IPT 5340 (PHYS 6840)

Quantum Optics

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IPT 5340 (PHYS 6840)

Time: T5T6F5 (01:10-03:00 PM, Tuesday; 01:10-02:00 PM, Friday) Course Description:

- The field of quantum optics has made a revolution on modern physics, from laser, precise measurement, Bose-Einstein condensates, quantum information process, to the fundamental issues in quantum mechanics.
- Through this course, I want to provide an in-depth and wide-ranging introduction to the fundamental concepts for quantum optics, including physical concepts, mathematical methods, simulation techniques, basic principles and applications.
- Current researches on non-classical state generation, quantum noise measurement, nonlinear quantum pulse propagation, quantum interference, quantum information science, Bose-Einstein condensates, and atom optics would also be stressed.
- Background requirements: Basics of quantum mechanics, electromagnetic theory, and nonlinear optics.

Teaching Method: in-class lectures with discussion and project studies.



Reference Books

- C. C. Gerry and P. L. Knight, "Introductory Quantum Optics," Cambridge (2005).
- Mark Fox, "Quantum Optics An Introduction," Oxford (2006).
- Marlan O. Scully and M. Suhail Zubairy, "Quantum Optics," Cambridge (1997).
- **Y**oshihisa Yamamoto and Atac Imamoglu, "Mesoscopic Quantum Optics," Wiley (1999).



Advanced Reference Books

Cavinder R. Put

Optics

ter terter

Quantum Information

Edited by Thomas Beth and Gord Louths

Processing

Second, revised and enlargest fielding

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Syllabus

- 1. A brief review about Quantum Mechanics, (02/18, 02/22, 02/26)
- 2. Quantum theory of Radiation, (02/39, 03/04, 03/07)
- 3. Coherent and Squeezed States, (03/11, 03/14, 03/18, 03/21)
- 4. Quantum Distribution Theory, (03/25, 03/28, 04/01, 04/08, 04/11)
- 5. Atom-field interaction, semi-classical and quantum theories, (04/15, 04/18, 04/22)
- 6. Quantum theory of Fluorescence, (04/25, 04/29, 05/02)
- 7. Cavity Quantum ElectroDynamics, Cavity-QED, (05/13, 05/16, 05/20)
- 8. Quantum theory of Lasers, (05/23, 05/27, 05/30)
- 9. Quantum theory of Nonlinear Optics, (06/03, 06/06)
- 10. Quantum Non-demolition Measurement (QND),
- 11. Quantum theory for Nonlinear Pulse Propagation,
- 12. Entangled source generation and Quantum Information, (06/10, 06/13)
- 13. Bose-Einstein Condensates (BEC) and Atom Optics,
- 14. Quantum optical test of Complementarity of Quantum Mechanics,

▶ 國 点 指.華Quantum optics in Semiconductors,

lational Tsing Hua University

16. Semester reports.

Evaluation

- Proposal 1:
 - 1. Homework $\times 4$ (monthly), 50%
 - 2. Midterm , 30%
 - 3. Semester Report, 20%
- Proposal 2:
 - 1. Homework $\times 8$ (biweekly), 80%

formula derivations, concept explanations, and preview.

- 2. Semester Report, 20%
- Other suggestions

Office hours: 13:30-15:30, Monday at Room 523, EECS bldg.



2005 Nobel Laureates





Roy J. Glauber: "for his contribution to the quantum theory of optical coherence,"

John L. Hall and Theodor W. Hänsch: "for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique."



from: http://nobelprize.org/

Coherent states and Comb lasers

coherent Glauber state:

$$|\alpha> = \sum_{n=0}^{\infty} \alpha^n \frac{e^{-\frac{|\alpha|^2}{2}}}{\sqrt{n!}} |n>$$

Self referencing of frequency combs:



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Nobel Laureats on Photonics: I













BEC(2001) BEC(2001) BEC(2001) Quant-Opt.(2005) Comb(2005) Comb(2005)













heterostructure(2000) heter.(2000) IC(2000) lasercooling(1997) L.C.(1997) L.C.(1997)

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spectroscopy(1981) spect.(1981) spect.(1981) holography(1971) laser(1964) laser(1964)

