Gap solitons in optical lattices: their classical and quantum properties

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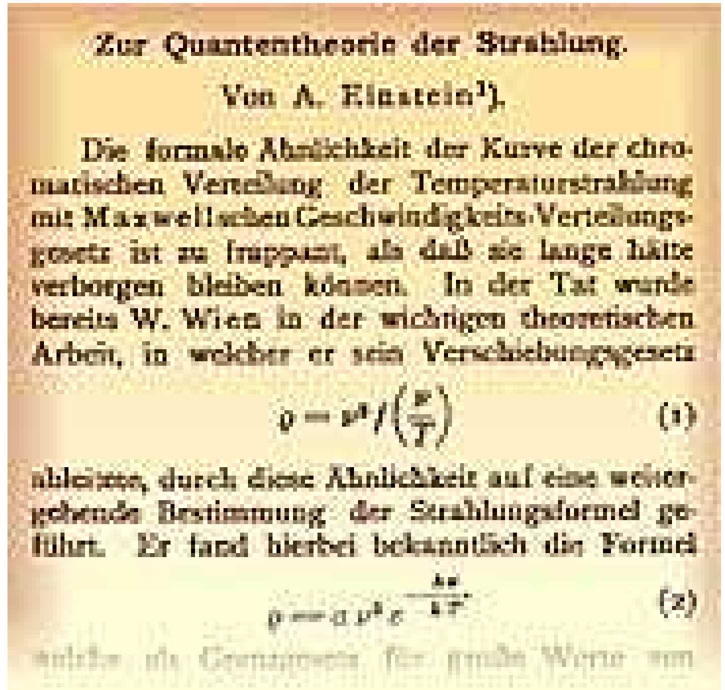
Outline

1. On the Shoulders of Giants
2. The Great Wave of Translation
3. Quantum Solitons
4. Quantum Bragg and Gap Solitons
5. Entangled Solitons for Quantum Information
6. Conclusions

Einstein on Radiation



Zur Quantentheorie der Strahlung.

Von A. Einstein[1]).

Die formale Ähnlichkeit der Kurve der chromatischen Verteilung der Temperaturstrahlung mit Maxwellschen Geschwindigkeits-Verteilungsgesetz ist zu frappant, als daß sie lange hätte verborgen bleiben können. In der Tat wurde bereits W. Wien in der wichtigen theoretischen Arbeit, in welcher er sein Verschiebungsgesetz

$$\varrho = \nu^3 f\left(\frac{\nu}{T}\right) \quad (1)$$

ableitete, durch diese Ähnlichkeit auf eine weitergehende Bestimmung der Strahlungsformel geführt. Er fand hierbei bekanntlich die Formel

$$\varrho = \alpha \nu^3 e^{-\frac{h\nu}{kT}} \quad (2)$$

welche als Grenzgesetz für große Werte von

"On the Quantum Theory of Radiation"

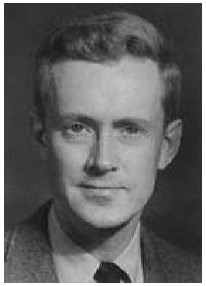
$$\rho(v_0) = \frac{A/B}{e^{hv_0/kT} - 1}$$

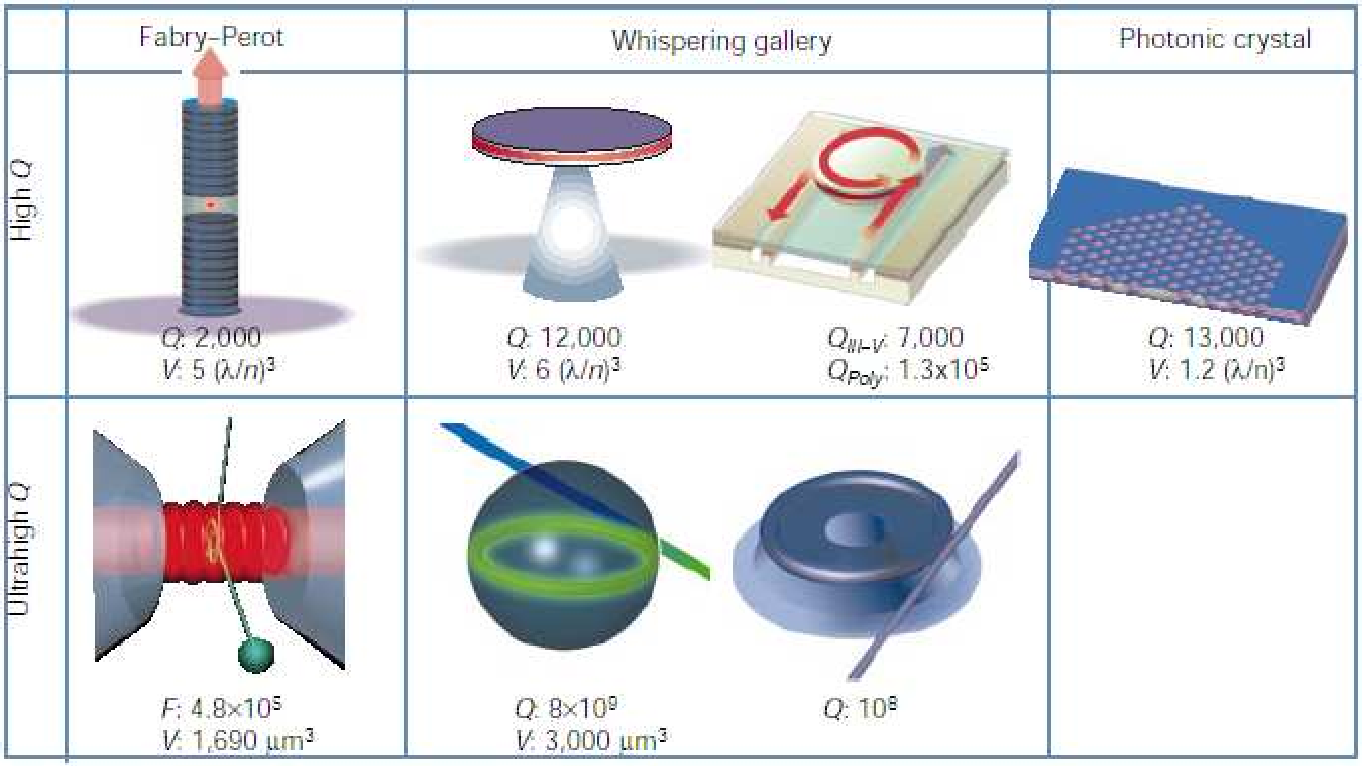
$$\frac{A}{B} = \frac{8\pi h v_0^3}{c^3}$$

A. Einstein, *Phys. Z.* **18**, 121 (1917).

D. Kleppner, "Rereading Einstein on Radiation," *Physics Today* **58**, 30 (Feb. 2005).

Purcell effect: Cavity-QED (Quantum ElectroDynamics)





E. M. Purcell, *Phys. Rev.* **69** (1946).

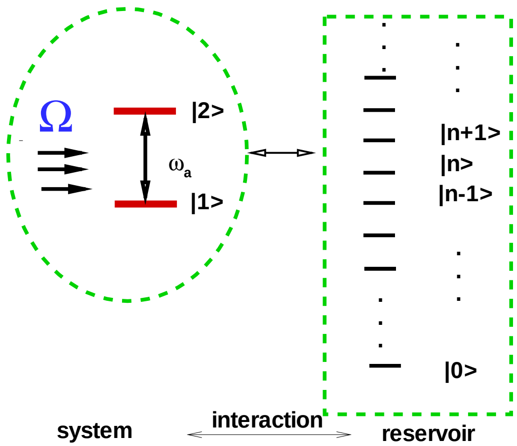
Nobel laureate Edward Mills Purcell (shared the prize with Felix Bloch) in 1952,

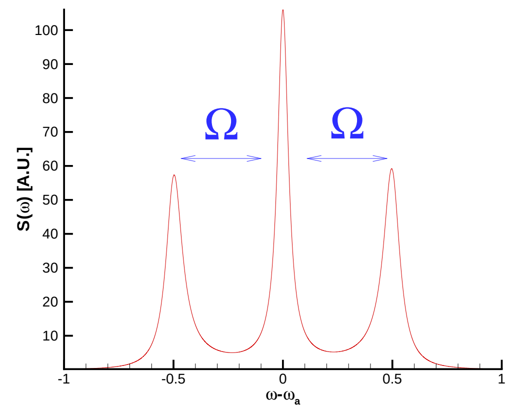
for their contribution to nuclear magnetic precision measurements.

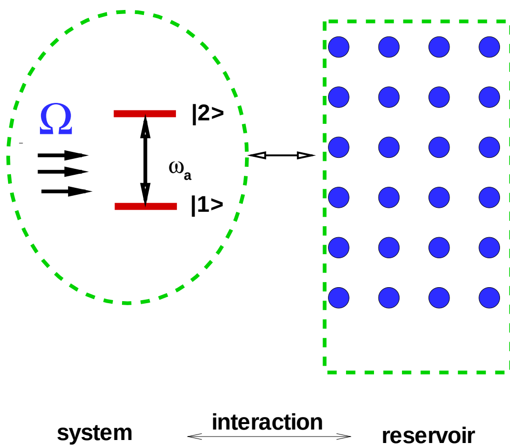
from: K. J. Vahala, *Nature* **424**, 839 (2003).

Photon-Atom Interaction in PhCs

Reservoir Theory

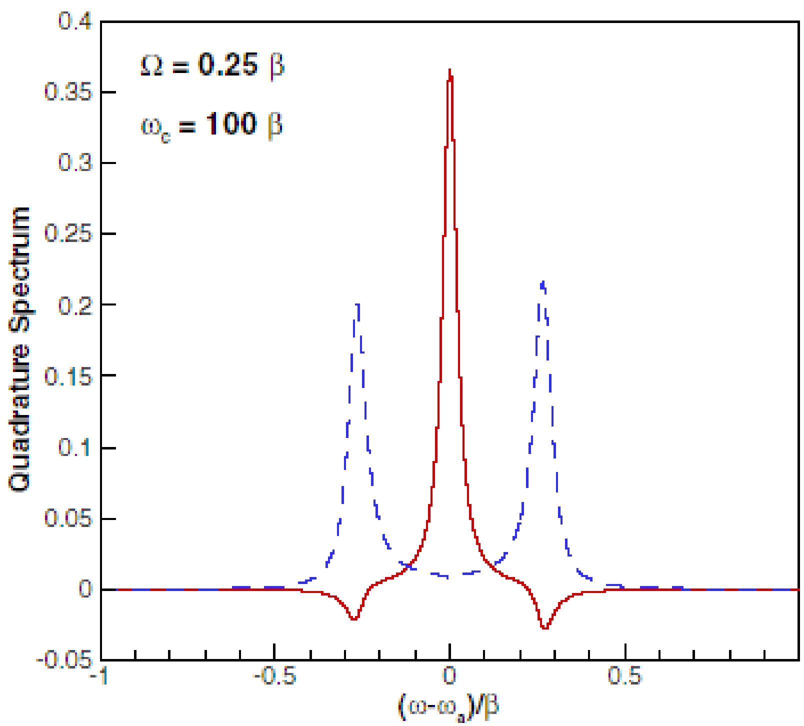


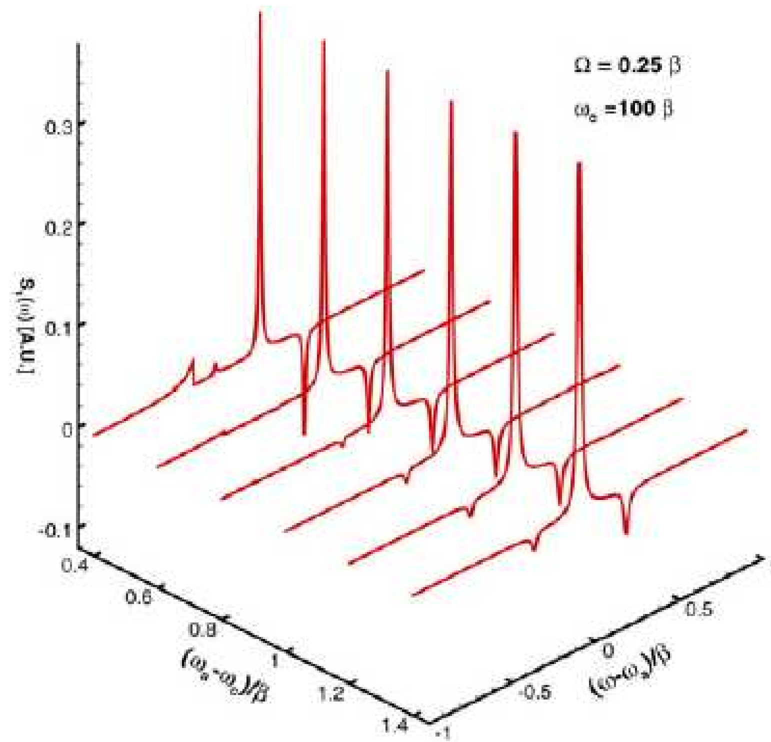




?

