11, Quantum theory for Nonlinear Pulse Propagation

1. Quantum Nonlinear Schrödinger Equation
2. Quadrature Squeezing of Optical Solitons
3. Amplitude Squeezing of Bragg Solitons
4. Quantum Correlation of Solitons
5. Quantum theory for Bound-State Solitons

Ref:

"Electromagnetic Noise and Quantum Optical Measurements," by H. Haus.

R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(**R**) (2004);

R.-K. Lee and Y. Lai, *J. Opt. B* **6**, S638 (2004);

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R.-K. Lee, Y. Lai and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004);

R.-K. Lee, Y. Lai and Yu. S. Kivshar, *Phys. Rev. A* **71**, 035801 (2005);

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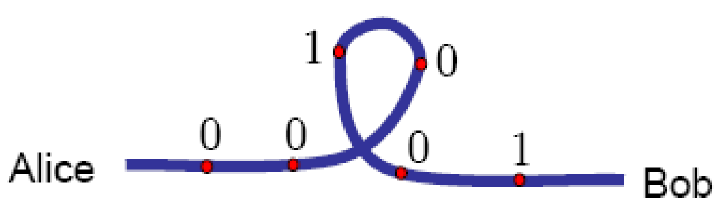
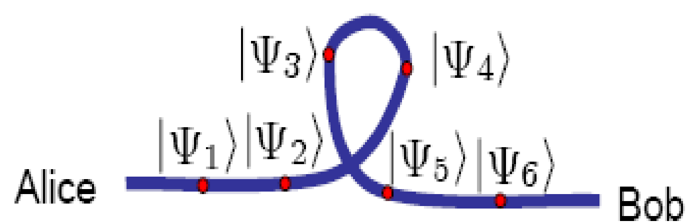
Communication

- classical communication



- quantum communication



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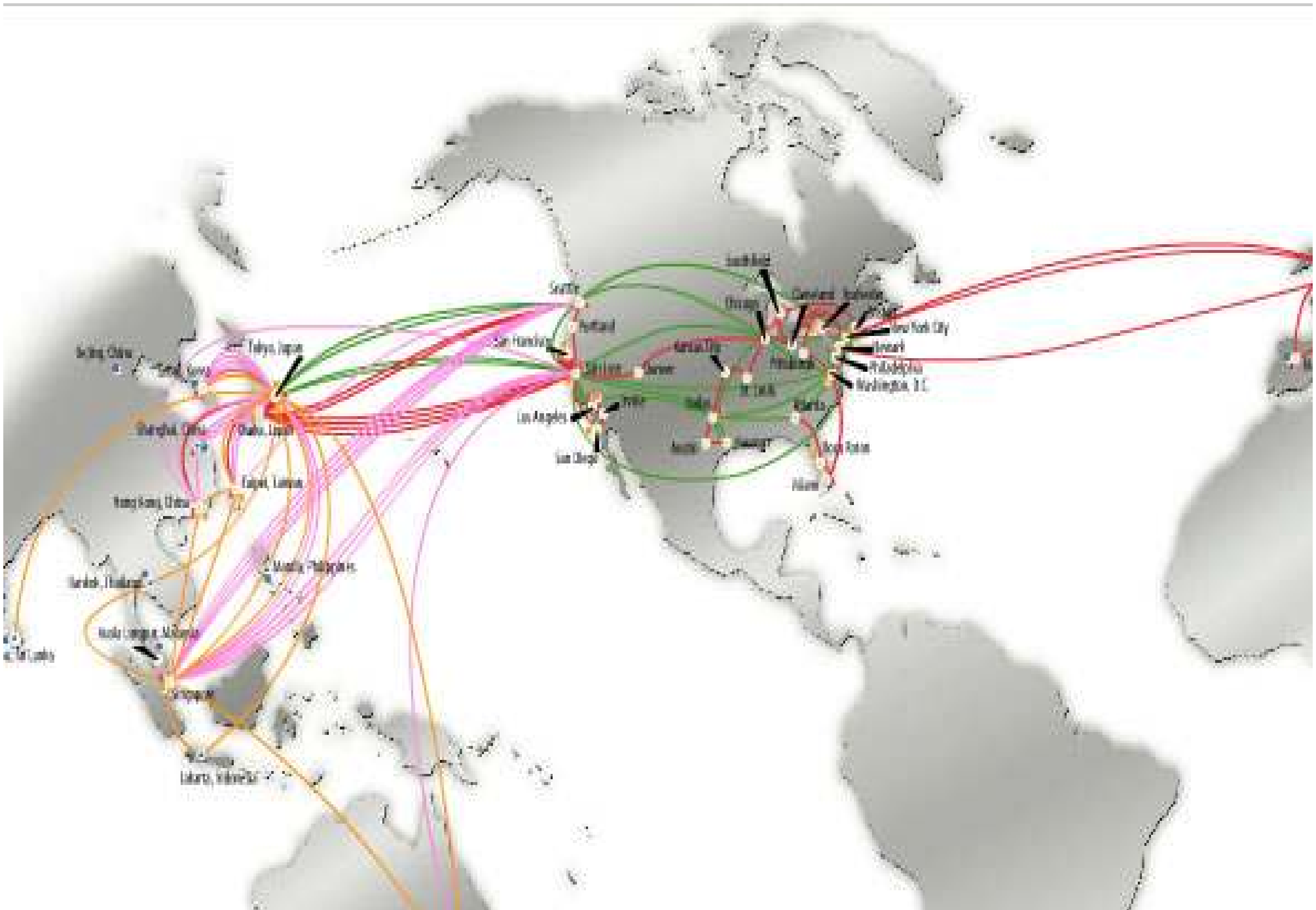
Global overseas fiber network



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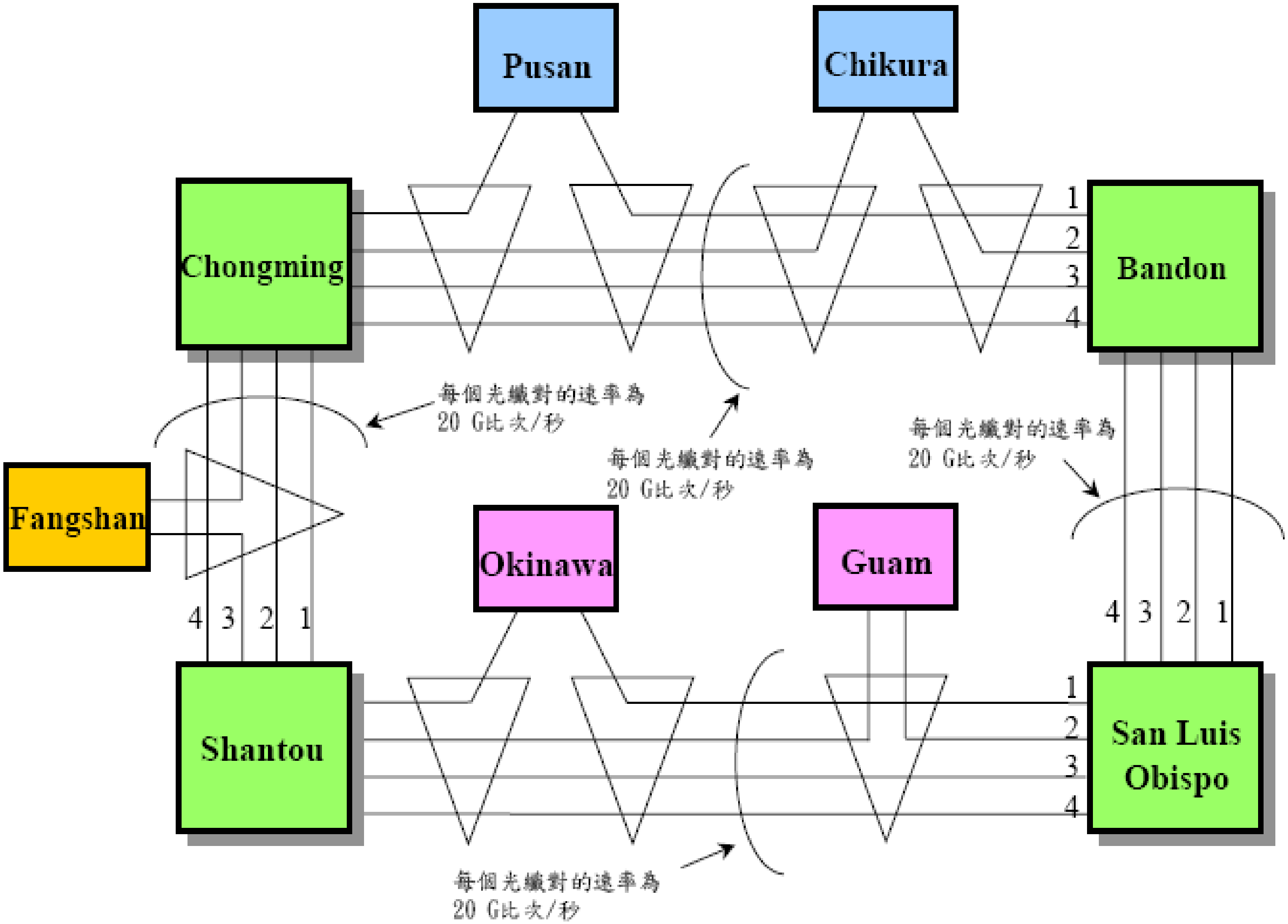
Taiwan-US overseas fiber network



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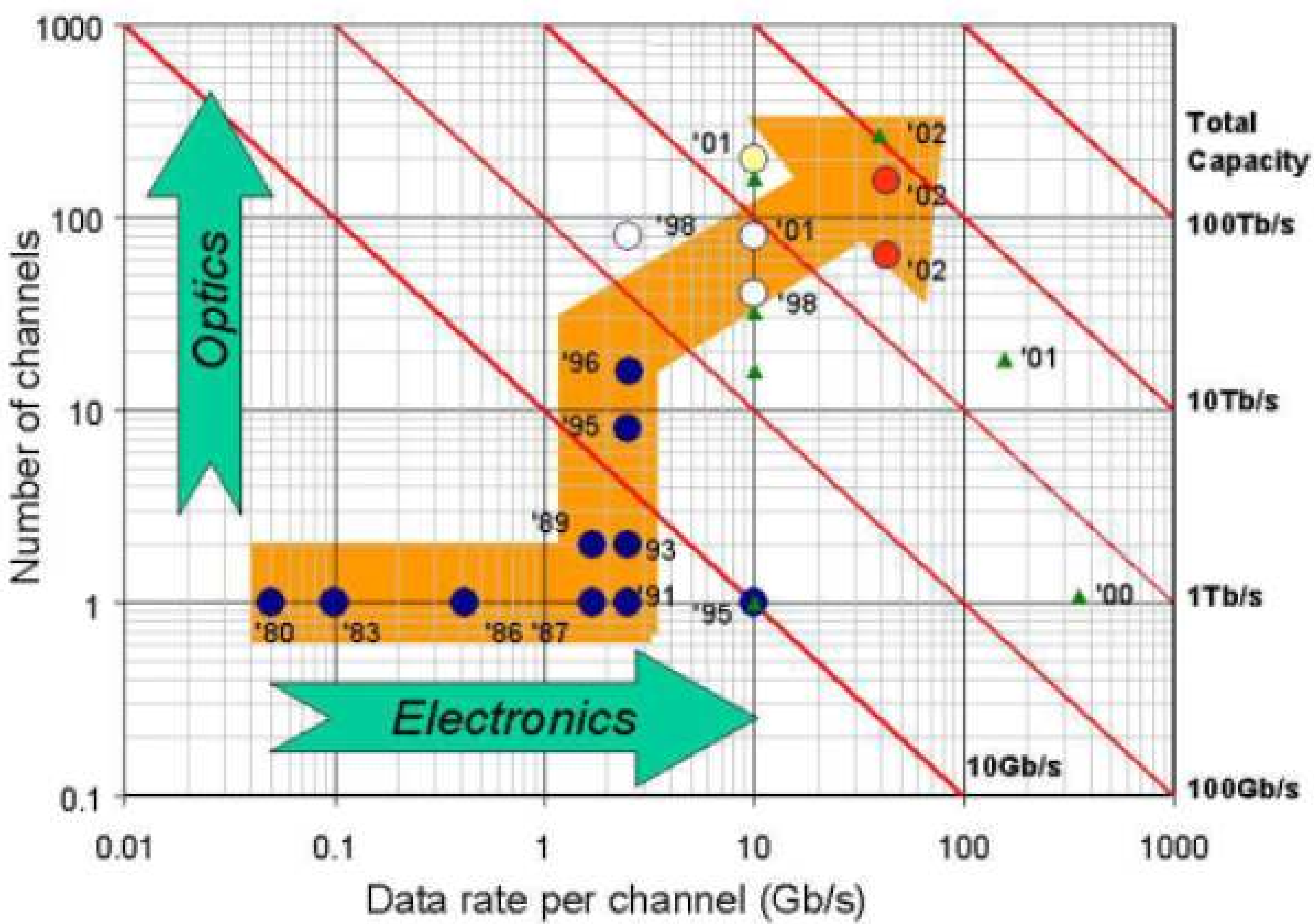
Capacities of optical network



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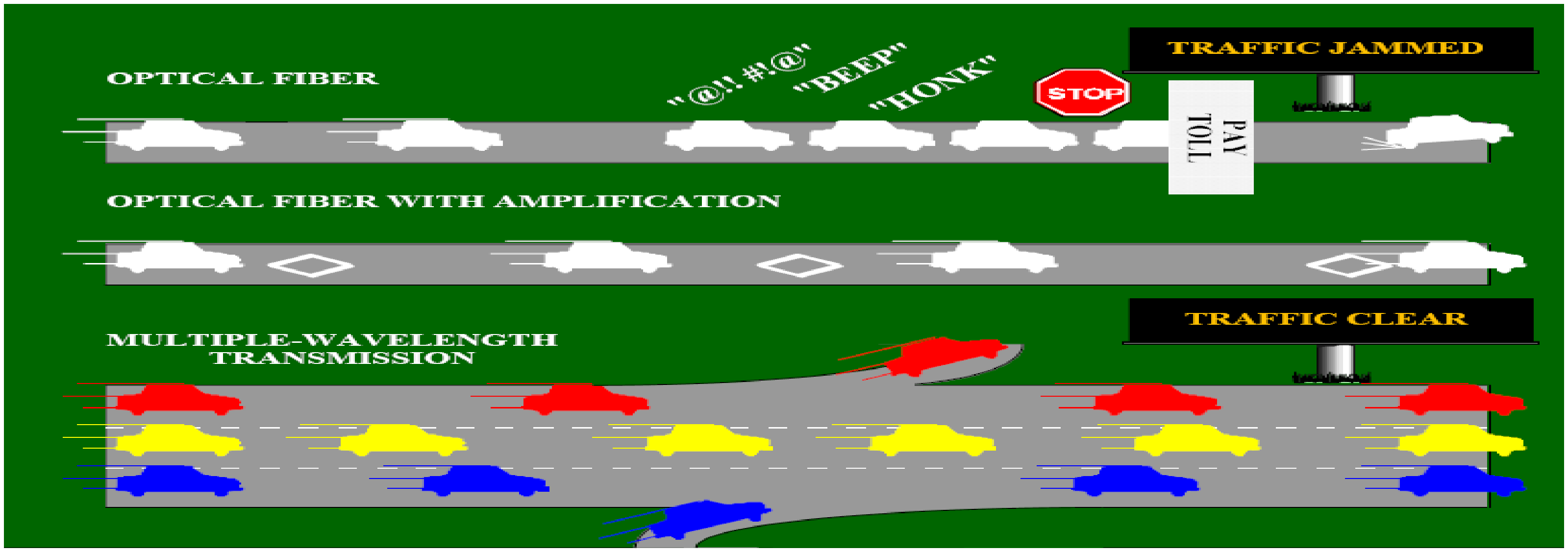
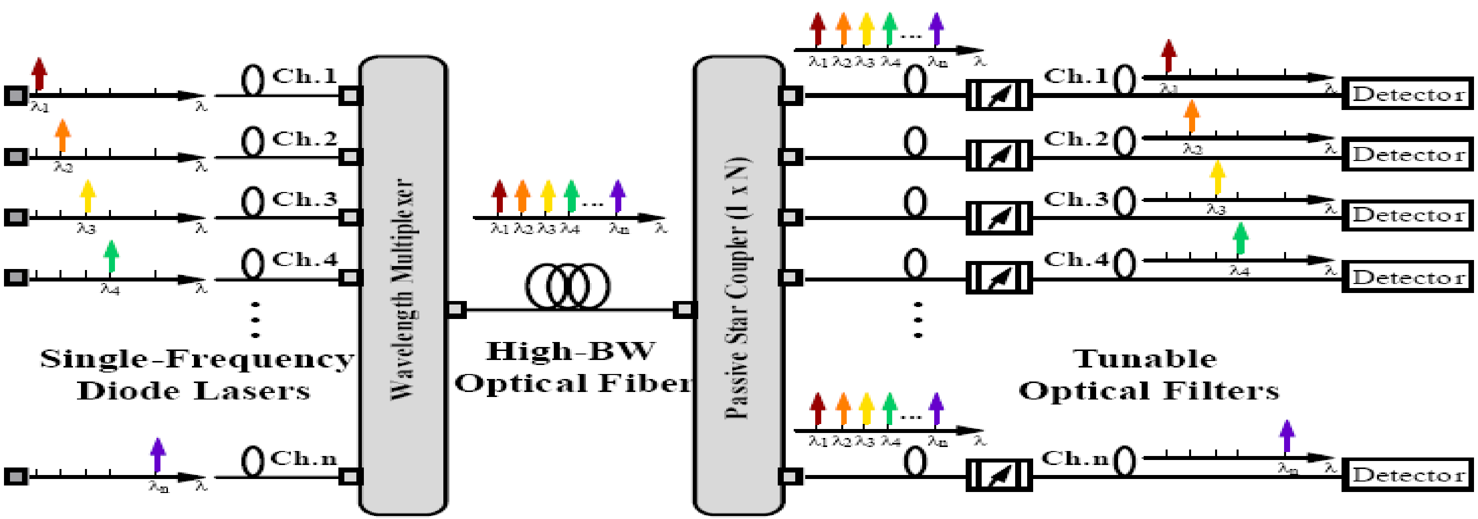
Wavelength-Division-Multiplex





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Fresnel diffraction

- A general plane-wave solution of the scalar wave equation in Cartesian coordinates is of the form,

$$e^{-jk_x x}e^{-jk_y y}e^{-jk_z z},$$

with

$$k_x^2 + k_y^2 + k_z^2 = k^2.$$

- in free space, the vector potential, A, is defined as

$$A(r,t) = \vec{n}\psi(x,y,z)e^{j\omega t},$$

which obeys the vector wave equation,

$$\nabla^2\psi + k^2\psi = 0.$$

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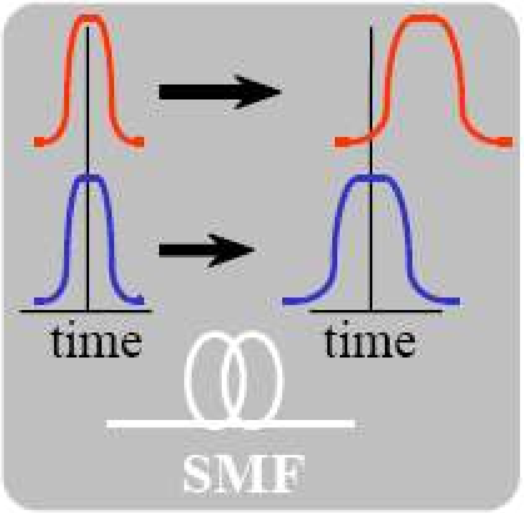
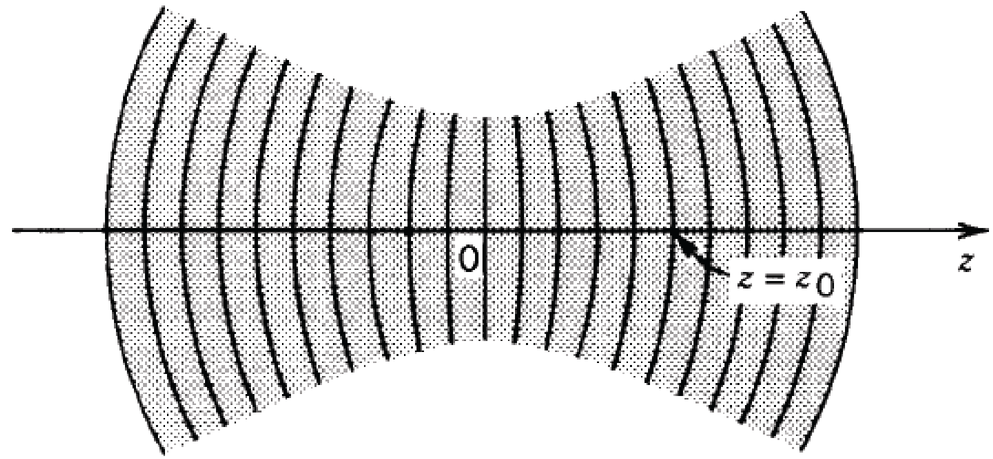
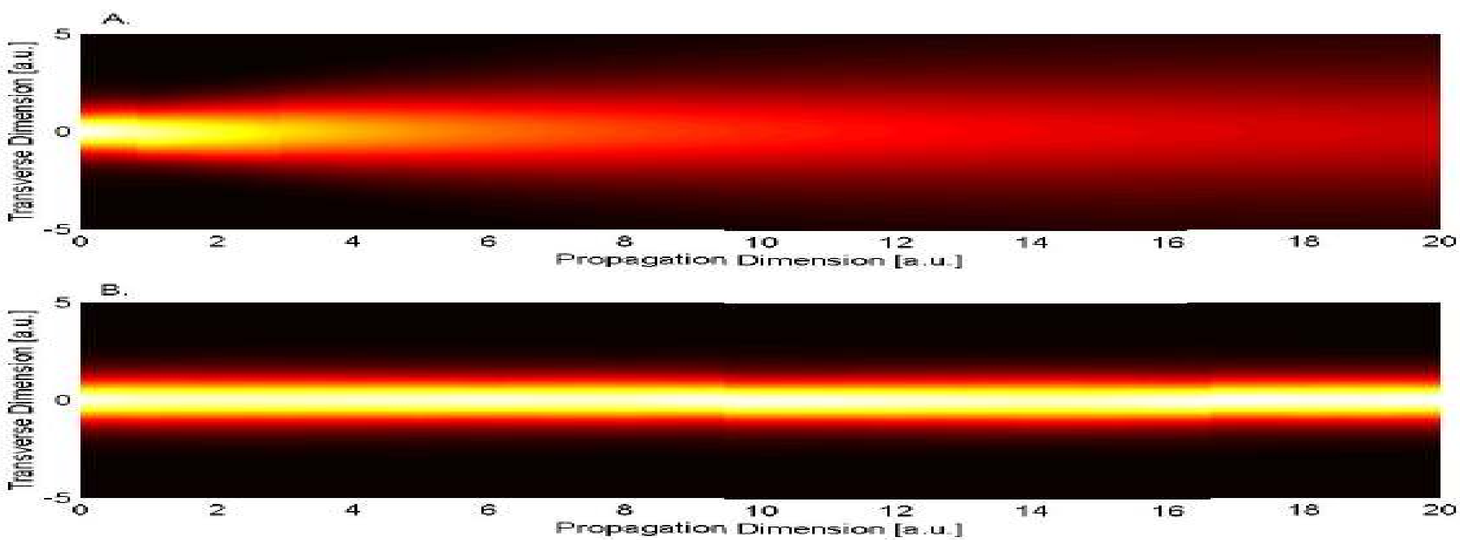
Dispersion/Diffraction effect







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Self-Focusing

- When a beam of finite transverse dimensions propagates through a nonlinear medium, with an index that depends on the optical intensity in the medium,

$$n = n_0 + n_2 I,$$

i.e. the index within the beam is different from that outside the beam.

- The vector potential A obeys approximately the wave equation,

$$\nabla^2 A + \omega^2 \mu_0 \epsilon A = 0,$$

where

$$\epsilon = \epsilon_0 n^2 = \epsilon_0 (n_0 + n_2 I)^2.$$

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Self-Focusing

- We assume that the vector potential is polarized at y direction,

$$A \propto \hat{y}\, u(x,y,z)e^{-jk_0 z},$$

where u varies slowly with z, and the propagation constant,

$$k_0 = \omega\sqrt{\mu_0\epsilon_0}\, n_0.$$

- Then the wave equation for the slowly varied envelope u becomes, in the paraxial limit,

$$\nabla_T^2 u - 2jk_0 \frac{\partial u}{\partial z} = -\omega^2\mu_0\epsilon_0(n^2 - n_0^2)u.$$

- In the one-dimension, the paraxial wave equation becomes,

$$-2jk_0 \frac{\partial u}{\partial z} + \frac{\partial^2}{\partial x^2} u + 2k_0^2 \frac{n_2}{n_0} |u|^2 u = 0.$$

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Self-Focusing

- In the one-dimension, the paraxial wave equation becomes,

$$-2jk_0\frac{\partial\, u}{\partial\, z}+\frac{\partial^2}{\partial\, x^2}u+2k_0^2\frac{n_2}{n_0}|u|^2u=0.$$

- If we introduce the variable,

$$q\equiv\frac{z}{2k_0},\quad \kappa\equiv 2k_0^2\frac{n_2}{n_0},$$

the paraxial wave equation with nonlinearity is put into the standard form of the *nonlinear Schrödinger equation*,

$$\frac{\partial^2\, u}{\partial\, x^2}-j\frac{\partial\, u}{\partial\, q}+\kappa|u|^2u=0.$$

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Soliton solutions for NLSE

- For $\kappa > 0$ (i.e. $n_2 > 0$), self-focusing, the nonlinear Schrödinger equation has the solution,

$$u = \sqrt{\frac{2}{\kappa}}\eta\,\text{sech}[\eta(x - x_0) + 2\eta\xi q]\text{exp}[j(\xi^2 - \eta^2)q + j\xi x - j\phi],$$

  with the arbitrary parameters, η, ξ, x_0, and ϕ. When $\xi = \phi = 0$, this solutions is simplified into,

$$u = \sqrt{\frac{2}{\kappa}}\eta\,\text{sech}[\eta(x - x_0)]\text{exp}[-j\eta^2 q].$$

- This is a beam with an x-dependent, but z-independent profile of width proportional to $1/\eta$. The area integral of the beam is independent of the beam parameters, i.e.

$$\sqrt{\frac{\kappa}{2}}\int_{-\infty}^{\infty}|u|dx = 2\pi.$$

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Soliton propagation in fiber

- A soliton is a pulse excitation of a nonlinear dispersive medium which propagates without distortion.
- The spreading of the pulse that would be caused by the dispersion acting alone is counteracted via the nonlinear phase modulation of the pulses by the nonlinearity of the medium.
- For the slowly varied envelope function of the pulse in optical fiber we have,

$$j\frac{\partial A}{\partial \zeta} + \frac{1}{2}\frac{d^2\beta}{d\omega^2}\frac{\partial^2 A}{\partial \tau^2} - \kappa |A|^2 A = 0,$$

where

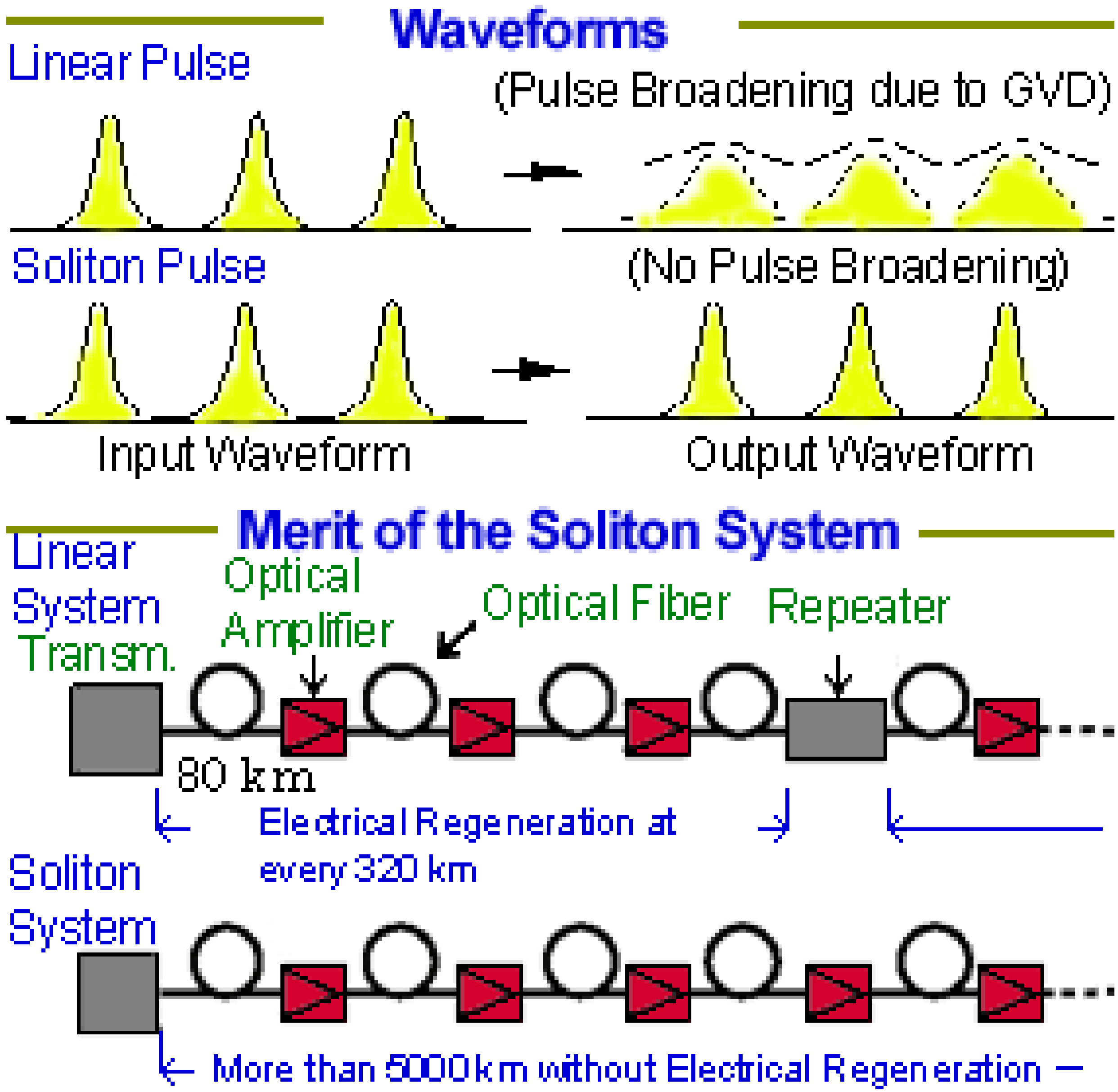
$$\kappa \equiv \frac{\omega_0^2 \mu_0 \epsilon_0}{\beta(\omega_0)} \frac{\int da\, n_0 n_2 |u|^4}{\int da\, |u|^2}$$

- This equation is identical in form with the equation of self-focusing, nonlinear Schrödinger equation, and thus must have identical solutions, provided that

$$\frac{d^2\beta}{d\omega^2} < 0,$$

the fiber has anomalous dispersion.