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Nobel Laureats on Photonics: II













laser(1964) transistor(1956) trans.(1956) trans.(1956) Rabi(1944) Raman(1930)













de Broglie(1929) X-ray spect.(1924) photoelectric(1923) P.O.(1921) quanta(1918) Bragg(1915)













Bragg(1915) photography(1908) Michelson(1907) cathode(1905) Rayleigh(1904) X-ray(1901)

Photonics

Modifying and Manipulating the properties of light, its classical and quantum properties



Phase diagram for EM waves

Electromagnetic waves can be represented by

$$\hat{E}(t) = E_0[\hat{X}_1 \sin(\omega t) - \hat{X}_2 \cos(\omega t)]$$

where

 $\hat{X}_1 =$ amplitude quadrature $\hat{X}_2 =$ phase quadrature





1, A brief review about Quantum Mechanics

- 1. Basic Quantum Theory
- 2. Time-Dependent Perturbation Theory
- 3. Simple Harmonic Oscillator
- 4. Quantization of the field
- 5. Canonical Quantization

Ref:

- Ch. 2 in "Introductory Quantum Optics," by C. Gerry and P. Knight.
- Ch. 2 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- Ch. 1 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 4 in "The Quantum Theory of Light," by R. Loudon.
- Ch. 1, 2, 3, 6 in "Mathematical Methods of Quantum Optics," by R. Puri
- Ch. 3 in "Elements of Quantum Optics," by P. Meystre and M. Sargent III.

Ch. 9 in *"Modern Foundations of Quantum Optics,"* by V. Vedral.

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2, Quantum theory of Radiation

- 1. Stimulated and Spontaneous Emission
- 2. Macroscopic theory of absorption
- 3. Microscopic theory of absorption
- 4. The Laser
- 5. Lamb shift
- 6. Quantum beats

Ref:

- Ch. 1 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 1 in "The Quantum Theory of Light," by R. Loudon.
- Ch. 2 in "Modern Foundations of Quantum Optics," by V. Vedral.



Einstein on Radiation



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Zur Quantentheorie der Strahlung.

Von A. Einstein¹).

Die formale Ahnlichkeit der Kurve der chromatischen Verteilung der Temperaturstrählung mit Maxwellschen Geschwindigkeits Verteilungsgesetz ist zu fruppant, als daß sie lange hätte verborgen bleiben können. In der Tat warde bereits W. Wien in der wichtigen theoretischen Arbeit, in welchter er sein Verschiebungsgesetz

$$q = \nu^{\mu} f\left(\frac{\nu}{T}\right)$$
 (1)

ableittte, durch diese Ahnlichkeit auf eine weittrgebende Bestimmung der Strahlungsformel geführt. Er fand hierbei bekanntlich die Formel

aidatas ata Germaneseta fila grada Werte ann

"On the Quantum Theory of Radiation"

$$\rho(v_0) = \frac{A/D}{e^{hv_0/kT} - 1}$$
$$\frac{A}{B} = \frac{8\pi h v_0^3}{c^3}$$

A. Einstein, *Phys. Z.* **18**, 121 (1917).

D. Kleppner, "Rereading Einstein on Radiation," *Physics Today* 58, 30 (Feb. 2005).

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3, Coherent and Squeezed States

- 1. Coherent states
- 2. Squeezed states
- 3. Field Correlation Functions
- 4. Hanbury Brown and Twiss experiment
- 5. Photon Antibunching
- 6. Quantum Phenomena in Simple Nonlinear Optics

Ref:

- Ch. 5, 7 in "Introductory Quantum Optics," by C. Gerry and P. Knight.
- Ch. 2, 4, 16 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 3, 4 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- **Ch. 6** in *"The Quantum Theory of Light,"* by R. Loudon.
- Ch 5,8 in "Quantum Optics," by D. Wall and G. Milburn.

Phase diagram for coherent states



Uncertainty Principle: $\Delta \hat{X}_1 \Delta \hat{X}_2 \ge 1$.

- 1. Coherent states: $\Delta \hat{X}_1 = \Delta \hat{X}_2 = 1$,
- 2. Amplitude squeezed states: $\Delta \hat{X}_1 < 1$,
- 3. Phase squeezed states: $\Delta \hat{X}_2 < 1$,
- 4. Quadrature squeezed states.