Fluorescence quadrature spectra near the band-edge





R.-K. Lee and Y. Lai, *J. Opt. B*, **6**, S715 (Special Issue 2004).

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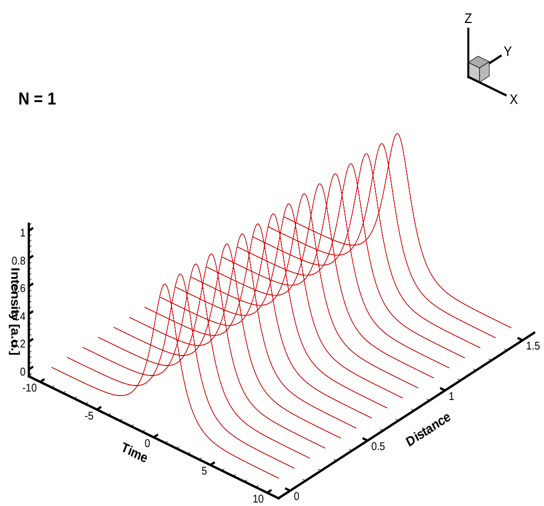
Solitons in optical fibers

Nonlinear Schrödinger Equation:

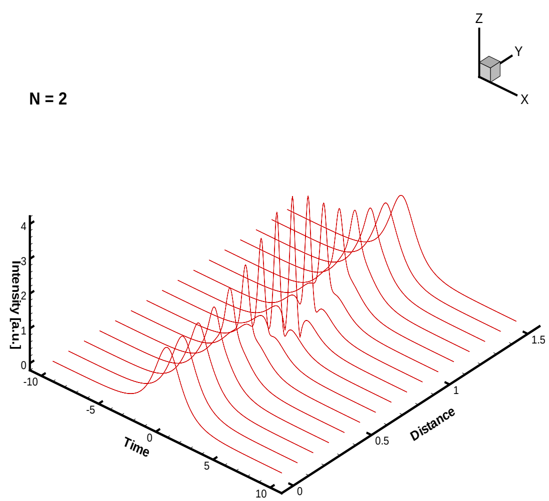
$$iU_z(z,t) \;=\; -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2 U(z,t)$$

Fundamental soliton:

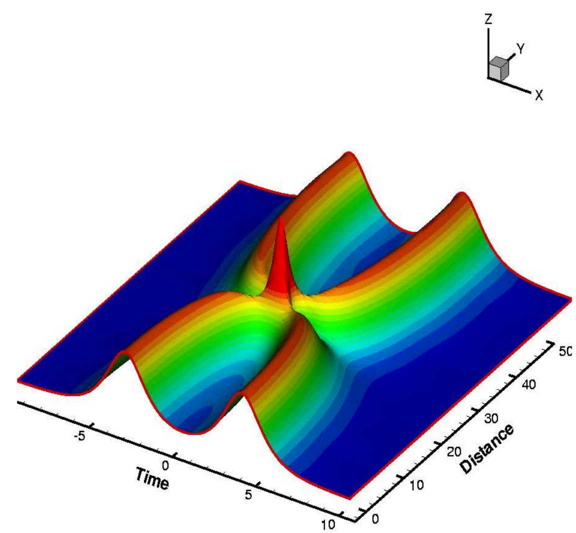
$$U(z,t) = \frac{n_0}{2} exp[i\frac{n_0^2}{8}z + i\theta_0] sech[\frac{n_0}{2}t]$$



Fundamental

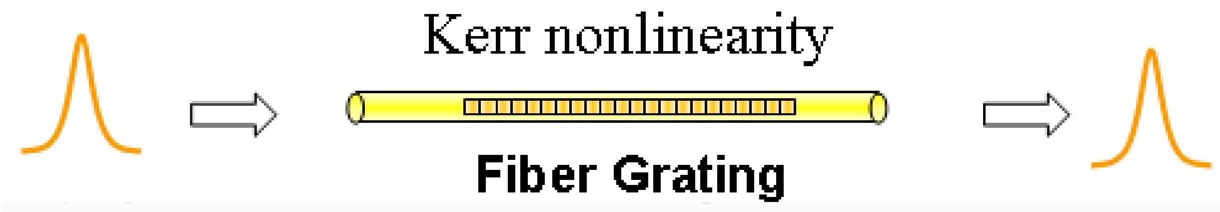


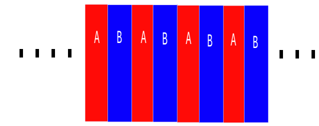
Higher-order ($N = 2$)

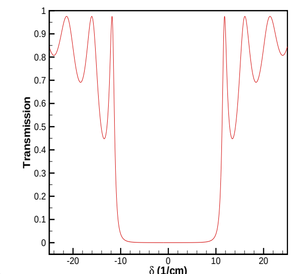


Soliton interaction

Fiber Bragg Grating Solitons





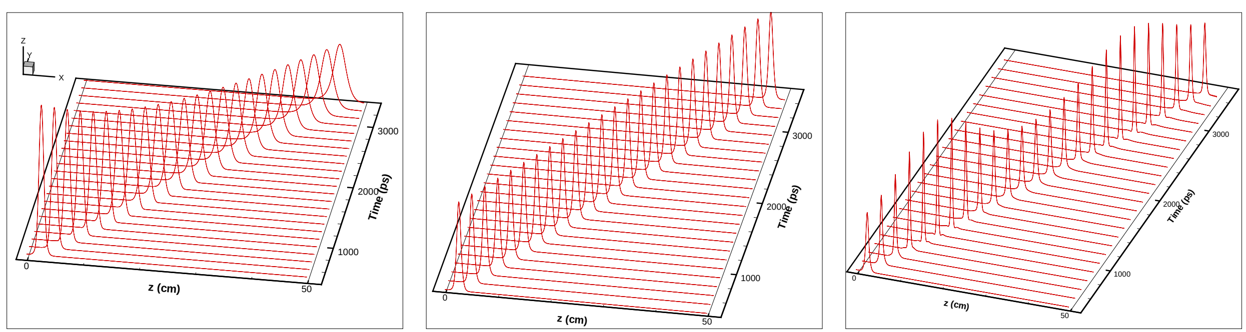


Nonlinear Coupled-Mode Equations:

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_a(z,t)+\frac{\partial}{\partial z}U_a=i\delta U_a+i\kappa U_b+i\Gamma|U_a|^2U_a+2i\Gamma|U_b|^2U_a$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_b(z,t)-\frac{\partial}{\partial z}U_b=i\delta U_b+i\kappa U_a+i\Gamma|U_b|^2U_b+2i\Gamma|U_a|^2U_b$$

decay stationary oscillate



A. Aceves and S. Wabnitz, *Phys. Lett. A* **141**, 37 (1986).

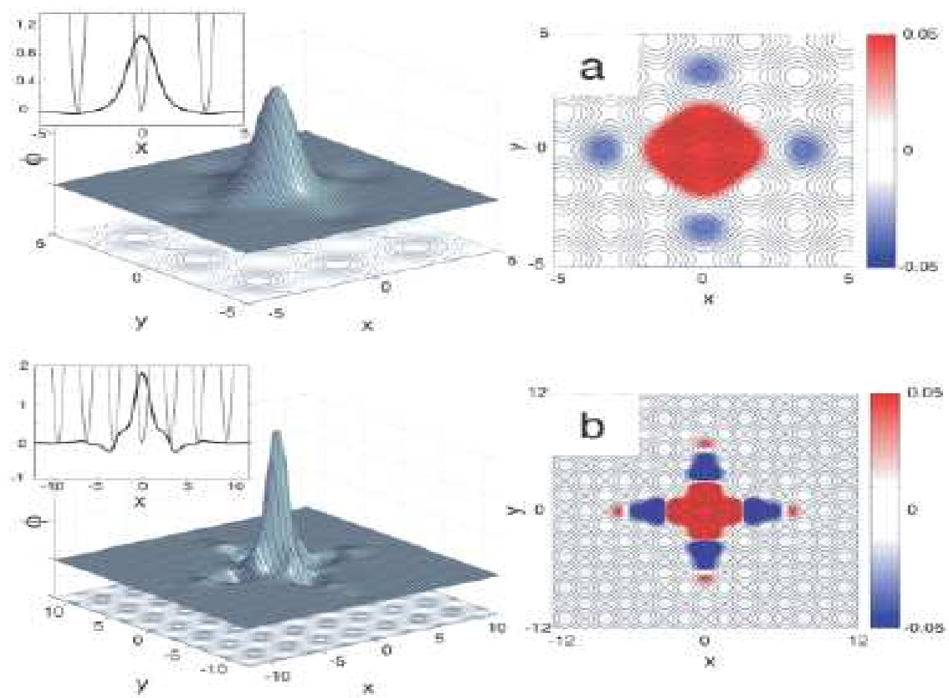
B. J. Eggleton, C. de Sterke, P. A. Krug, and J. E. Sipe,*Phys. Rev. Lett.* **76**, 1627 (1996).

BEC in optical lattices

Gross-Pitaevskii equation with periodic potentials,

$$i\hbar\frac{\partial}{\partial t}\Phi = -\frac{1}{2}\nabla^2\Phi + V(t)\Phi + g|\phi|^2\phi$$

which has gap soliton solutions in 1D, 2D, and 3D.



E. A. Ostrovskaya and Yu. S. Kivshar, *Phys. Rev. Lett.* **90**, 160407 (2003).

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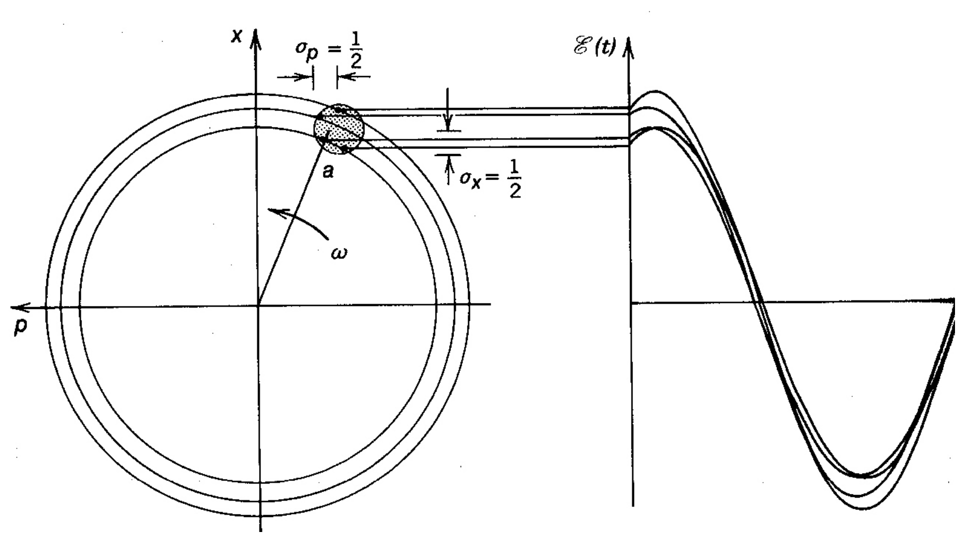
Phase diagram for EM waves