Soliton communication system



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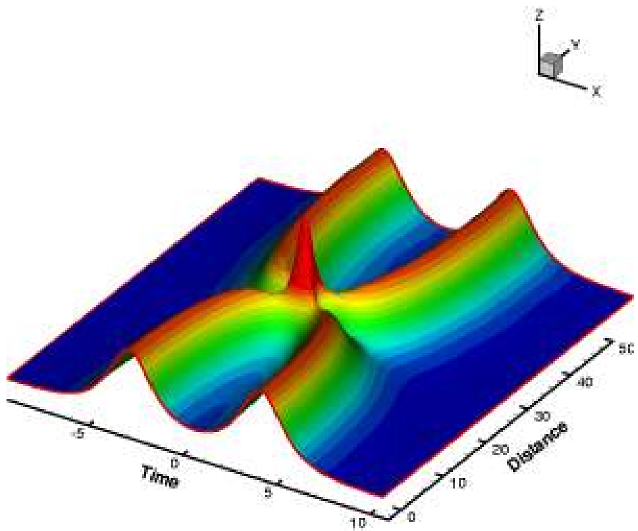
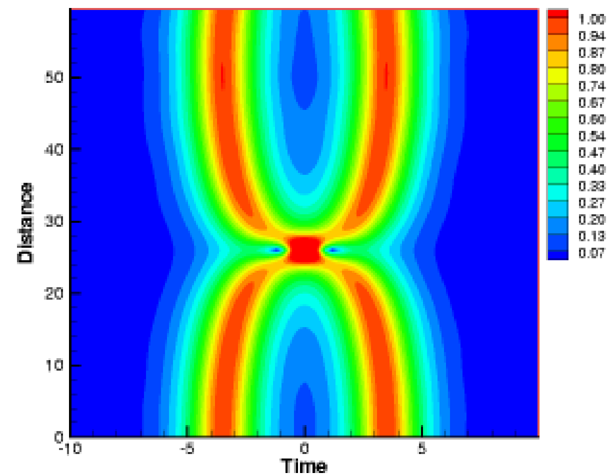
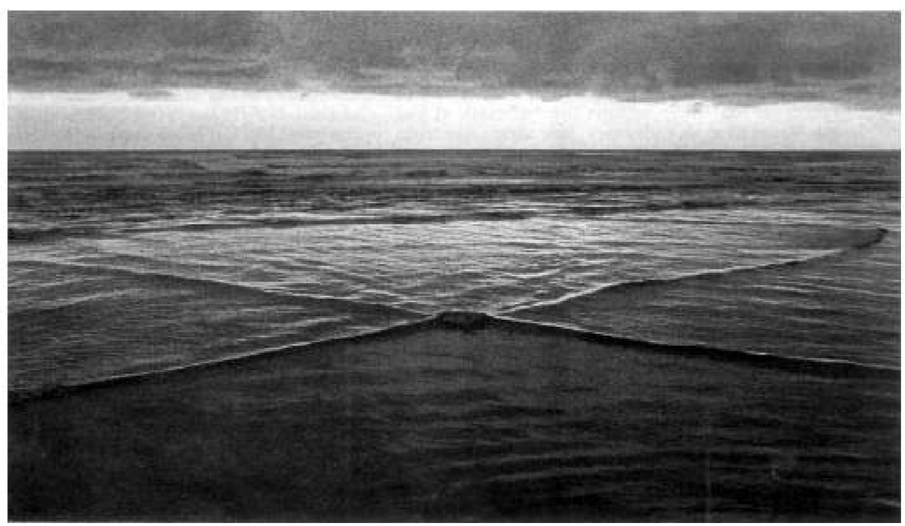
Wave-particle characteristics of solitons

Collision between solitons







Courtesy of T. Toedterneier

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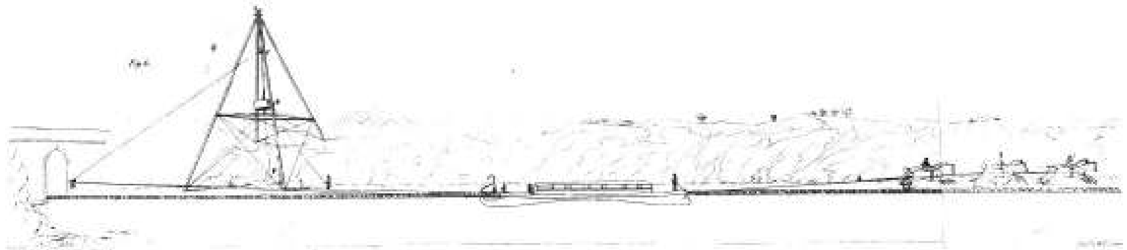
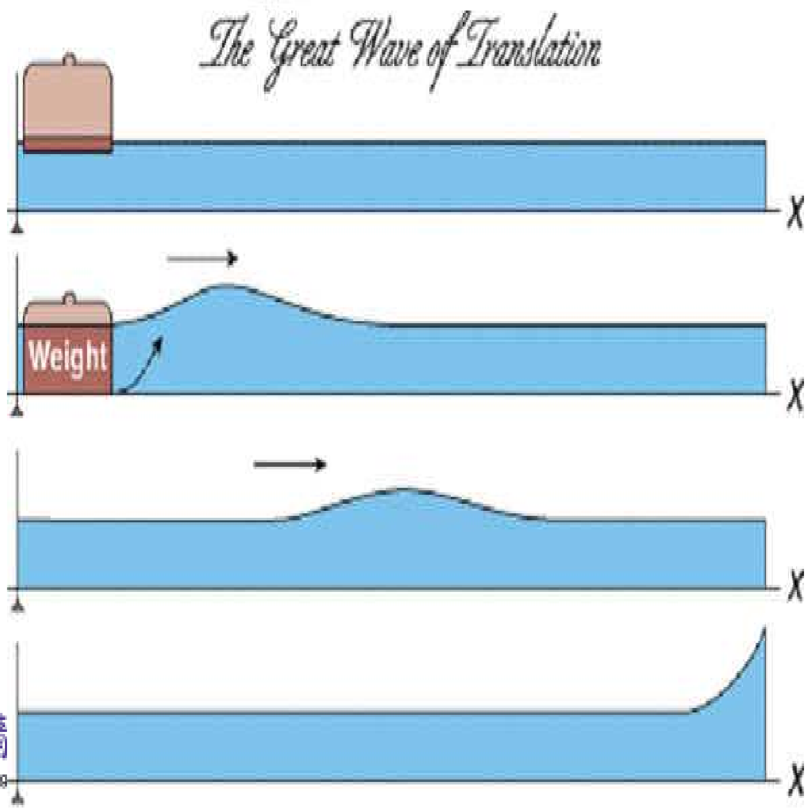
The Great Wave of Translation

Scottish engineer John Scott Russell (1808-1882), *fourteenth meeting of the British Association for the Advancement of Science*, York, September 1844 (London 1845).

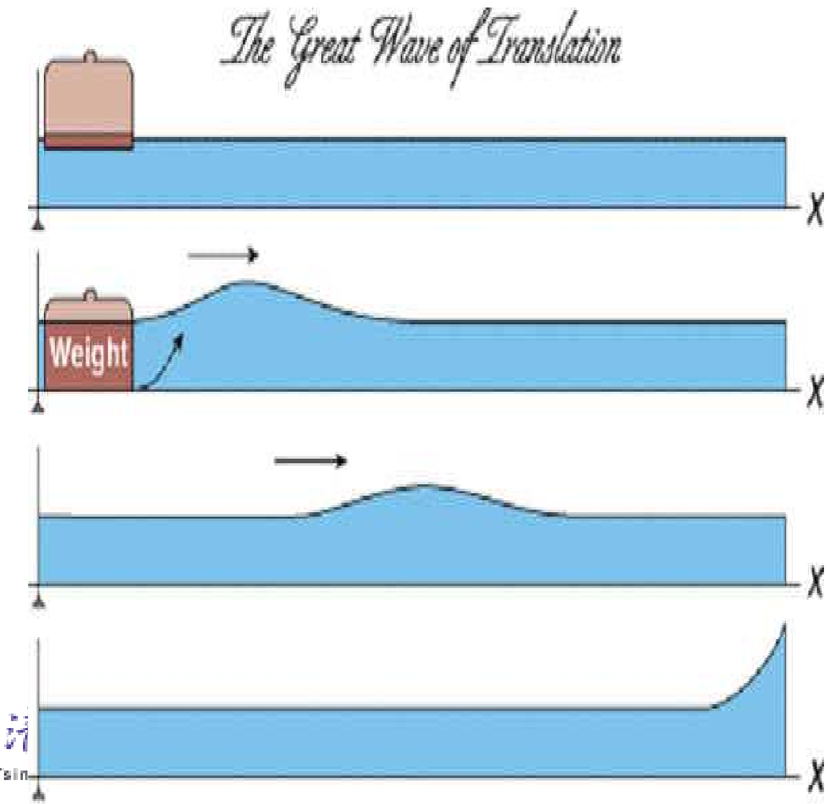




The Great Wave of Translation

Scottish engineer John Scott Russell (1808-1882), *fourteenth meeting of the British Association for the Advancement of Science*, York, September 1844 (London 1845).

Soliton on the Scott Russell Aqueduct on the Union Canal near Heriot-Watt University, 12 July 1995.



Tsunami

The Great Wave of Kanag’awa is an example of a soliton.

Hokusai, 1879, Japanese woodcut.

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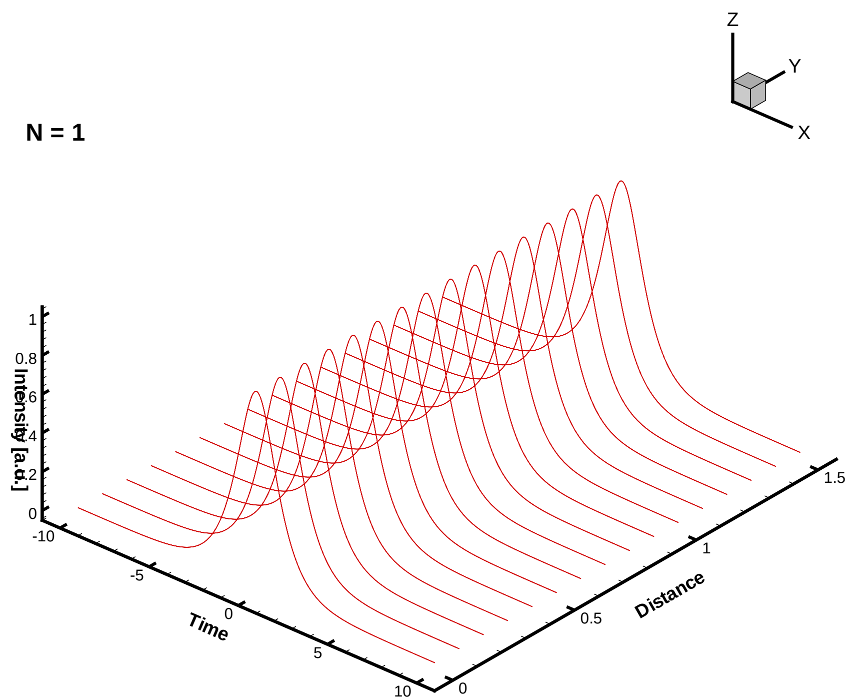
Solitons in optical fibers

Nonlinear Schrödinger Equations: Hermitian System

$$iU_z = -\frac{D}{2}U_{tt} - |U|^2U \qquad \text{, i.e.}$$

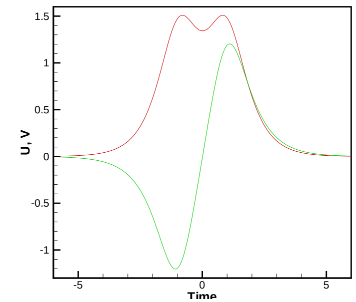
$$i\hbar\Psi_t = -\frac{\hbar^2}{2m}\nabla^2\Psi + \mathcal{V}\Psi = \mathcal{H}\Psi$$



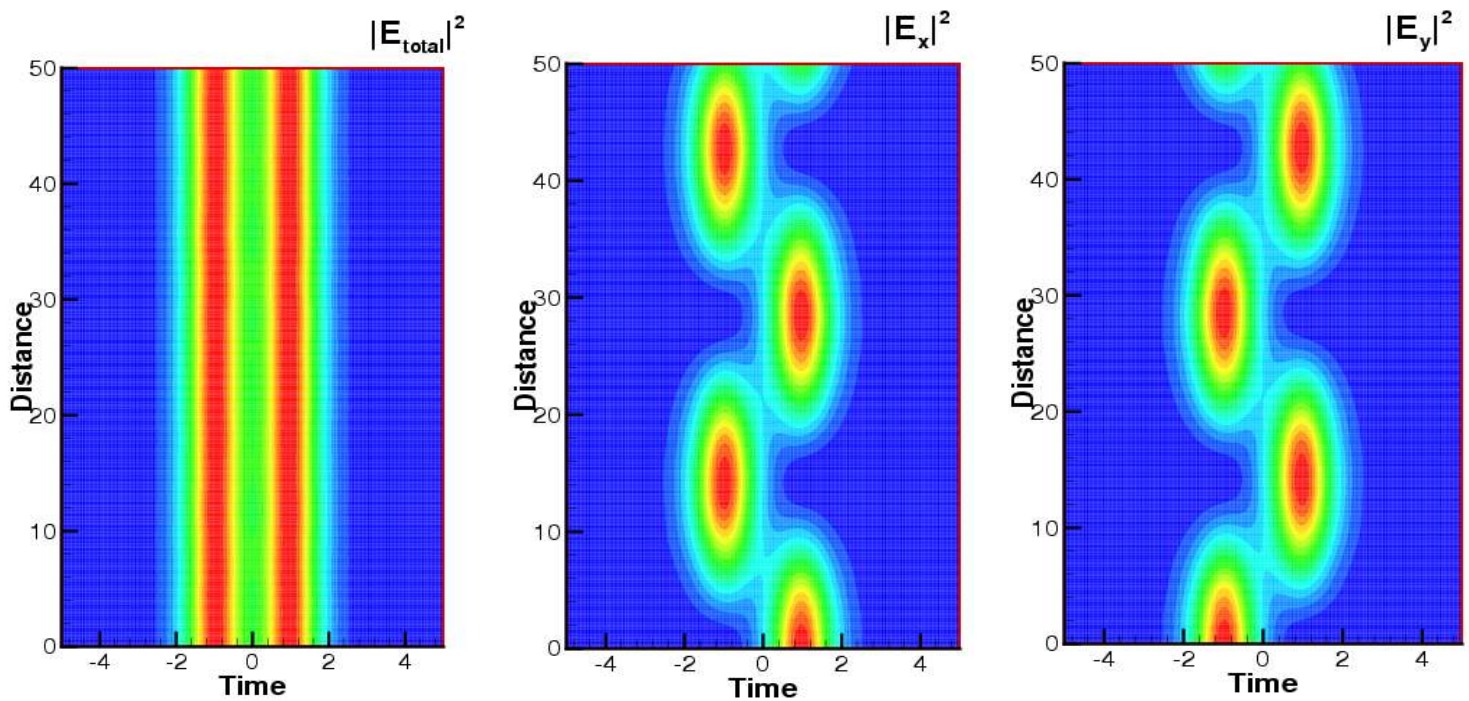
Vector bound solitons

Coupled Nonlinear Schrödinger Equations:

$$
\begin{aligned}
i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2U + B|V|^2U &= 0 \\
i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2V + B|U|^2V &= 0
\end{aligned}
$$



where $A = 1/3$, $B = 2/3$; and U, V are circular polarization fields.

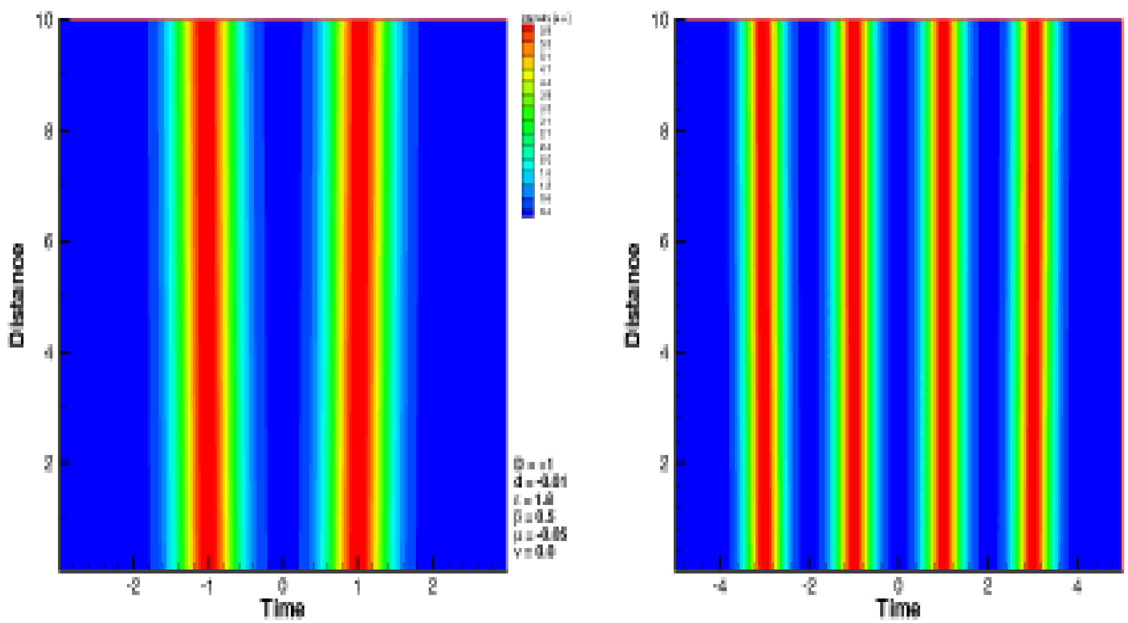


M. Haelterman, A. P. Sheppard, and A. W. Snyder, *Opt. Lett.* **18**, 1406 (1993).

Bounded-Solitons

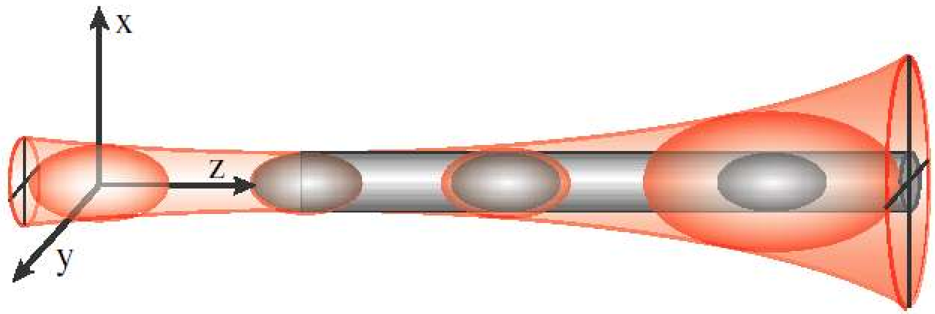
Complex Ginzburg-Lanau Equation:

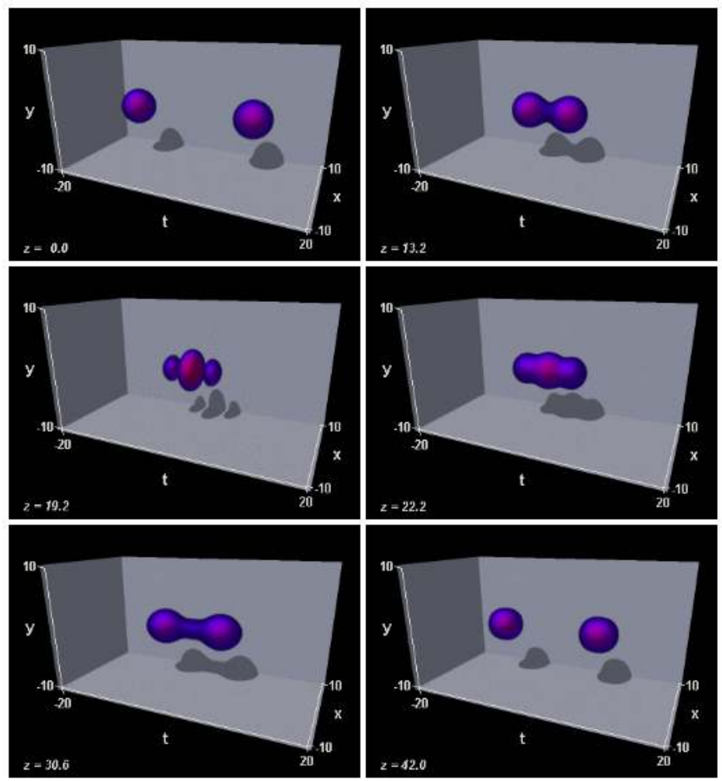
$$
\begin{aligned}
iU_z + \frac{D}{2}U_{tt} + |U|^2 U &= i\delta U + i\epsilon |U|^2 U + i\beta U_{tt} \\
&+ i\mu |U|^4 U - v|U|^4 U
\end{aligned}
$$



N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, *J. Opt. Soc. Am. B* **15**, 515 (1998).

Spatio-temporal solitons: light bullet





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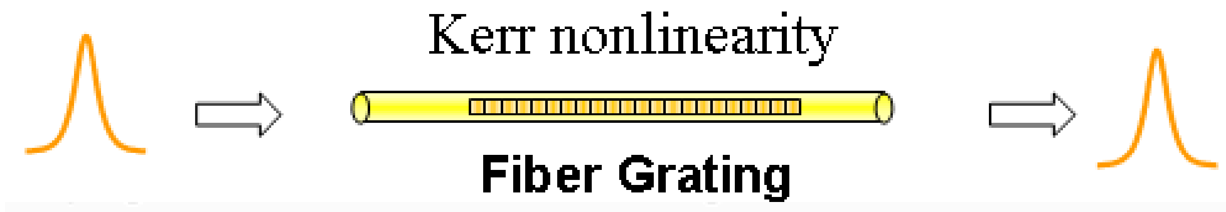
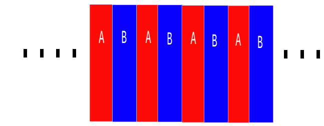
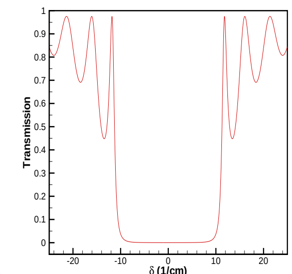
B. A. Malomed, D. Mihalache, F. Wise, and L. Torner, *J. Op. B* **7**, R53-R72 (2005).

Fiber Bragg Grating Solitons





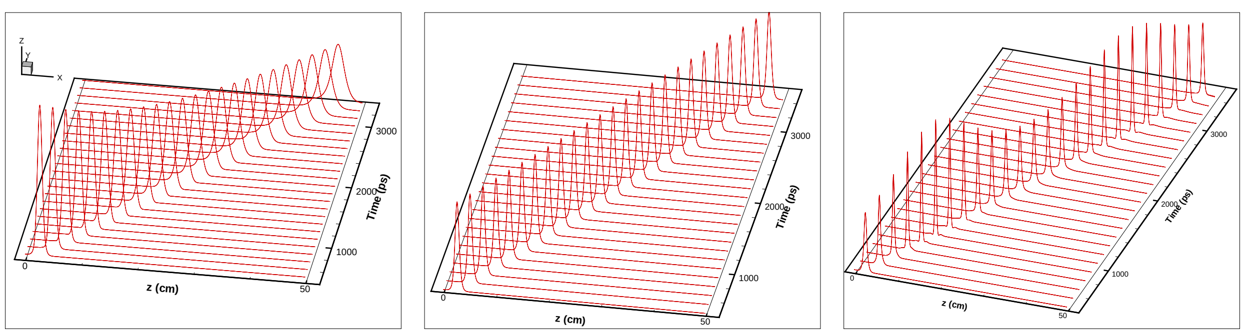


Nonlinear Coupled-Mode Equations:

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_a(z,t)+\frac{\partial}{\partial z}U_a=i\delta U_a+i\kappa U_b+i\Gamma|U_a|^2U_a+2i\Gamma|U_b|^2U_a$$

$$\frac{1}{v_g}\frac{\partial}{\partial t}U_b(z,t)-\frac{\partial}{\partial z}U_b=i\delta U_b+i\kappa U_a+i\Gamma|U_b|^2U_b+2i\Gamma|U_a|^2U_b$$

decay stationary oscillate



A. Aceves and S. Wabnitz, *Phys. Lett. A* **141**, 37 (1986).

B. J. Eggleton, C. de Sterke, P. A. Krug, and J. E. Sipe, *Phys. Rev. Lett.* **76**, 1627 (1996).

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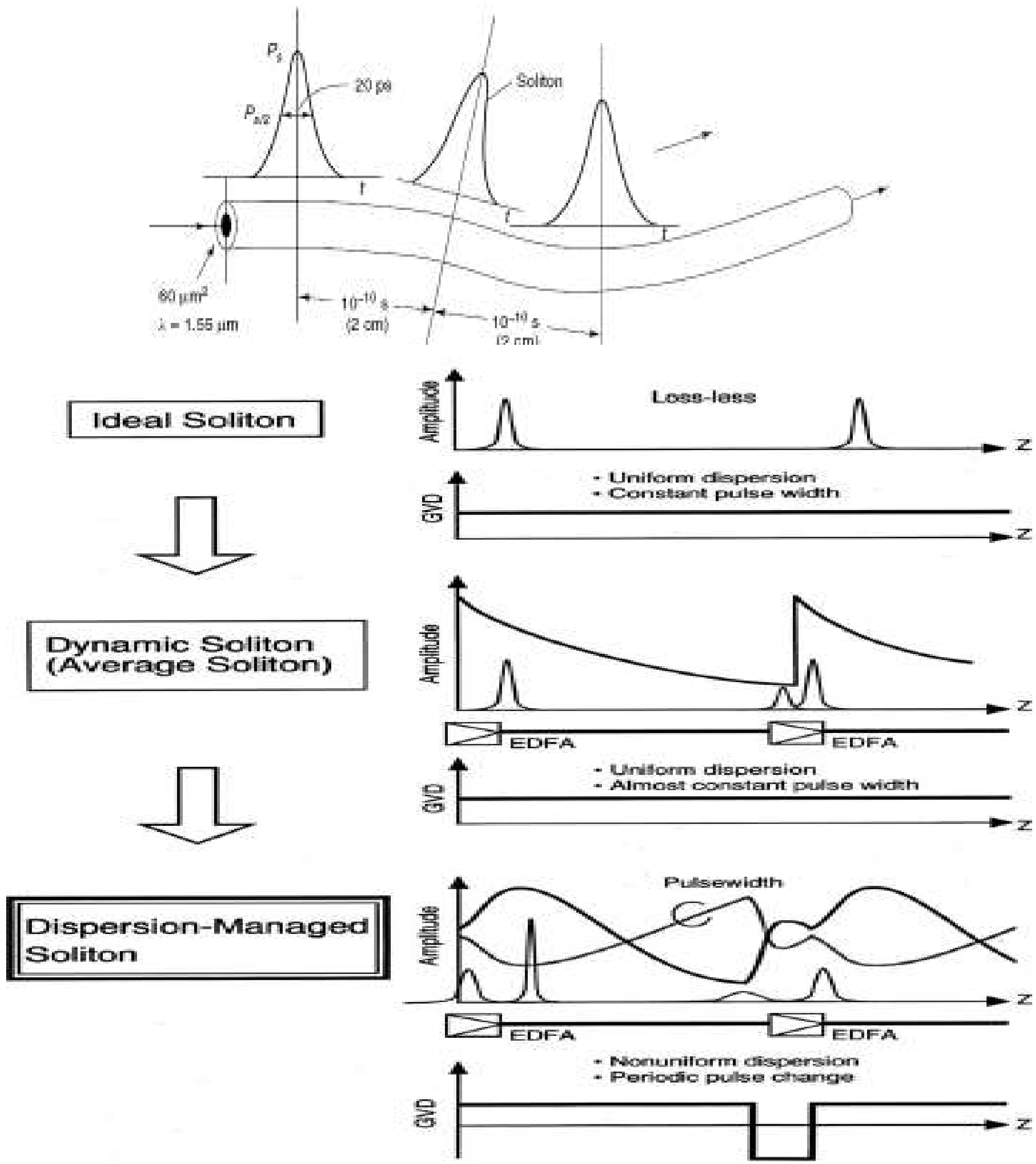
Practical Soliton Communication



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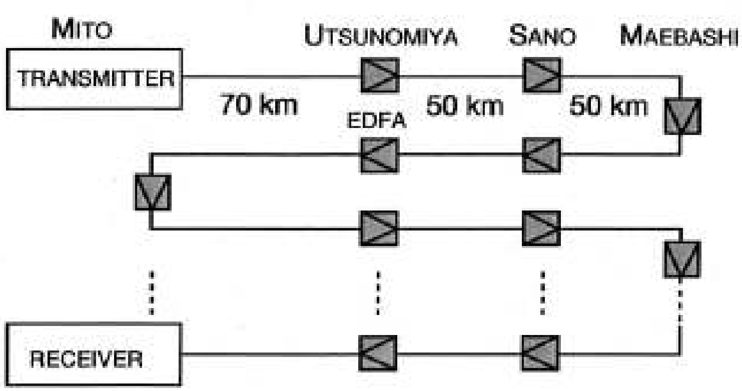
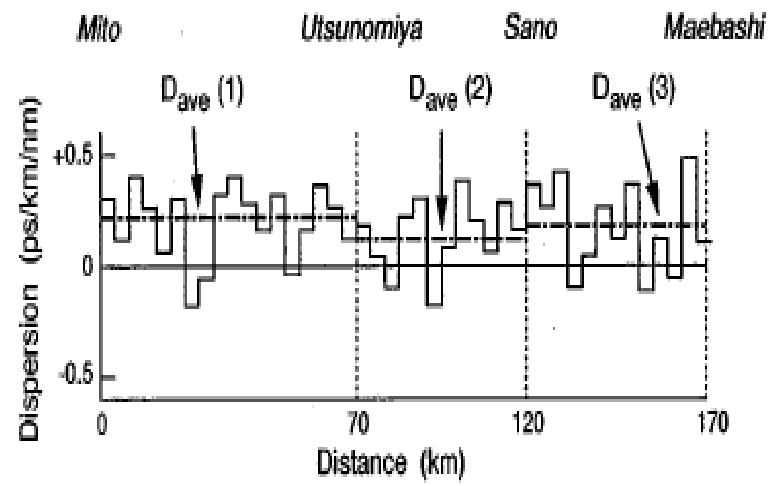
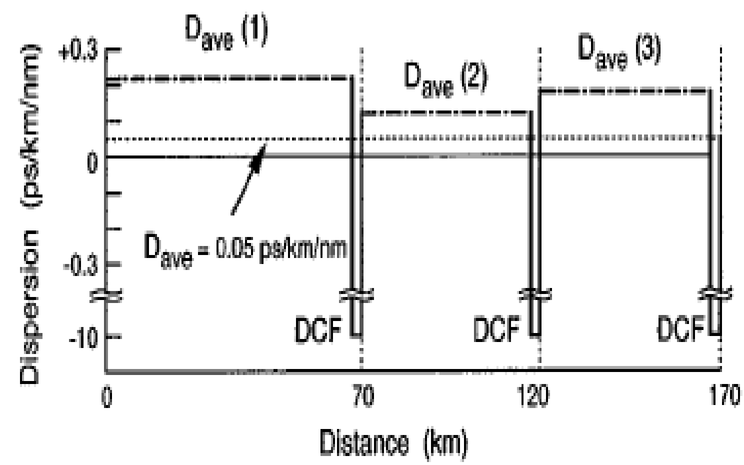
Dispersion-Management Soliton System in Tokyo

(a)



(b)





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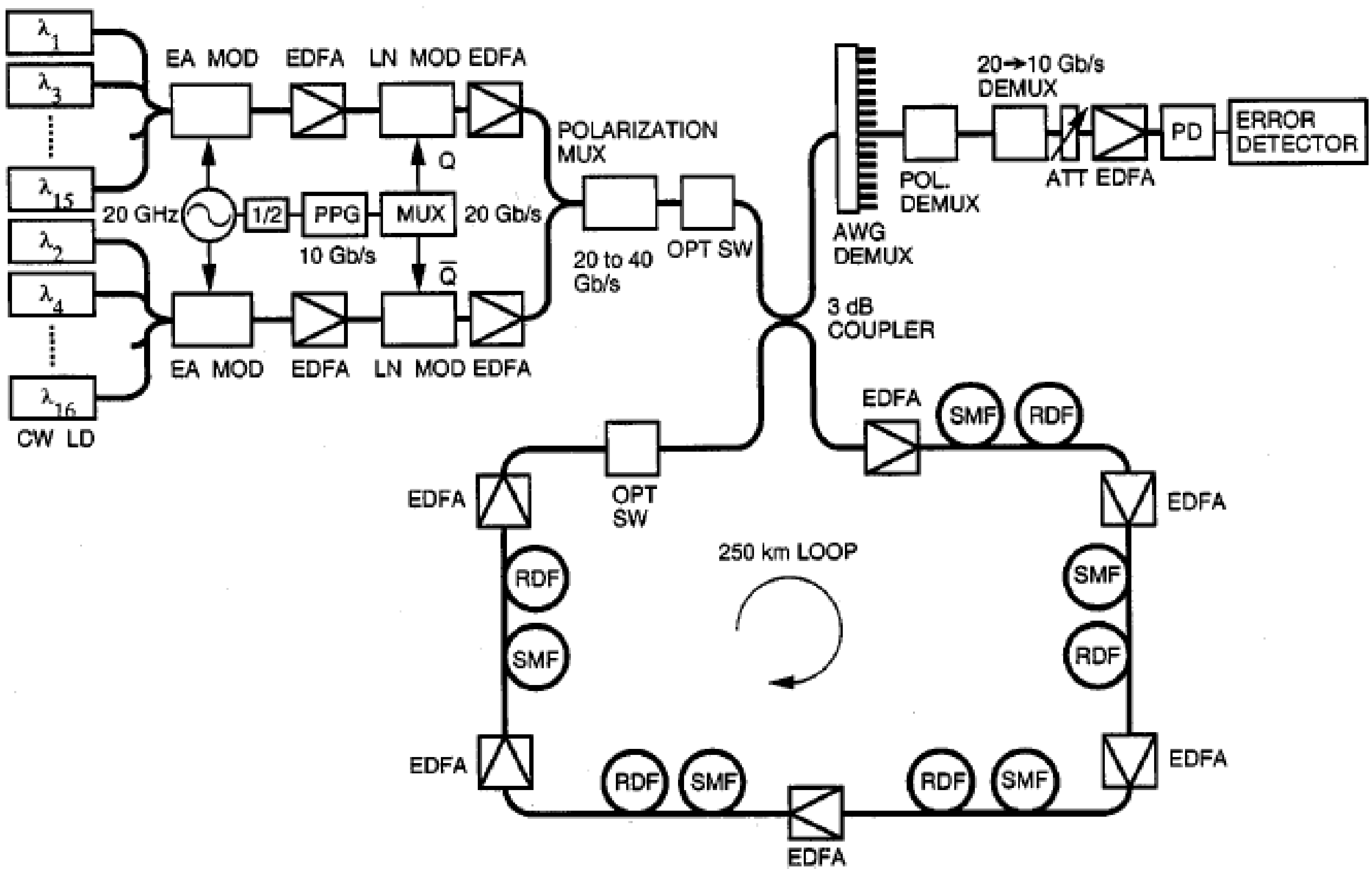
State of Art Soliton Communication System



160 Gbit/s (20 Gbit/s X 8 channels) WDM soliton transmission in a 250 km dispersion-shifted fiber loop.

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Universal Solitons

A Universal phenomenon of self-trapped wave packets.

