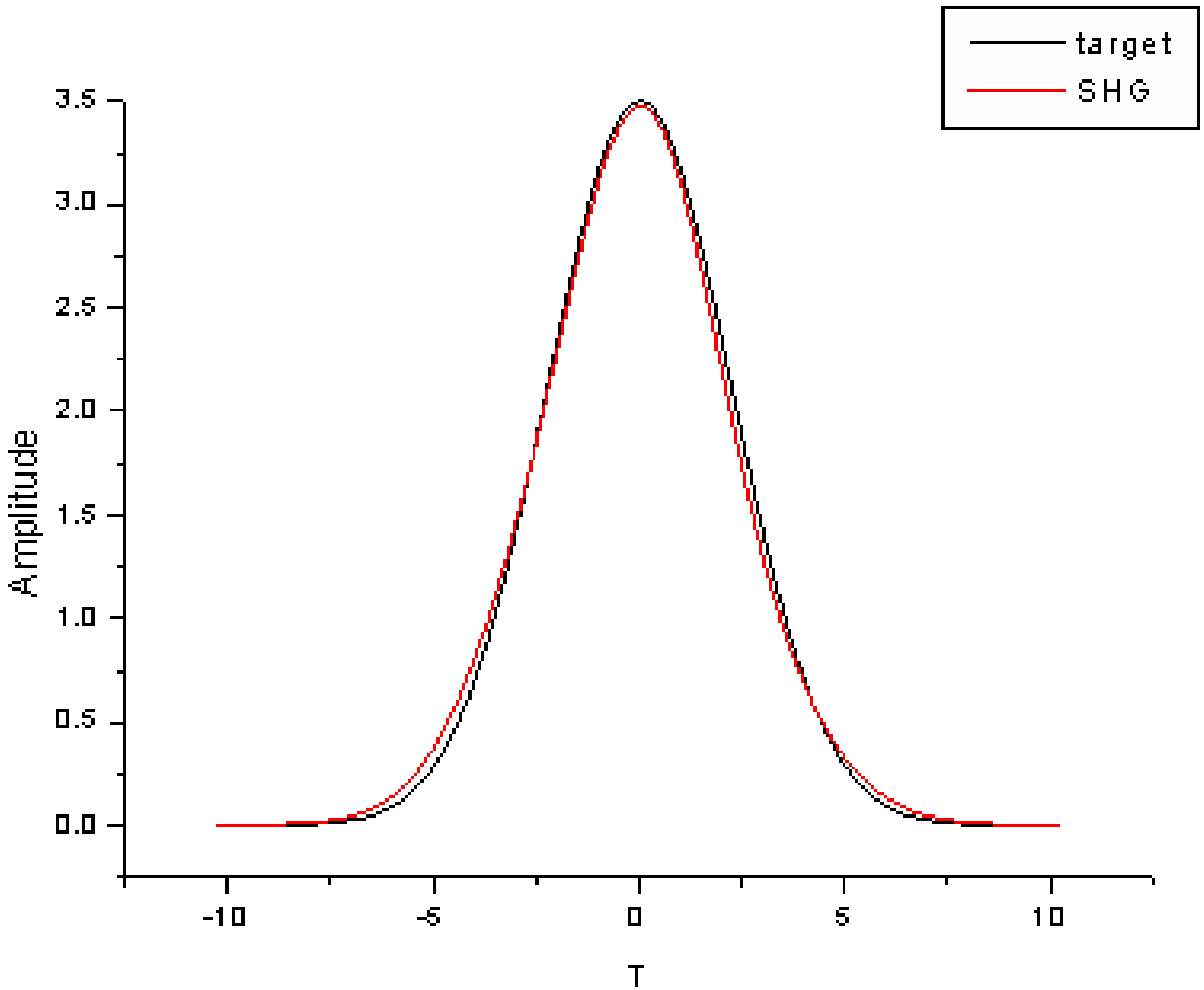
$T = T_f$

Calculation finish, get SHG spectrum and best structure

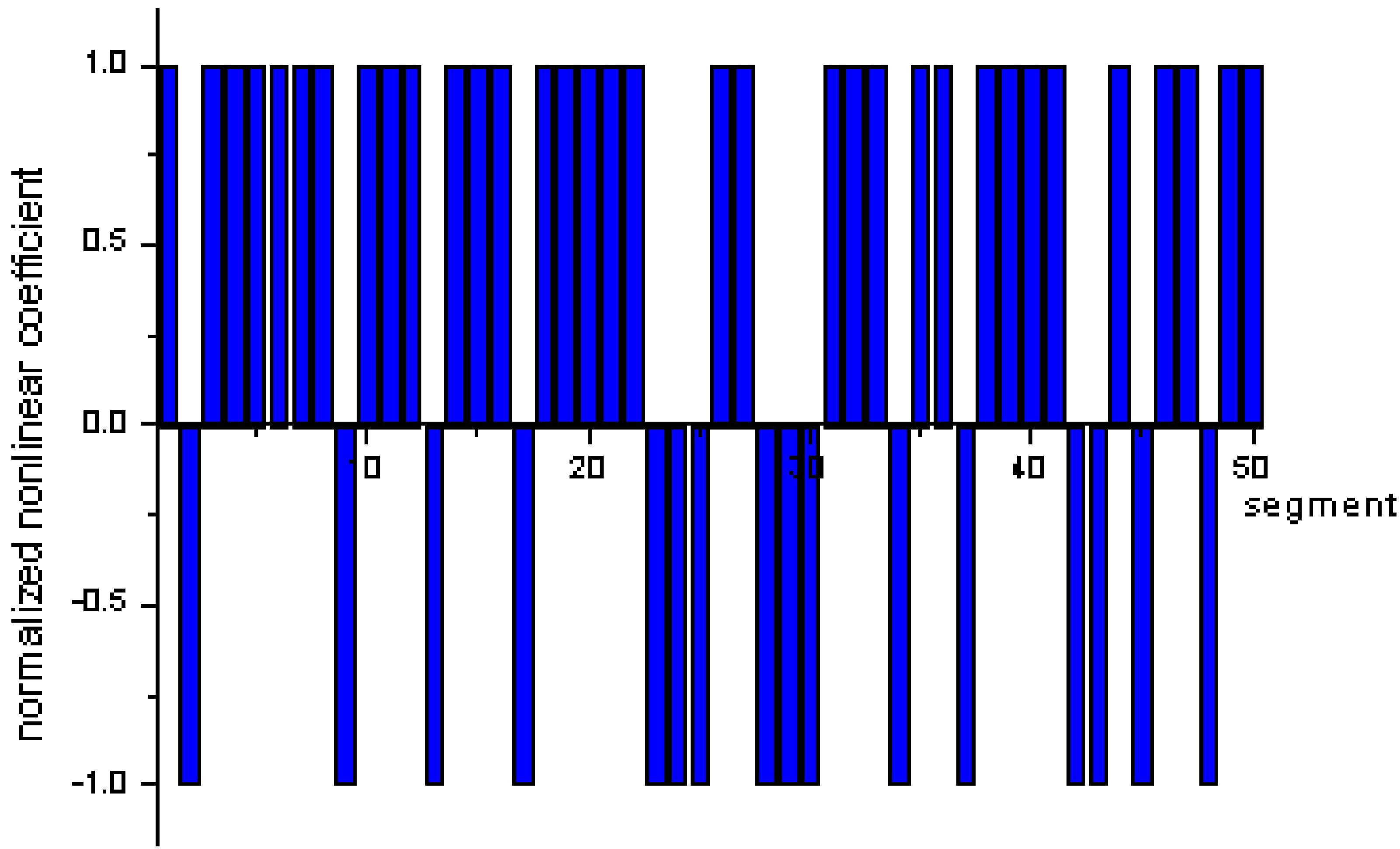
Optimization of SHG pulse