Vacuum, Coherent, and Squeezed states

quad-squeezed

phase-squeezed

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Generations of Squeezed States

Nonlinear optics:

4, Quantum Distribution Theory

- 1. Expansion in Number states
- 2. Expansion in Coherent states
- 3. Q-representation
- 4. Wigner-Weyl distribution
- 5. Master Equation
- 6. Stochastic Differential Equation

Ref:

- Ch. 3 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 6 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 8 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- Ch. 4, 5 in "Mathematical Methods of Quantum Optics," by R. Puri.

"Quantum Optics in Phase Space," by W. Schleich.

Wigner function for a Kerr state

M. Stobinska et al., quant-ph/0605166

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5, Atom-field interaction, semi-classical and quantum theories

- 1. Semiclassical theory
- 2. Jaynes-Cummings Hamiltonian
- 3. Multi-mode squeezing
- 4. Rabi Oscillation
- 5. Superradiance

Ref:

- Ch. 5, 6 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 3, 4 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- Ch. 5 in "The Quantum Theory of Light," by R. Loudon.
- Ch. 10 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 13 in "Elements of Quantum Optics," by P. Meystre and M. Sargent III.

photon-atom bound state

S. John and J. Wang, Phys. Rev. Lett. 64, 2418 (1990).

6, Quantum theory of Fluorescence

- 1. Quantum theory of Damping: Density operator
- 2. Quantum theory of Damping: Langevin equation
- 3. System-Reservoir Interaction
- 4. Resonance Fluorescence
- 5. Decoherence

Ref:

- Ch. 8 in "Introductory Quantum Optics," by C. Gerry and P. Knight.
- Ch. 8, 9, 10 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 7 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- Ch. 8 in "The Quantum Theory of Light," by R. Loudon.
- Ch. 14, 15 in "Elements of Quantum Optics," by P. Meystre and M. Sargent III.

Mollow's triplet: Resonance Fluorescence Spectrum

Theory: B. R. Mollow, *Phys. Rev.* 188, 1969 (1969).

國 立清 華大學 National Tsing Hua University Exp: F. Y. Wu, R. E. Grove, and S. Ezekiel, *Phys. Rev. Lett.* 35, 1426 (1975).

Reservoir Theory

Hamiltonian of our system: Jaynes-Cummings model

$$H = \frac{\hbar}{2}\omega_a \sigma_z + \hbar \sum_k \omega_k a_k^{\dagger} a_k + \frac{\Omega}{2}\hbar(\sigma_- e^{i\omega_L t} + \sigma_+ e^{-i\omega_L t})$$

+
$$\hbar \sum_k (g_k \sigma_+ a_k + g_k^* a_k^{\dagger} \sigma_-)$$

And we want to solve the generalized Bloch equations:

$$\begin{split} \dot{\sigma}_{-}(t) &= i\frac{\Omega}{2}\sigma_{z}(t)e^{-i\Delta t} + \int_{-\infty}^{t} dt'G(t-t')\sigma_{z}(t)\sigma_{-}(t') + n_{-}(t) \\ \dot{\sigma}_{+}(t) &= -i\frac{\Omega}{2}\sigma_{z}(t)e^{i\Delta t} + \int_{-\infty}^{t} dt'G_{c}(t-t')\sigma_{+}(t')\sigma_{z}(t) + n_{+}(t) \\ \dot{\sigma}_{z}(t) &= i\Omega(\sigma_{-}(t)e^{i\Delta t} - \sigma_{+}(t)e^{-i\Delta t}) + n_{z}(t) \\ &\approx 2\int_{-\infty}^{t} dt'[G(t-t')\sigma_{+}(t)\sigma_{-}(t') + G_{c}(t-t')\sigma_{+}(t')\sigma_{-}(t)] \\ &\approx 2\int_{-\infty}^{t} dt'[G(t-t')\sigma_{+}(t)\sigma_{-}(t')\sigma_{-}(t') + G_{c}(t-t')\sigma_{+}(t')\sigma_{-}(t)] \\ &\approx 2\int_{-\infty}^{t} dt'[G(t-t')\sigma_{+}(t)\sigma_{-}(t')\sigma_{-}$$

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Fluorescence quadrature spectra near the band-edge

R.-K. Lee and Y. Lai, J. Opt. B, 6, S715 (Special Issue 2004).