- EM waves in nonlinear optical materials;
- shallow- and deep-water waves;
- charge-density waves in plasmas;
- sound waves in liquid ^{3}He;
- matter waves in Bose-Einstein condensates;
- excitations on DNA chains;
- domain walls in supergravity, and
- "branes" at the end of open strings in superstring theory; to name only a few.

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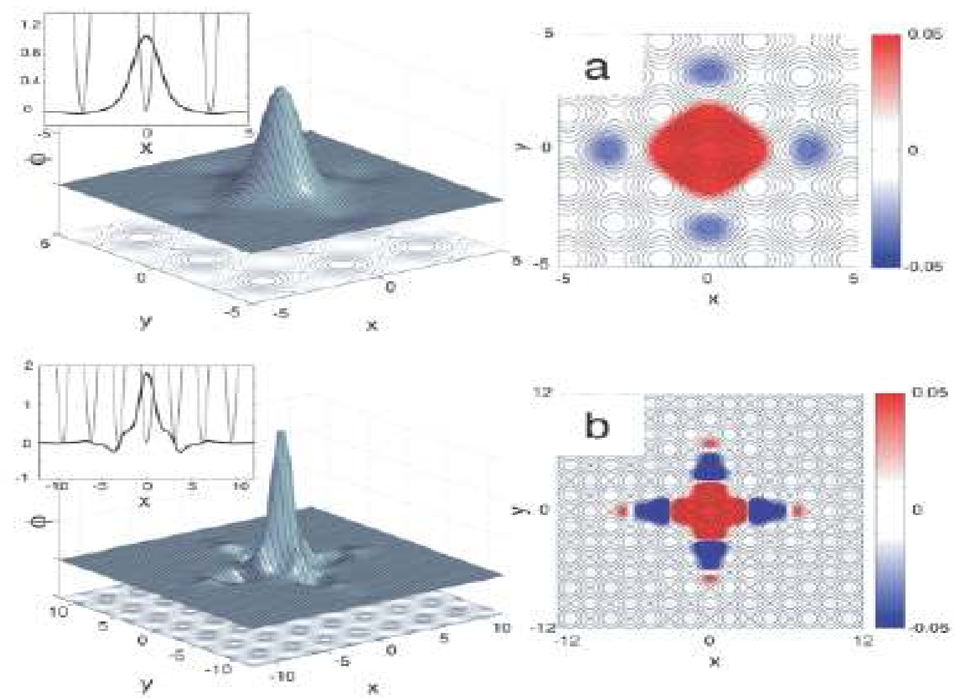
M. Segev, *Optics & Photonics News*, pp. 27 (Feb. 2002).

BEC in optical lattices

Gross-Pitaevskii equation with periodic potentials,

$$i\hbar\frac{\partial}{\partial t}\Phi = -\frac{1}{2}\nabla^2\Phi + V(t)\Phi + g|\phi|^2\phi$$

which has gap soliton solutions in 1D, 2D, and 3D.



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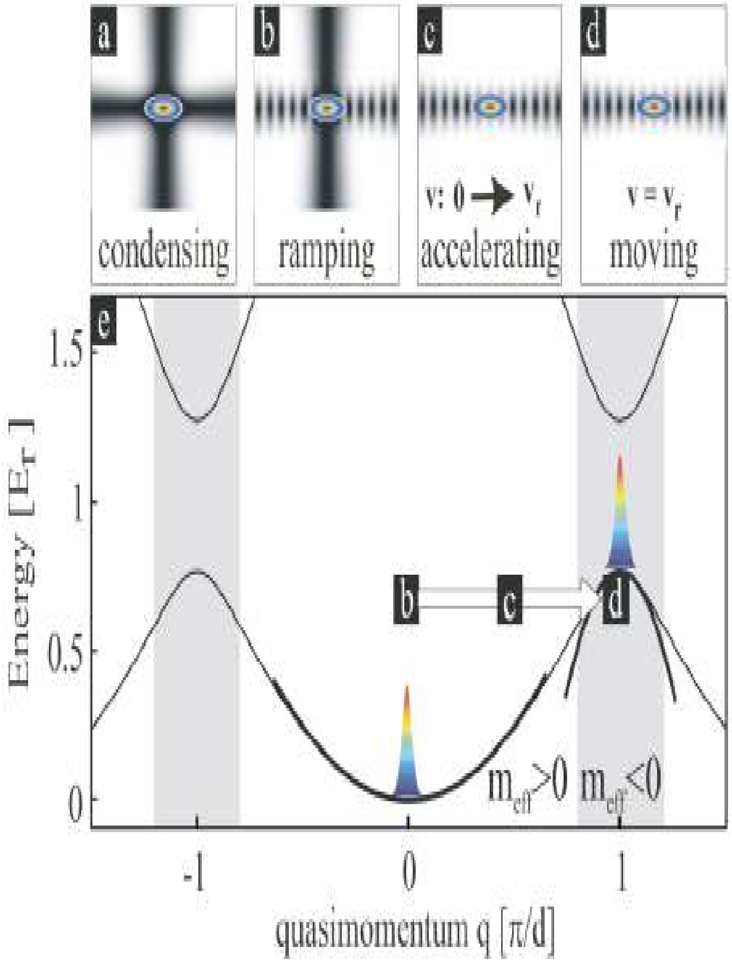
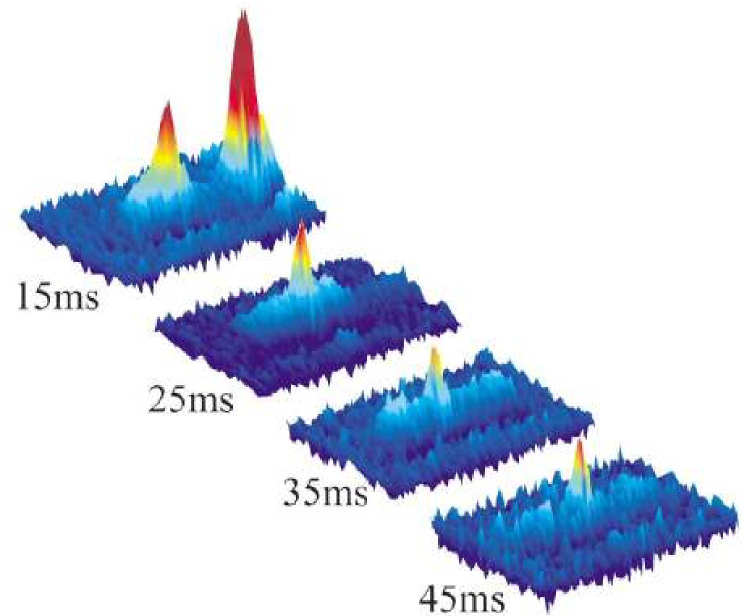
E. A. Ostrovskaya and Yu. S. Kivshar, *Phys. Rev. Lett.* **90**, 160407 (2003).

Gap soliton in optical lattices with repulsive interaction





Exp: B. Eiermann, Th. Anker, M. Albiez, M. Taglieber, P. Treutlein, K.-P. Marzlin, and M. K. Oberthaler, *Phys. Rev. Lett.* **92**, 230401 (2004).

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Entanglement

- Entanglement is ...

  "the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical mechanics."

  E. Schrödinger

- Entangled states are a non factorisable sum of product states, i.e.

  Entangled:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2)$$

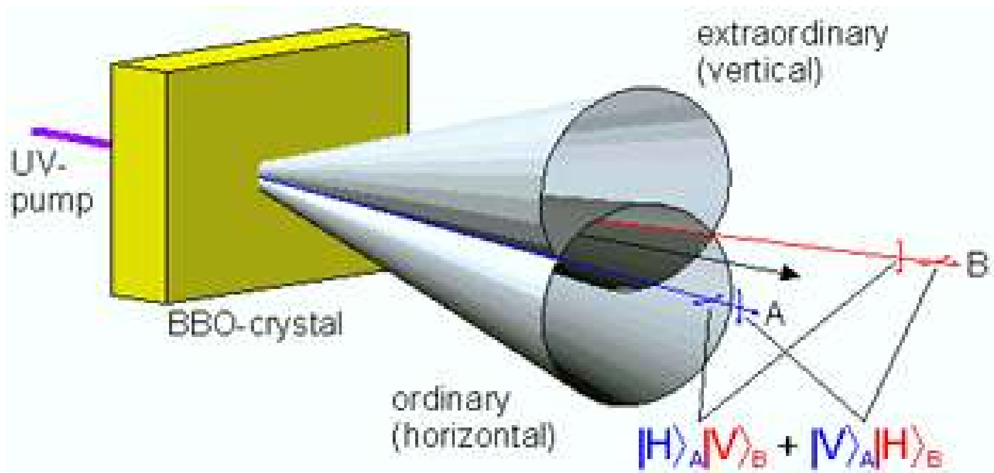
  Not Entangled:

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 - |1\rangle_1|0\rangle_2) \\ &= \frac{1}{\sqrt{2}}(|0\rangle_1 - |1\rangle_1)|0\rangle_2) \end{aligned}$$

Polarization-entangled

Non-collinear type II down-conversion:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|V\rangle_1|H\rangle_2 + e^{i\phi}|H\rangle_1|V\rangle_2)$$



with additional half-wave plate, we can also have,

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|V\rangle_1|V\rangle_2 + e^{i\phi}|H\rangle_1|H\rangle_2)$$

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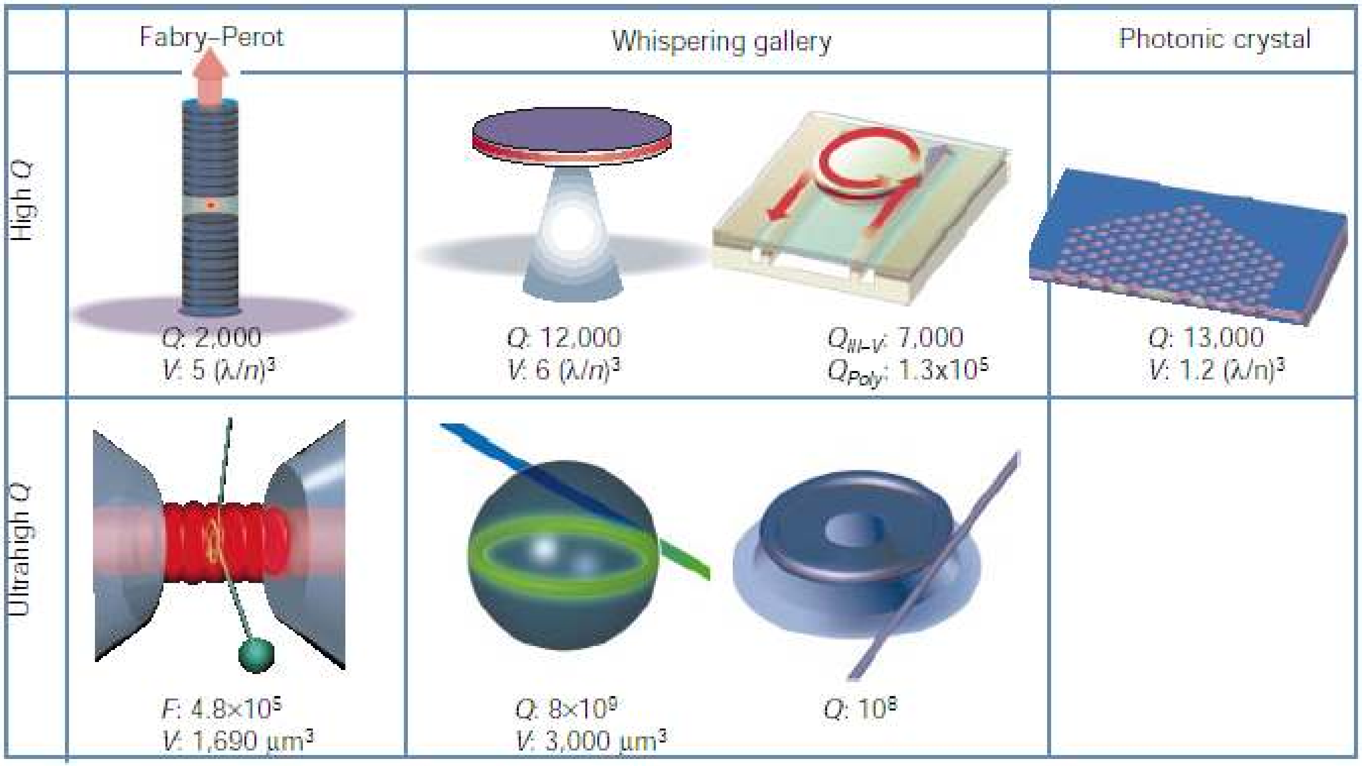
Courtesy of *Paul. Kwiat*

Single Photon Source

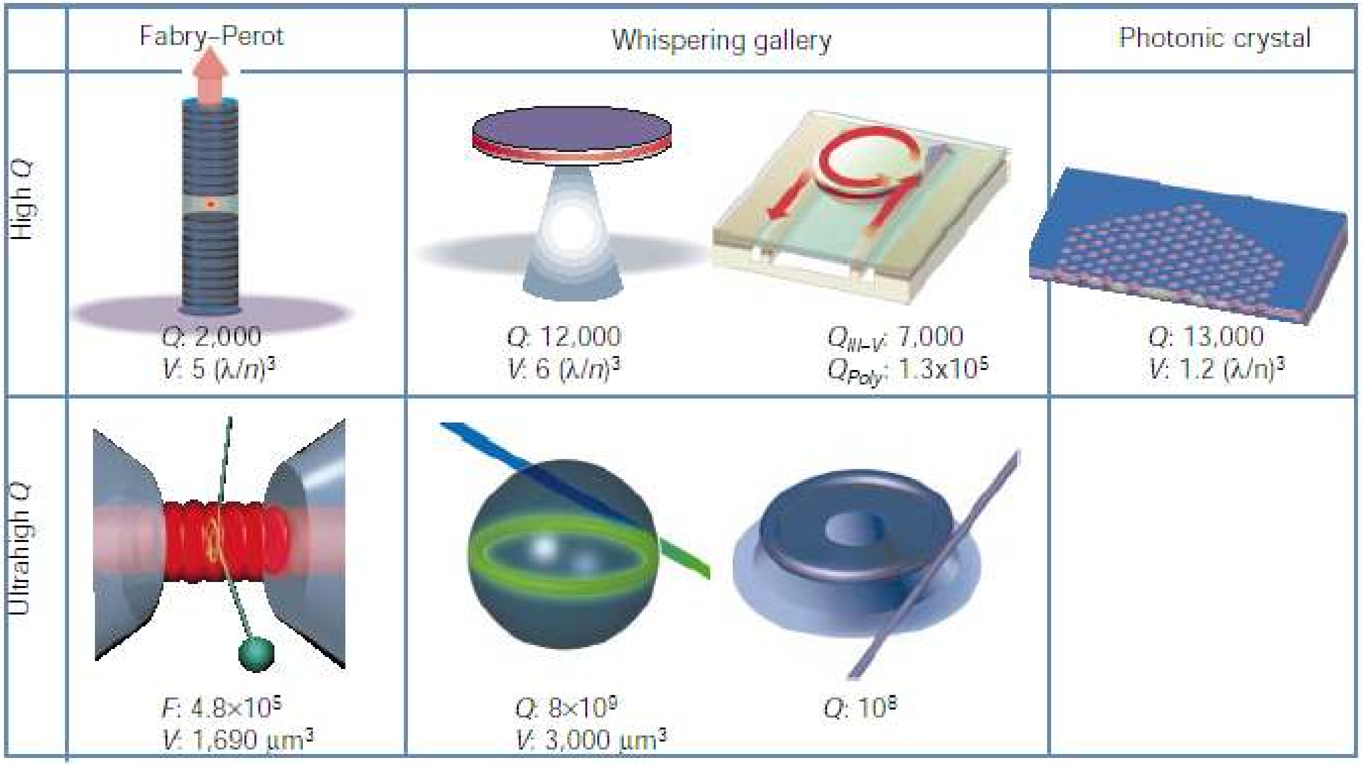
with Cavity-QED (Quantum ElectroDynamics) technologies



from: K. J. Vahala, *Nature* **424**, 839 (2003).

Single Photon Source

with Cavity-QED (Quantum ElectroDynamics) technologies



from: K. J. Vahala, *Nature* **424**, 839 (2003).

Problems with Single Photon Source

- Non-deterministic: you can't tell if it contains only a single photon before detection.
- Too fragile: the energy of single photon is too small.

Continuous Variables Entanglement

- Quantum teleportation is not restricted to discrete quantum states.

Continuous Variables Entanglement

- Quantum teleportation is not restricted to discrete quantum states.
- Most of entangled pairs are followed by Bohm's suggestion[1],
  ex: *spin*, *polarization*, *single photon ...*,

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Continuous Variables Entanglement

- Quantum teleportation is not restricted to discrete quantum states.
- Most of entangled pairs are followed by Bohm's suggestion[1],
  ex: *spin*, *polarization*, *single photon* ...,
- But in the original EPR paradox[2],
  Einstein, Podolsky and Rosen used position and momentum as entanglement sources.

1: D. Bohm, *"Quantum Theory,"* (Prentice-Hall, 1951).

2: A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).

Entangled sources for Quantum Information Science

In QIS, you need *non-classical* states as *qbits*.

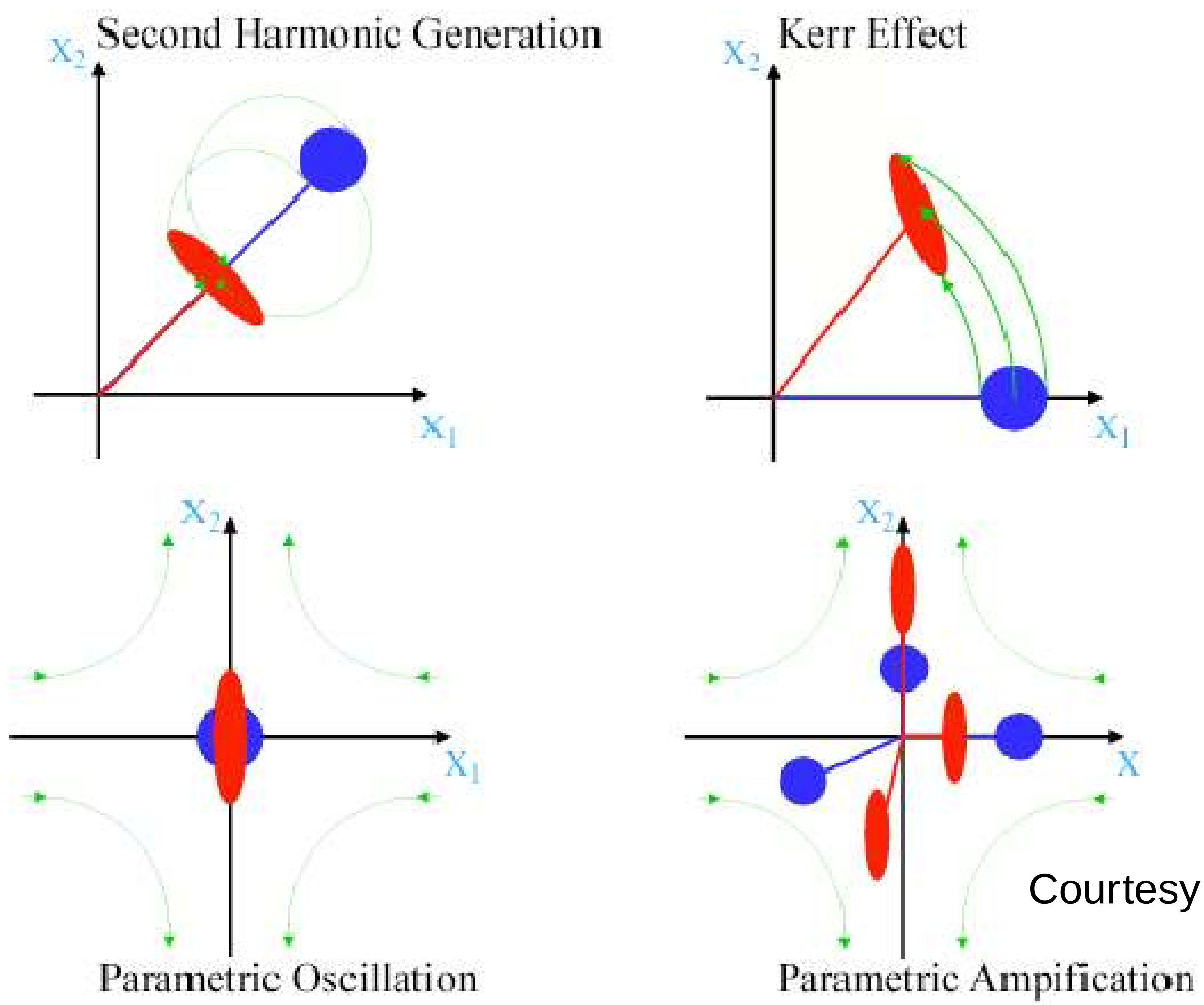
- Low-intensity limit:
  Single photon sources, with definite *photon number* but largest fluctuation in phase, which is intrinsic *non-classical* states.
- High-intensity limit:
  Squeezed states, which are *macroscopic*, continuous-variables, i.e.

$$\hat{M} = M_0 + \Delta\hat{M},$$

where M_0 is the classical (mean-field) variables, such as *photon-number*, *phase*, *position*, and *momentum* etc.

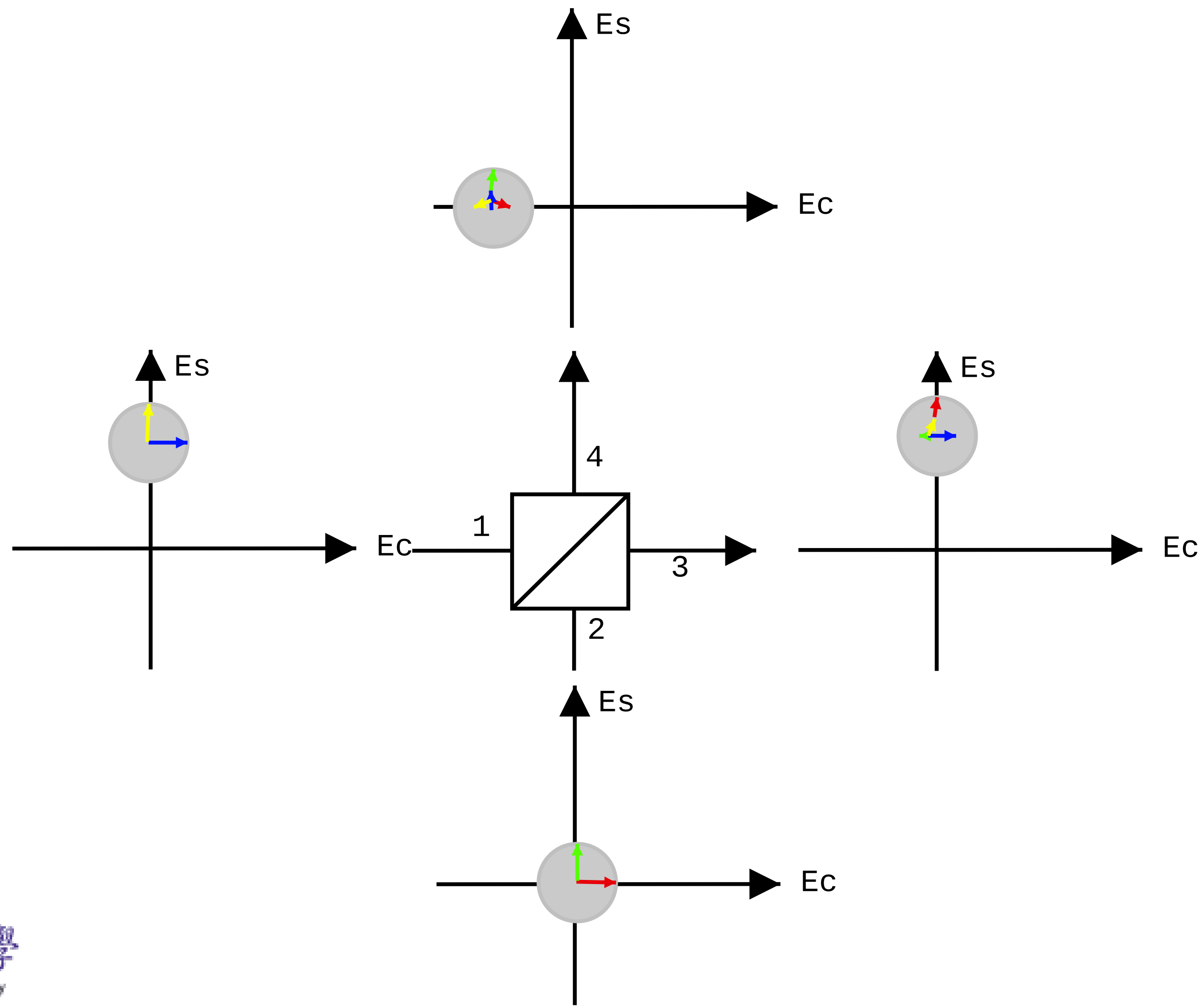
Generations of Squeezed States

Nonlinear optics:



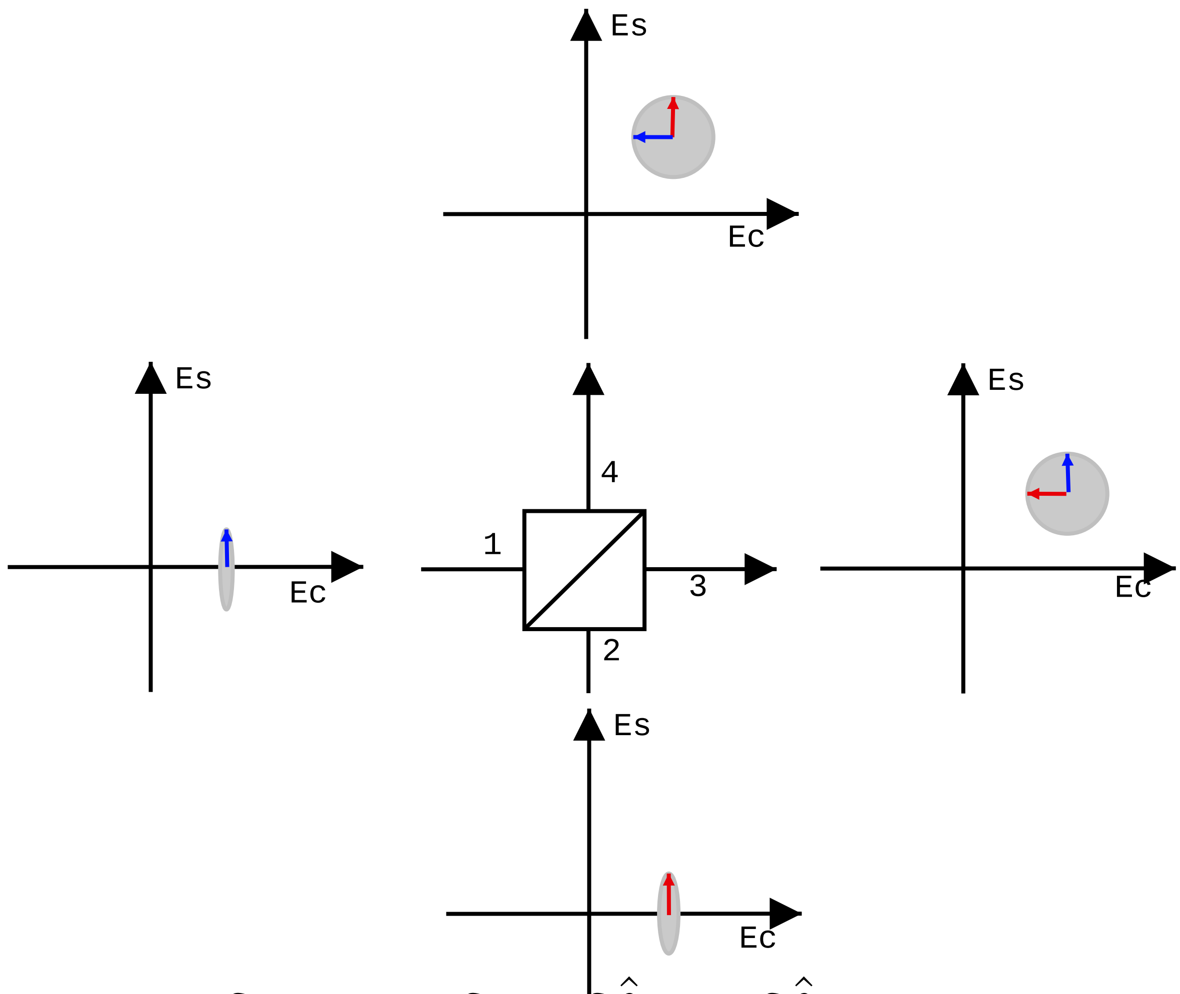
Courtesy of P. K. Lam

Coherent States



Generation of Continuous Variables Entanglement

Preparation EPR pairs by Squeezed Sates



$$\delta\hat{n}_3 = -\delta\hat{n}_4, \delta\hat{\theta}_3 = \delta\hat{\theta}_4.$$

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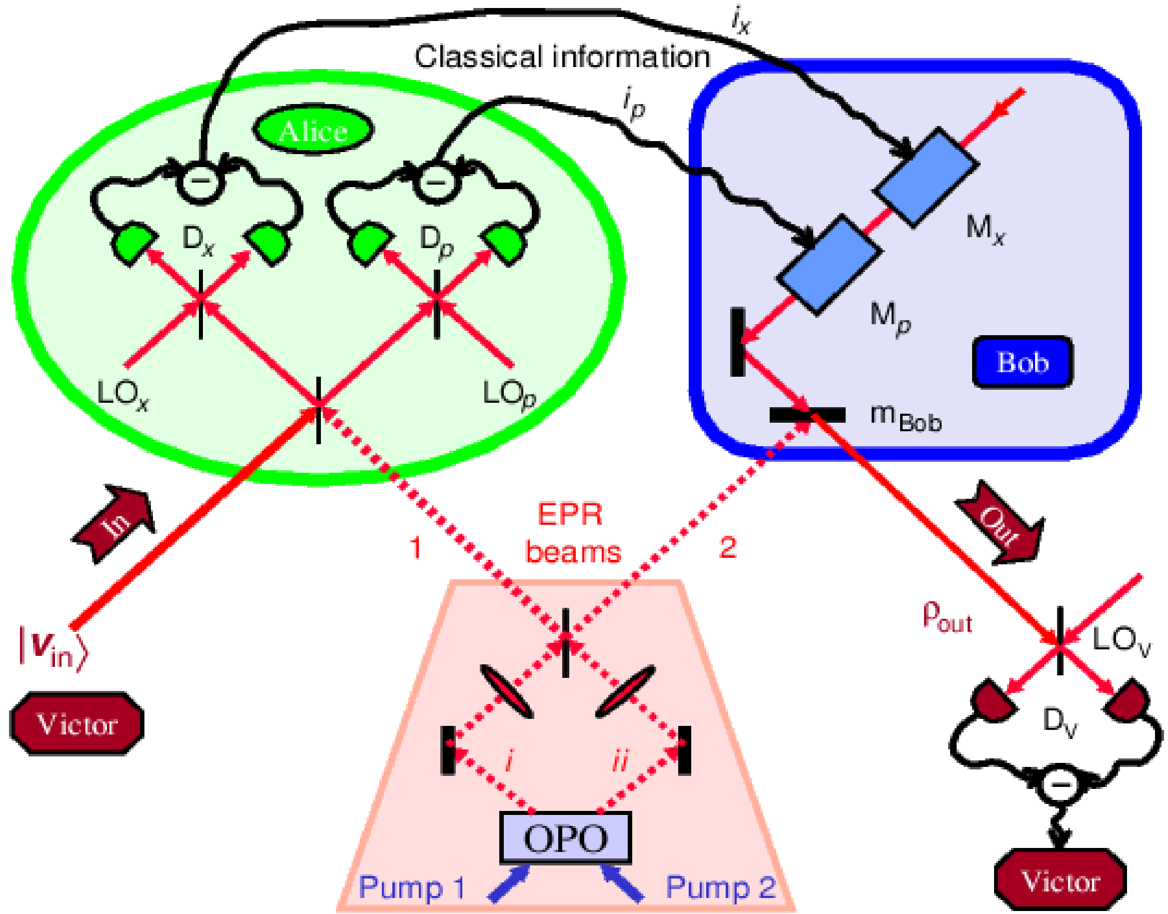
Experimental CV teleportation



A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble,

and E. S. Polzik, *Science* **282**, 706 (1998).