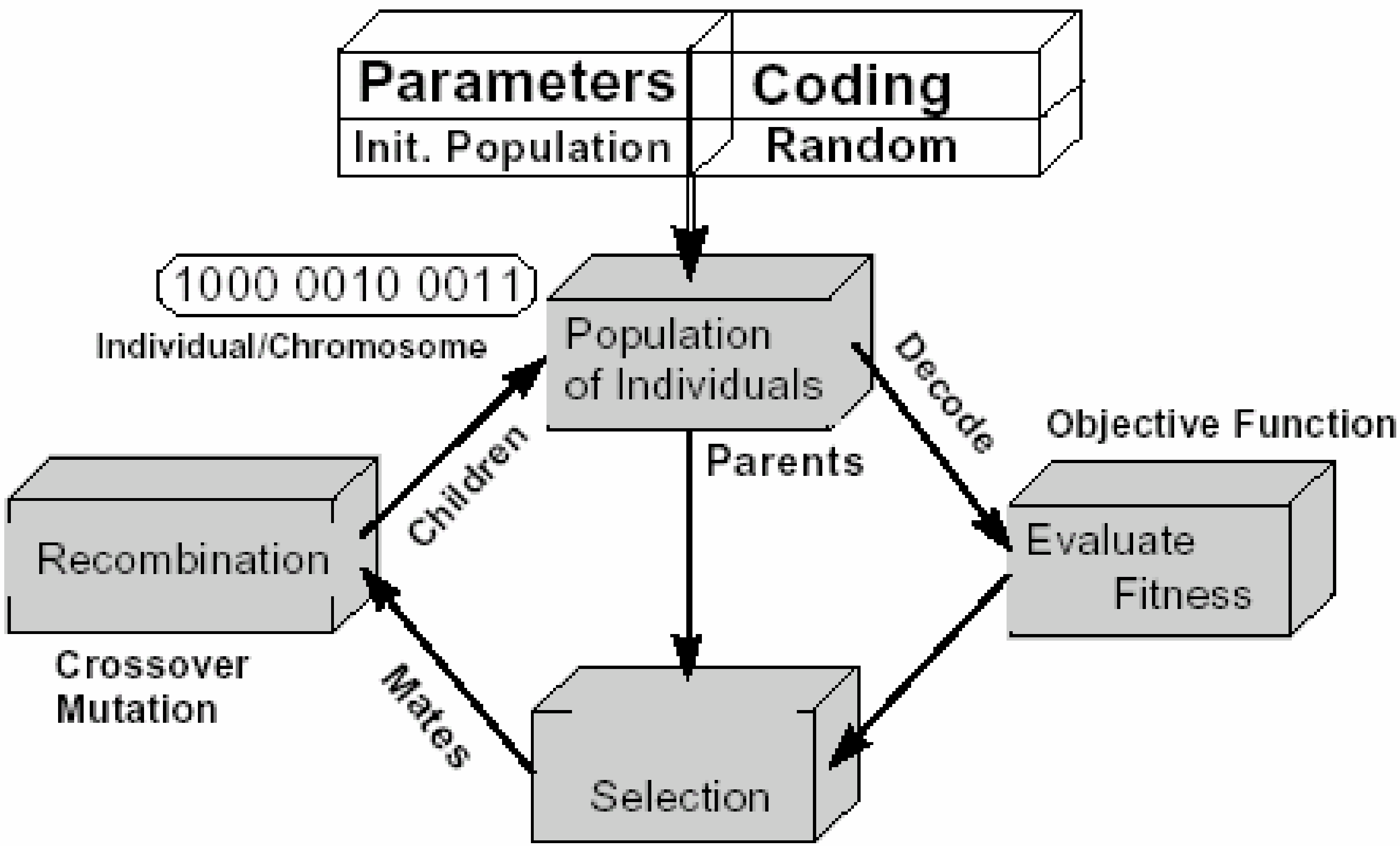
Optimization of SHG pulse



Optimization of SHG pulse



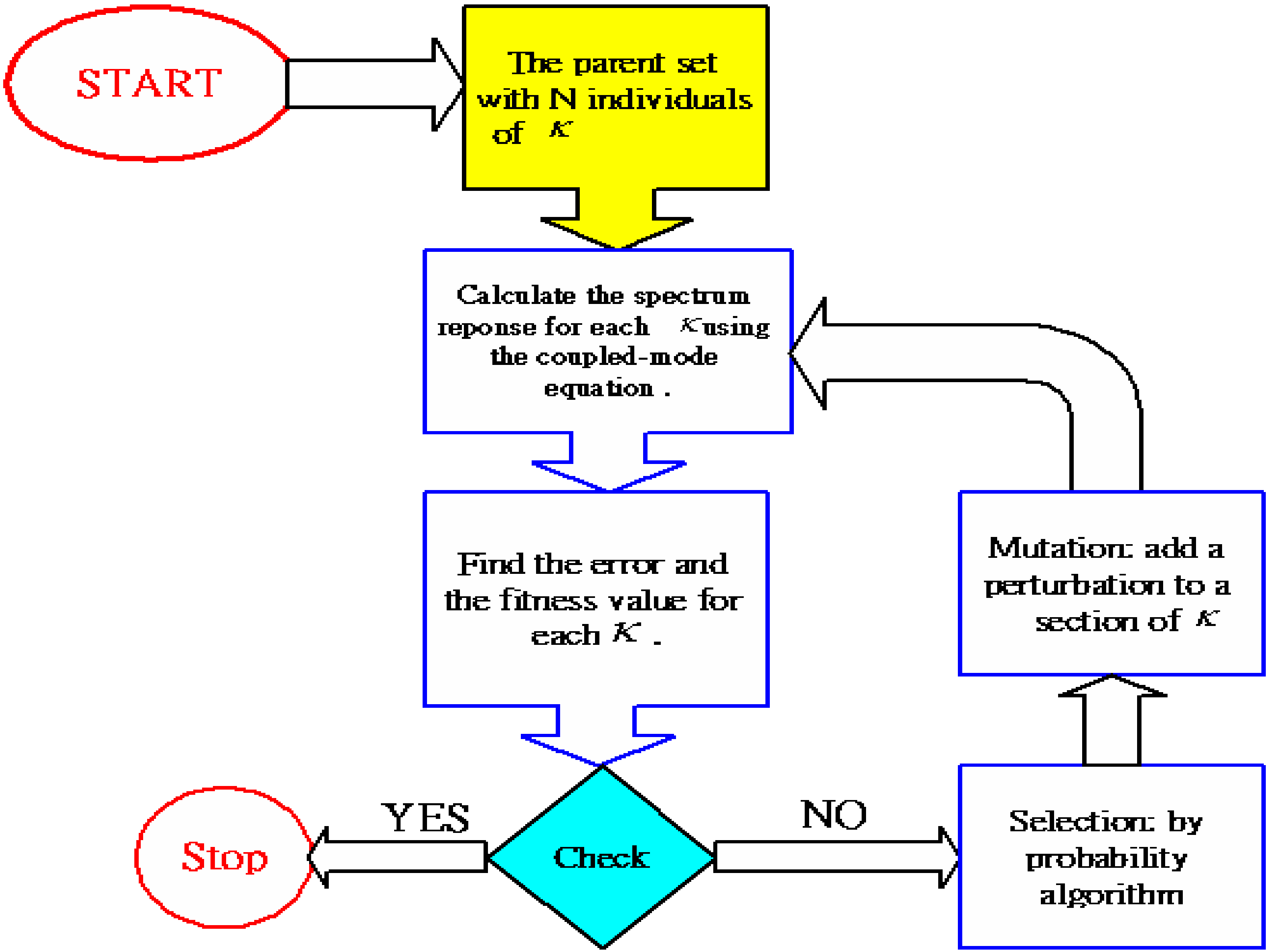
Genetic Algorithm



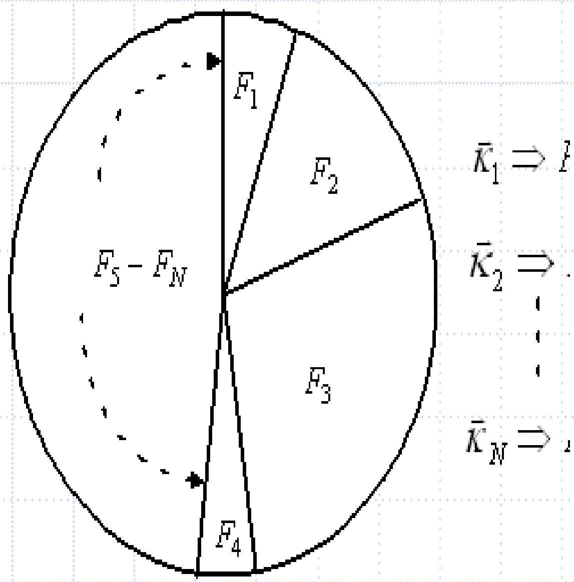
Genetic Algorithm

- Initialize the Population
- Reproduction by Selection
- Crossover and/or Mutation
- Evalution