Electromagnetic waves can be represented by

$$\hat{E}(t) = E_0[\hat{X}_1 \sin(\omega t) - \hat{X}_2 \cos(\omega t)]$$

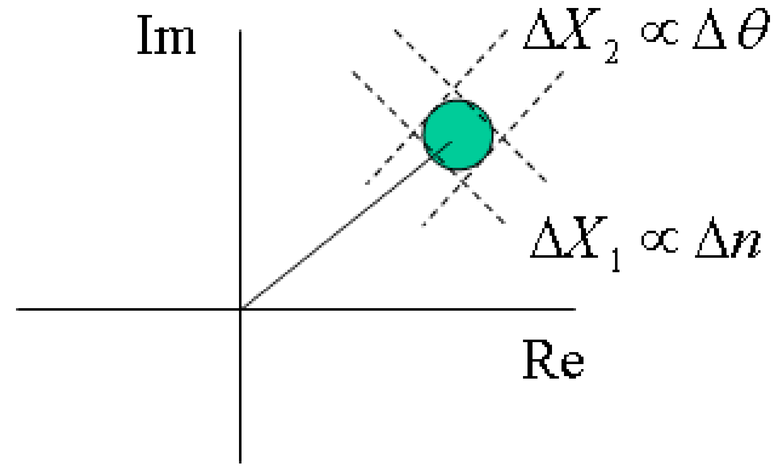
where

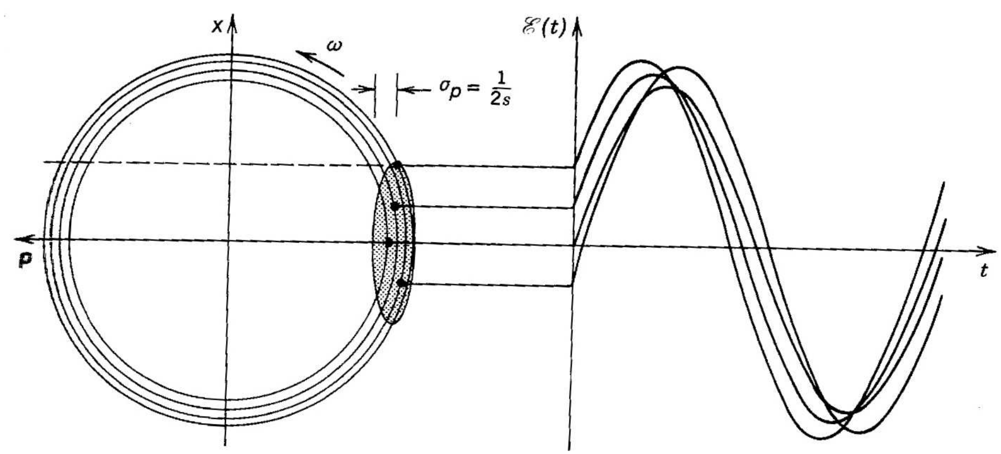
$$\hat{X}_1 = \text{amplitude quadrature}$$
$$\hat{X}_2 = \text{phase quadrature}$$



Uncertainty Principle: $\Delta\hat{X}_1\Delta\hat{X}_2 \geq 1$.

1. Coherent states: $\Delta\hat{X}_1 = \Delta\hat{X}_2 = 1$,
2. Amplitude squeezed states: $\Delta\hat{X}_1 < 1$,
3. Phase squeezed states: $\Delta\hat{X}_2 < 1$,
4. Quadrature squeezed states.





Definition of Squeezing and Correlation

Squeezing Ratio

$$\hat{M} = M + \Delta\hat{M}$$

$$\text{SR} = \frac{\langle \Delta\hat{M}^2 \rangle}{\langle \Delta\hat{M}^2 \rangle_{\text{c.s.}}}$$

SR $< 1 : Squeezing$

SR $> 1 : Anti - Squeezing$

Correlation

$$C = \frac{\langle : \Delta\hat{A}\Delta\hat{B} : \rangle}{\sqrt{\langle \Delta\hat{A}^2 \rangle \langle \Delta\hat{B}^2 \rangle}}$$

$0 \leq C \leq 1$: *Positive Correlation*

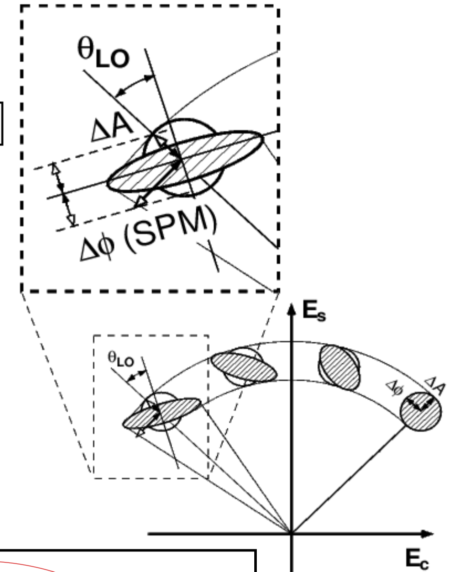
$C = 0$: *No Correlation*

$-1 \leq C \leq 0$: *Negative Correlatior*

Quadrature Squeezing of Solitons

For $N = 1$ soliton:

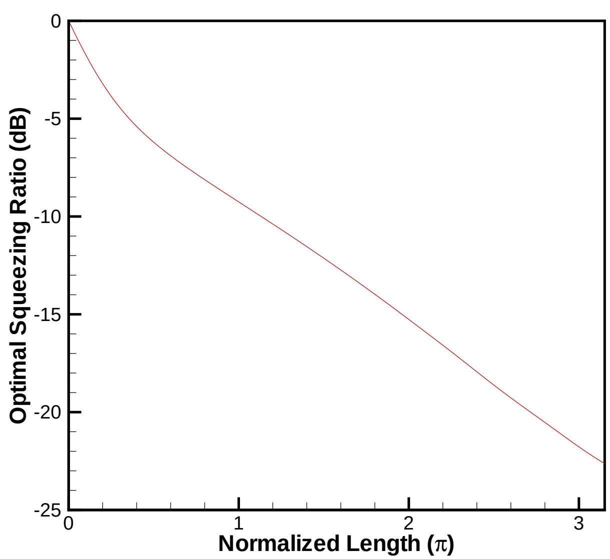
$$U(z,t) = \frac{n_0}{2} exp[i\frac{n_0^2}{8}z + i\theta_0] sech[\frac{n_0}{2}t]$$

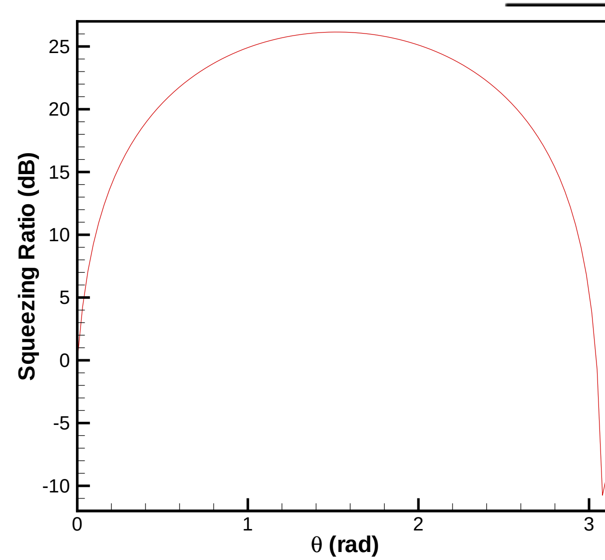


$$\Delta\hat{n}(z) = \Delta\hat{n}(0)$$

$$\Delta\hat{\theta}(z) = \Delta\hat{\theta}(0) + \frac{n_0}{4} z \Delta\hat{n}(0)$$

$$\Delta\hat{X}_\theta(z) = \alpha_1 \Delta\hat{n}(z) + \alpha_2 \Delta\hat{\theta}(z)$$



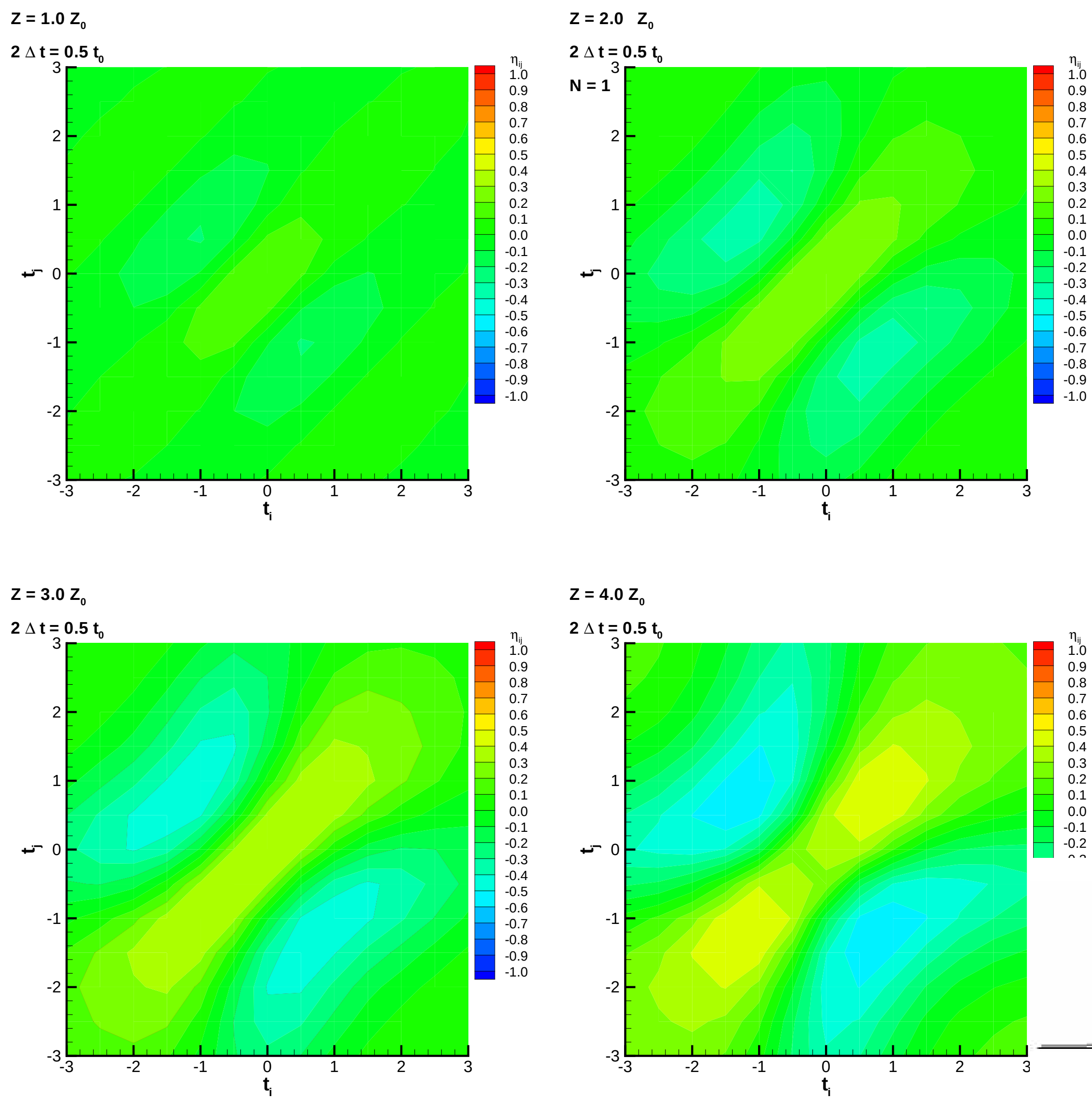


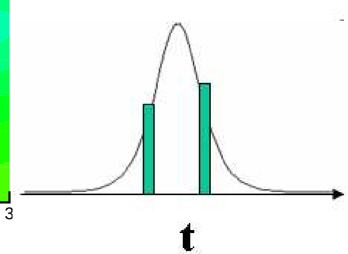
Optimal Squeezing Ratio $\equiv$ min $\frac{\mathrm{var}[\Delta\hat{X}_\theta(z)]}{\mathrm{var}[\Delta\hat{X}_\theta(0)]}$

Y. Lai and H. A. Haus, *Phys. Rev. A* **40**, 844 (1989); *ibid* **40**, 854 (1989).

Evolutions of Quantum correlation Spectra

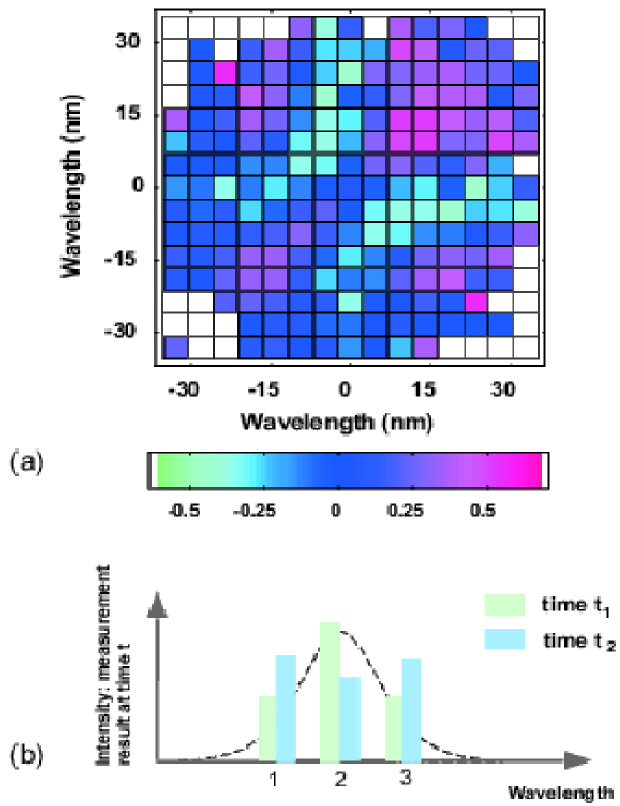
Time-domain intra-pulse photon-number correlations, for $N = 1$ soliton,