Definition of Squeezing and Correlation

Squeezing Ratio

$$\hat{M} = M + \Delta\hat{M}$$

$$\text{SR} = \frac{\langle\Delta\hat{M}^2\rangle}{\langle\Delta\hat{M}^2\rangle_{\text{c.s.}}}$$

SR < 1 : $Squeezing$

SR > 1 : $Anti - Squeezing$

Correlation

$$C = \frac{\langle:\Delta\hat{A}\Delta\hat{B}:\rangle}{\sqrt{\langle\Delta\hat{A}^2\rangle\langle\Delta\hat{B}^2\rangle}}$$

$0 \leq C \leq 1$: *Positive Correlation*

$C = 0$: *No Correlation*

$-1 \leq C \leq 0$: *Negative Correlation*

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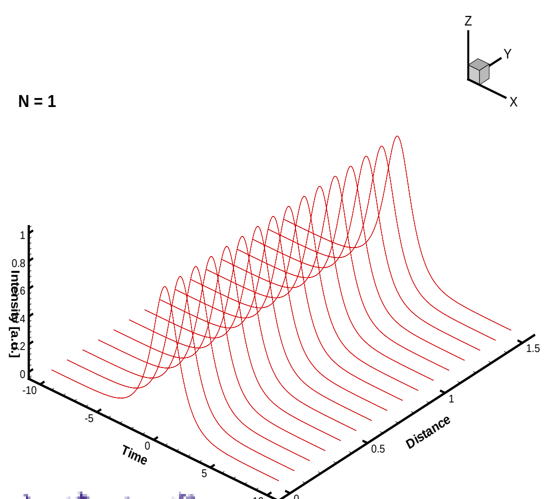
Solitons in optical fibers

Classical nonlinear Schrödinger Equation

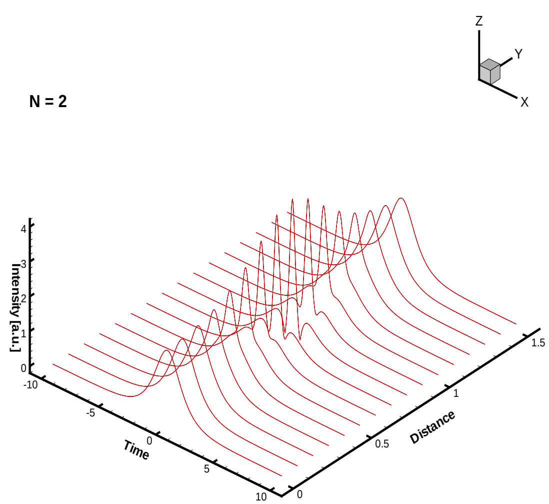
$$iU_z(z,t) \;=\; -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2 U(z,t)$$

Fundamental soliton:

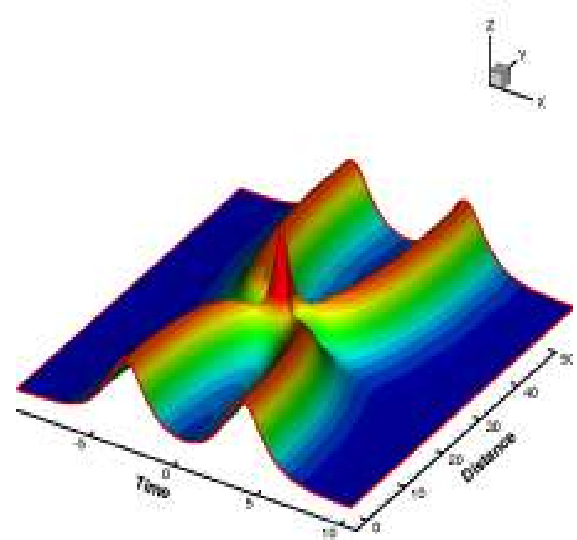
$$U(z,t) = \frac{n_0}{2} exp[i\frac{n_0^2}{8}z + i\theta_0] sech[\frac{n_0}{2}t]$$



Fundamental



Higher-order ($N = 2$)



Soliton interaction

1D Quantum nonlinear Schrödinger equation

Quantum nonlinear Schrödinger equation

$$i\frac{\partial}{\partial t}\hat{\phi}(t,x) = -\frac{\partial^2}{\partial x^2}\hat{\phi}(t,x) + 2\,c\hat{\phi}^\dagger(t,x)\hat{\phi}(t,x)\hat{\phi}(t,x)$$

where $\hat{\phi}(t,x)$ and $\hat{\phi}^\dagger(t,x)$ are annihilation and creation field operators and satisfy Bosonic commutation relations:

$$[\hat{\phi}(t,x'),\hat{\phi}^\dagger(t,x)] = \delta(x-x')$$
$$[\hat{\phi}(t,x'),\hat{\phi}(t,x)] = [\hat{\phi}^\dagger(t,x'),\hat{\phi}^\dagger(t,x)] = 0$$

and in classical (mean-field) solution, i.e. $\hat{\phi} \rightarrow \phi$,
for attractive case ($a_s < 0$), $c < 0$, bright soliton exists;
for repulsive case ($a_s > 0$), $c > 0$, dark soliton exists.

1-D Bose gas with δ-interaction

Expand the quantum state in Fock space

$$|\psi> = \sum_n a_n \int d^n x \frac{1}{\sqrt{n!}} f_n(x_1, \ldots, x_n, t) \hat{\phi}^\dagger(x_1) \ldots \hat{\phi}^\dagger(x_n)|0>$$

then, QNLSE corresponds to 1-D Bosons with δ-interaction

$$i\frac{d}{dt} f_n(x_1, \ldots, x_n, t) = [-\sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1\leq i<j\leq n} \delta(x_j - x_i)] f_n(x_1, \ldots, x$$

and can be solved by

1. Bethe's ansatz (exact solution);
2. Hatree approximation (N is large);
3. Quantum inverse scattering method (exact solution).

Solutions of Bethe's ansatz

The exact solution for 1-D Bose gas with δ-interaction

$$f_n(x_1,\ldots,x_n,t) = e^{-iE_nt}\sum_Q A_Q exp[i\sum_{j=1}^{n} k_{Q(j)}x_j]$$

for $x_1 \le x_2 \le \cdots \le x_n$ with the energy $E_n = \sum_{j=1}^{n} k_j^2$, and

$$A_{Q'} = \frac{k_{Q(j+1)} - k_{Q(j)} + ic}{k_{Q(j+1)} - k_{Q(j)} - ic} A_Q$$

In general, k_j must be real, and only $c < 0$ makes bound states possible.

One can construct exact solution of quantum solitons based on this solution.

E. Lieb and W. Liniger, *Phys. Rev.* **130**, 1605 (1963).

Y. Lai and H. A. Haus, *Phys. Rev. A* **40**, 854 (1989).

Hatree approximation

when the number of particles is large

$$f_n^{(H)}(x_1, \ldots, x_n, t) = \prod_{j=1}^{n} \Phi_n(x_j, t)$$

The functions Φ_n satisfy

$$i\frac{\partial}{\partial t}\Phi_n = -\frac{\partial^2}{\partial x^2}\Phi_n + 2(n-1)c\Phi_n^*\Phi_n\Phi_n$$

which have soliton solutions.

This fact is one of the connection between quantum theory and classical theory

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Y. Lai and H. A. Haus, *Phys. Rev. A* **40**, 844 (1989).

Quantum solitons in the Hatree approximation

The Hatree product eigenstates can be constructed by

$$|n,p,t>=\frac{1}{\sqrt{n!}}[\int dx\Phi_{np}(x,t)\hat{\phi}^{\dagger}(x)]^n|0>$$

And a superposition of these states with a Possonian distribution of n gives the soliton state

$$|\psi>_H=\sum_n \frac{\alpha_0^n}{\sqrt{n!}}e^{-|\alpha_0|^2/2}|n,p,t>_H$$

and its expection value

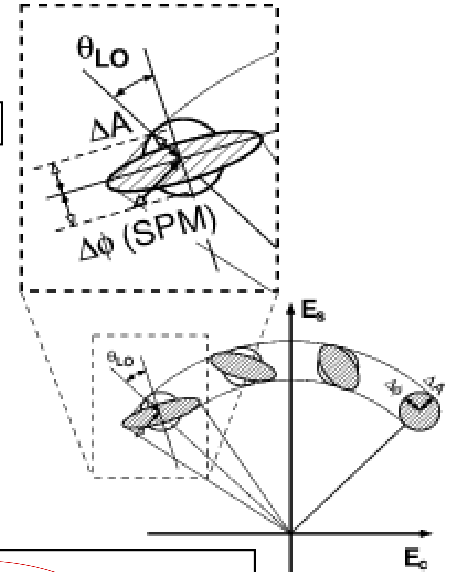
$${}_H<\psi|\hat{\phi}^{\dagger}(x)|\psi>_H \propto \sum_n \frac{\sqrt{n}|\alpha_0|^{2n}}{n!}e^{i\frac{n^2}{4}|c|^2t}sech[\frac{n}{2}|c|(x-x_0-2pt)]$$

just gives the classical soliton solution.

Quadrature Squeezing of Solitons

For $N = 1$ soliton:

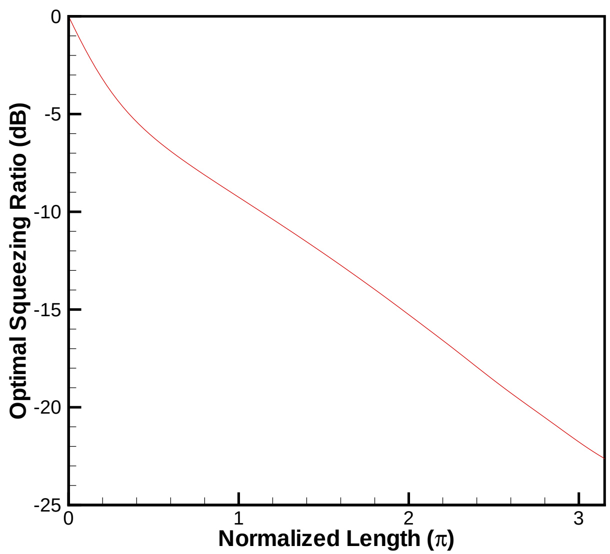
$$U(z,t) = \frac{n_0}{2} exp[i\frac{n_0^2}{8}z + i\theta_0] sech[\frac{n_0}{2}t]$$

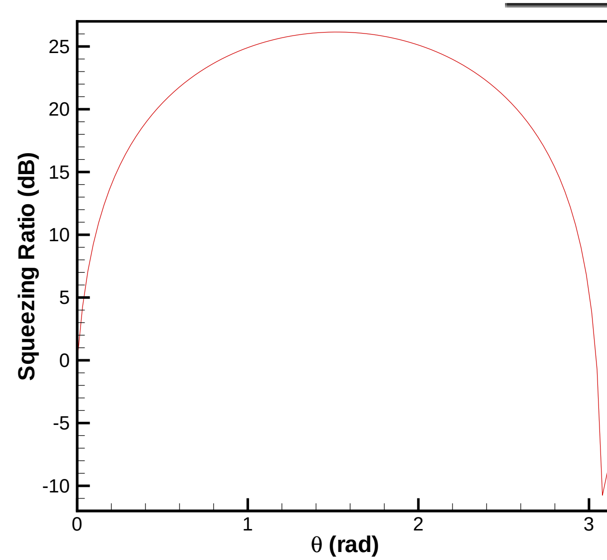


$$\Delta\hat{n}(z) = \Delta\hat{n}(0)$$

$$\Delta\hat{\theta}(z) = \Delta\hat{\theta}(0) + \frac{n_0}{4} z \Delta\hat{n}(0)$$

$$\Delta\hat{X}_\theta(z) = \alpha_1 \Delta\hat{n}(z) + \alpha_2 \Delta\hat{\theta}(z)$$





Optimal Squeezing Ratio $\equiv$ min $\frac{\mathrm{var}[\Delta\hat{X}_\theta(z)]}{\mathrm{var}[\Delta\hat{X}_\theta(0)]}$

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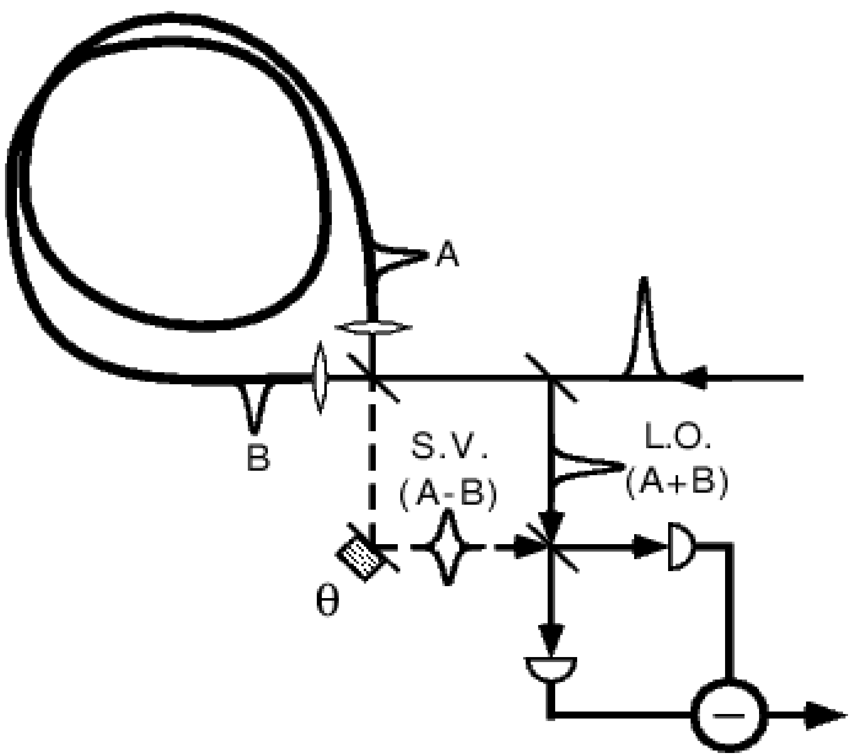
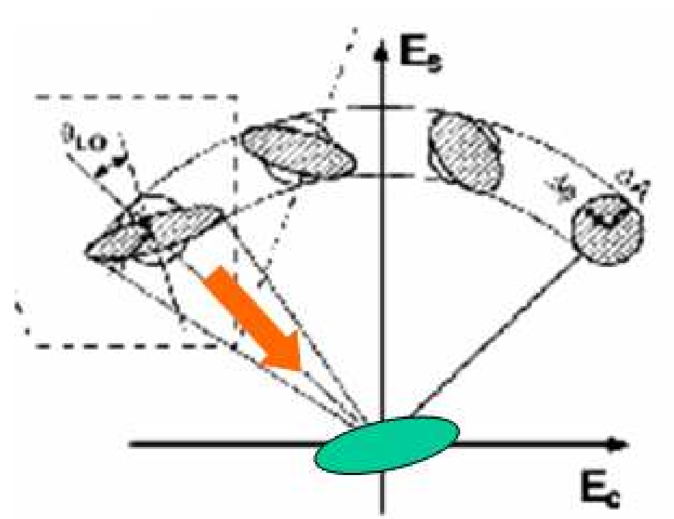
Y. Lai and H. A. Haus, *Phys. Rev. A* **40**, 844 (1989); *ibid* **40**, 854 (1989).

Generation and Detection of Squeezed Vacuum

1. Balanced Sagnac Loop (to cancel the mean field),

2. Homodyne Detection.

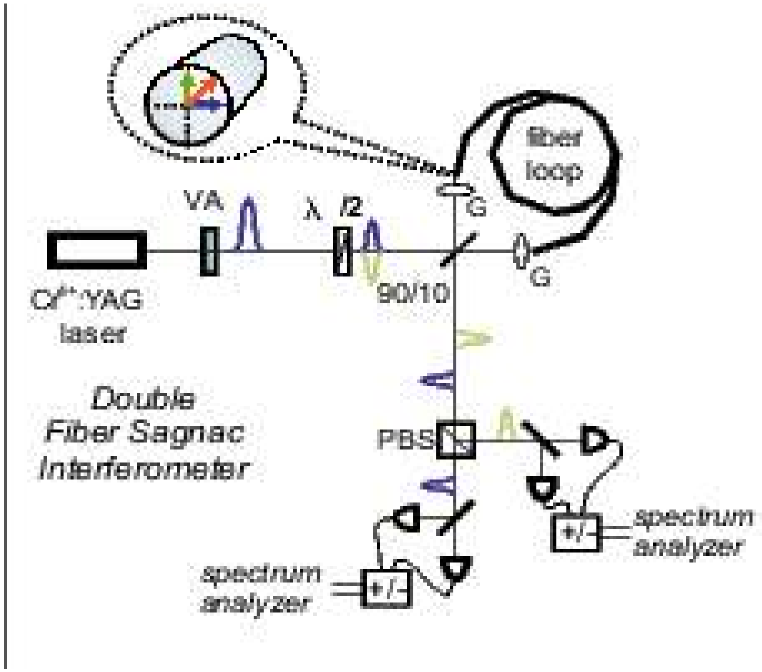


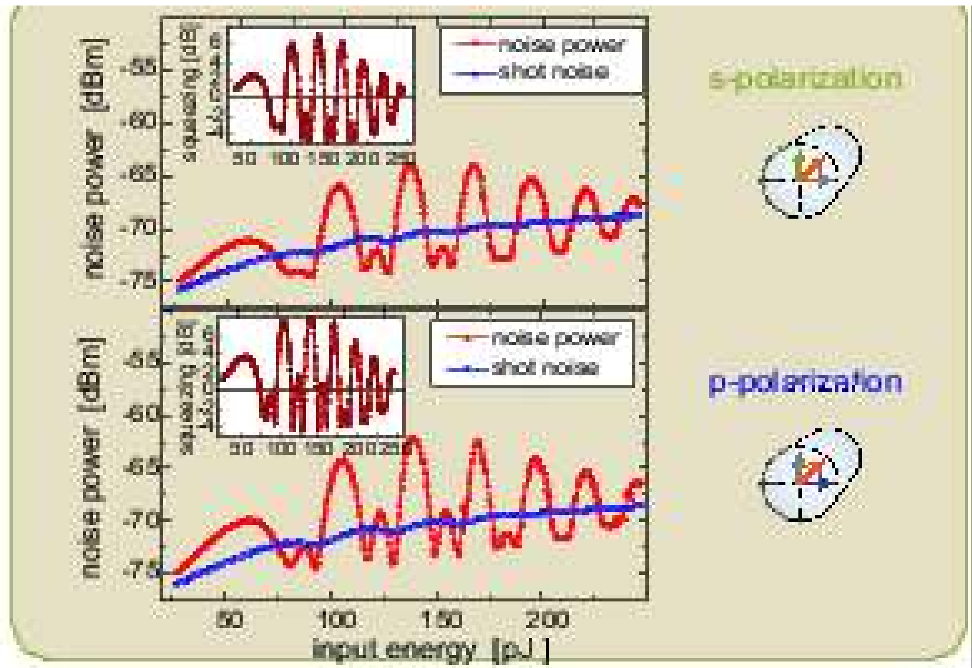


M. Rosenbluh and R. M. Shelby, *Phys. Rev. Lett.* **66**, 153(1991).

Generation and Detection of Amplitude Squeezed States

By asymmetric Sagnac Loop



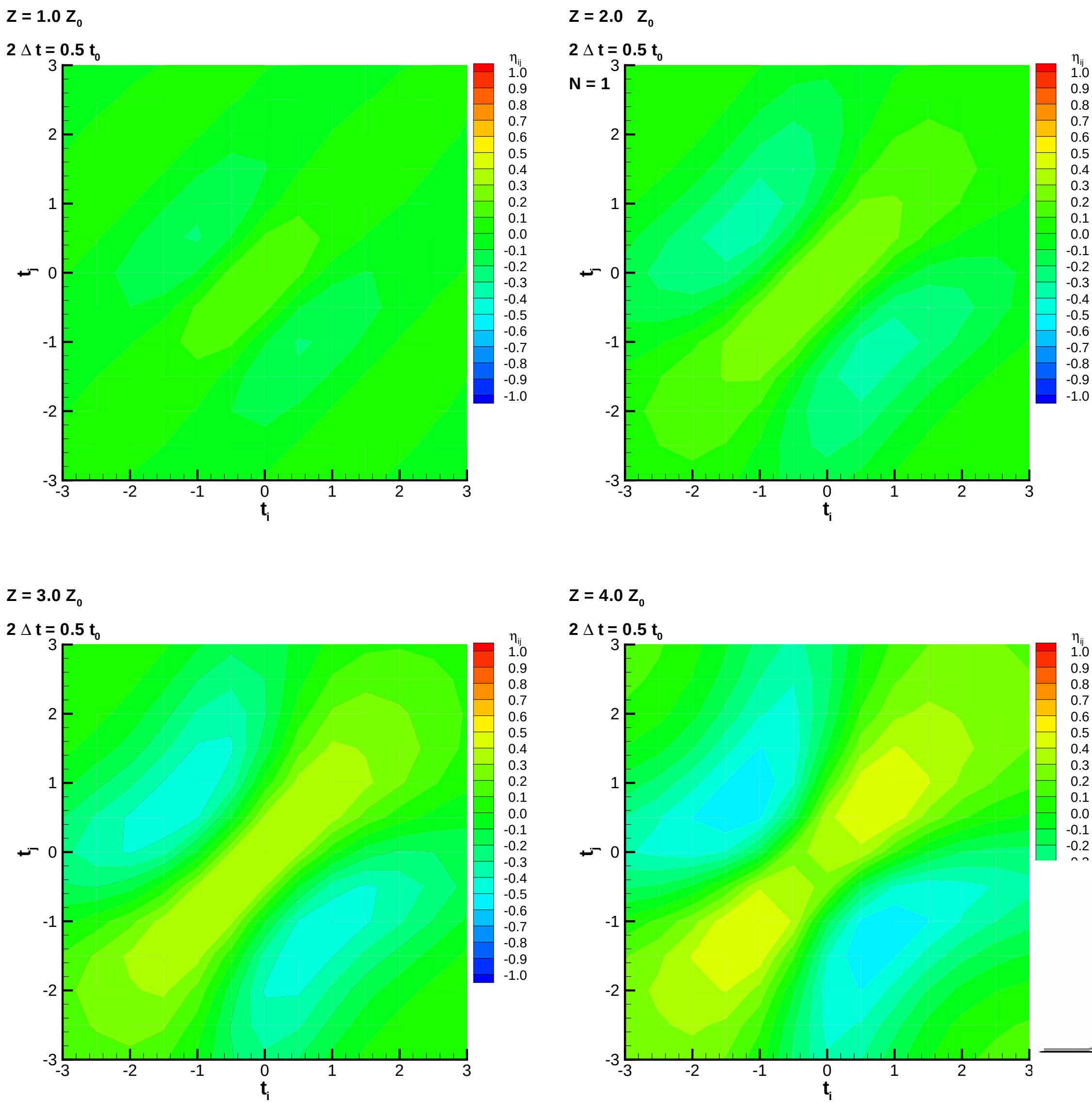


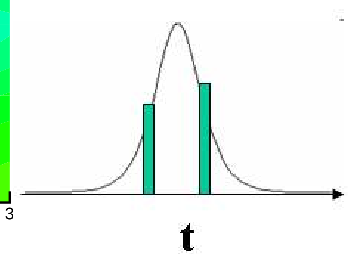
Ch. Silberhorn, P. K. Lam, O. Weis, F. Konig, N. Korolkova, and G. Leuchs,

Phys. Rev. Lett. **86**, 4267 (2001).