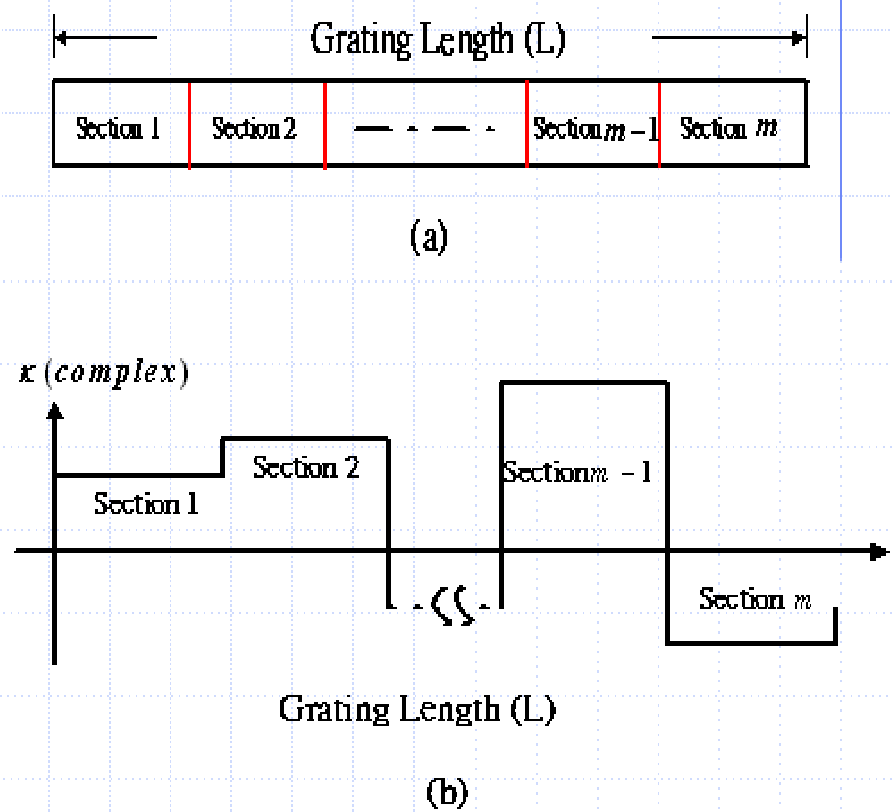
Genetic Algorithm for FBG



Genetic Algorithm for FBG



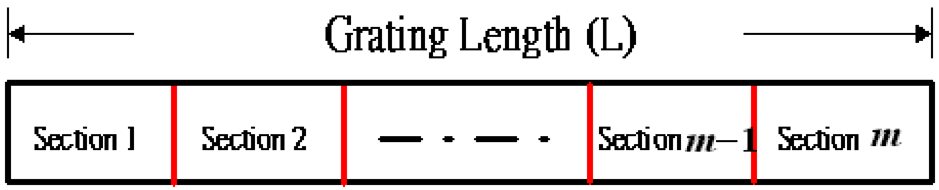
The distribution chart for making selection.



(a) and (b) show the proposed leveled sectional mutation process

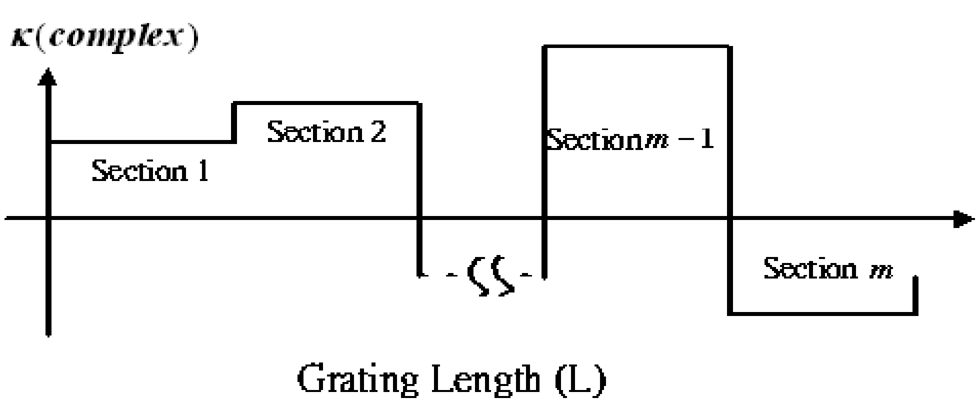