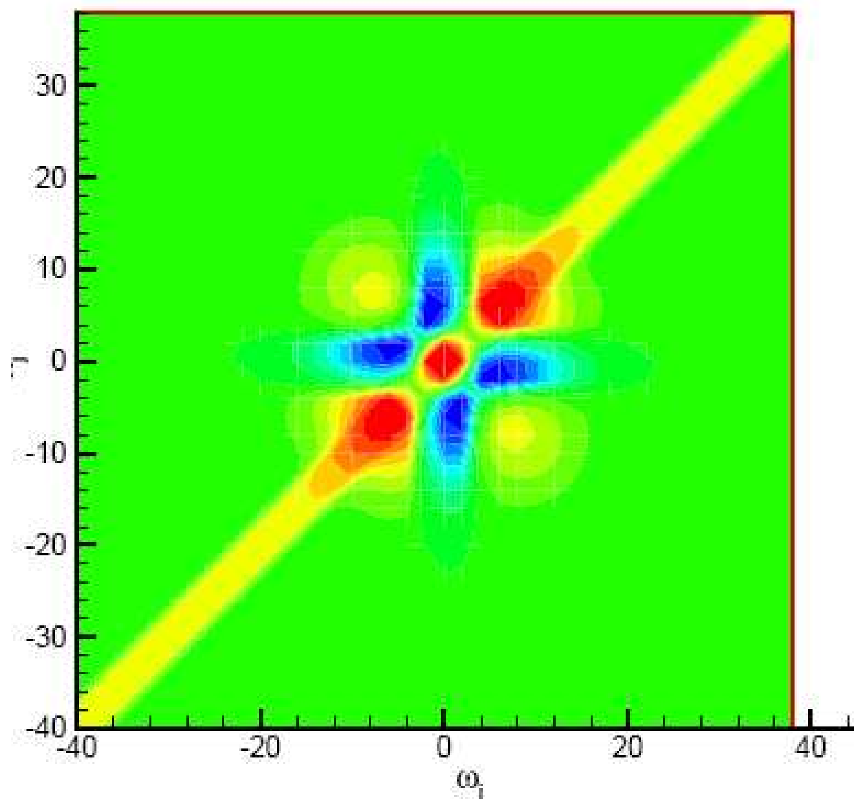




Multimode Quantum Correlations

With Spatral Filters, $C_{i,j} = \frac{\langle:\Delta\hat{n}_i\Delta\hat{n}_j:\rangle}{\sqrt{\langle\Delta\hat{n}_i^2\rangle\langle\Delta\hat{n}_j^2\rangle}}, i \neq j$





S. Spälter, N. Korolkova, F. Konig, A. Sizmann, and G. Leuchs,

Phys. Rev. Lett. **81**, 786 (1998).

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The Hamiltonian for Bragg solitons

The Hamiltonian for Bragg Solitons is

$$
\begin{aligned}
\mathcal{H} &= v_g\{-i\int dz\,(\hat{U}_a^\dagger \frac{\partial}{\partial z}\hat{U}_a - \hat{U}_b^\dagger \frac{\partial}{\partial z}\hat{U}_b) \\
&+ \int dz[\delta(\hat{U}_a^\dagger \hat{U}_a + \hat{U}_b^\dagger \hat{U}_b) - \kappa(\hat{U}_a^\dagger \hat{U}_b + \hat{U}_b^\dagger \hat{U}_a)] \\
&- \frac{\Gamma}{2}\int dz\,(\hat{U}_a^\dagger \hat{U}_a^\dagger \hat{U}_a \hat{U}_a + \hat{U}_b^\dagger \hat{U}_b^\dagger \hat{U}_b \hat{U}_b) \\
&- \Gamma\int dz\,(\hat{U}_a^\dagger \hat{U}_b^\dagger \hat{U}_b \hat{U}_a + \hat{U}_b^\dagger \hat{U}_a^\dagger \hat{U}_a \hat{U}_b)\}
\end{aligned}
$$

where $\hat{U}_a$, $\hat{U}_b$ represent forward/backward fields, satisfying Bosonic commutation relations:

$$[\hat{U}_a(z_1,t),\hat{U}_a^\dagger(z_2,t)] = \delta(z_1-z_2), \quad [\hat{U}_b(z_1,t),\hat{U}_b^\dagger(z_2,t)] = \delta(z_1-z_2),$$

$$[\hat{U}_a(z_1,t),\hat{U}_a(z_2,t)] = [\hat{U}_a^\dagger(z_1,t),\hat{U}_a^\dagger(z_2,t)] = [\hat{U}_b(z_1,t),\hat{U}_b(z_2,t)] = 0$$

$$[\hat{U}_b^\dagger(z_1,t),\hat{U}_b^\dagger(z_2,t)] = [\hat{U}_a(z_1,t),\hat{U}_b(z_2,t)] = [\hat{U}_a(z_1,t),\hat{U}_b^\dagger(z_2,t)] = 0$$

Linearization Approach

By setting $\hat{U}(x,t) = U_0(z,t) + \hat{u}(z,t)$, we can linearize the QNLCME as follows:

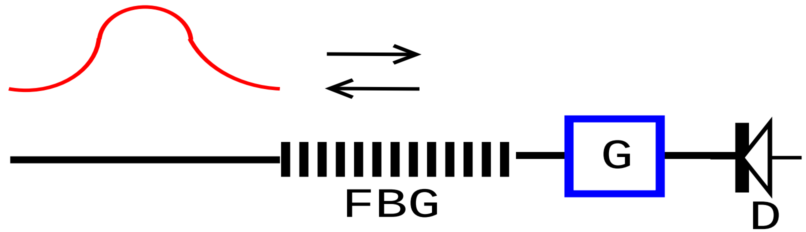
$$\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix} \hat{u}_a \\ \hat{u}_b \end{pmatrix} = \begin{pmatrix} i\Gamma U_{a0}^2 & 2i\Gamma U_{a0}U_{b0} \\ 2i\Gamma U_{a0}U_{b0} & +i\Gamma U_{b0}^2 \end{pmatrix}\begin{pmatrix} \hat{u}_a^\dagger \\ \hat{u}_b^\dagger \end{pmatrix} +$$

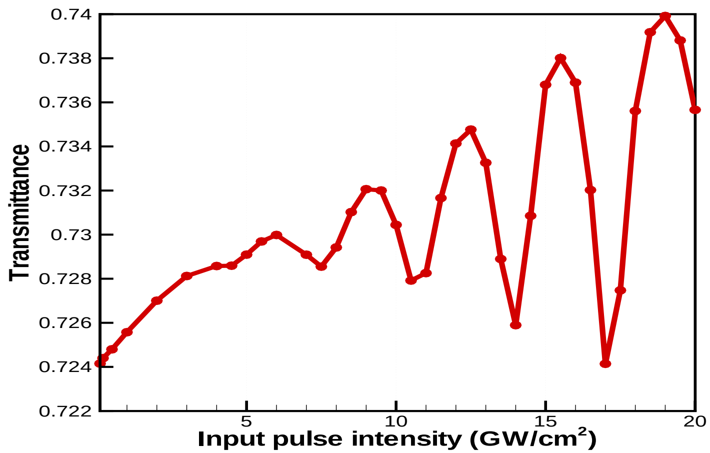
$$\begin{pmatrix} -\frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 & i\kappa + 2i\Gamma U_{a0}U_{b0}^* \\ +i\kappa + 2i\Gamma U_{a0}^* U_{b0} & \frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 \end{pmatrix} \cdot \begin{pmatrix} \hat{u}_a \\ \hat{u}_b \end{pmatrix}$$

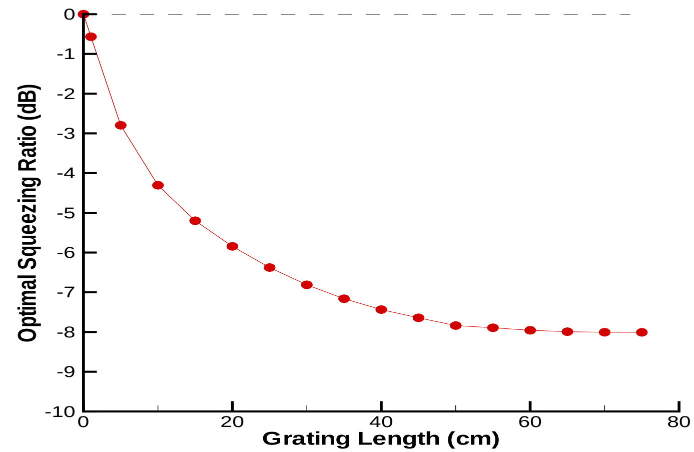
where the perturbation fields $\hat{u}_a(z,t)$ and $\hat{u}_b(z,t)$ also have to satisfy the same Bosonic commutation relations.

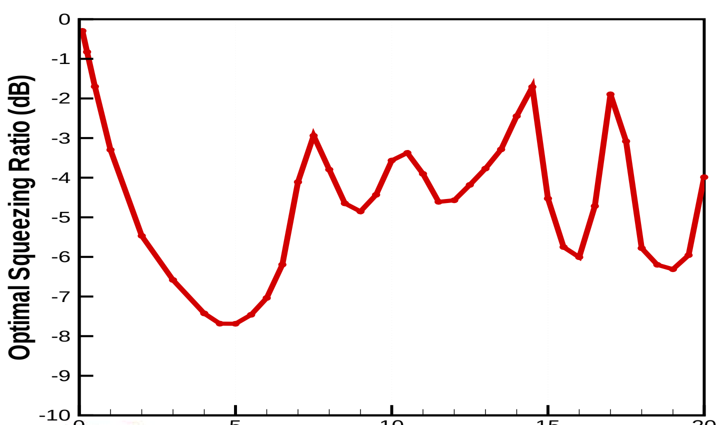
R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