Evolutions of Quantum correlation Spectra

Time-domain intra-pulse photon-number correlations, for $N = 1$ soliton,





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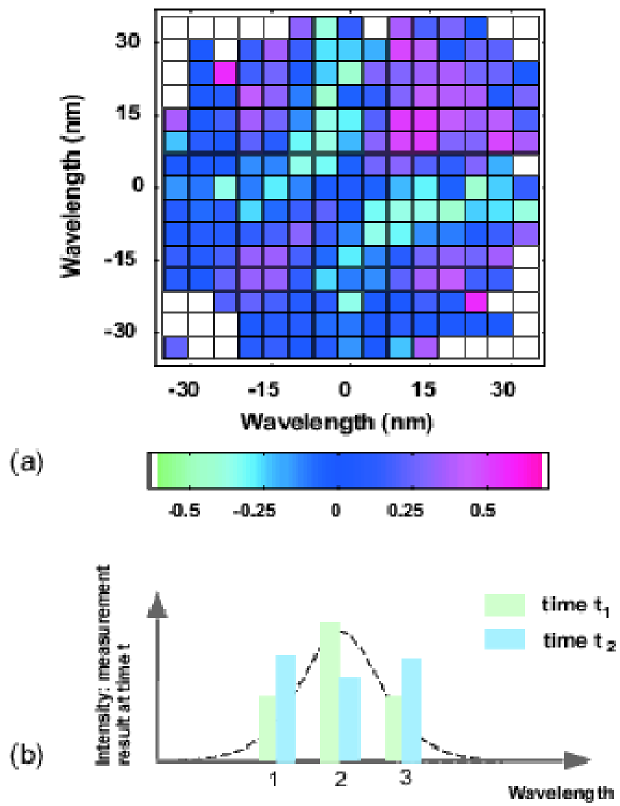
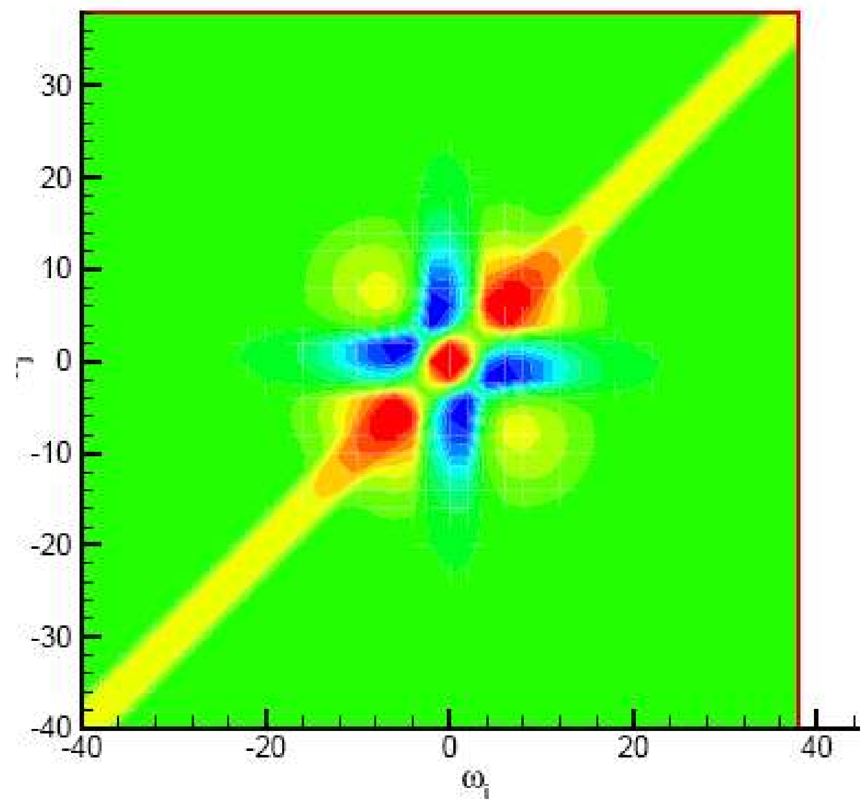
Multimode Quantum Correlations

With Spatral Filters, $C_{i,j} = \frac{\langle : \Delta \hat{n}_i \Delta \hat{n}_j : \rangle}{\sqrt{\langle \Delta \hat{n}_i^2 \rangle \langle \Delta \hat{n}_j^2 \rangle}}$, $i \neq j$

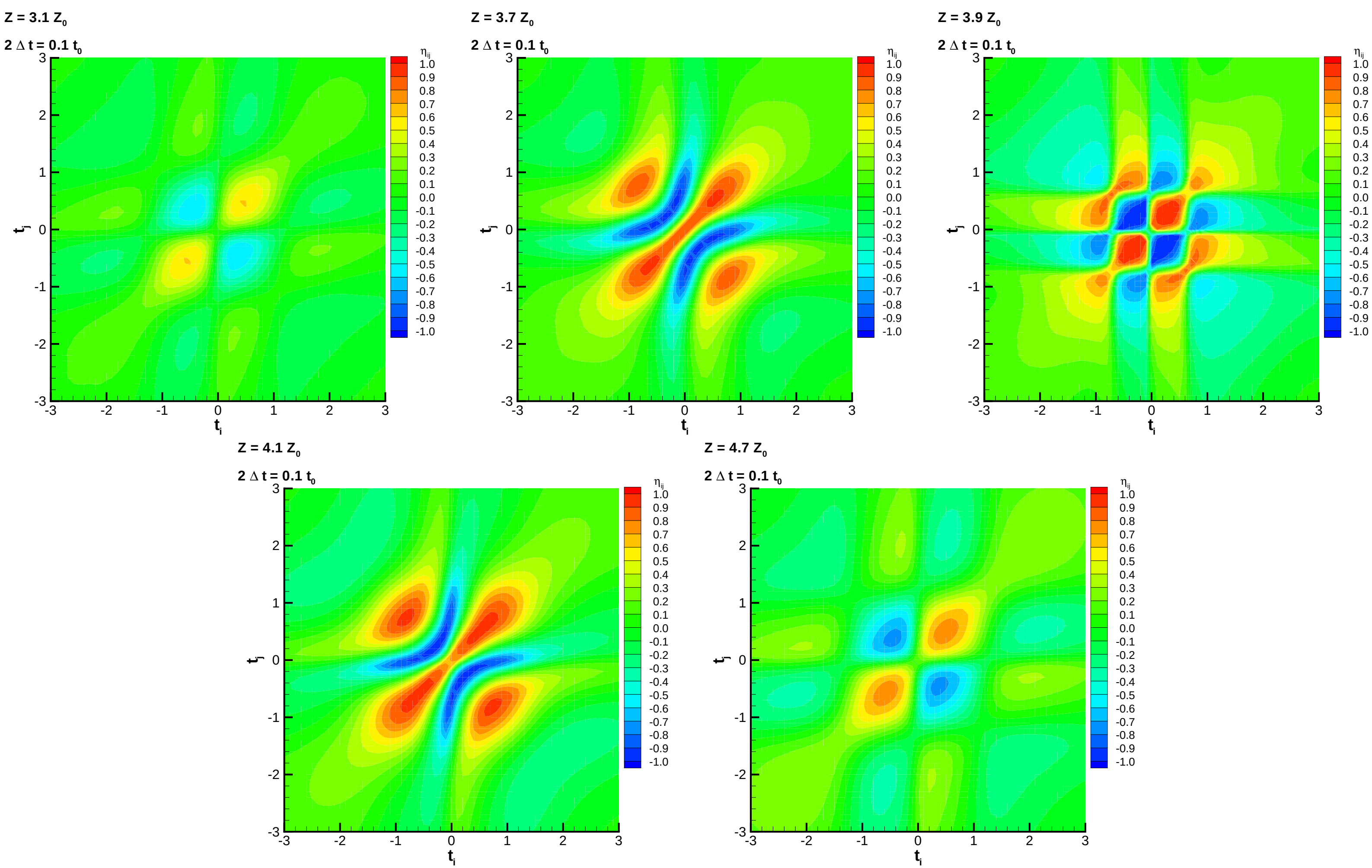




S. Spälter, N. Korolkova, F. Konig, A. Sizmann, and G. Leuchs,

Phys. Rev. Lett. **81**, 786 (1998).

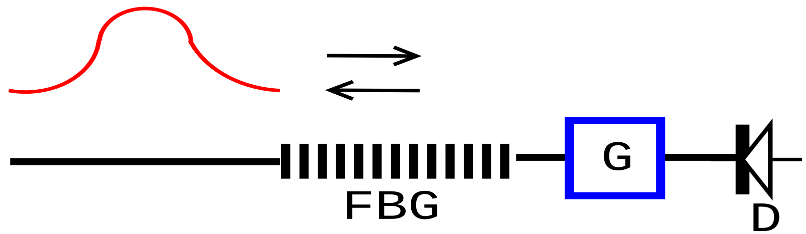
Quantum Correlation of $N = 2$ Solitons

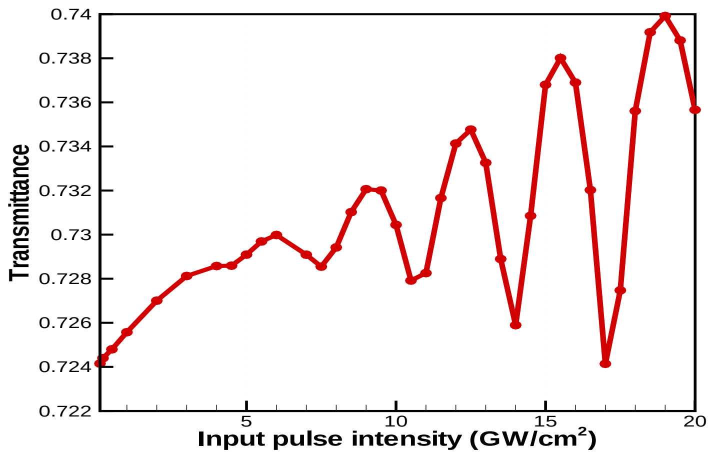


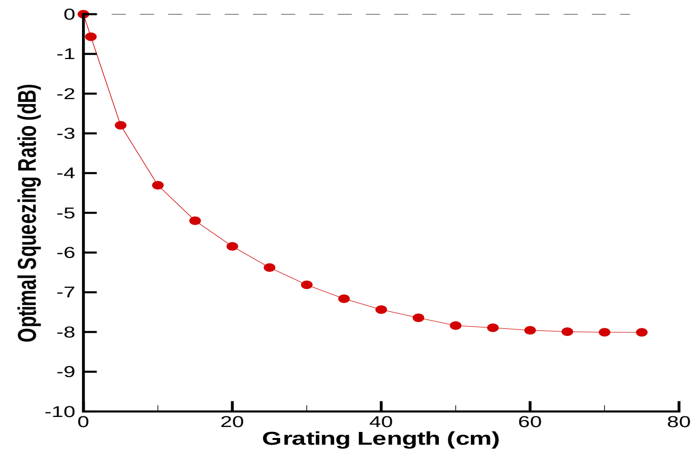
E. Schmidt, L. Knöll, and D. Welsch, *Opt. Comm.* **179**, 603 (2000).

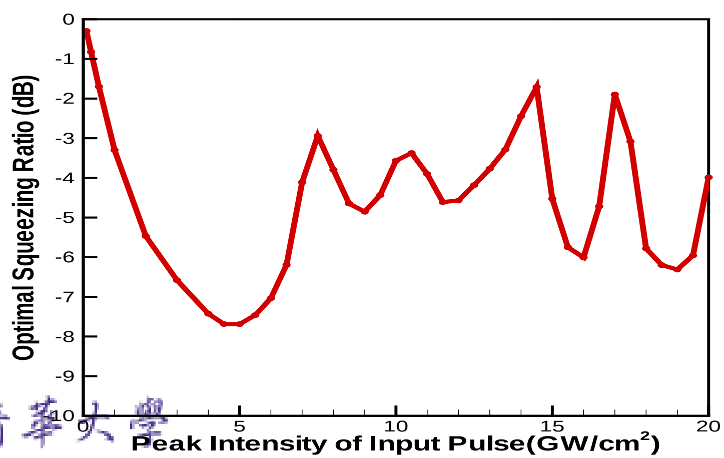
R.-K. Lee, and Y. Lai, Yu. S. Kivshar, *Phys. Rev. A* **71**, 035801 (2005).

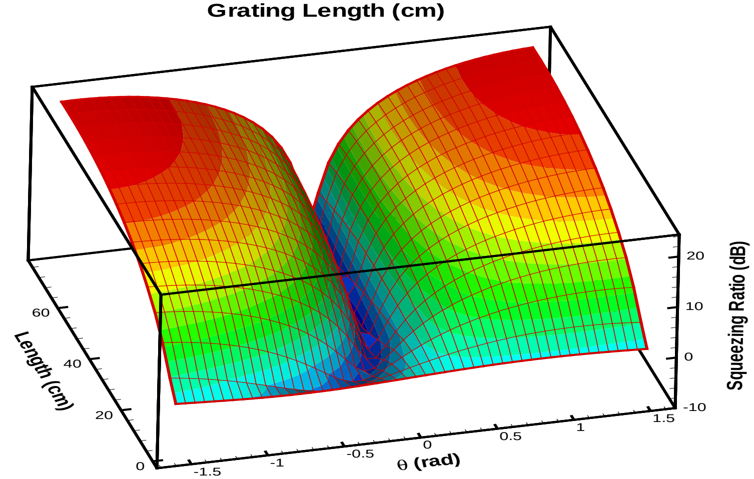
Amplitude Squeezing of FBG solitons











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R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

The Hamiltonian for Bragg solitons

The Hamiltonian for Bragg Solitons is

$$
\begin{aligned}
\mathcal{H} &= v_g\{-i\int dz\,(\hat{U}_a^\dagger \frac{\partial}{\partial z}\hat{U}_a - \hat{U}_b^\dagger \frac{\partial}{\partial z}\hat{U}_b) \\
&+ \int dz[\delta(\hat{U}_a^\dagger\hat{U}_a + \hat{U}_b^\dagger\hat{U}_b) - \kappa(\hat{U}_a^\dagger\hat{U}_b + \hat{U}_b^\dagger\hat{U}_a)] \\
&- \frac{\Gamma}{2}\int dz\,(\hat{U}_a^\dagger\hat{U}_a^\dagger\hat{U}_a\hat{U}_a + \hat{U}_b^\dagger\hat{U}_b^\dagger\hat{U}_b\hat{U}_b) \\
&- \Gamma\int dz\,(\hat{U}_a^\dagger\hat{U}_b^\dagger\hat{U}_b\hat{U}_a + \hat{U}_b^\dagger\hat{U}_a^\dagger\hat{U}_a\hat{U}_b)\}
\end{aligned}
$$

where $\hat{U}_a$, $\hat{U}_b$ represent forward/backward fields, satisfying Bosonic commutation relations:

$$
\begin{aligned}
&[\hat{U}_a(z_1,t),\hat{U}_a^\dagger(z_2,t)] = \delta(z_1-z_2), \quad [\hat{U}_b(z_1,t),\hat{U}_b^\dagger(z_2,t)] = \delta(z_1-z_2), \\
&[\hat{U}_a(z_1,t),\hat{U}_a(z_2,t)] = [\hat{U}_a^\dagger(z_1,t),\hat{U}_a^\dagger(z_2,t)] = [\hat{U}_b(z_1,t),\hat{U}_b(z_2,t)] = 0 \\
&[\hat{U}_b^\dagger(z_1,t),\hat{U}_b^\dagger(z_2,t)] = [\hat{U}_a(z_1,t),\hat{U}_b(z_2,t)] = [\hat{U}_a(z_1,t),\hat{U}_b^\dagger(z_2,t)] = 0
\end{aligned}
$$

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Linearization Approach

By setting $\hat{U}(x,t) = U_0(z,t) + \hat{u}(z,t)$, we can linearize the QNLCME as follows:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix}\hat{u}_a \\ \hat{u}_b\end{pmatrix} = \begin{pmatrix} i\Gamma U_{a0}^2 & 2i\Gamma U_{a0}U_{b0} \\ 2i\Gamma U_{a0}U_{b0} & +i\Gamma U_{b0}^2\end{pmatrix}\begin{pmatrix}\hat{u}_a^\dagger \\ \hat{u}_b^\dagger\end{pmatrix} +$$

$$\begin{pmatrix} -\frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 & i\kappa + 2i\Gamma U_{a0}U_{b0}^* \\ +i\kappa + 2i\Gamma U_{a0}^* U_{b0} & \frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2\end{pmatrix} \cdot \begin{pmatrix}\hat{u}_a \\ \hat{u}_b\end{pmatrix}$$

where the perturbation fields $\hat{u}_a(z,t)$ and $\hat{u}_b(z,t)$ also have to satisfy the same Bosonic commutation relations.

R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

Back-Propagation Method

With a set of adjoint equations

$$\frac{1}{v_g}\frac{\partial}{\partial t}\begin{pmatrix} u_a^A \\ u_b^A \end{pmatrix} = \begin{pmatrix} -i\Gamma U_{a0}^2 & -2i\Gamma U_{a0}U_{b0} \\ -2i\Gamma U_{a0}U_{b0} & -i\Gamma U_{b0}^2 \end{pmatrix}\begin{pmatrix} u_a^{A*} \\ u_b^{A*} \end{pmatrix} +$$

$$\begin{pmatrix} -\frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 & i\kappa + 2i\Gamma U_{a0}U_{b0}^* \\ i\kappa + 2i\Gamma U_{a0}^* U_{b0} & \frac{\partial}{\partial z} + i\delta + 2i\Gamma|U_{a0}|^2 + 2i\Gamma|U_{b0}|^2 \end{pmatrix}\begin{pmatrix} u_a^A \\ u_b^A \end{pmatrix}$$

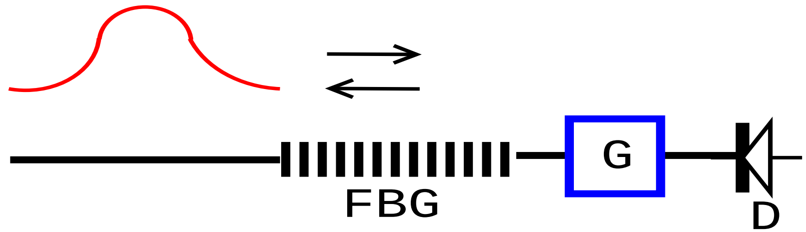
which satisfy

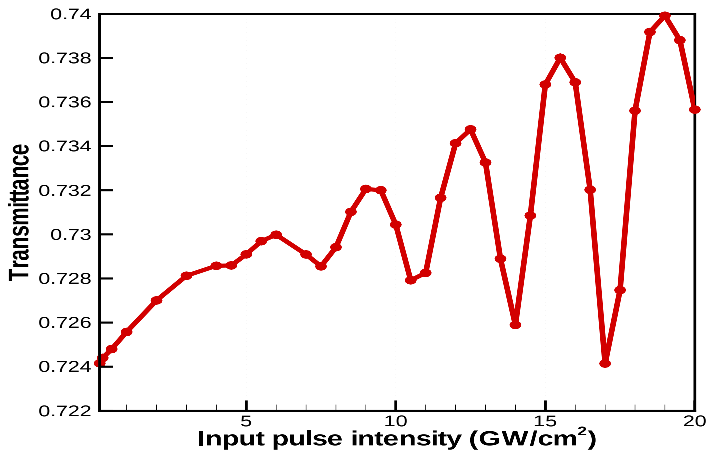
$$\frac{d}{dt}\langle \vec{u}^A | \vec{\hat{u}} \rangle = 0$$

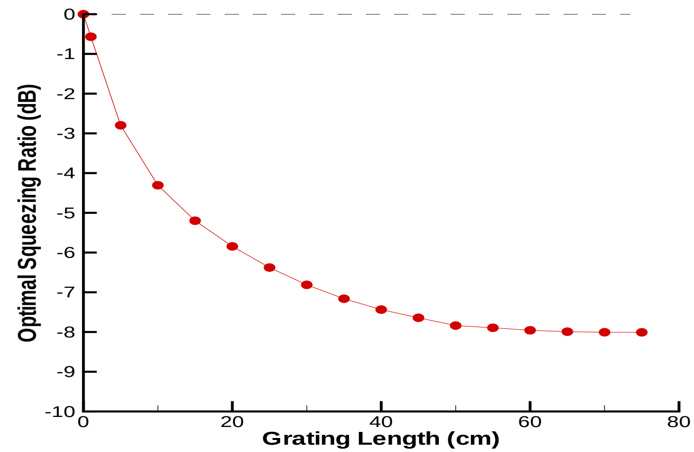
Due to the conservation of inner product, then we can calculate the measurement at t_1 by back-propagating the inner product to t_0.

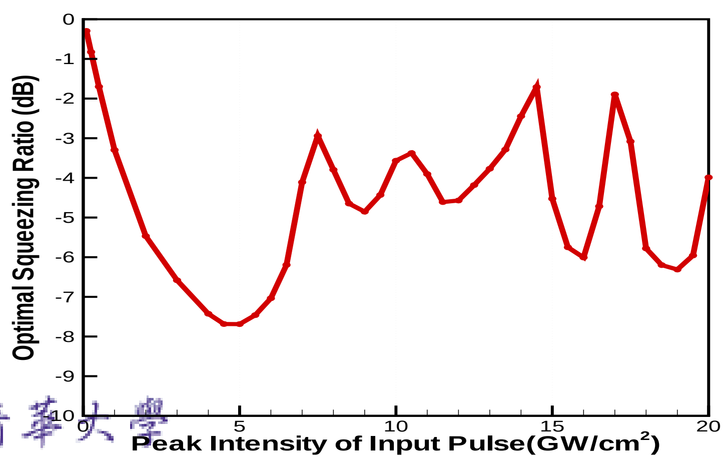
Y. Lai and S.-S. Yu, *Phys. Rev. A* **51**, 817 (1995).

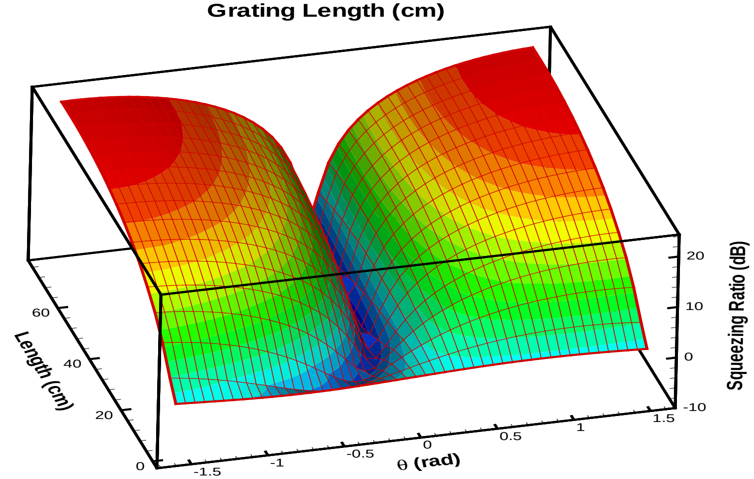
Amp. Squeezing of FBG solitons











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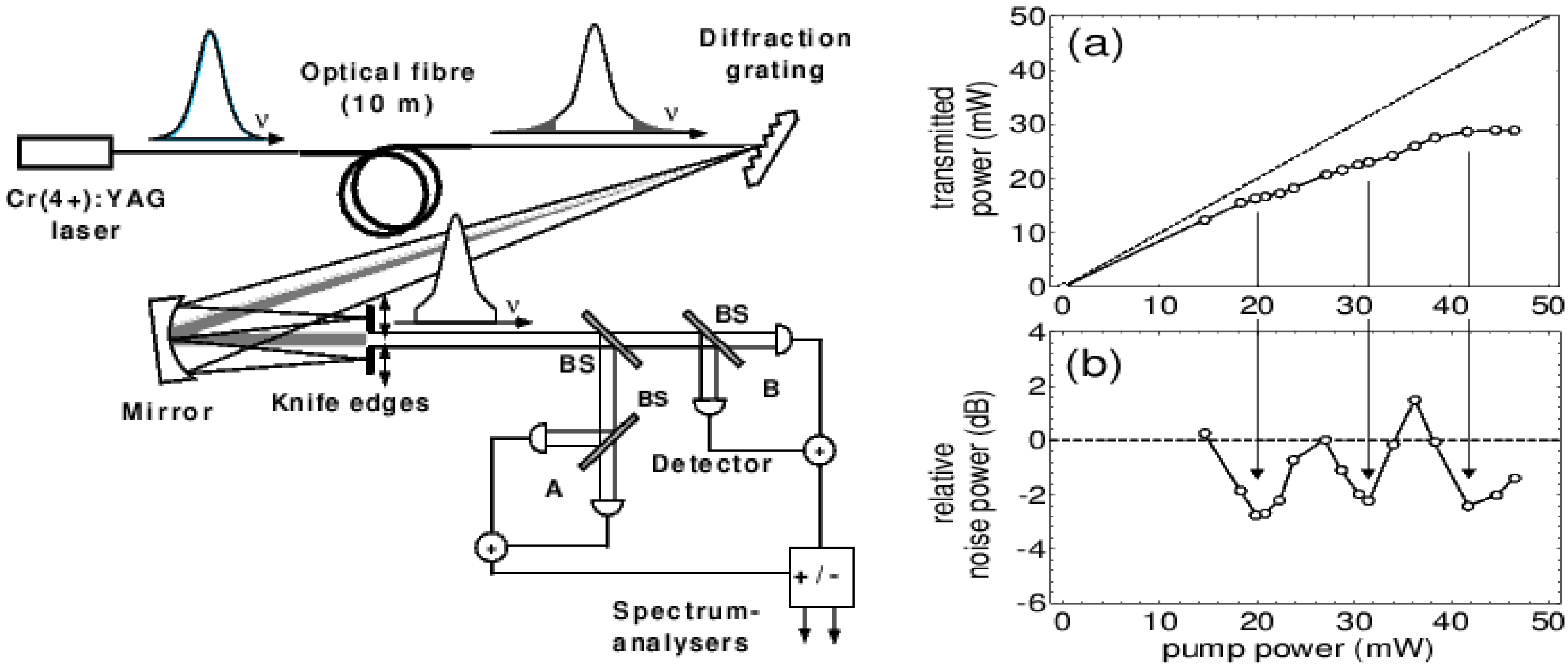
R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(R) (2004).

Amplitude Squeezing by Spectral Filter



S. R. Friberg, S. Machida, M. J. Werner, A. Levanon, and Takaaki Mukai,

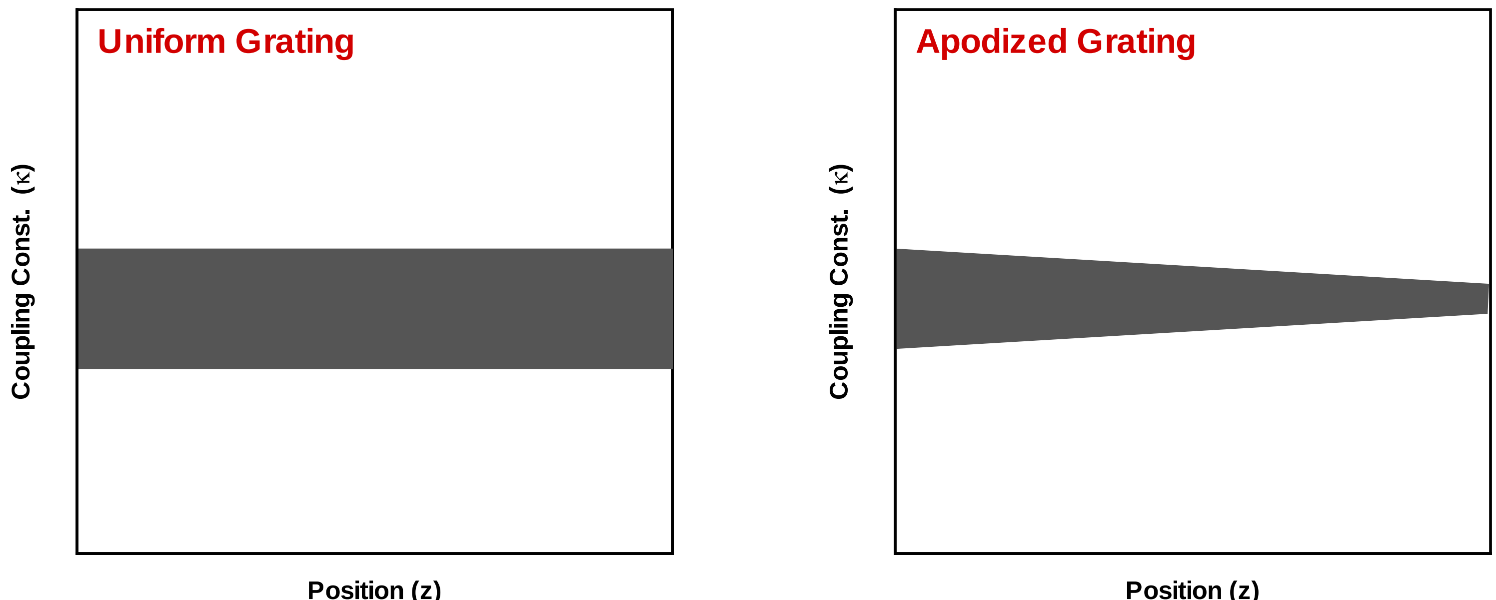
Phys. Rev. Lett. **77**, 3775 (1996).

Apodized Fiber Bragg Gratings

We consider a nonuniform FBG which has a position dependent coupling coefficient described by

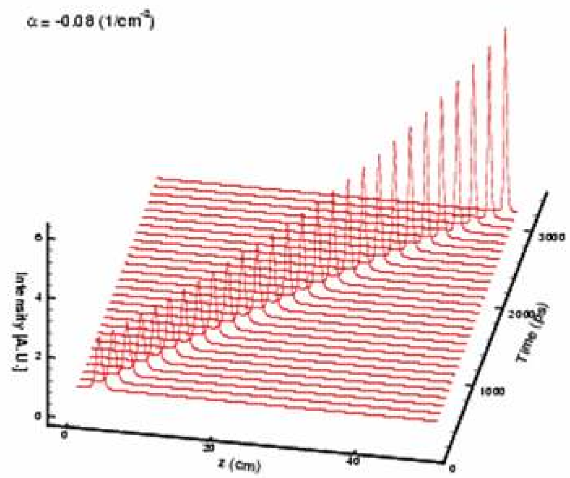
$$\kappa(z) = \kappa_0 + \alpha z$$

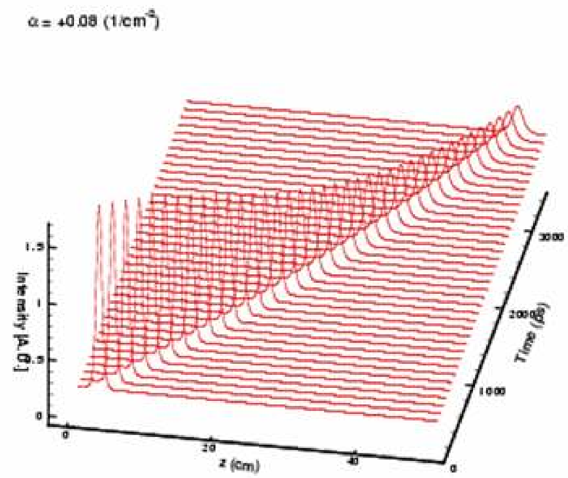
where κ_0 is the initial coupling coefficient and α is the slope of the coupling coefficient.

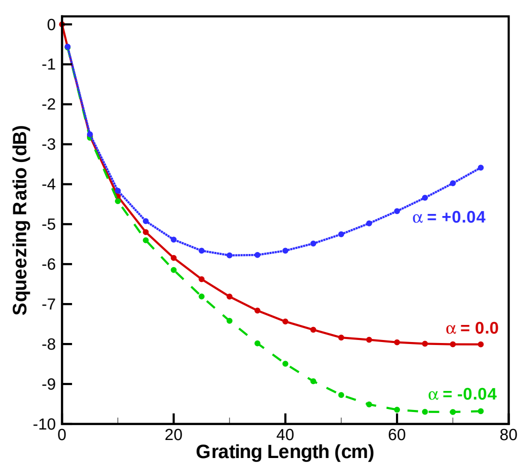


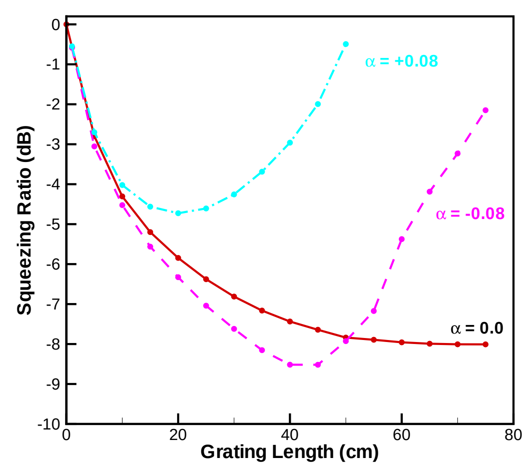
Tailor the Noise by Apodized Fiber Bragg Gratings

$\alpha < 0$ $\alpha > 0$









$\alpha = \pm 0.04 (1/cm^2)$ $\alpha = \pm 0.08 (1/cm^2)$

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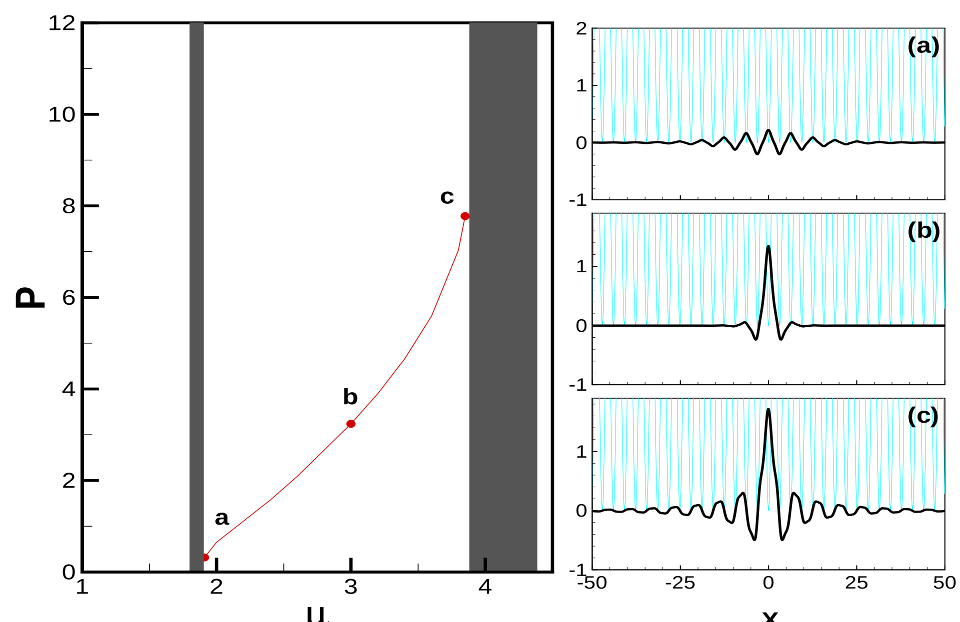
R.-K. Lee and Y. Lai, *J. Opt. B* **6**, S638 (Special Issue 2004).