7, Cavity Quantum ElectroDynamics (Cavity-QED)

- 1. Cavity Modes
- 2. Purcell effect
- 3. Input-Output Formulation
- 4. Intracavity Atomic Systems
- 5. Squeezed state generation

Ref:

Ch. 10 in "Introductory Quantum Optics," by C. Gerry and P. Knight.

- Ch. 7, 13 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 13 in "Elements of Quantum Optics," by P. Meystre and M. Sargent III.
- Ch. 16 in "Quantum Optics," by M. Scully and M. Zubairy.

"Theoretical Problems in Cavity Nonlinear Optics," by P. Mandel.

Purcell effect: Cavity-QED (Quantum ElectroDynamics)

E. M. Purcell, Phys. Rev. 69 (1946).

Nobel laureate Edward Mills Purcell (shared the prize with Felix Bloch) in 1952,

for their contribution to nuclear magnetic precision measurements.

from: K. J. Vahala, *Nature* **424**, 839 (2003).

Quantum State Transfer as a Quantum Repeater

J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi1, Phys. Rev. Lett. 78, 3221 (1997).

8, Quantum theory of Lasers

- 1. Quantum theory of Laser: Density operator
- 2. Quantum theory of Laser: Langevin equation
- 3. Micromaser
- 4. Sub-Poissonian Laser

Ref:

- Ch. 11, 12, 13, 14 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 15 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- Ch. 12 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 17 in "Elements of Quantum Optics," by P. Meystre and M. Sargent III.

Recently, formation of stable double-, triple-, and multisoliton bound states has been observed experimentally in various passively mode-locked fiber lasers.

Total photon-number fluctuations

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R.-K. Lee, Y. Lai, and B. A. Malomed, Opt. Lett. 34, 3084 (2005).

9, Quantum theory of Nonlinear Optics

- 1. Degenerate Parametric Amplification
- 2. Optical Parametric Oscillator
- 3. Third-Harmonic Generation
- 4. Four-Wave Mixing
- 5. Stimulated Raman effect

Ref:

Ch. 16 in "Quantum Optics," by M. Scully and M. Zubairy.

Ch. 8 in "Quantum Optics," by D. Wall and G. Milburn.

Ch. 9 in "The Quantum Theory of Light," by R. Loudon.

Generations of Squeezed States

Nonlinear optics:

10, Quantum Non-demolition Measurement (QND)

- 1. Backaction Evasion
- 2. Condition for QND measurements
- 3. QND measurement via Optical Kerr effect
- 4. QND measurement via Optical Parametric Process

Ref:

- **Ch. 19** in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 9 in "Mesoscopic Quantum Optics," by Y. Yamamoto and A. Imamoglu.
- Ch. 15 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 17 in "Elements of Quantum Optics," by P. Meystre and M. Sargent III.

QND with Optical Solitons

J. Schneider et al., Opt. Lett., 31, 2628 (2006).

11, Quantum theory for Nonlinear Pulse Propagation

- 1. Quantum Nonlinear Schrödinger Equation
- 2. Quadrature Squeezing of Optical Solitons
- 3. Amplitude Squeezing of Bragg Solitons
- 4. Quantum Correlation of Solitons
- 5. Quantum theory for Bound-State Solitons

Ref:

"Electromagnetic Noise and Quantum Optical Measurements," by H. Haus.

R.-K. Lee and Y. Lai, *Phys. Rev. A* 69, 021801(R) (2004);

R.-K. Lee and Y. Lai, J. Opt. B 6, S638 (2004);

- R.-K. Lee, Y. Lai and B. A. Malomed, J. Opt. B 6, 367 (2004);
- R.-K. Lee, Y. Lai and B. A. Malomed, Phys. Rev. A 70, 063817 (2004);
- R.-K. Lee, Y. Lai and Yu. S. Kivshar, Phys. Rev. A 71, 035801 (2005);

Solitons in optical fibers

Classical nonlinear Schrödinger Equation

$$iU_z(z,t) = -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2 U(z,t)$$

Fundamental soliton:

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Quantum nonlinear Schrödinger equation

$$i\frac{\partial}{\partial t}\hat{\phi}(t,x) = -\frac{\partial^2}{\partial x^2}\hat{\phi}(t,x) + 2c\hat{\phi}^{\dagger}(t,x)\hat{\phi}(t,x)\hat{\phi}(t,x)$$

where $\hat{\phi}(t, x)$ and $\hat{\phi}^{\dagger}(t, x)$ are annihilation and creation field operators and satisfy Bosonic commutation relations:

$$\begin{aligned} &[\hat{\phi}(t,x'),\hat{\phi}^{\dagger}(t,x)] = \delta(x-x')\\ &[\hat{\phi}(t,x'),\hat{\phi}(t,x)] = [\hat{\phi}^{\dagger}(t,x'),\hat{\phi}^{\dagger}(t,x)] = 0 \end{aligned}$$

and in classical (mean-field) solution, i.e. $\hat{\phi} \rightarrow \phi$, for attractive case ($a_s < 0$), c < 0, bright soliton exists; for repulsive case ($a_s > 0$), c > 0, dark soliton exists. Expand the quantum state in Fock space

$$|\psi\rangle = \sum_{n} a_n \int d^n x \frac{1}{\sqrt{n!}} f_n(x_1, \dots, x_n, t) \hat{\phi}^{\dagger}(x_1) \dots \hat{\phi}^{\dagger}(x_n) |0\rangle$$

then, QNLSE corresponds to 1-D Bosons with δ -interaction

$$i\frac{d}{dt}f_n(x_1,\ldots,x_n,t) = \left[-\sum_{j=1}^n \frac{\partial^2}{\partial x_j^2} + 2c\sum_{1\le i< j\le n} \delta(x_j-x_i)\right]f_n(x_1,\ldots,x_n)$$

and can be solved by

- 1. Bethe's ansatz (exact solution);
- 2. Hatree approximation (N is large);

^w 図 立 う 準 Quantum inverse scattering method (exact solution).

Amplitude Squeezing of FBG solitons

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S. Spälter, N. Korolkova, F. Konig, A. Sizmann, and G. Leuchs,

Phys. Rev. Lett. 81, 786 (1998).

12, Entangled source generation and Quantum Information

- 1. The Einstein-Podolsky-Rosen paradox
- 2. Bell's Inequality
- 3. Violations of Bell's Inequality using OPA
- 4. Quantum Teleportation
- 5. Quantum Cryptography via Optics

Ref:

Ch. 11 in "Introductory Quantum Optics," by C. Gerry and P. Knight.

- Ch. 18 in "Quantum Optics," by M. Scully and M. Zubairy.
- Ch. 14 in "Quantum Optics," by D. Wall and G. Milburn.
- Ch. 11 in "Modern Foundations of Quantum Optics," by V. Vedral.
- R.-K. Lee, Y. Lai and B. A. Malomed, Phys. Rev. A 71, 013816 (2005);

Quantum Information Processing," by G. Leuchs and T. Beth.

Generation of Continuous Variables Entanglement

Preparation EPR pairs by Squeezed Sates

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Experiment of CV Teleportation

A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble,

and E. S. Polzik, Science 282, 706 (1998).

Evolutions of Photon Number Correlation Spectra

 $Z = 2.0Z_0$,

 $Z = 4.0Z_0$,

 $Z = 6.0Z_0.$

國 点清 華大學 $Z = 30.0Z_0,$ $Z = 50.0Z_0$ National Tsing Hua UniversityR.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* 71, 013816 (2005).

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Quantum State Transfer

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Quantum State Transfer with Spin of Atoms

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Electromagnetically Induced Transparency

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Slow-light in QD VCSELs

● 図 点清 華大 學C.-S. Chou, R.-K. Lee, P. C. Peng, H. C. Kuo, G. Lin, H. P. Yang and J. Y. Chi, National TEEE/LEOS Optical MEMS and Nanophotoincs (2007); arXiv: 0710.0136 to special issue in J. Opt. A.

SIT soliton in micro-structured fiber

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Courtesy: G. Leuchs (Erlangen, Germany).

Soi-Chan Lai and Ray-Kuang Lee, *arXiv*:0709.1352 (quant-ph) (2007), to appear in *Phys. Rev. A*.

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Squeezing and Entanglement with EIT

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Thanks for your attention !!

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Thanks for your attention !!

■ 或清華大學 Imagination is more important than knowledge.

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