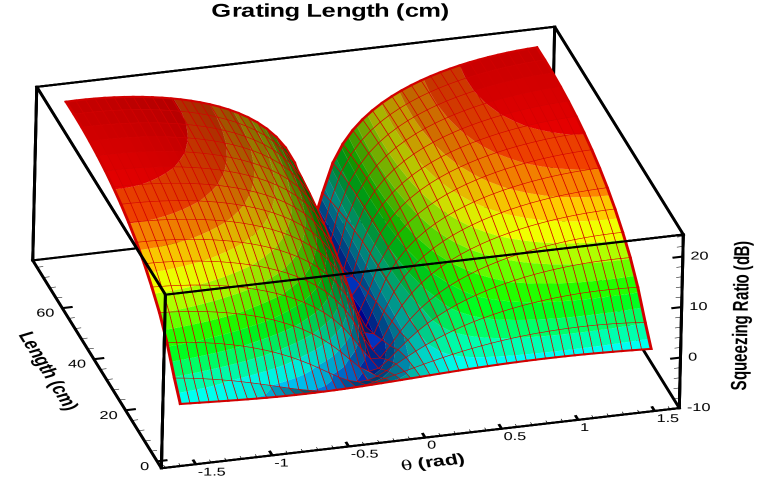
Amp. Squeezing of FBG solitons











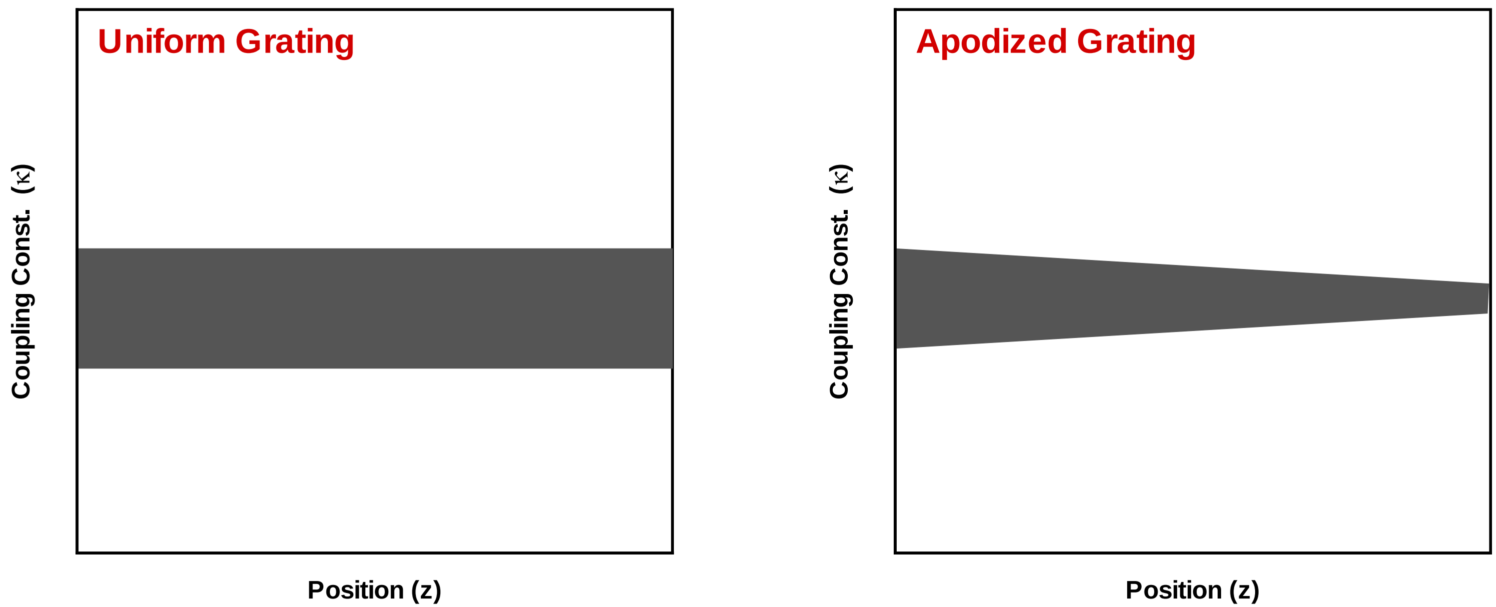
R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

Apodized Fiber Bragg Gratings

We consider a nonuniform FBG which has a position dependent coupling coefficient described by

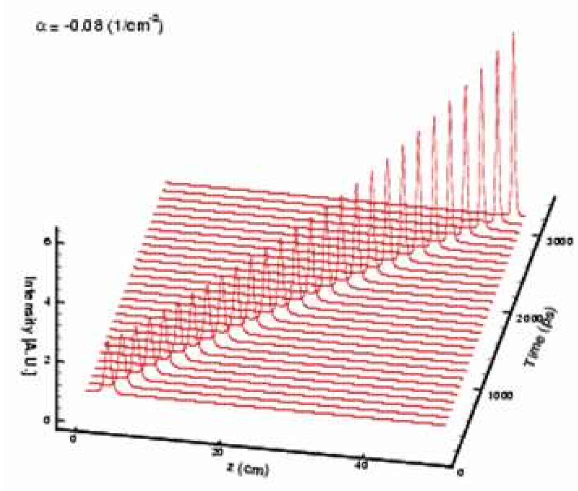
$$\kappa(z) = \kappa_0 + \alpha z$$

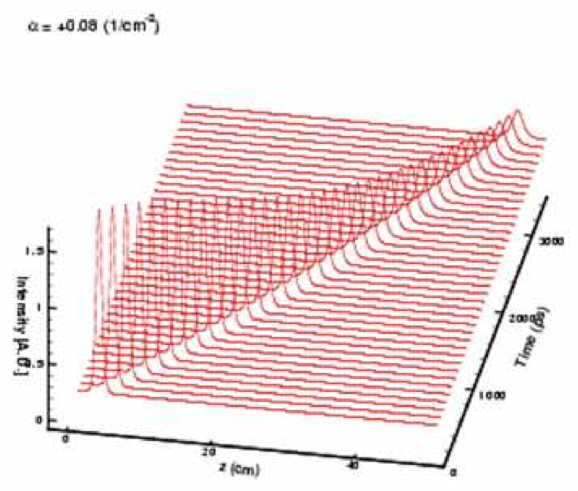
where κ_0 is the initial coupling coefficient and α is the slope of the coupling coefficient.

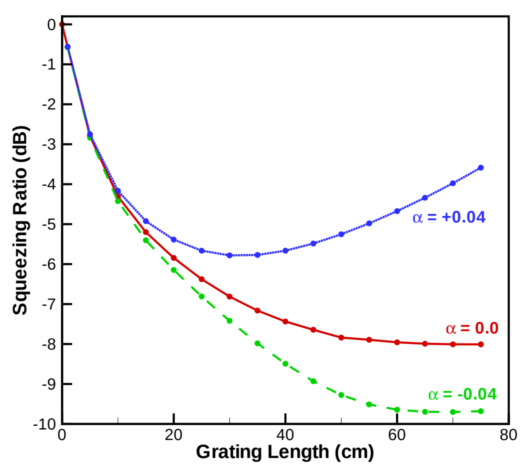


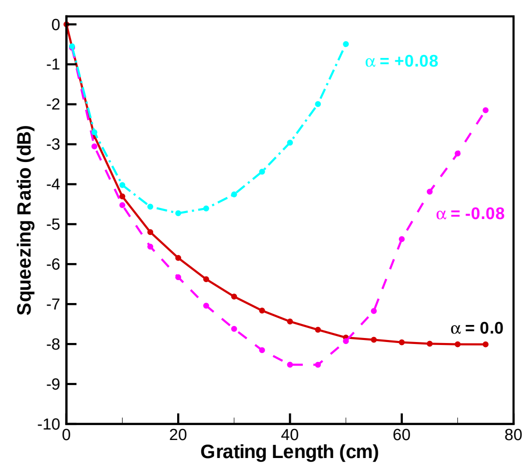
Tailor the Noise by Apodized Fiber Bragg Gratings

$\alpha < 0$ $\alpha > 0$









$\alpha = \pm 0.04(1/cm^2)$ $\alpha = \pm 0.08(1/cm^2)$

R.-K. Lee and Y. Lai, *J. Opt. B* **6**, S638 (Special Issue 2004).

Mater-wave gap soliton in optical lattices

In terms of field operators, the Hamiltonian for the BEC is

$$H = \int dr[-\hat{\Phi}^{\dagger}(r)\frac{1}{2}\nabla^2\hat{\Phi}(r)+V(r)\hat{\Phi}^{\dagger}(r)\hat{\Phi}(r)+g\hat{\Phi}^{\dagger}(r)\hat{\Phi}^{\dagger}(r)\hat{\Phi}(r)\hat{\Phi}(r$$

In the Heisenberg picture, for 1-D,

$$i\hbar\frac{\partial}{\partial t}\hat{\Phi}(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\hat{\Phi}(t,x)+V(x)\hat{\Phi}(t,x)+g_{1D}\hat{\phi}^{\dagger}(t,x)\hat{\phi}(t,x)\hat{\phi}(t,x$$

where $\hat{\Phi}(t,x)$ and $\hat{\Phi}^{\dagger}(t,x)$ are field operators with Bosonic commutation relations:

$$[\hat{\Phi}(t,x'),\hat{\Phi}^{\dagger}(t,x)] = \delta(x-x')$$

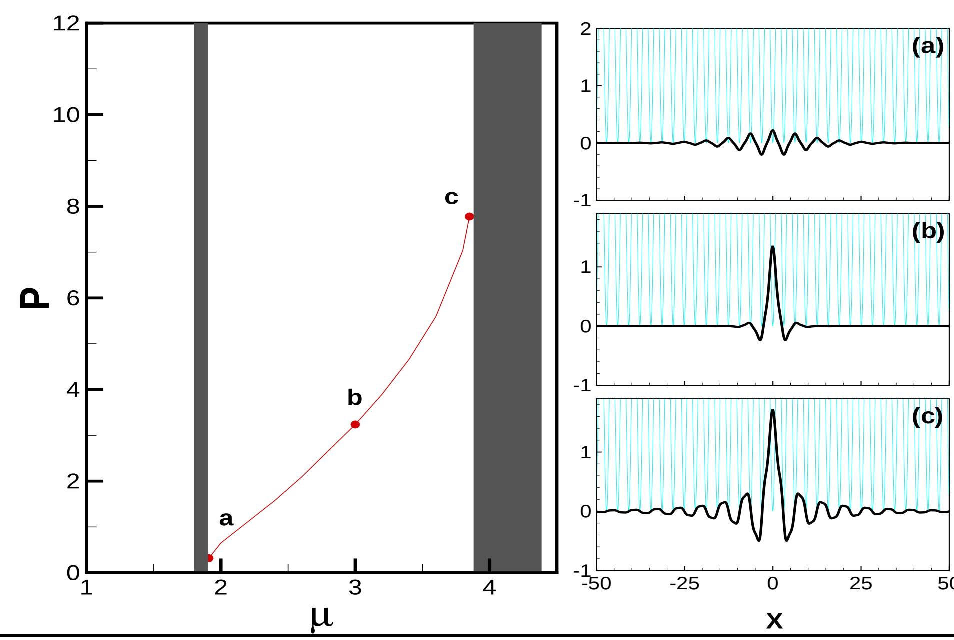
$$[\hat{\Phi}(t,x'),\hat{\Phi}(t,x)] = [\hat{\Phi}^{\dagger}(t,x'),\hat{\Phi}^{\dagger}(t,x)] = 0$$

Linearization approach for large atom number

Using $\hat{\Phi}(t,x) = \Phi_0(t,x) + \hat{\phi}(t,x)$ for large atom number, where $\Phi_0(t,x)$ is the mean-field solution of 1-D Gross-Pitaevskii equation,

$$i\hbar\frac{\partial}{\partial}\Phi_0(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi_0(t,x)+V(x)\Phi_0(t,x)+g_{1D}|\phi_0(t,x)|^2\phi_0(t,x)$$

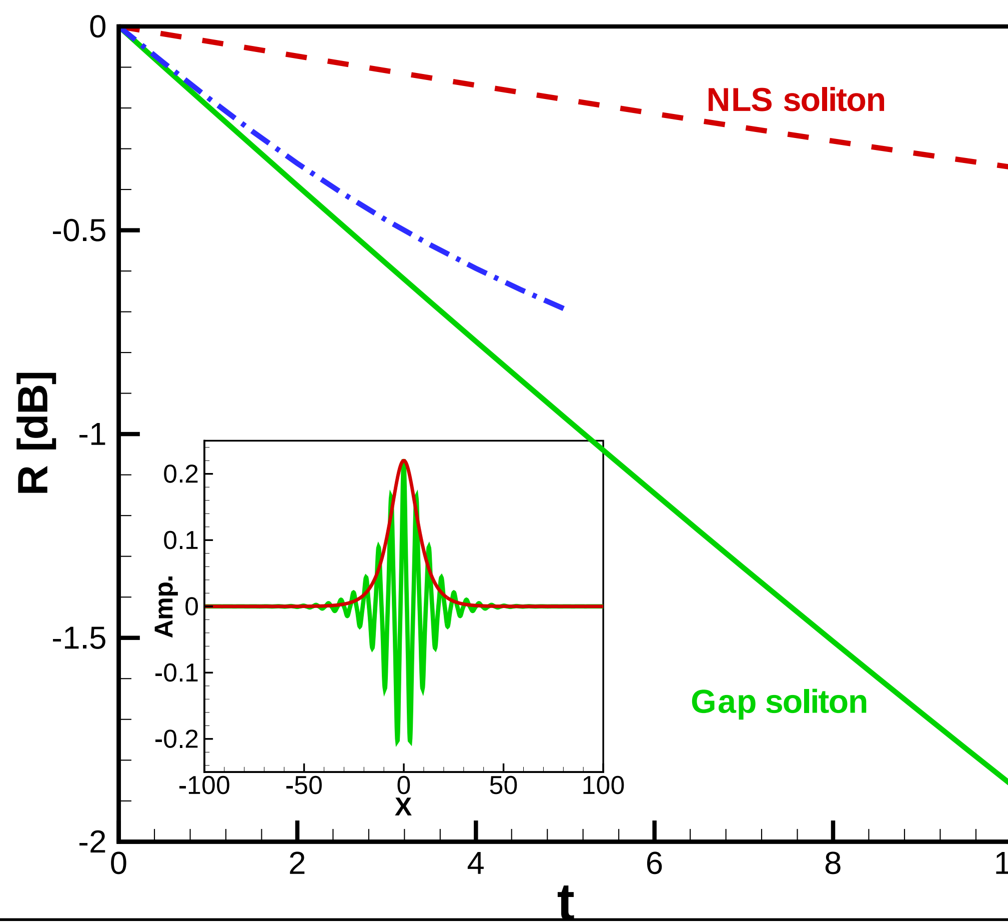
which has gap soliton solutions.