Mater-wave gap soliton in optical lattices

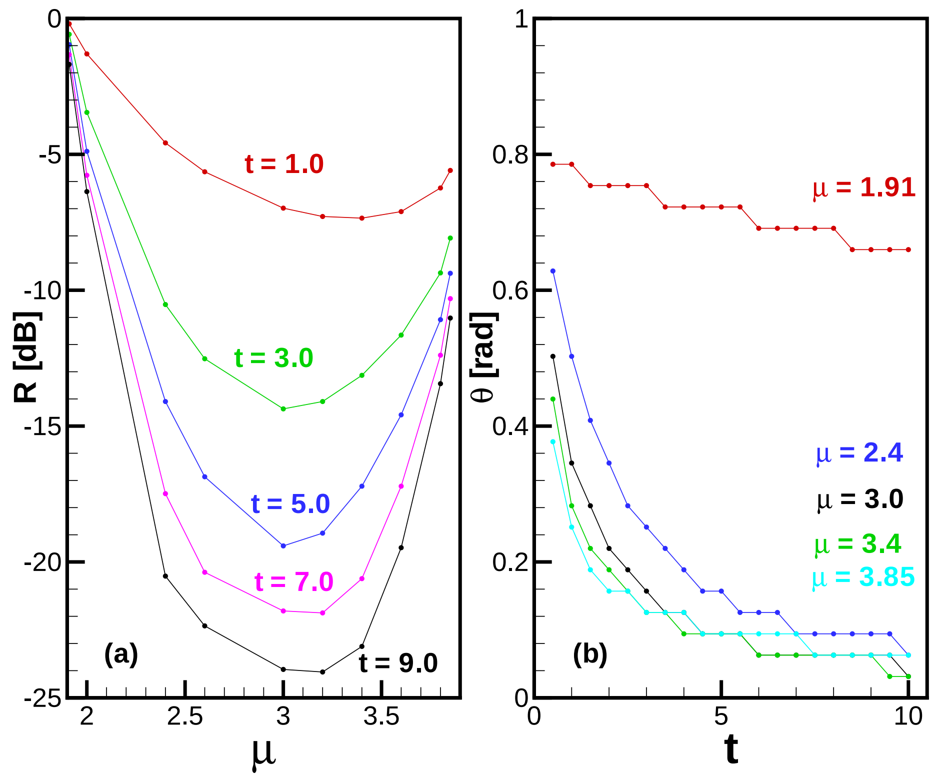
Using $\hat{\Phi}(t,x) = \Phi_0(t,x) + \hat{\phi}(t,x)$ for large atom number, where $\Phi_0(t,x)$ is the mean-field solution of 1-D Gross-Pitaevskii equation,

$$i\hbar\frac{\partial}{\partial t}\Phi_0(t,x) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi_0(t,x) + V(x)\Phi_0(t,x) + g_{1D}|\phi_0(t,x)|^2\phi_0(t,x$$

which has gap soliton solutions.



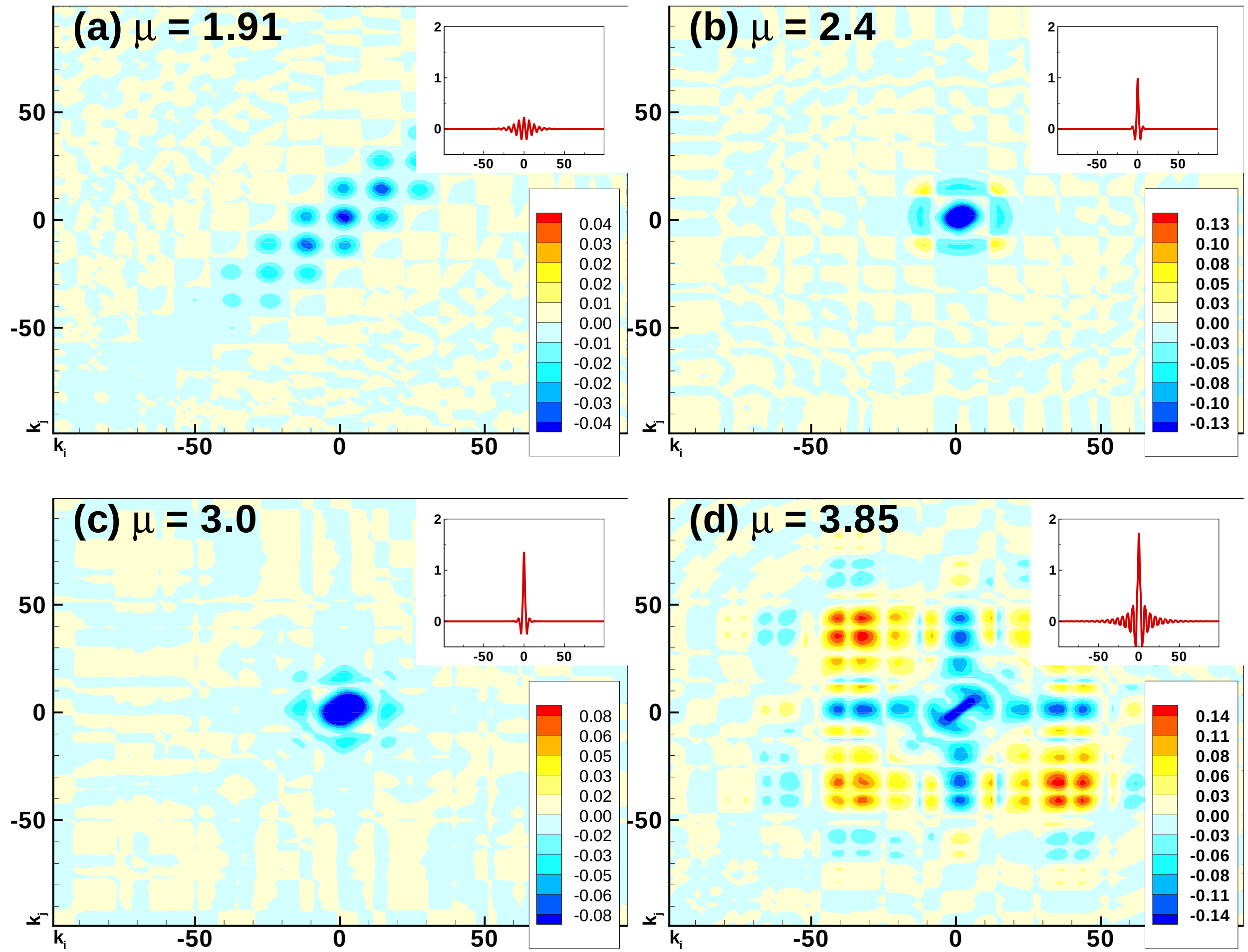
Squeezing ratio v.s. chemical potential



Squeezing effect is most profound in the depth of the gap and reduced near the band edges.

Quantum correlation patterns v.s. chemical potential

x-domain

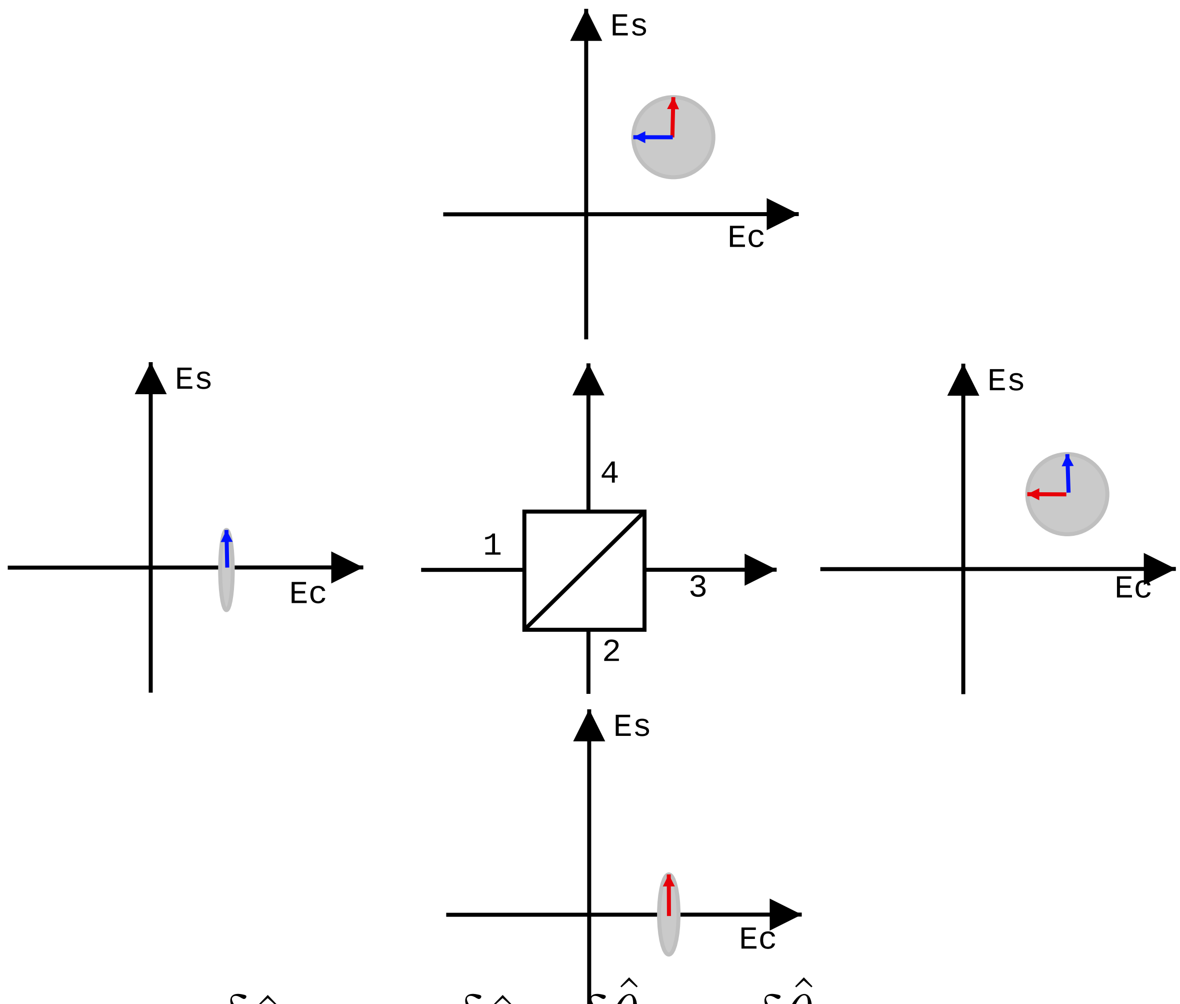


R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai,

Phys. Rev. A **72**, 033607 (2005).

Generation of Continuous Variables Entanglement

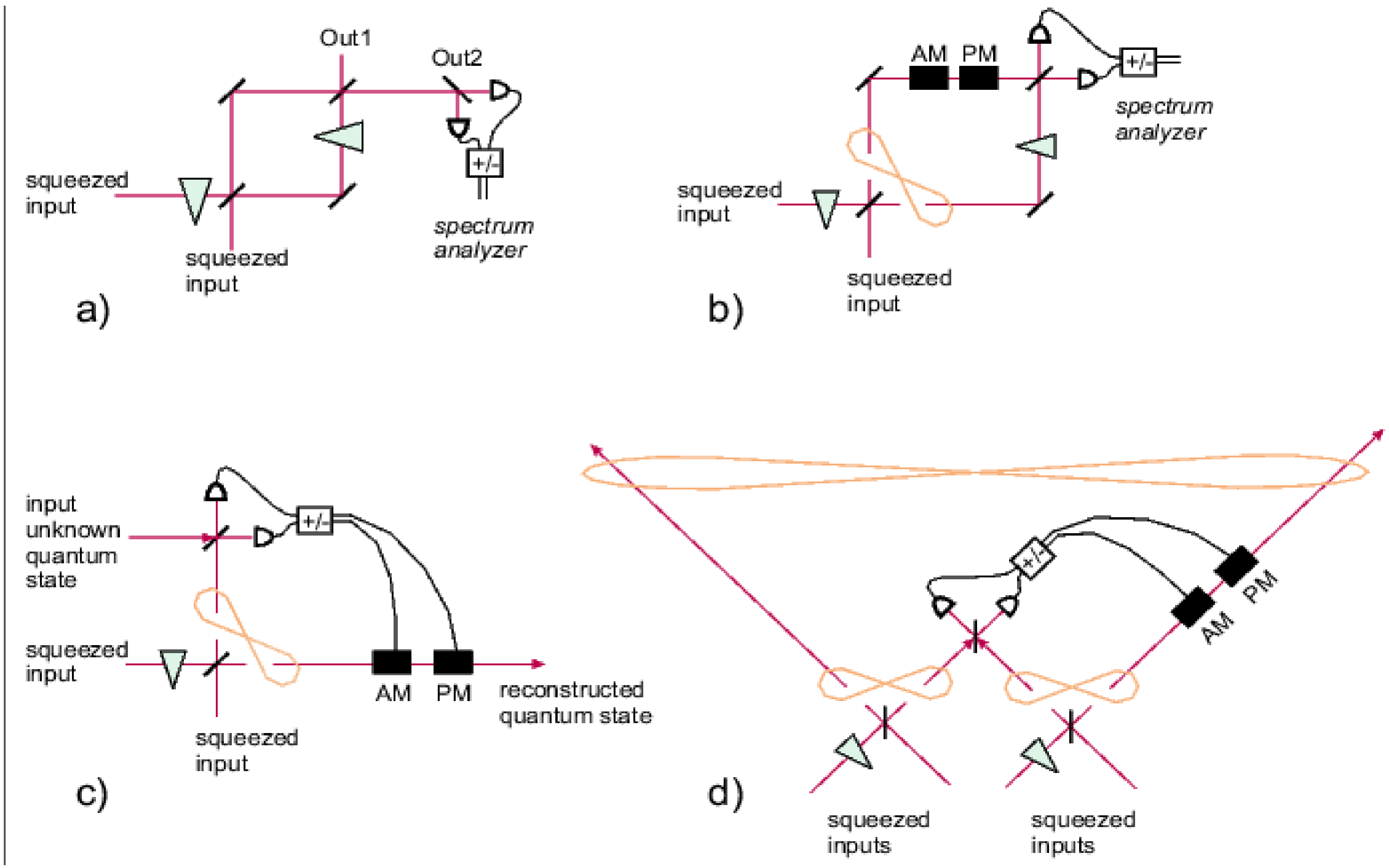
Preparation EPR pairs by Squeezed Sates



$$\delta\hat{n}_3 = -\delta\hat{n}_4, \delta\hat{\theta}_3 = \delta\hat{\theta}_4.$$

Applications of EPR Pairs by Using Squeezed States

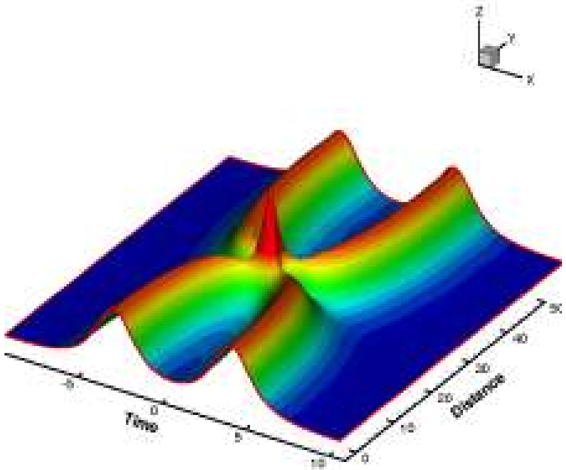
(a)entanglement; (b)quantum dense coding; (c) teleportation; (d)entangle swapping.



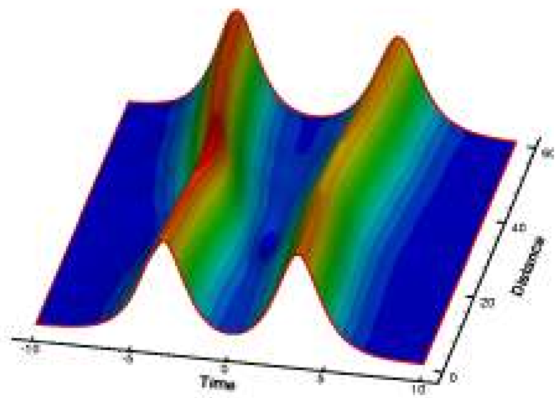
G. Leuchs and N. Korolkova, *Optics & Photonics News* **Feb.**, 64 (2002).

Photon Number Correlation of 2-Solitons Interaction

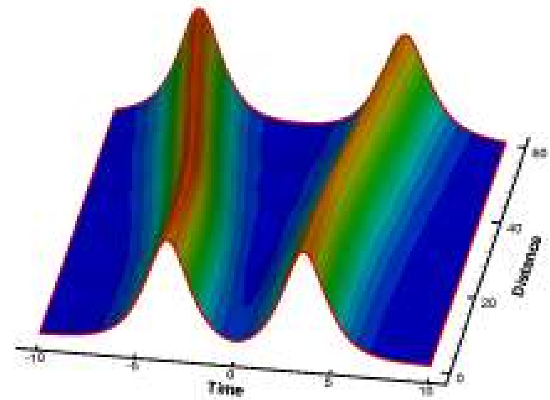
$$U(z,t) = sech(z, t+\rho) + r\, sech(z, t_\rho)e^{i\theta}$$



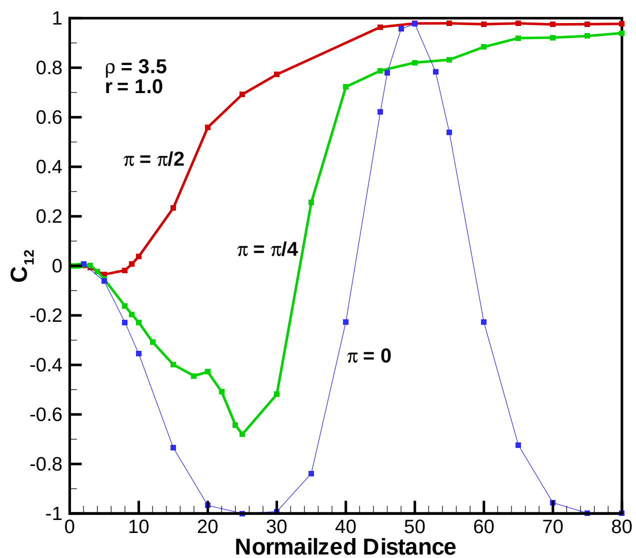
$\theta = 0$



$\theta = \pi/4$



$\theta = \pi/2$



$$C_{1,2} = \frac{\langle :\Delta\hat{n}_1 \Delta\hat{n}_2 : \rangle}{\sqrt{\Delta\hat{n}_1^2 \Delta\hat{n}_2^2}}$$

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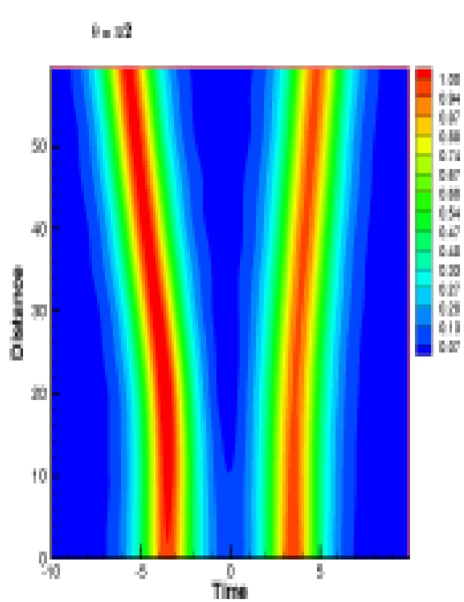
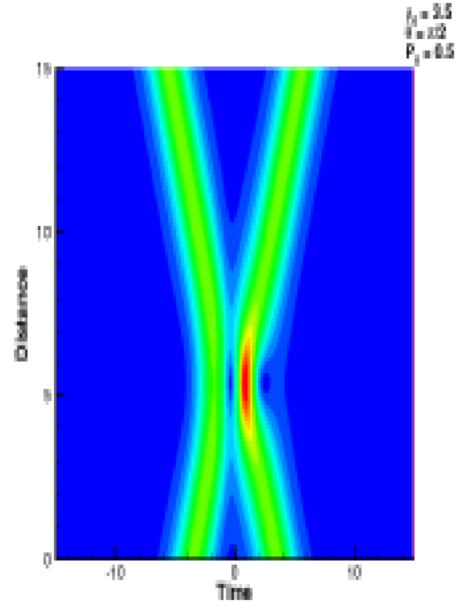
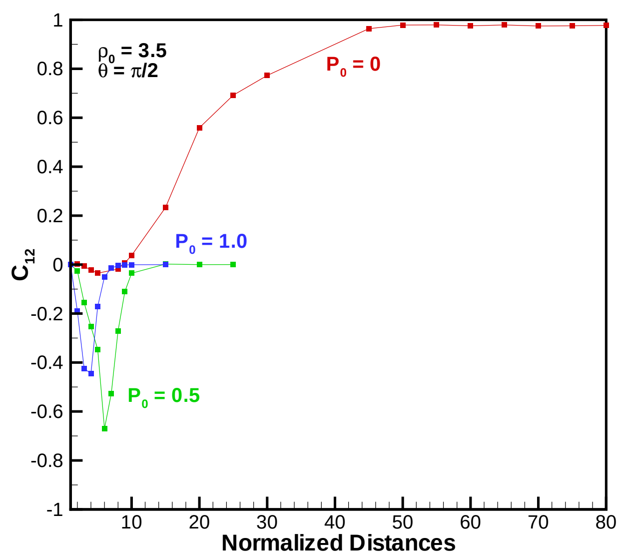
Quantum Correlation of 2-NLSE Solitons after Collision

Solitons move with the same (right) and different (left) velocities.







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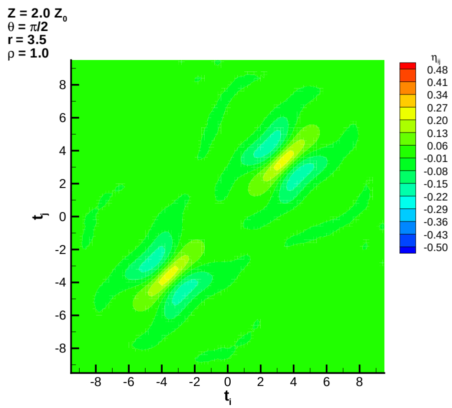
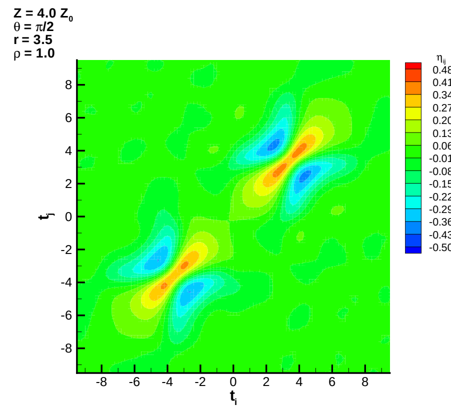
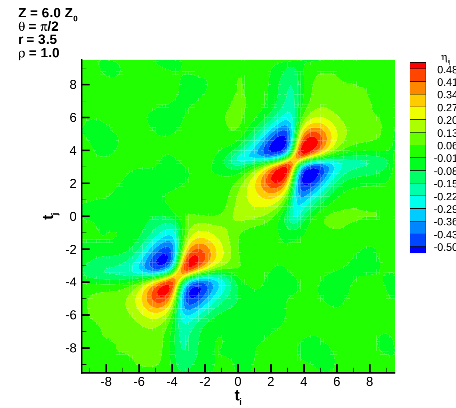
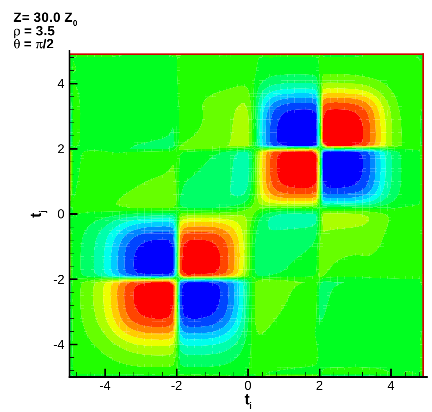
Evolutions of Photon Number Correlation Spectra

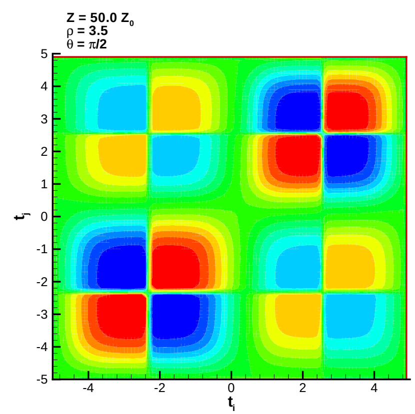
$Z = 2.0Z_0$, $Z = 4.0Z_0$, $Z = 6.0Z_0$.











$Z = 30.0Z_0$, $Z = 50.0Z_0$

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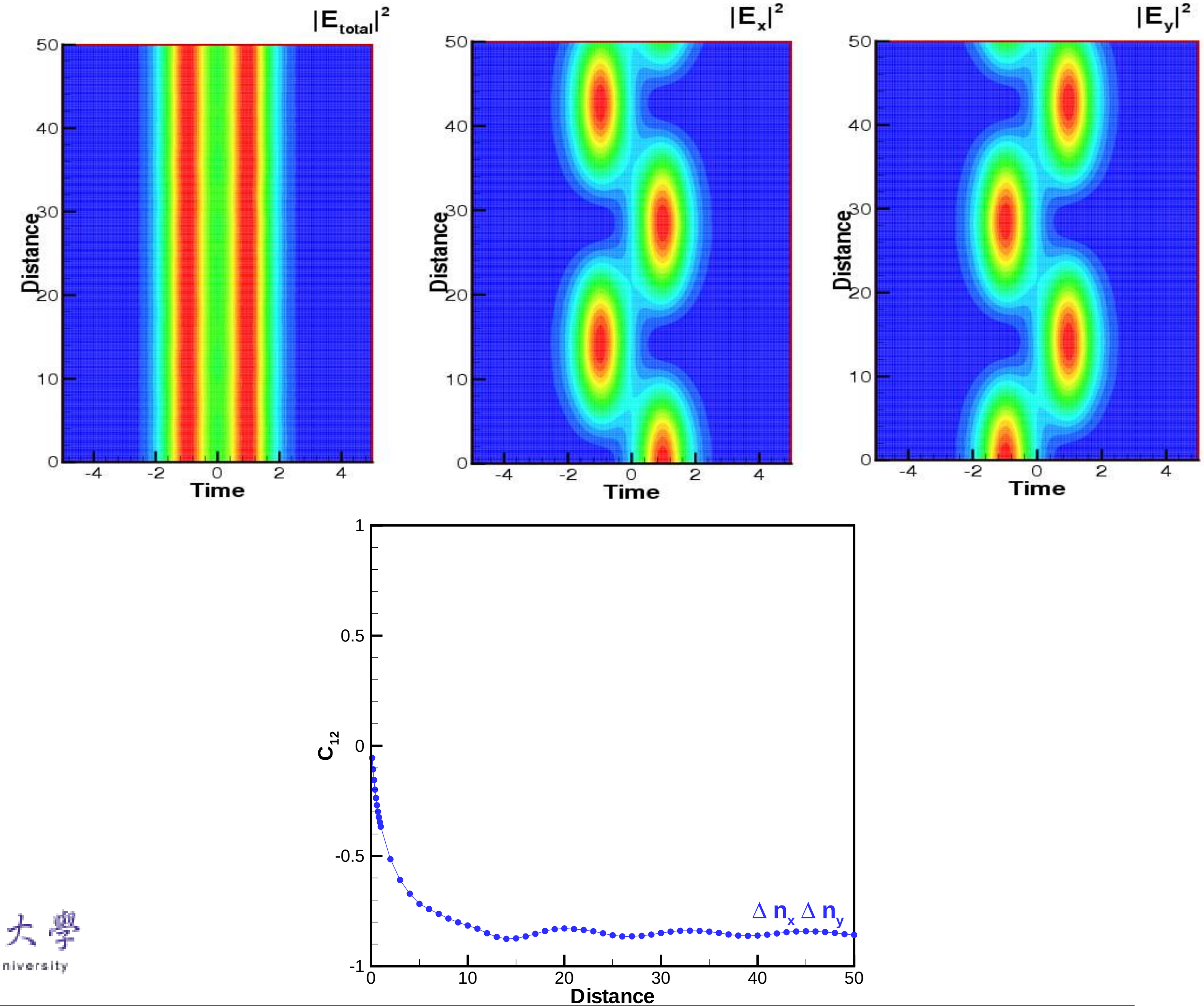
R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **71**, 013816 (2005).