Genetic Algorithm for FBG

An adaptive mutation process : κ with higher fitness value have lower Perturbation (vice versa)



(a)

The grating length is divided into m uniform sections with equal section width of Δz



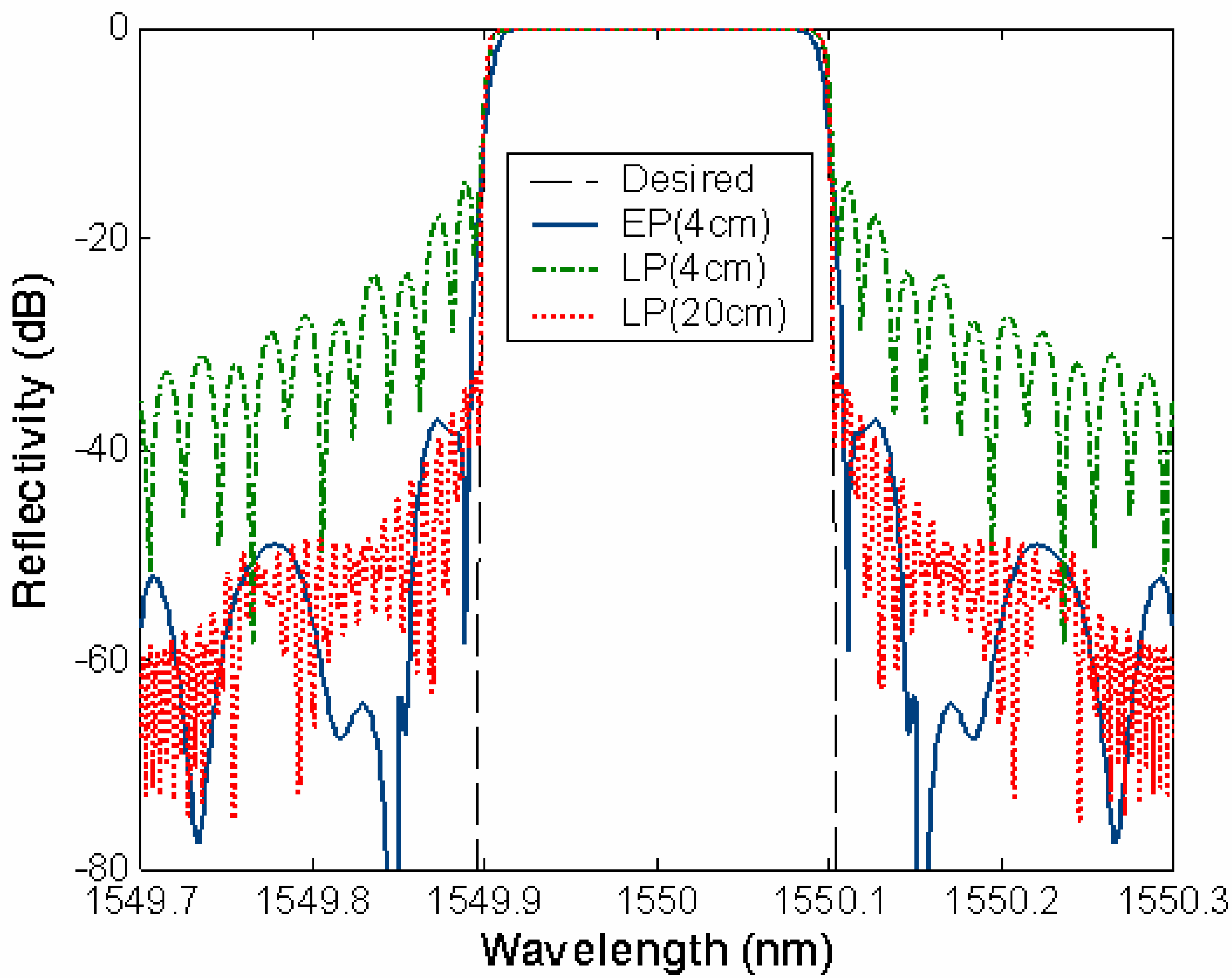
(b)