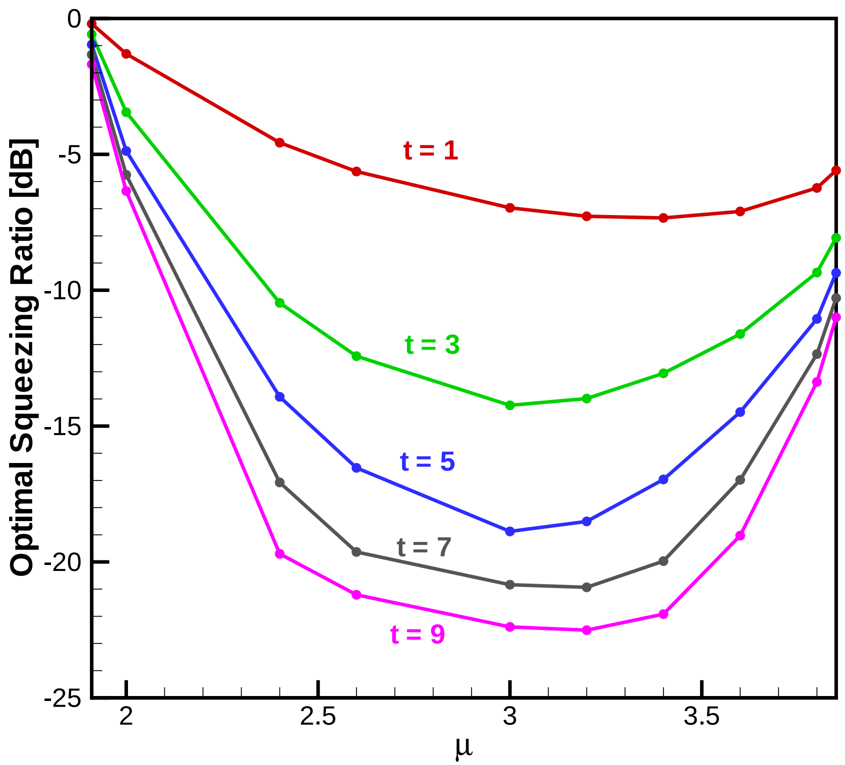
Comparison of envelope function near the bandedge

Near the bottom edge of the gap, one can use envelope approximation for the gap solitons

$$\psi(t,x) = AF(x)\phi(t,x).$$



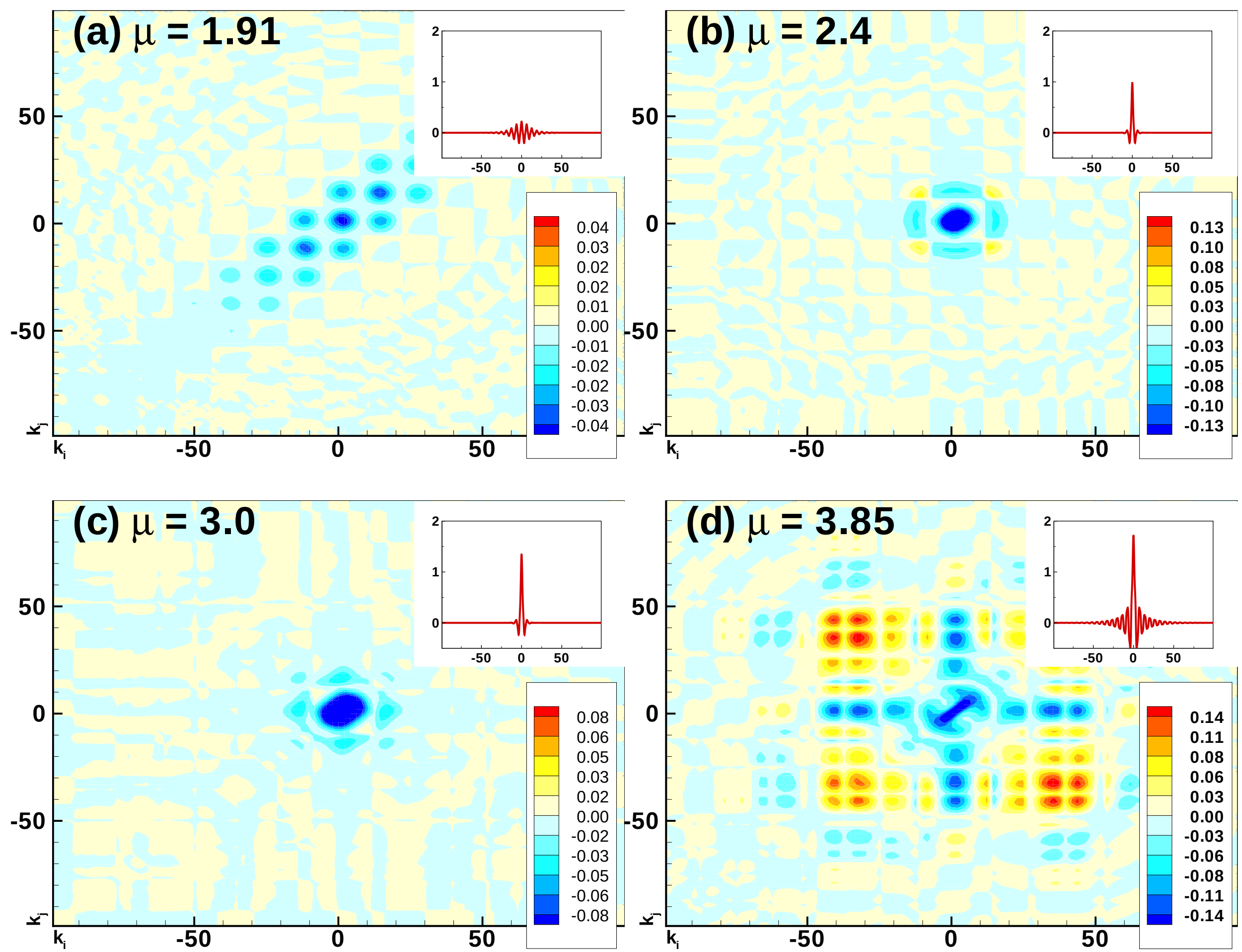
Squeezing ratio v.s. chemical potential



Squeezing effect is most profound in the depth of the gap and reduced near the band edges.

Quantum correlation patterns v.s. chemical potential

x-domain



R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai,

Phys. Rev. A **72**, 033607 (2005).

Outline

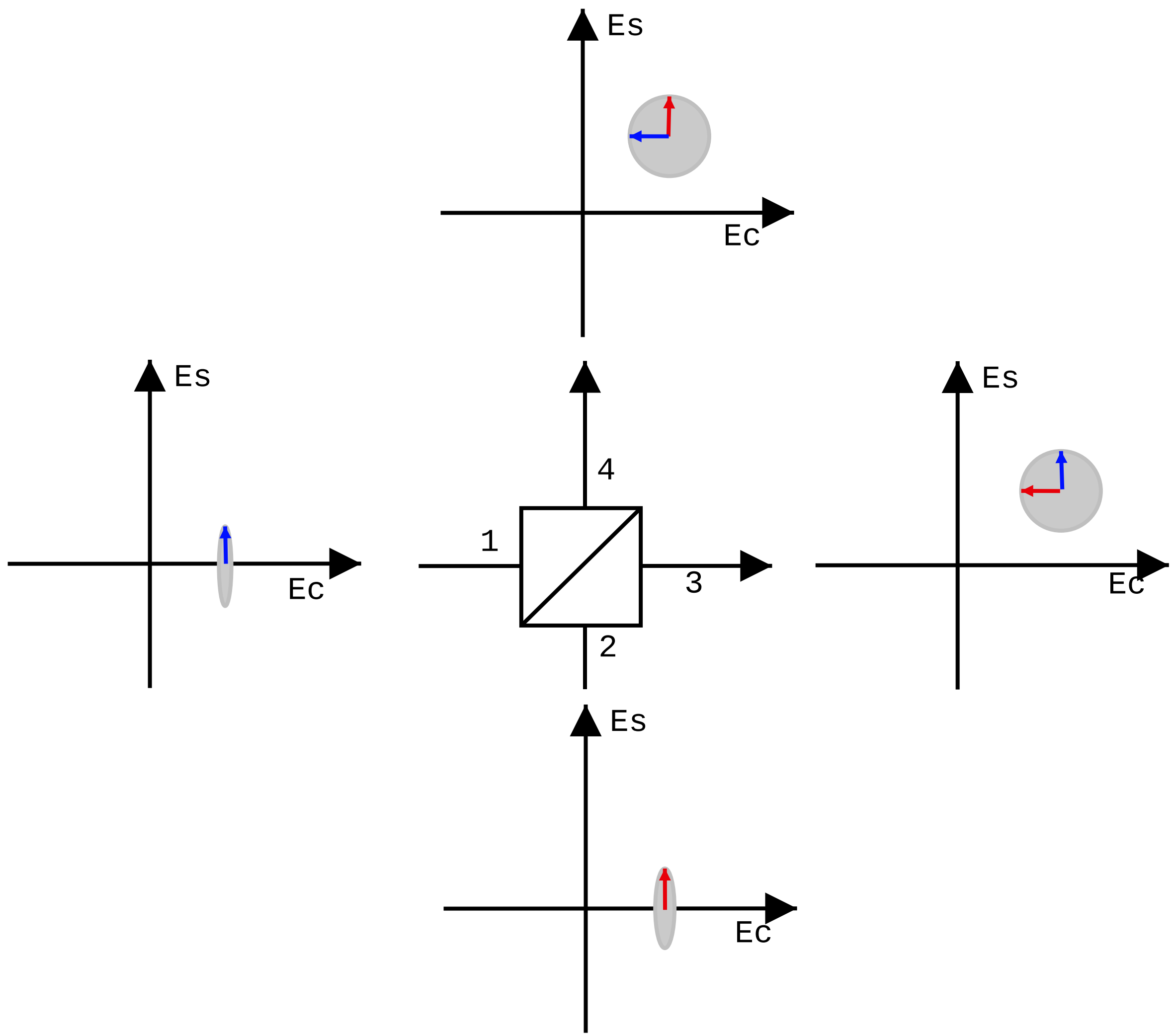
1. On the Shoulders of Giants
2. The Great Wave of Translation
3. Quantum Solitons
4. Quantum Bragg and Gap Solitons
5. Entangled Solitons for Quantum Information
6. Conclusions

Applications of Squeezed Light

- Gravitational Waves Detection
- Quantum Non-Demolition Measurement (QND)
- Super-Resolved Images (Quantum Images)
- Generation of EPR Pairs

Generation of Continuous Variables Entanglement

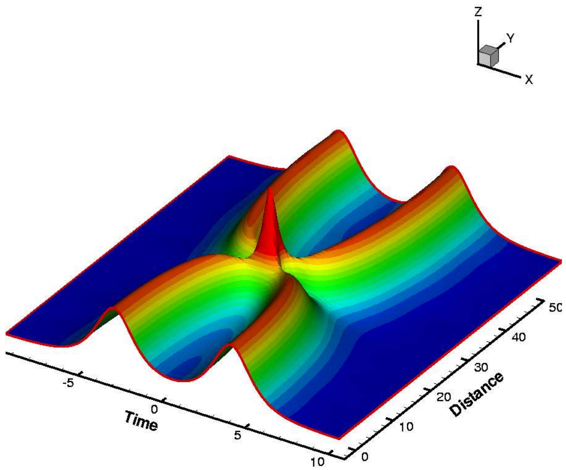
Preparation EPR pairs by Squeezed Sates



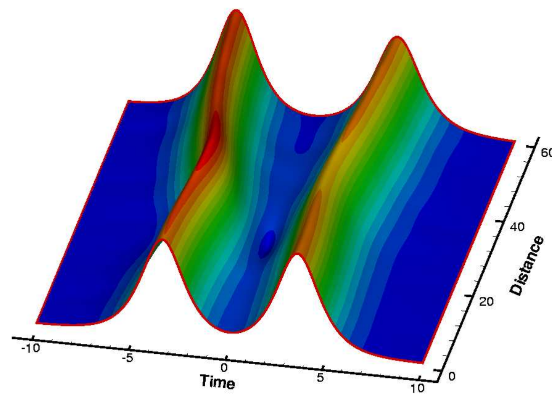
$$\delta\hat{n}_3 = -\delta\hat{n}_4, \delta\hat{\theta}_3 = \delta\hat{\theta}_4.$$