Photon Number Correlations of Vector-Bound Solitons



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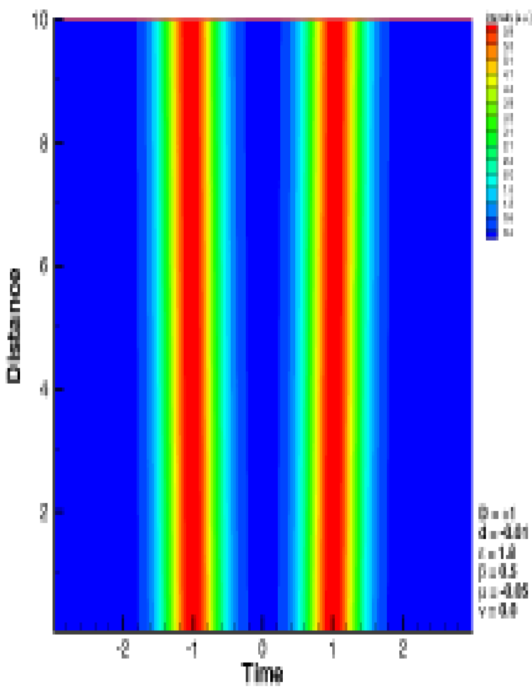
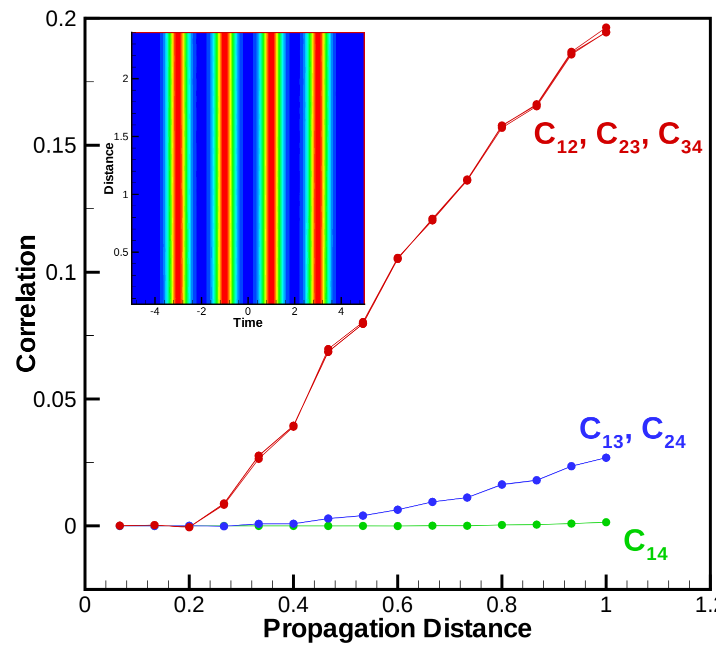
Quantum Correlations of Bound-States of Solitons

Complex Ginzburg-Lanau Equation:

$$
\begin{aligned}
iU_z + \frac{D}{2}U_{tt} + |U|^2 U &= i\delta U + i\epsilon |U|^2 U + i\beta U_{tt} \\
&+ i\mu |U|^4 U - v|U|^4 U
\end{aligned}
$$





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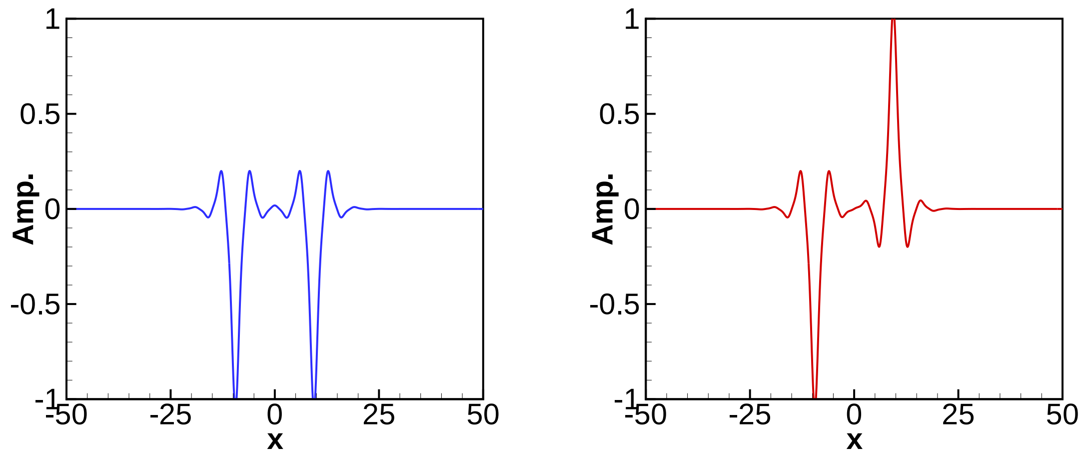
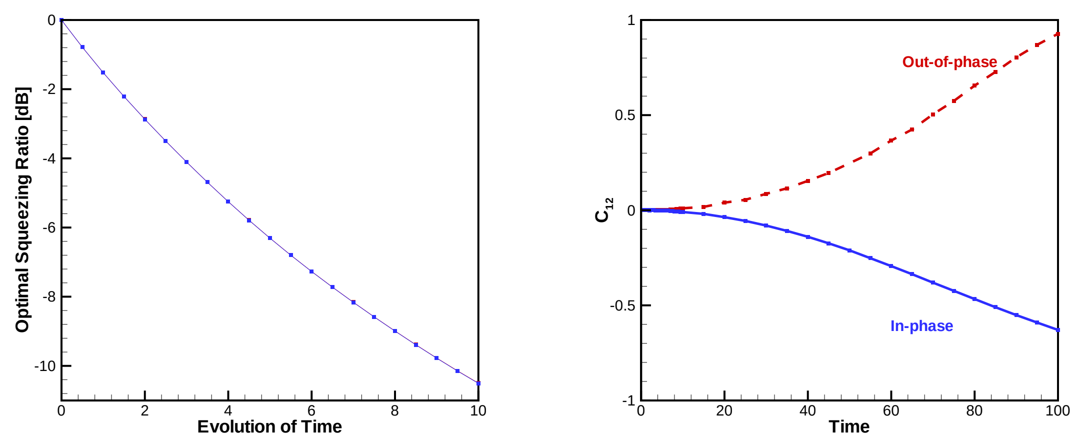
R.-K. Lee, Y. Lai, and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004).

Bound gap solitons and high correlated EPR pairs



The noise fluctuations of bound gap soliton pairs are the same, but with different photon-number correlation parameter.



R.-K. Lee, E. A. Ostrovskaya, Yu. S. Kivshar, and Y. Lai, *Phys. Rev. A* **72**, 033607 (2005).

From temporal solitons to spatial solitons

Temporal solitons in optical fibers:

$$iU_z(z,t) \;=\; -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2 U(z,t)$$

Spatial solitons in the CW cases:

$$iU_z(z,x) \;=\; -\frac{D}{2}U_{xx}(z,x) - |U(z,x)|^2 U(z,x)$$

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From temporal solitons to spatial solitons

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$$iU_z(z,t) \;=\; -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2U(z,t)$$

Spatial solitons in the CW cases:

$$iU_z(z,x) \;=\; -\frac{D}{2}U_{xx}(z,x) - |U(z,x)|^2U(z,x)$$

Both have the same mathematical models.

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From temporal solitons to spatial solitons

Temporal solitons in optical fibers:

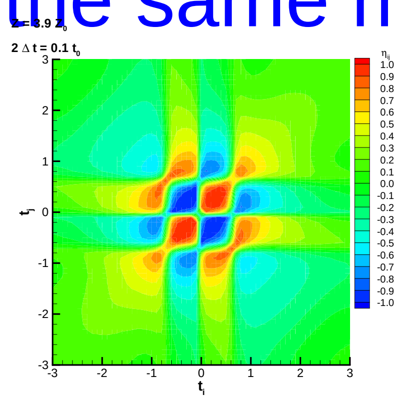
$$iU_z(z,t) \;=\; -\frac{D}{2}U_{tt}(z,t) - |U(z,t)|^2U(z,t)$$

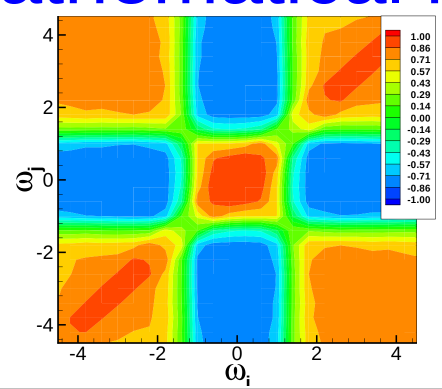
Spatial solitons in the CW cases:

$$iU_z(z,x) \;=\; -\frac{D}{2}U_{xx}(z,x) - |U(z,x)|^2U(z,x)$$

Both have the same mathematical models.

t/x domain





ω/k domain

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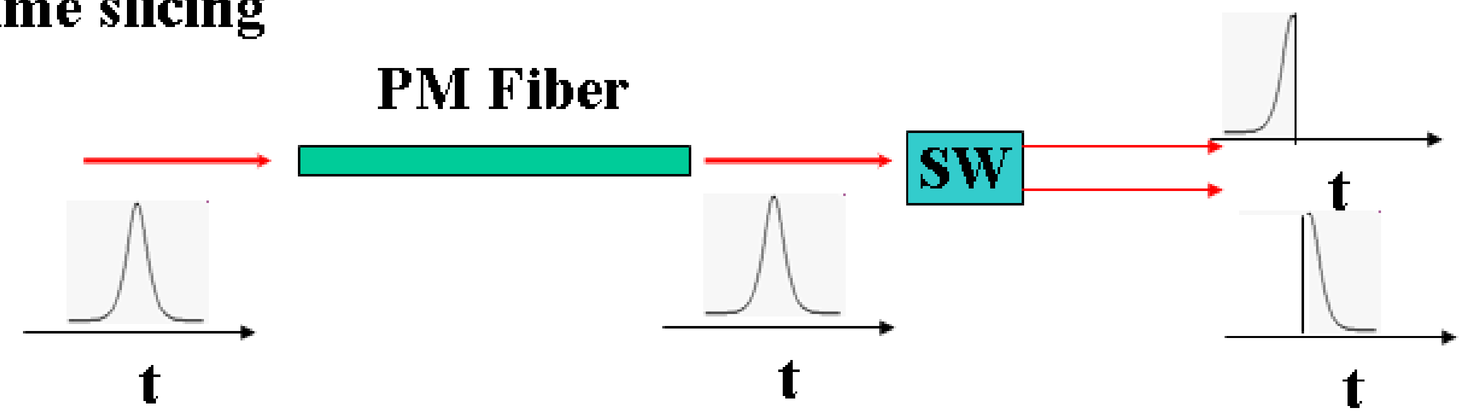
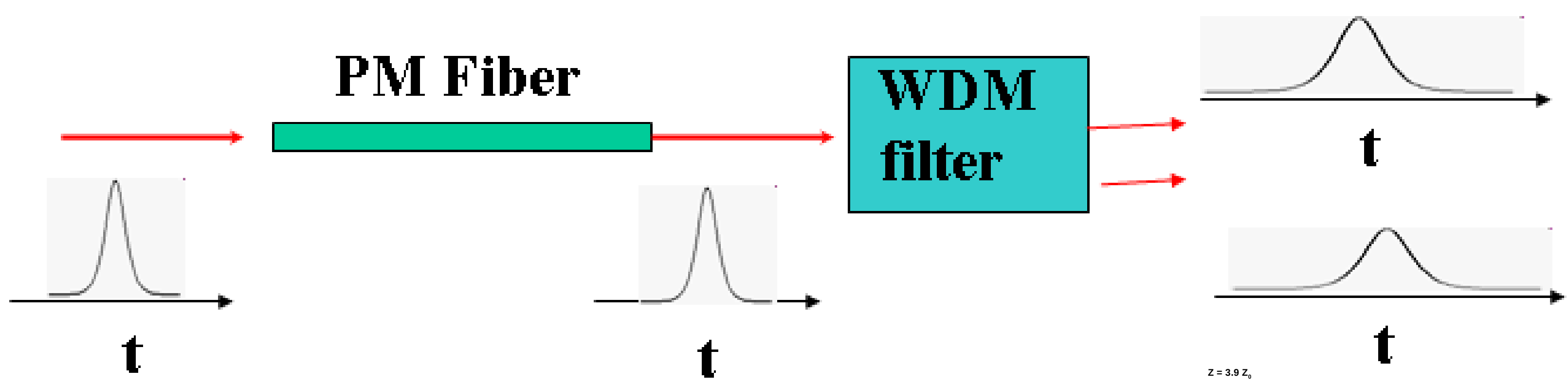
Entangled States by Time or Wavelength Slicing

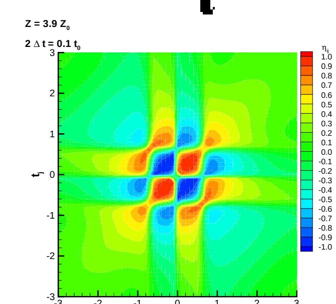
(1) time slicing



(2) Wavelength slicing



Quantum Images !



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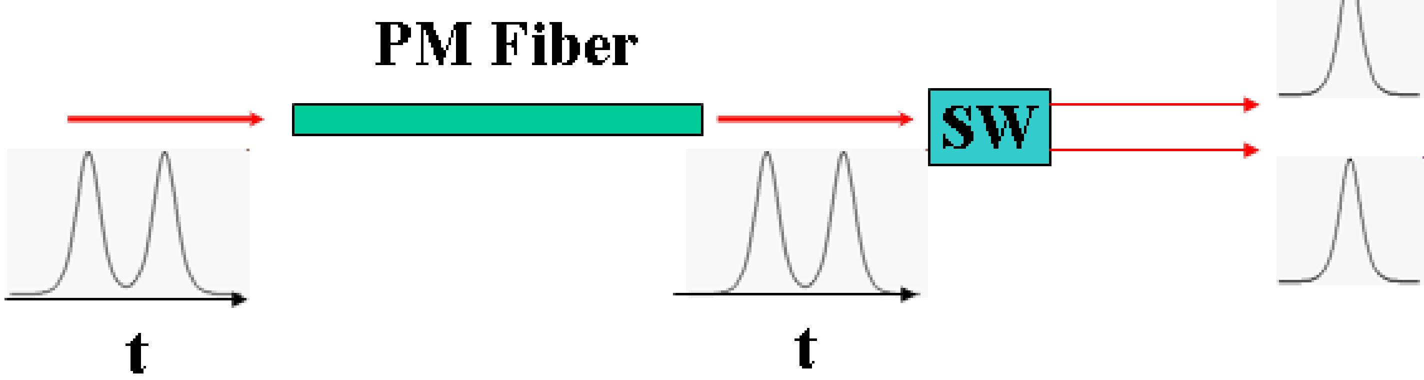
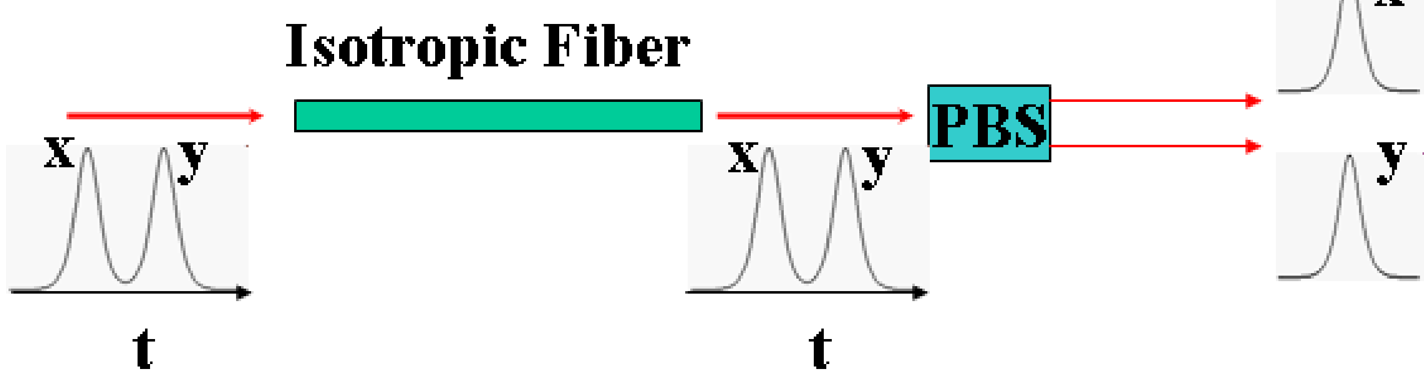
Entangled Soliton Pairs

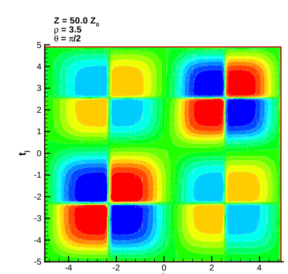
(1) TDM soliton pair



(2) PDM soliton pair



If necessary, the Sagnac loop configuration also can be used.



Continuous Variable Entanglement

- Non-separability criterion

$$V_{sq}^{\pm}(X) + V_{sq}^{\mp}(Y) < 2$$

- Squeezed-state entanglement

$$V_{sq}^{\pm}(X) = \frac{V(\hat{X}_1 \pm g\hat{X}_2)}{V(\hat{X}_{1,CS} + g\hat{X}_{2,CS})} < 1,$$

$$V_{sq}^{\mp}(Y) = \frac{V(\hat{Y}_1 \pm g\hat{Y}_2)}{V(\hat{Y}_{1,CS} + g\hat{Y}_{2,CS})} < 1$$

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G. Leuchs, et al. in *"Quantum Information Theory with Continuous Variables"* (2003).

Continuous Variable Entanglement

- EPR-entanglement

$$V_{cd}^{\pm}(X_1|X_2) = \frac{V(\hat{X}_1 \pm g\hat{X}_2)}{V(\hat{X}_{1,CS})} < 1,$$

$$V_{cd}^{\mp}(Y_1|Y_2) = \frac{V(\hat{Y}_1 \mp g\hat{Y}_2)}{V(\hat{Y}_{1,CS})} < 1$$

- QND-entanglement (obeys at least one of the inequalities)

$$V_{cd}^{\pm}(X_1|X_2) < 1,$$

$$V_{cd}^{\mp}(Y_1|Y_2) < 1,$$

$$V_{cd}^{\pm}(X_1|X_2)V_{cd}^{\mp}(Y_1|Y_2) < 1,$$

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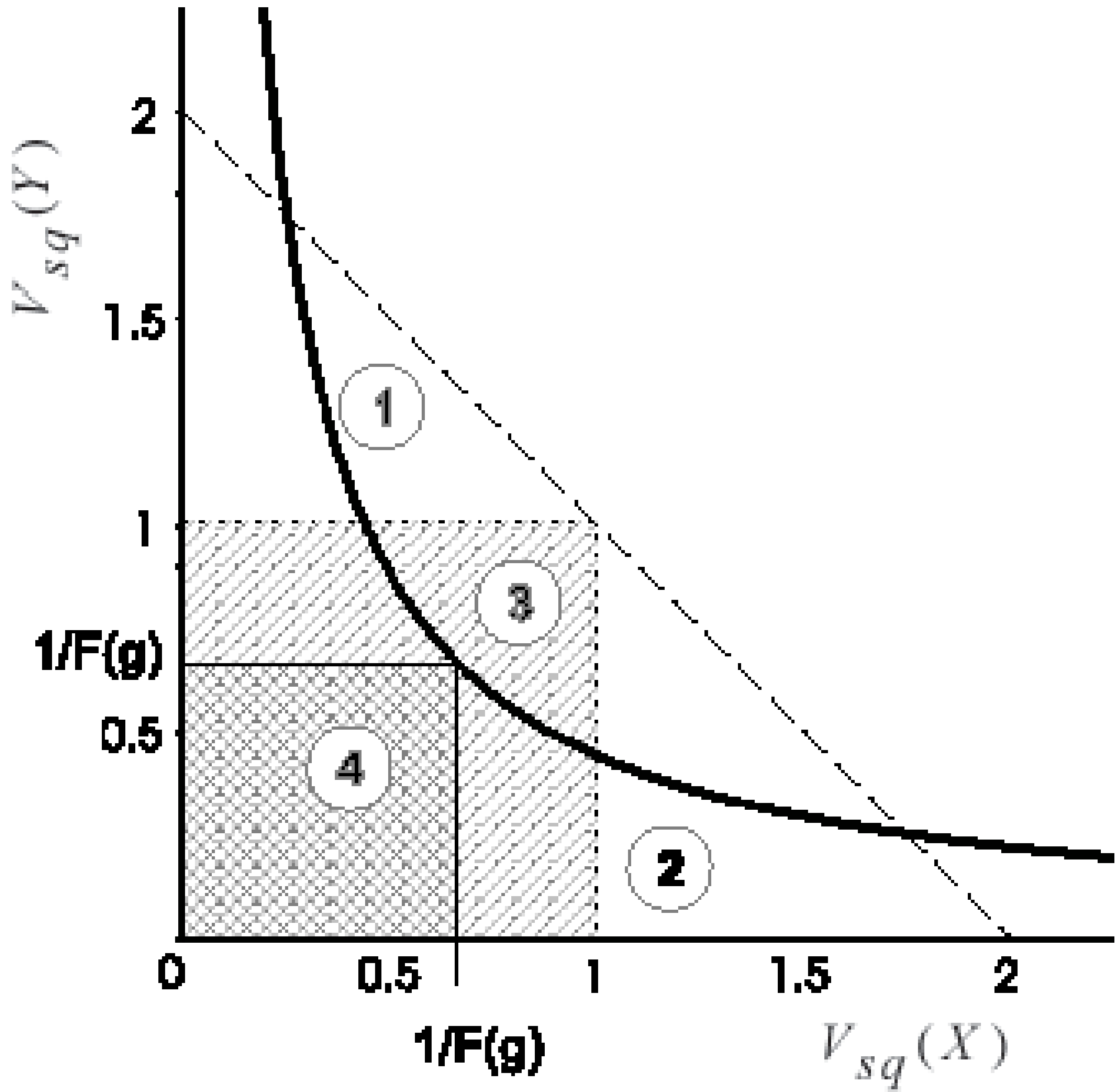
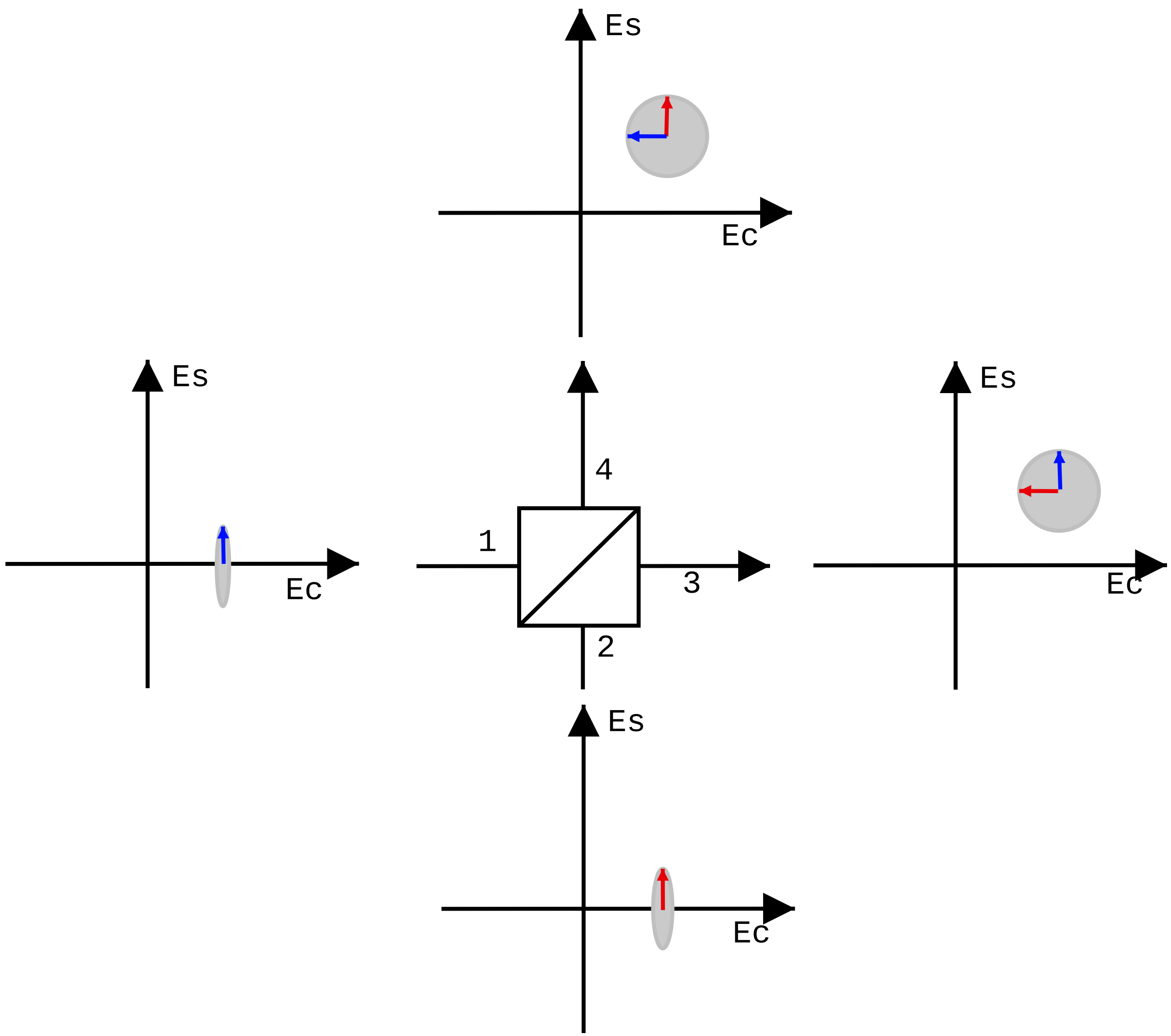
G. Leuchs, et al. in *"Quantum Information Theory with Continuous Variables"* (2003).

Continuous Variable Entanglement



Entanglement via Beamsplitter

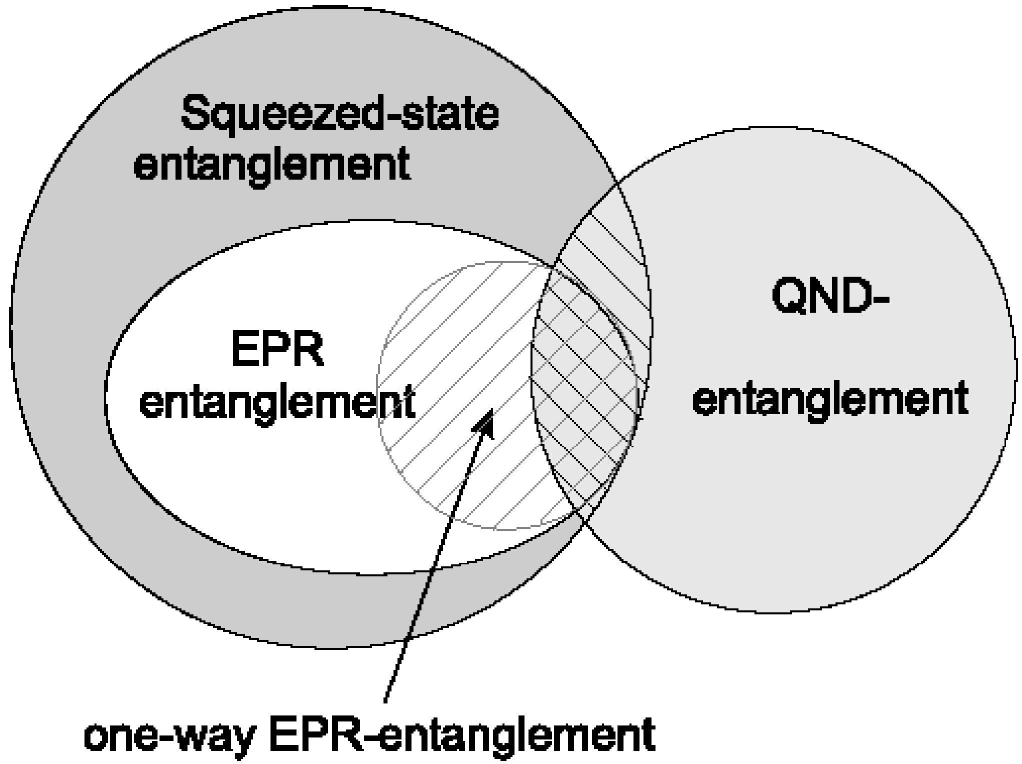


$$\delta\hat{n}_3 = -\delta\hat{n}_4, \delta\hat{\theta}_3 = \delta\hat{\theta}_4.$$

Continuous Variable Entanglement

Generating process	*Inter-channel coupling*	*Correlated variables*	*Correlation type*
OPO type II; Ou et al. [10].	non-linear	$\delta X_1 \propto \delta X_2$ $\delta Y_1 \propto -\delta Y_2$	EPR and/or SSE
OPO type I; Furusawa et al. [22].	linear	$\delta X_1 \propto \delta X_2$ $\delta Y_1 \propto -\delta Y_2$	EPR and/or SSE
Kerr nonlinearity in fibre; Silberhorn et al. [11].	linear	$\delta X_1 \propto -\delta X_2$ $\delta Y_1 \propto \delta Y_2$	EPR and/or SSE
QND (phase shift); Friberg et al. [26].	non-linear	$\Delta\phi_p \propto n_s$ $\Delta\phi_s \propto n_p$	QND; one-way EPR
QND (spectral shift); König et al. [14].	non-linear	$\Delta f_p \propto n_s$ ($\hat{p}_p \propto \hat{n}_s$) $\Delta f_s \propto n_p$ ($\hat{p}_s \propto \hat{n}_p$)	QND
QND (spectral filtering); König et al. [14].	non-linear	$\delta n_p \propto \delta n_s$	QND
QND (squeezed light beam splitter $V(X_p^{in}) < 1$); Bruckmeier et al. [34].	linear	$\hat{X}_p \propto \hat{X}_s$	QND

Continuous Variable Entanglement



one-way EPR-entanglement

$$V_{cd}(X_1|X_2) < 1 \quad \wedge \quad V_{cd}(Y_2|Y_1) < 1$$

or

$$V_{cd}(X_1|Y_2) < 1 \quad \wedge \quad V_{cd}(X_2|Y_1) < 1$$

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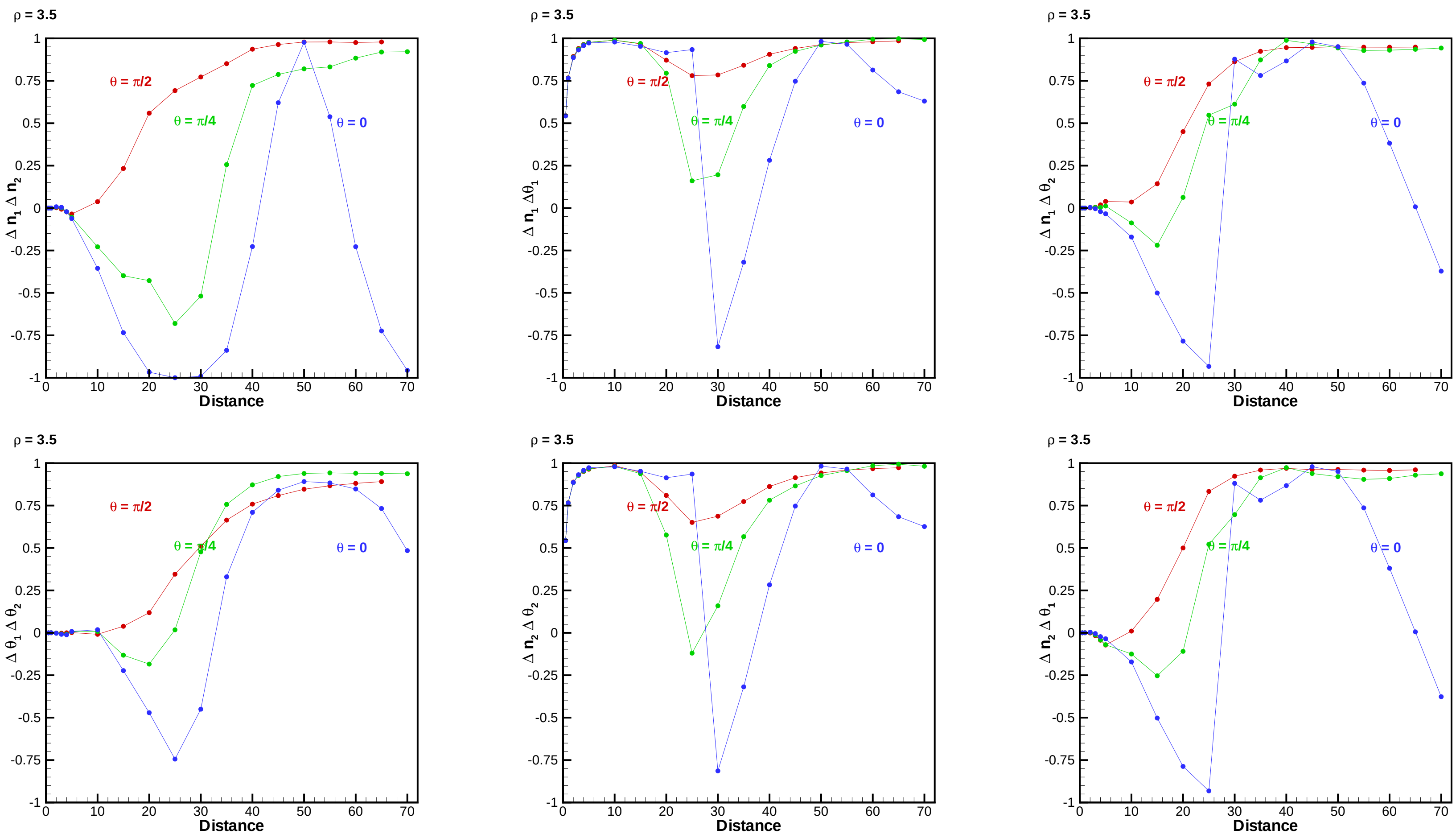
Quantum Correlations of Interacting Soliton Pairs

$\Delta n_1 \Delta n_2,$ $\Delta n_1 \Delta \theta_1,$ $\Delta n_1 \Delta \theta_2.$



$\Delta \theta_1 \Delta \theta_2,$ $\Delta n_2 \Delta \theta_2,$ $\Delta n_2 \Delta \theta_1.$

QND-Entanglement

- In our studies,

$$V^{\pm}_{cd}(X_1|X_2) < 1,$$

where $+$ $(-)$ sign depends on anti-correlated (correlated) photon-number correlations.