Randomly choose one of its (spatial) components $\kappa_{i,j} = \kappa_{i,j} e^{i\varphi_{i,j}}$ (sections)and change it into $\hat{\kappa}_{i,j}$

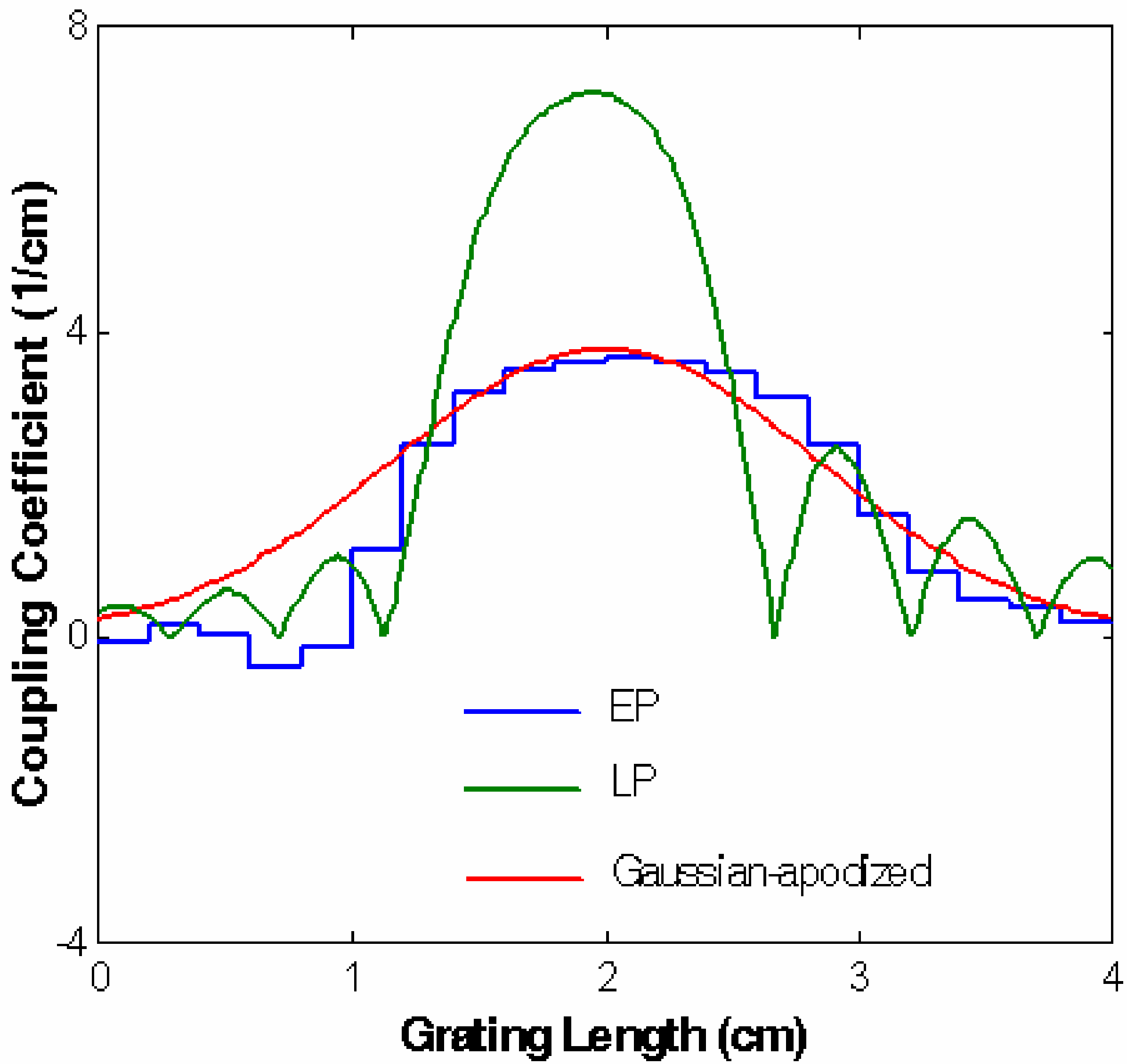
$$\hat{\kappa}_{i,j} = \left|\kappa_{i,j} \pm \delta\kappa_{i,j}\right| e^{i(\varphi_{i,j} \pm \delta\varphi_{i,j})}$$