Photon Number Correlation of 2-Solitons Interaction

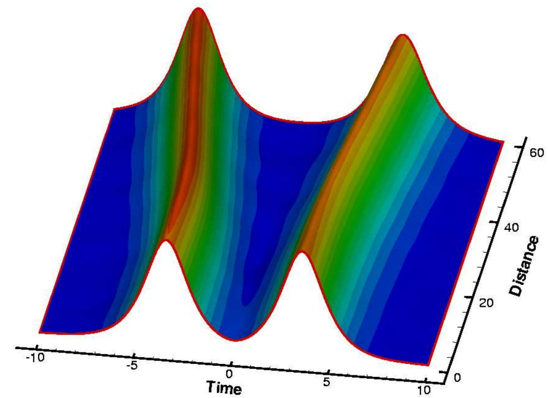
$$U(z,t) = sech(z, t+\rho) + r\,sech(z, t_\rho)e^{i\theta}$$



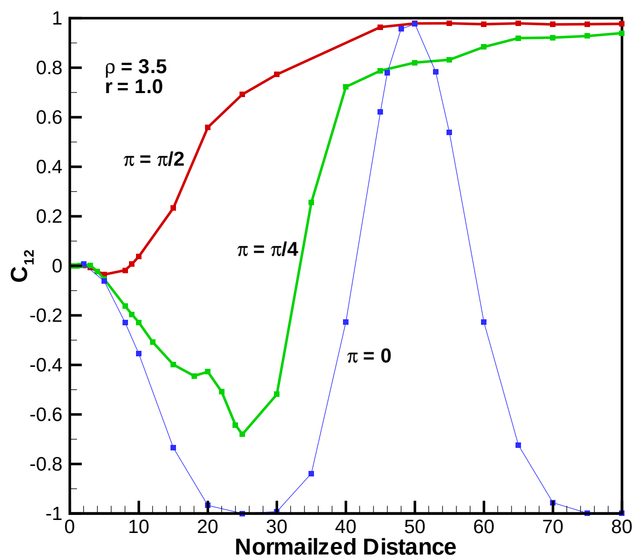
$\theta = 0$



$\theta = \pi/4$



$\theta = \pi/2$



$$C_{1,2} = \frac{\langle :\Delta\hat{n}_1 \Delta\hat{n}_2 : \rangle}{\sqrt{\Delta\hat{n}_1^2 \Delta\hat{n}_2^2}}$$

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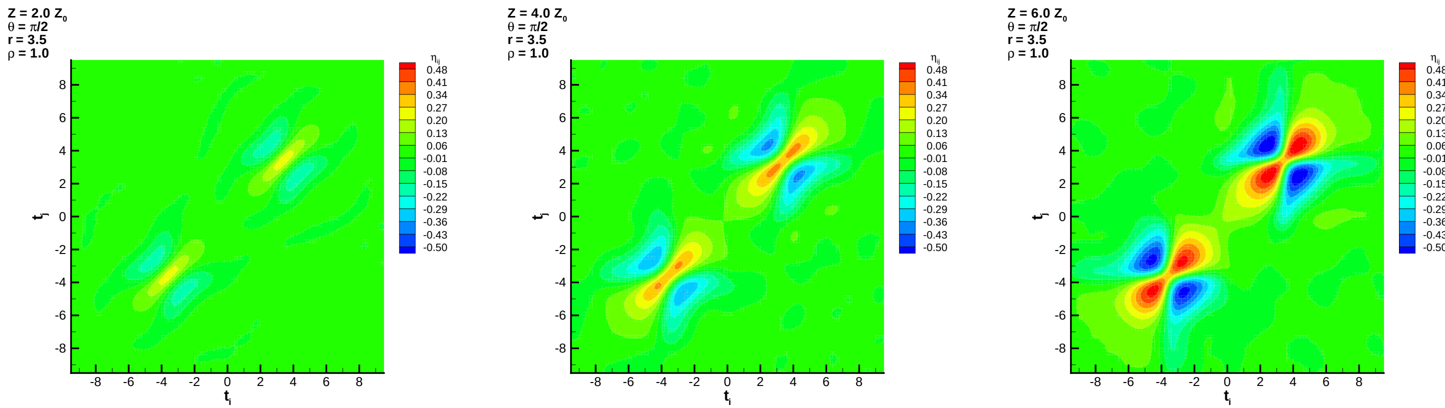
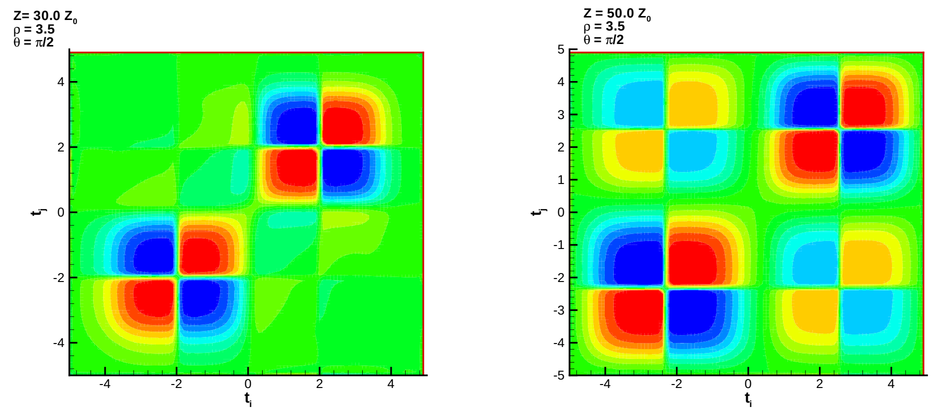
Evolutions of Photon Number Correlation Spectra

$Z = 2.0 Z_0$, $Z = 4.0 Z_0$, $Z = 6.0 Z_0$.



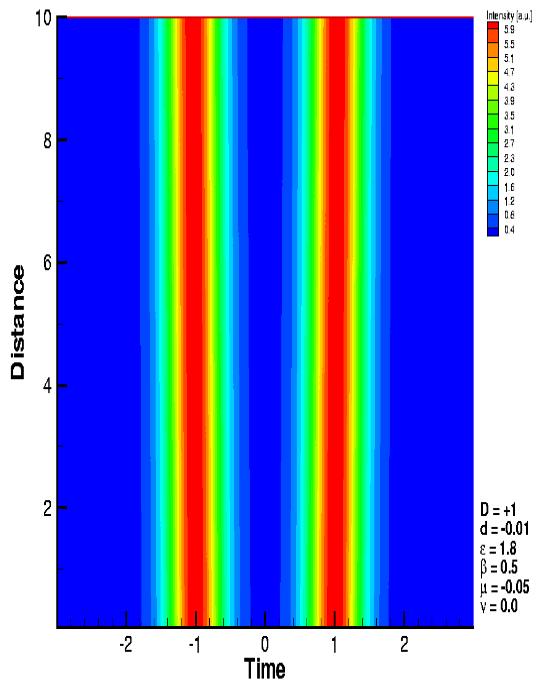


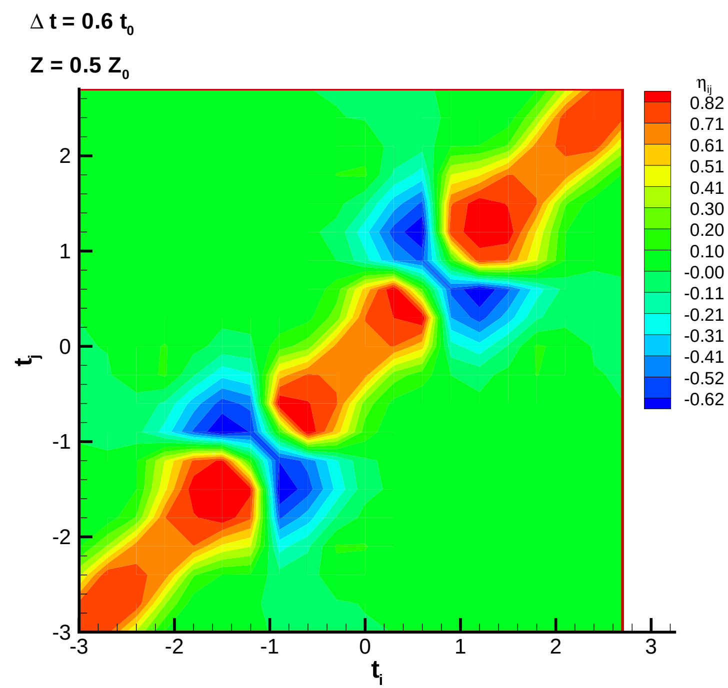
$Z = 30.0 Z_0$, $Z = 50.0 Z_0$

R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **71**, 013816 (2005).

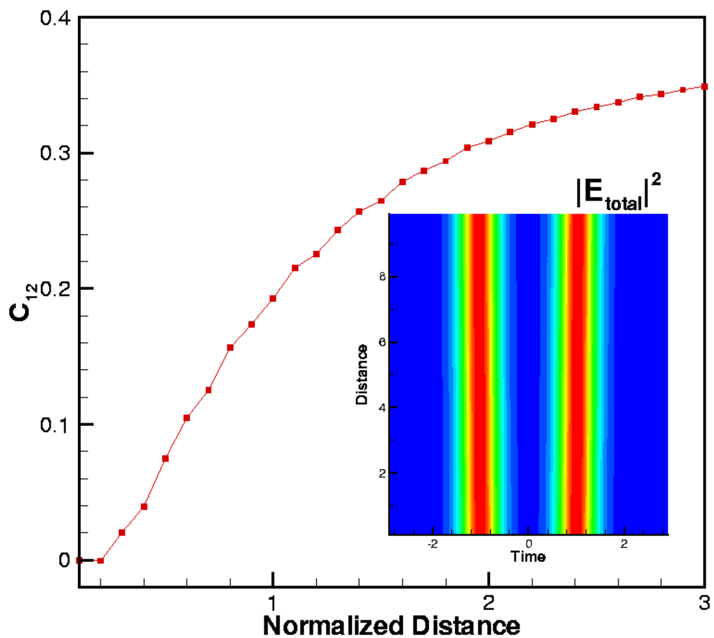
Quantum Correlations of Bound-States of Solitons

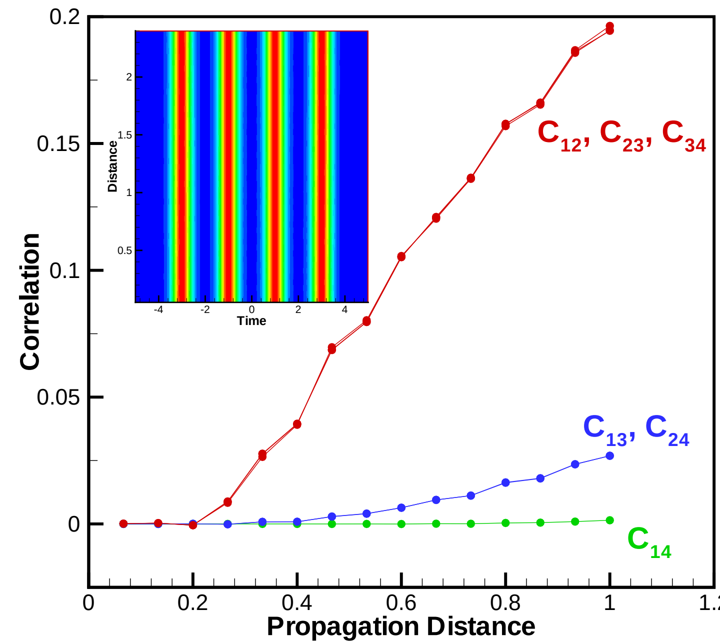
$$
\begin{aligned}
iU_z + \frac{D}{2}U_{tt} + |U|^2U &= i\delta U + i\epsilon|U|^2U + i\beta U_{tt} \\
&+ i\mu|U|^4U - v|U|^4U
\end{aligned}
$$





Photon Number Correlations of Bounded Soliton Trains

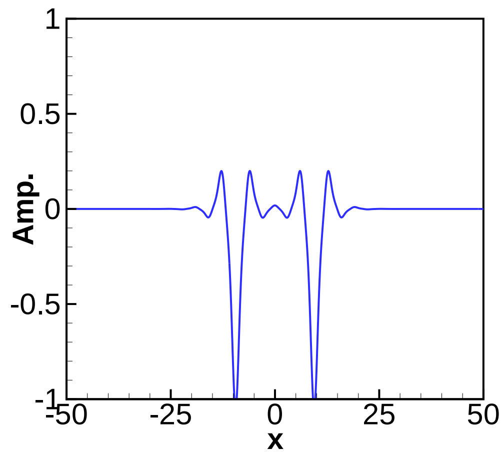


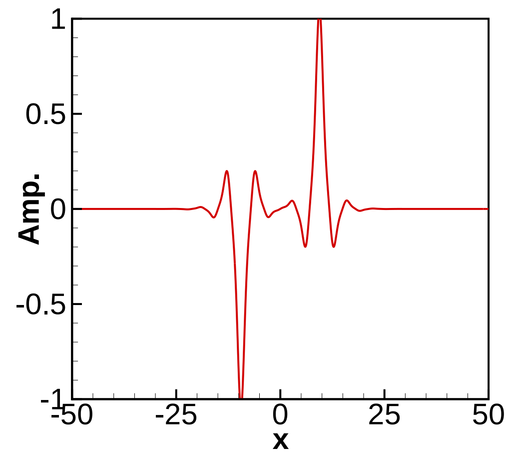


Photon-number correlated bound soliton trains offer novel possibilities to produce multipartite entangled sources for quantum communication and computation.