- We never have

$$V^{\pm}_{cd}(Y_1|Y_2) < 1, or, V^{\pm}_{cd}(X_1|Y_2) < 1,$$

for phase fluctuations are anti-squeezed.

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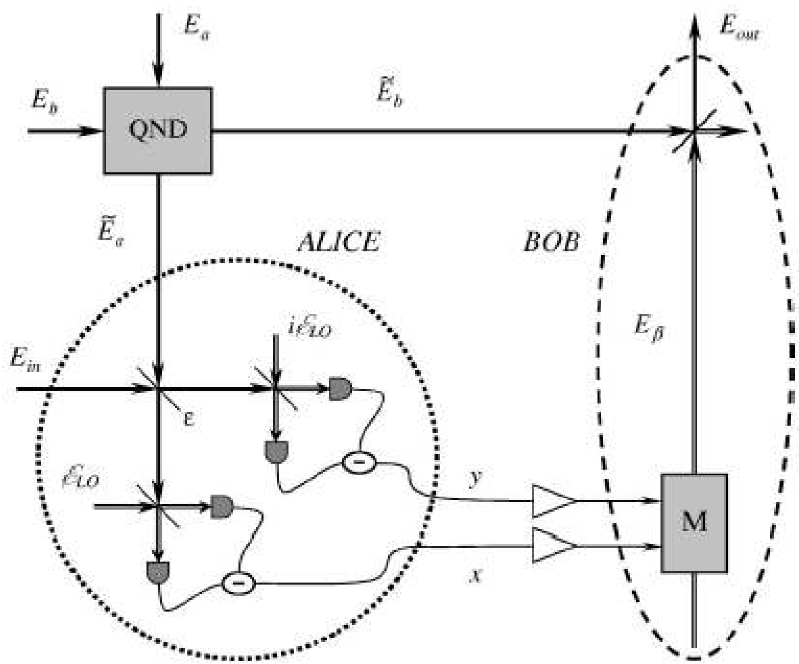
Future works

- Multipartite QND-entanglement in bit-parallel soliton systems?
- Quantum teleportation using QND



D. B. Horoshko and S. Ya. Kilin, *Phys. Rev. A* **61**, 032304 (2000).

Teleportation with Continuous Variables I

- Consider Alice wants to teleport $|\Phi\rangle_1$ to Bob.
  $|\Phi\rangle_1$ has a certain position x_1 and momentum p_1.
- Due to the Heisenberg uncertainty relation between x and p,
  Alice cannot measure both x_1 and p_1 with arbitrary precision.
- By using the EPR source shared by Alice and Bob, in which the entanglement is by

$$\begin{aligned} x_2 + x_3 &= 0 \\ p_2 - p_3 &= 0 \end{aligned}$$

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Teleportation with Continuous Variables II

- The properties of the individual particles, x_2, x_3, p_2, p_3 are completely undetermined.
- The operator $(x_2 + x_3)$ and $(p_2 + p_3)$ commute.
- Next Alice performs BSM on particles 1 and 2, and the measurement yields

$$
\begin{aligned}
x_1 + x_2 &= a \\
p_1 - p_2 &= b
\end{aligned}
$$

where a and b both are continuous real values.

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Teleportation with Continuous Variables III

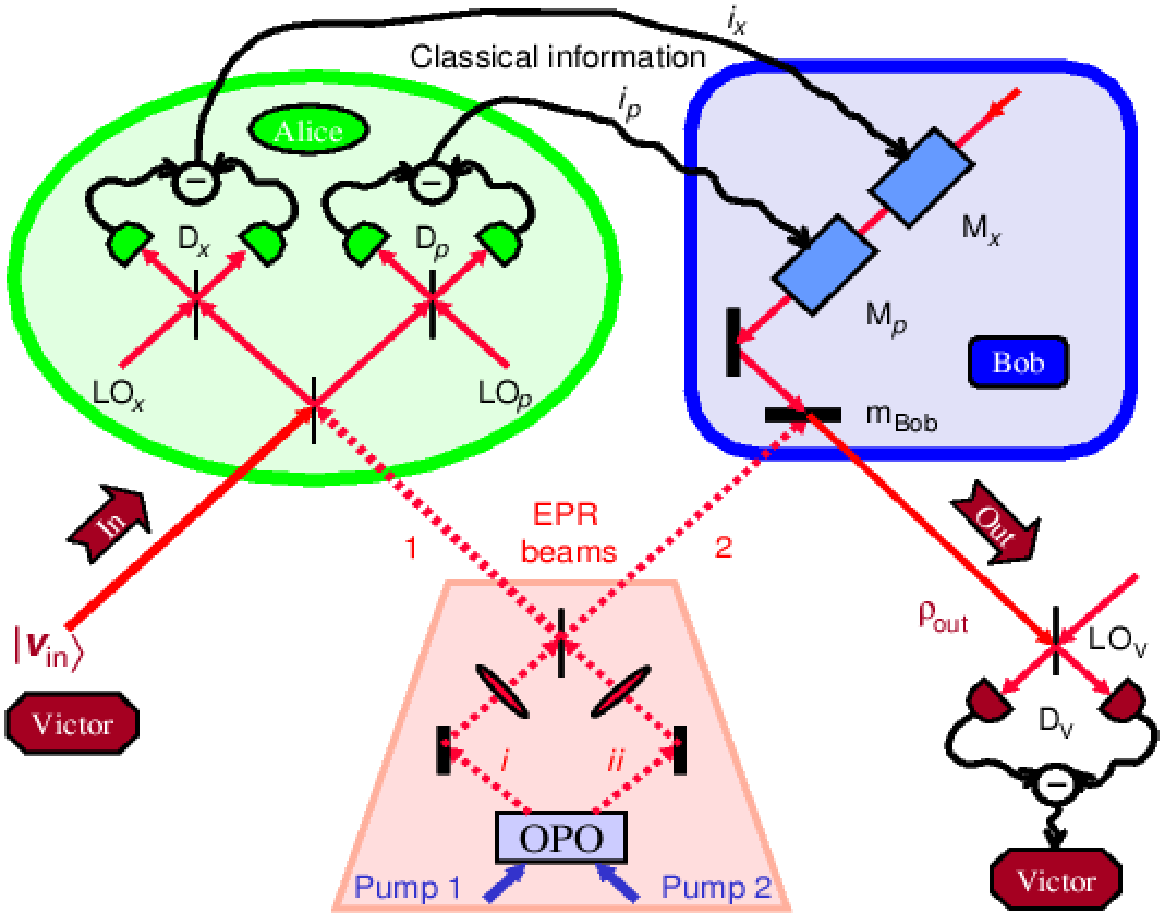
- The quantum state of Bob is

$$\begin{aligned} x_3 &= x_1 - a \\ p_3 &= p_1 - b \end{aligned}$$

- All alice has to do is to sent the results of her measurement via a classical channel.
- Then Bob just displaces the position and momentum of his particle by a and b.
- The final result is that Bob has particle 3 in the initial quantum state of particle 1.

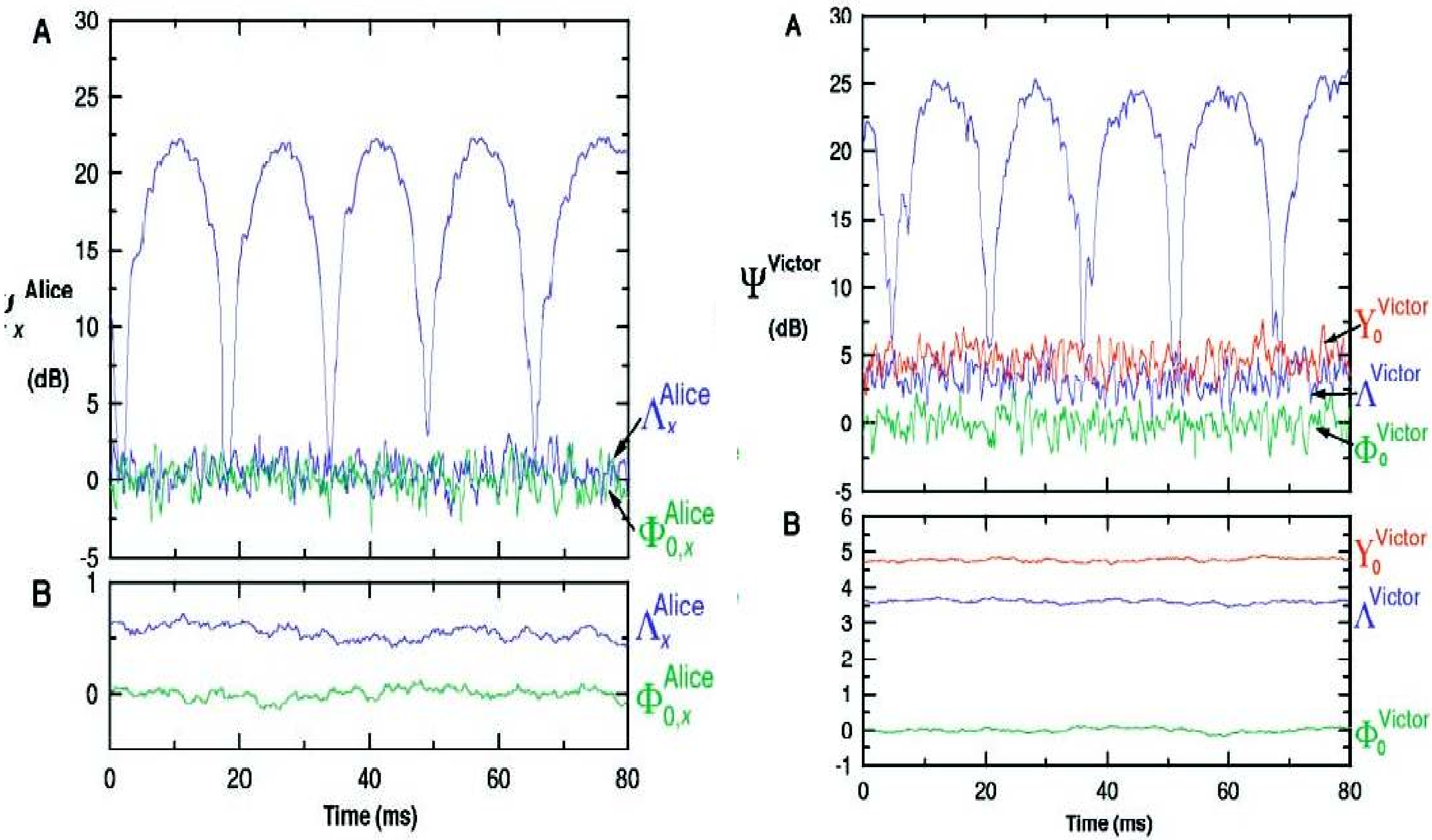
1: Z. Y. Ou, et al., *Phys. Rev. Lett.* **68**, 3663 (1992).
2: L. Vaidman, *Phys. Rev. A* **49**, 1473 (1994).
3: S. L. Braunstein and H. J. Kimble, *Phys. Rev. Lett.* **80**, 869 (1998).

Experimental CV teleportation I



A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, *Science* **282**, 706 (1998).

Experimental CV teleportation II



A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, *Science* **282**, 706 (1998).

Long-distance teleportation

1. quantum cryptography, runs under the lake between Nyon, about 23km north of Geneva, and the center of the city.
2. quantum teleportation, at telecommunication wavelengths and separated by 2 km.

I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, N. Gisin, *nature* **421**, 509 (2003).

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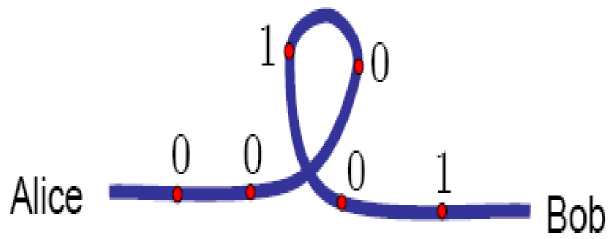
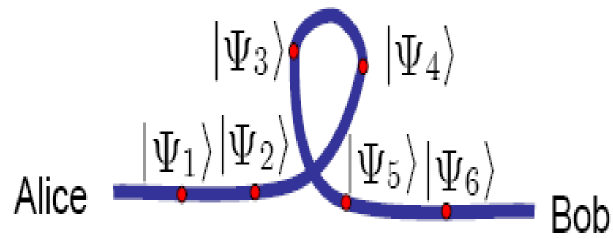
Classical and Quantum Communication

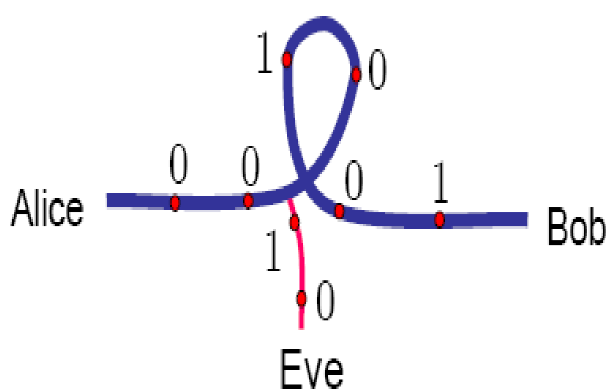
- classical communication

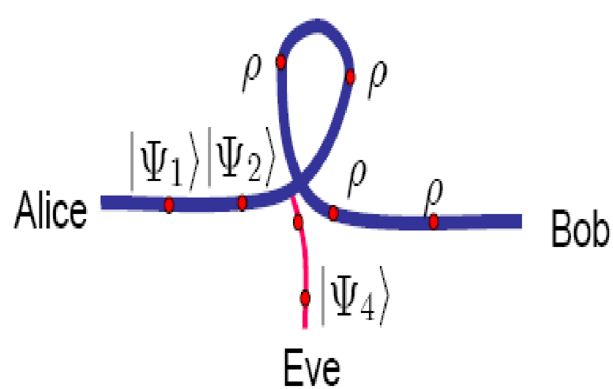


- quantum communication



✓ quantum networks
✓ cryptopgraphy





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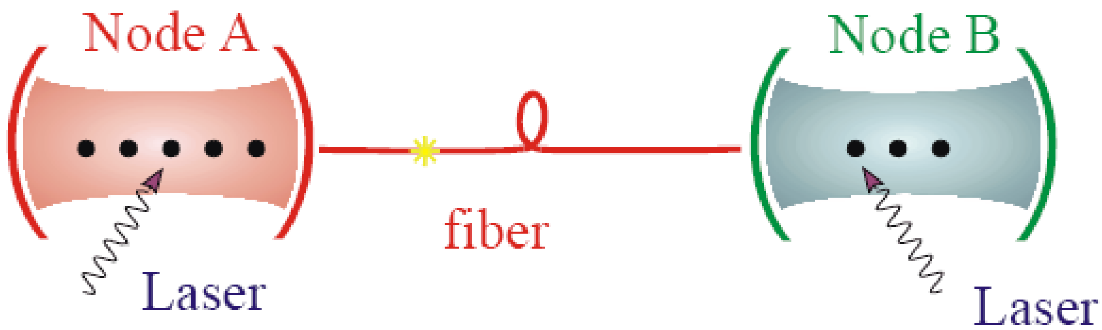
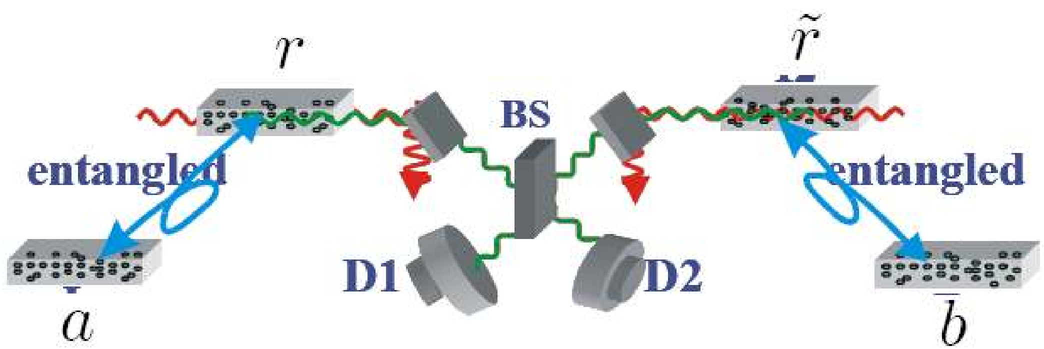
Quantum State Transfer



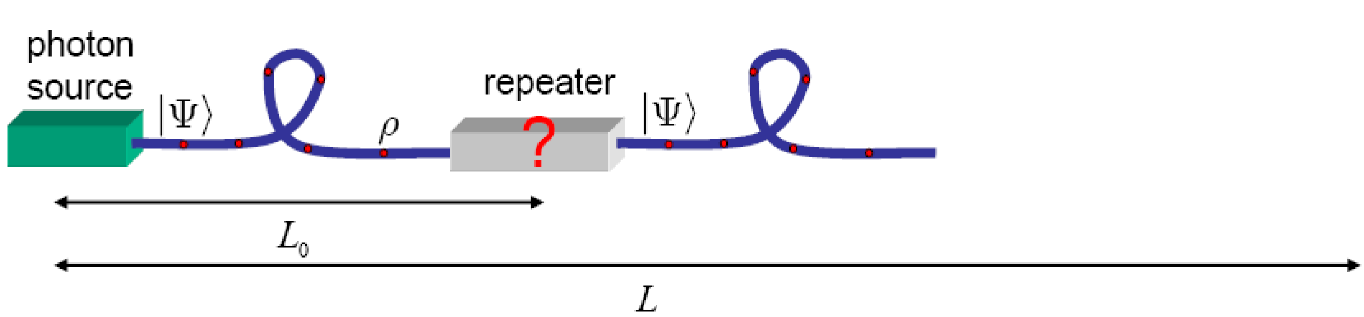
$|\phi\rangle$ $\xrightarrow{\text{transmit}}$

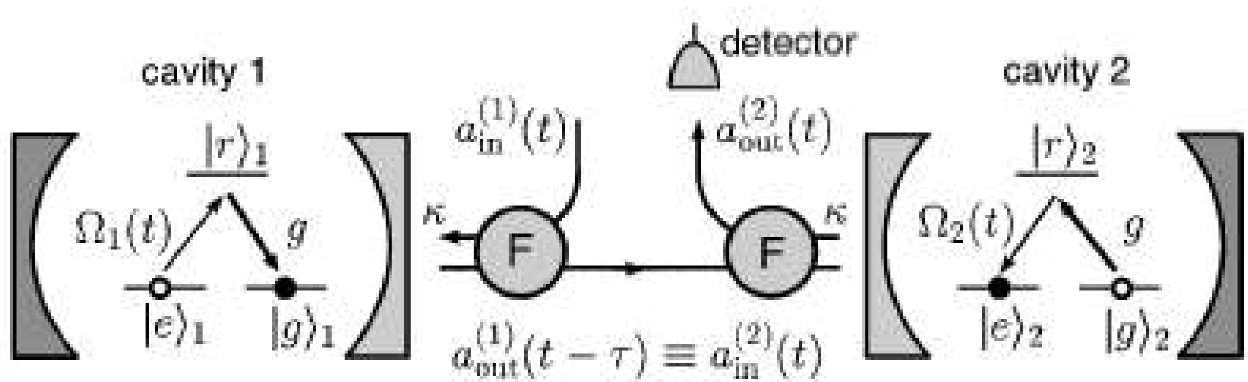
memory: atoms → databus: photons → memory: atoms



L.-M. Duan, M. D. Lukin, J. I. Cirac, P. Zoller, *nature* **414**, 413 (2001).

Quantum State Transfer as a Quantum Repeater





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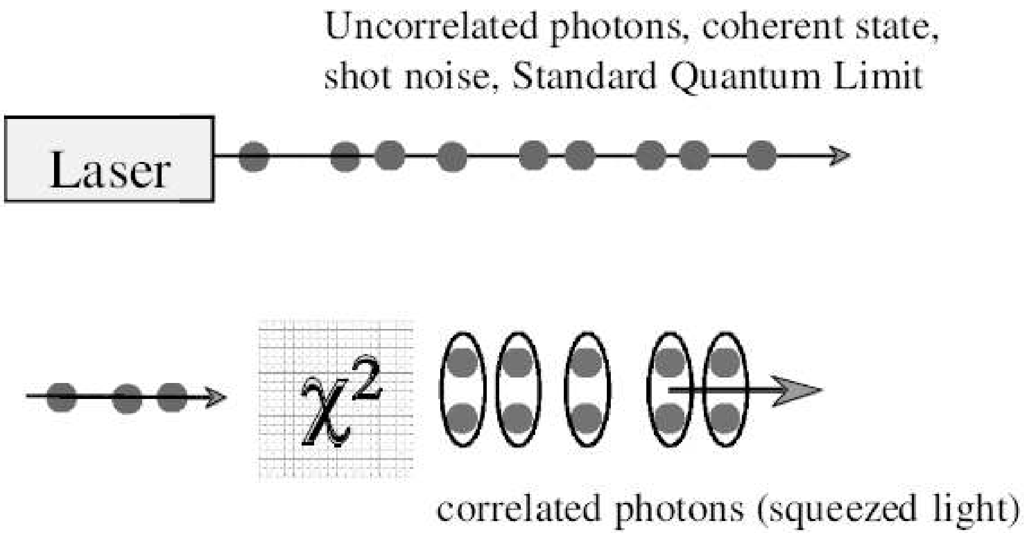
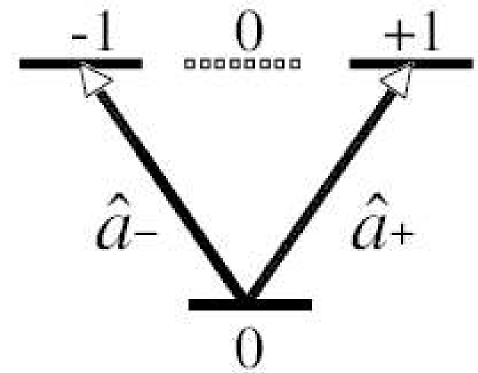
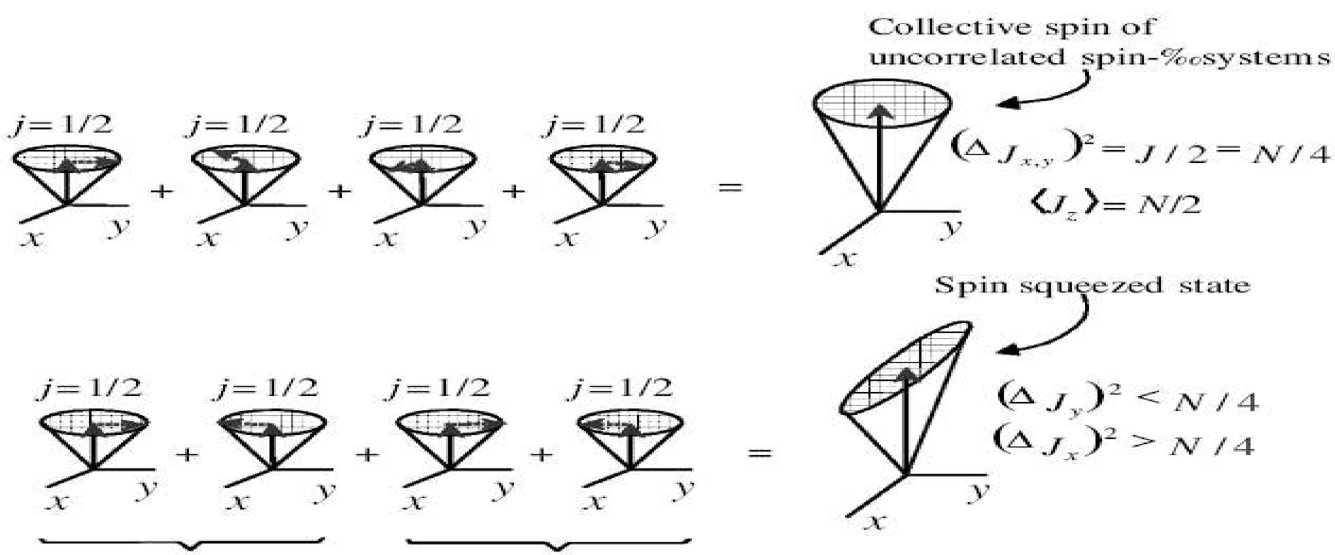
Quantum State Transfer with Spin of Atoms







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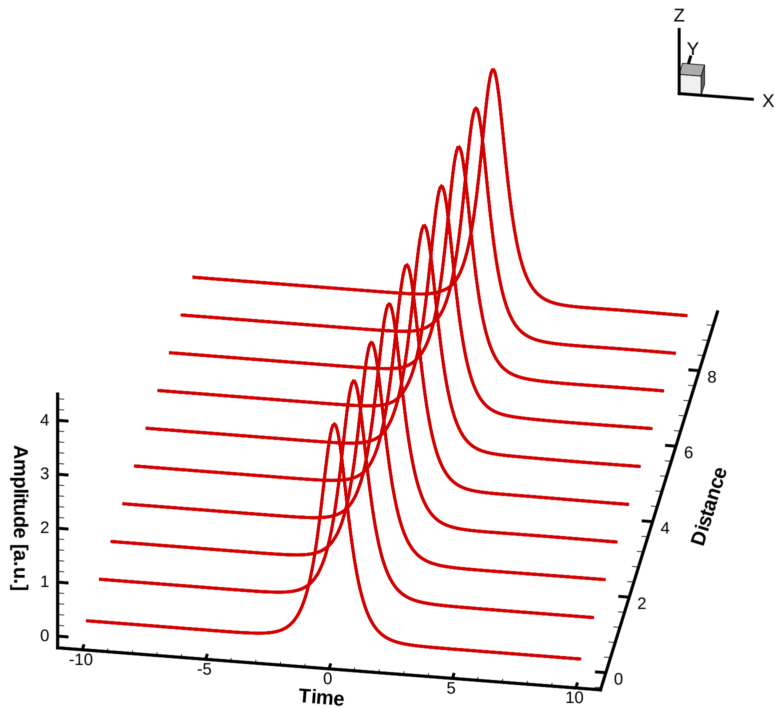
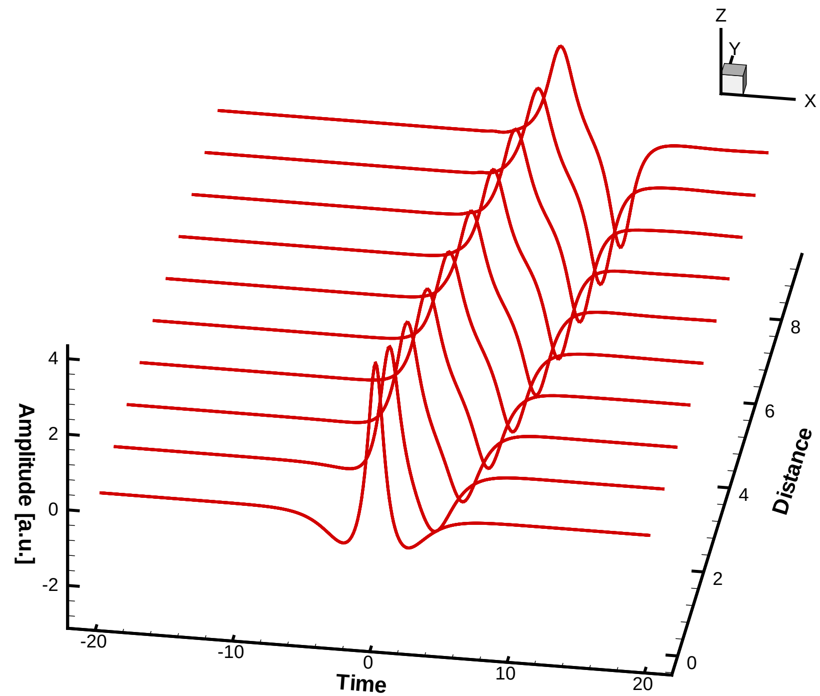
Solitons in SIT/EIT media





2π-pulse 0π-pulse

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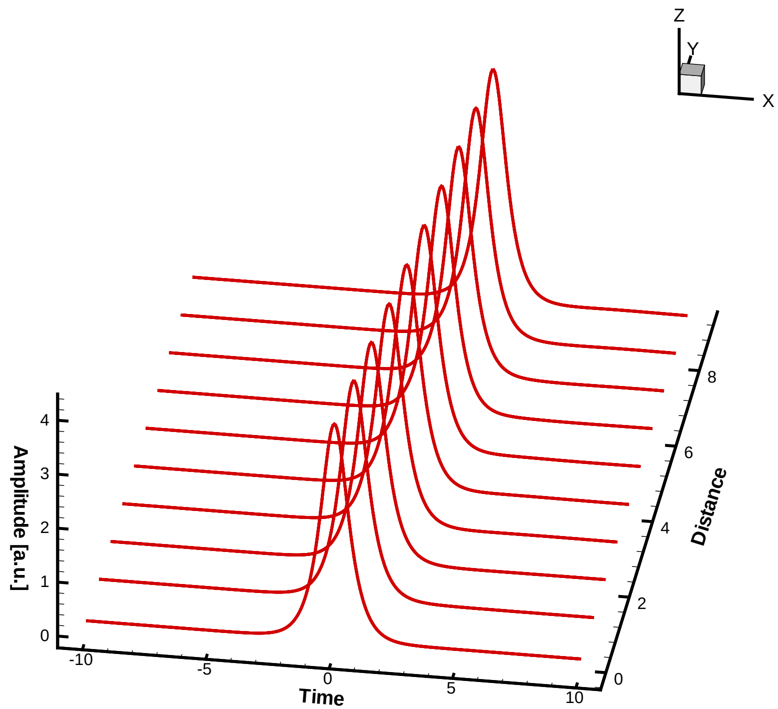
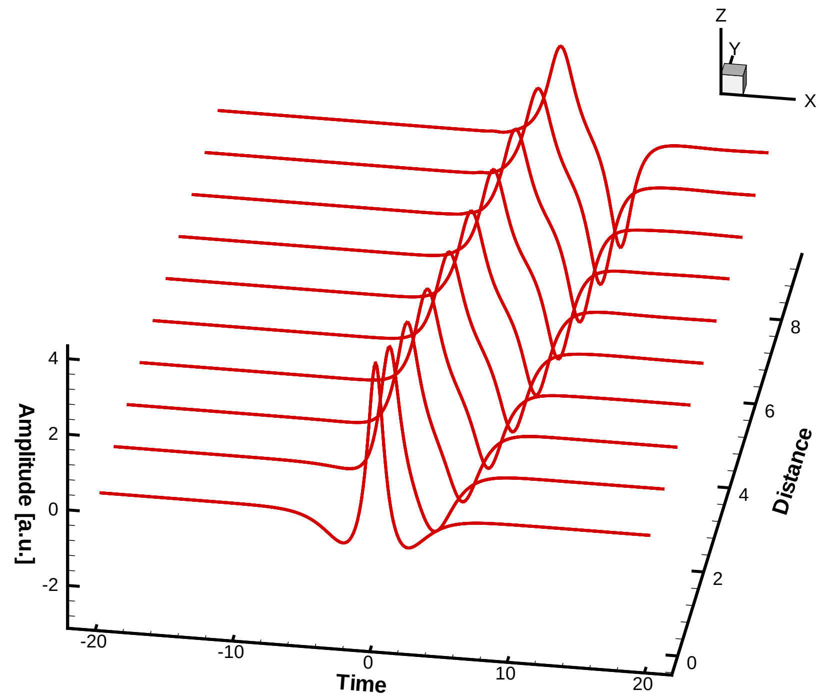
Solitons in SIT/EIT media





2π-pulse 0π-pulse

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