[δκ , δφ: **Perturbations**] are randomly chosen numbers based on an **adaptive** mutation process.

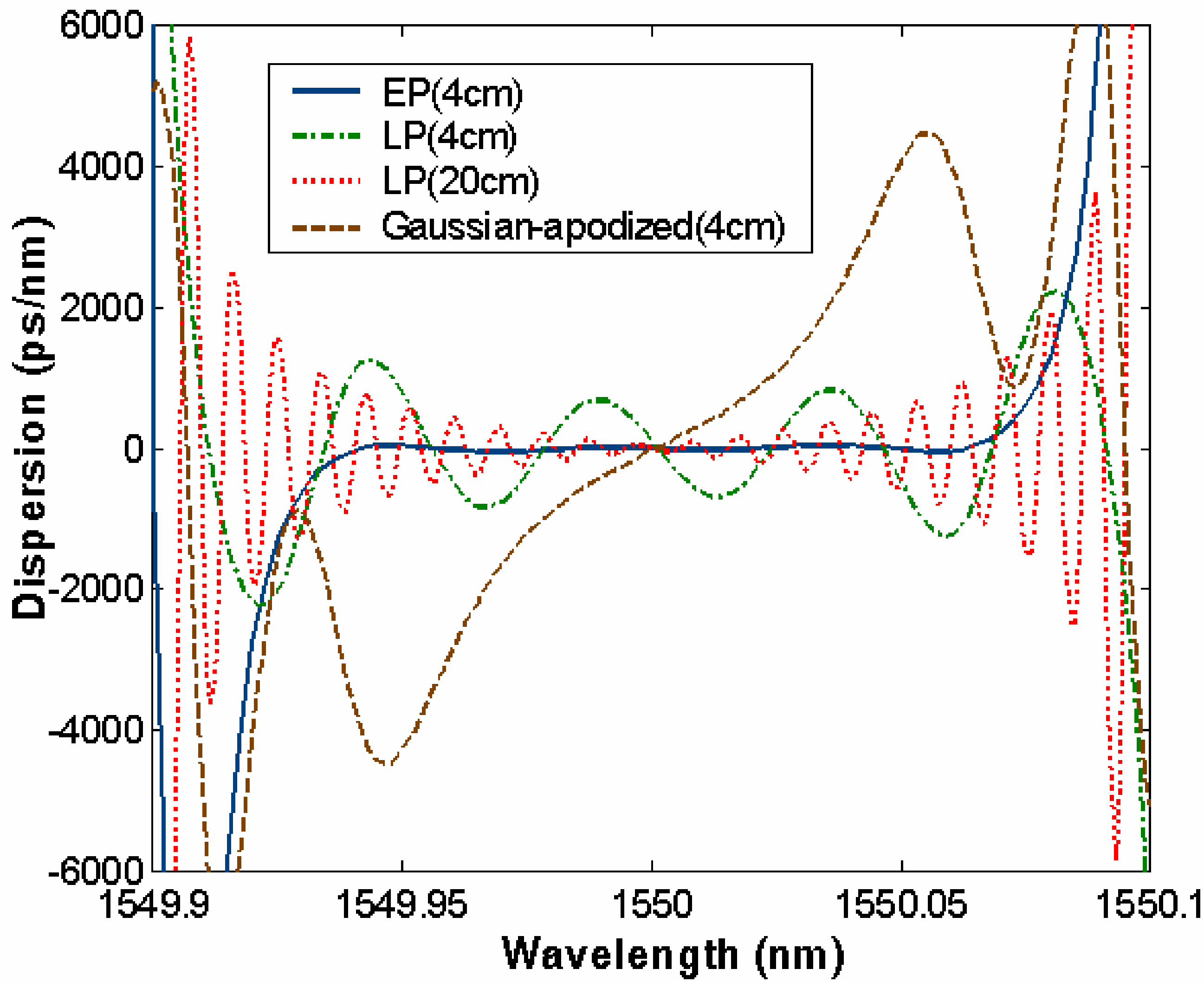
Genetic Algorithm for FBG



Genetic Algorithm for FBG



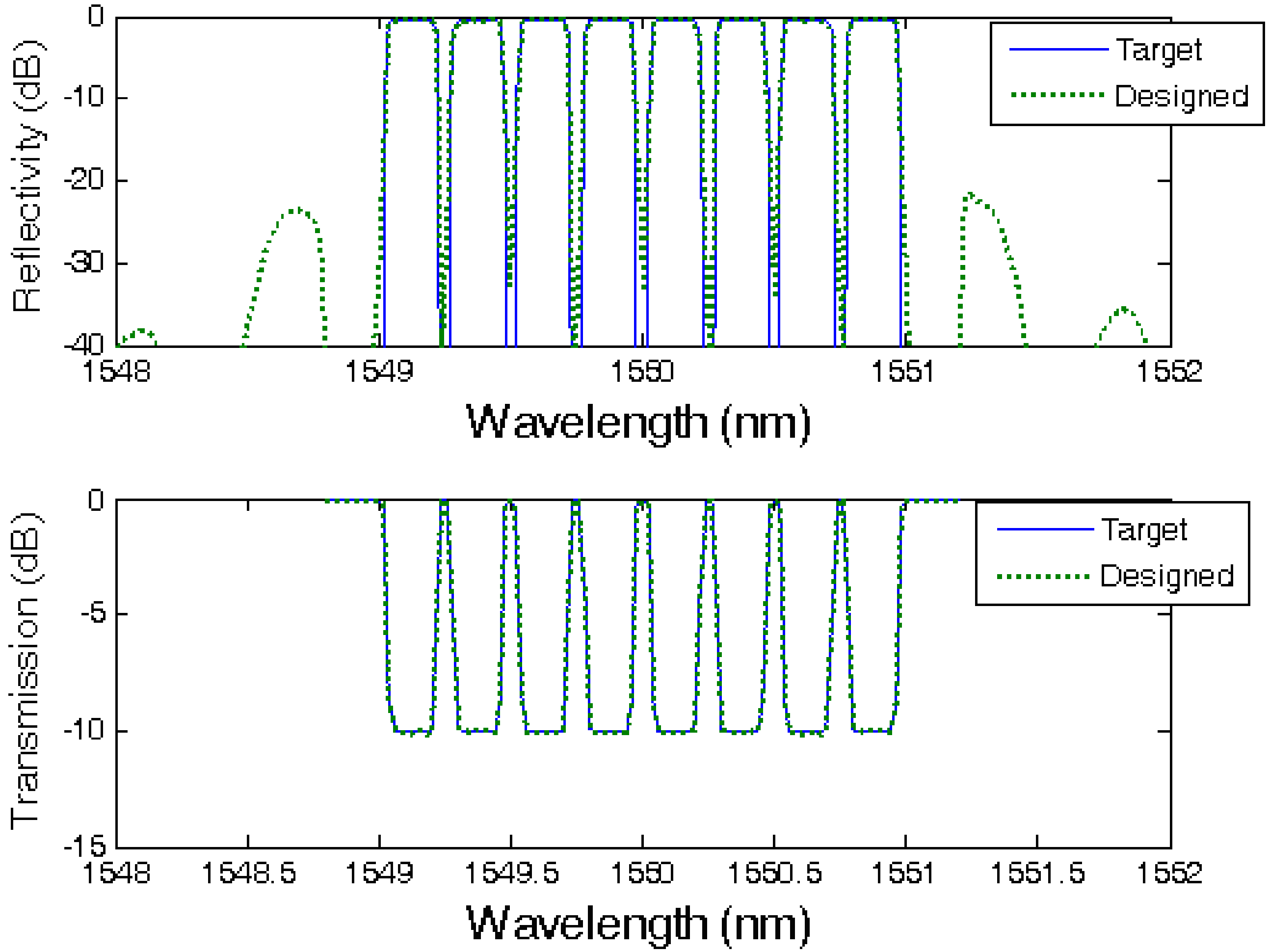
Genetic Algorithm for FBG



Constrained optimization

- Lagrange Multiplier method
- Penalty Function method

Lagrange Multiplier method for FBG



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Lagrange Multiplier method for FBG