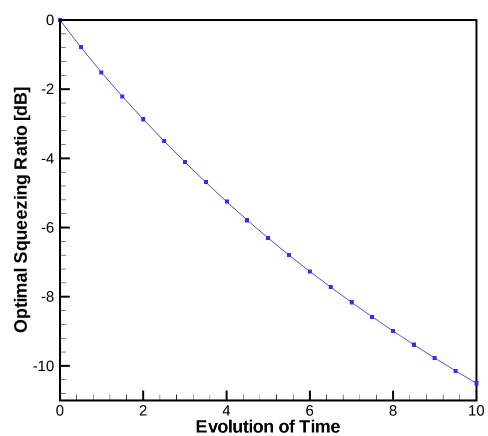
R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004).

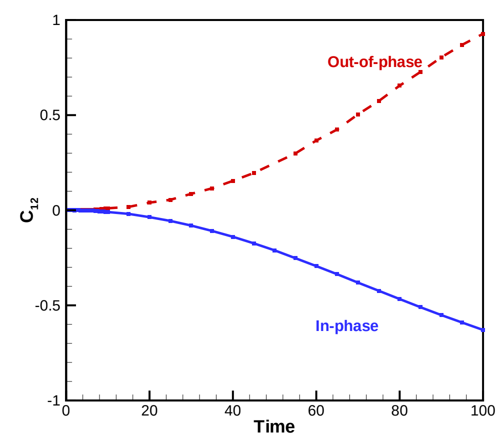
Bound gap solitons and high correlated EPR pairs





The noise fluctuations of bound gap soliton pairs are the same, but with different photon-number correlation parameter.

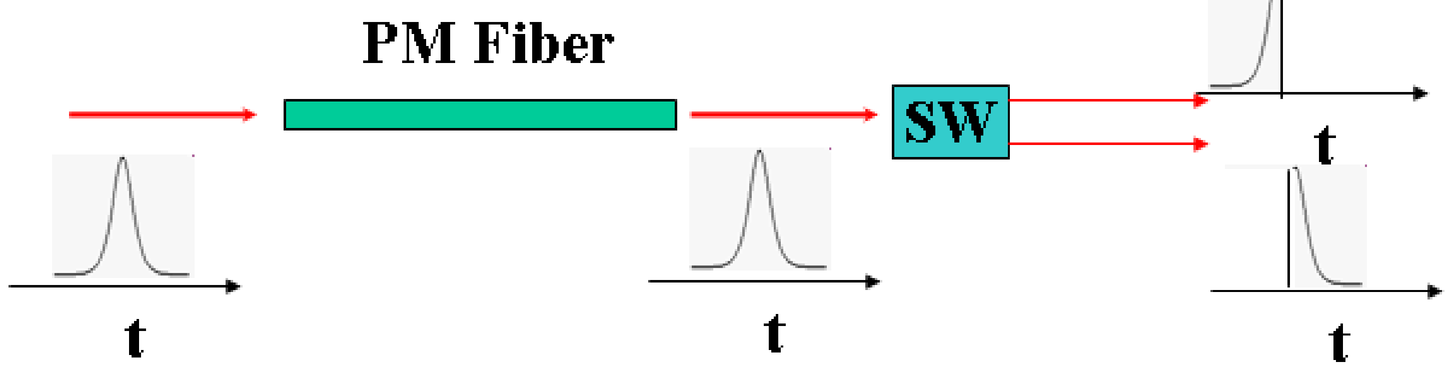




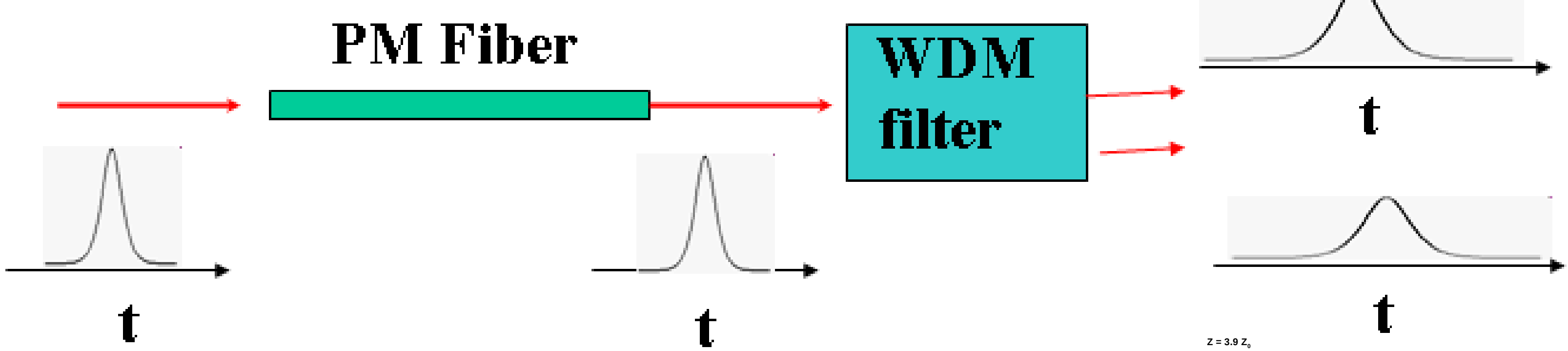
R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72**, 033607 (2005).

Entangled States by Time or Wavelength Slicing

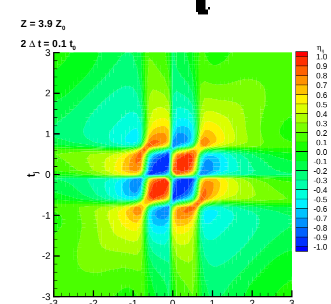
(1) time slicing



(2) Wavelength slicing

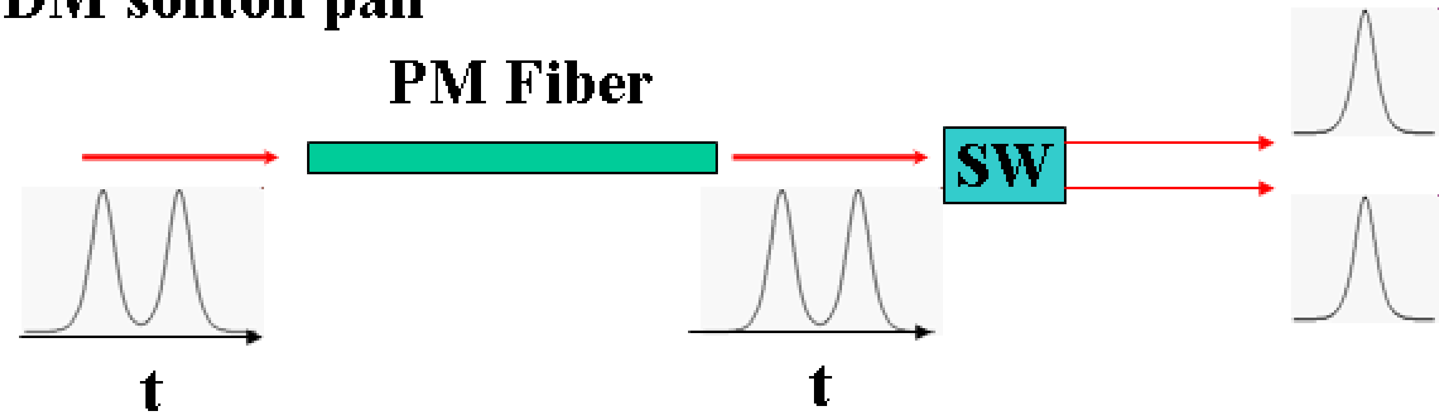


Quantum Images !

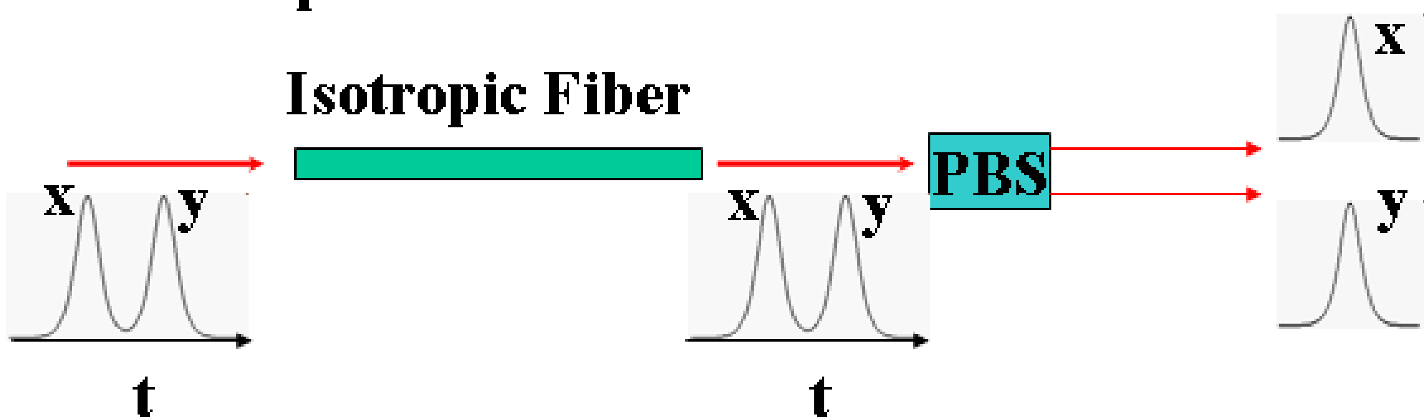


Entangled Soliton Pairs

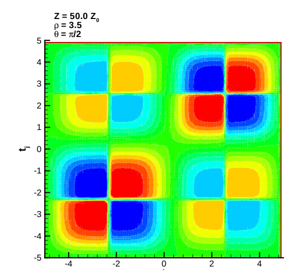
(1) TDM soliton pair



(2) PDM soliton pair



If necessary, the Sagnac loop configuration also can be used.



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Conclusions

1. Optical lattice offers a new way to stabilize optical/matter-wave solitons in high dimensions.
2. Quantum properties and theories of gap solitons are reviewed.
3. Possible applications of quantum optical solitons in quantum information are needed to be explored more.

Ref: R.-K. Lee *et al.*,
q-Bragg-soliton: *Phys. Rev. A* **69**, 021801(**R**) (2004);
apodization: *J. Opt. B* **6**, S638 (2004);
q-bistable-soliton: *J. Opt. B* **6**, 367 (2004);
squeezed-spectra: *J. Opt. B* **6**, S715 (2004);
q-bound-soliton: *Phys. Rev. A* **70**, 063817 (2004);
entangled-soliton: *Phys. Rev. A* **71**, 013816 (2005);
q-N-2-soliton: *Phys. Rev. A* **71**, 035801 (2005);
q-gap-soliton: *Phys. Rev. A* **72**, 033607 (2005).
q-π-bound-soliton: *Opt. Lett.* **30**, xxxx (2005, accepted).

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