

Collusion and Renegotiation in Principal-Supervisor-Agent Models*

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Abstract

We aim at examining the interactions between collusion and renegotiation in a principal-supervisor-agent framework, in which the supervisor and agent can collude while the principal can subsequently renegotiate. Despite common sense and the findings in the literature that collusion and renegotiation are usually costly when considered separately, we find that they play a weakly positive role when considered jointly. The proposed framework unifies and reexamines the claims of some important studies in the literature, suggesting that, in organizational design, the issues of collusion and renegotiation should be studied in tandem.

1 Introduction

Side contracting among agents, known as collusion, and side contracting including the principal, known as renegotiation, coexist in organizations. For example, the union can be viewed as a form of collusion among workers, and

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firms often renegotiate compensation and benefits plans with their workers when facing financial problems. While there are many studies that consider collusion or renegotiation separately, very few take both into account. Therefore, the interactions that occur between collusion and renegotiation remain mostly unstudied. In particular, the following questions deserve careful examination: Do collusion and renegotiation strengthen or weaken each other? Do collusion, if observable by the principal, convey the agent's private information that the principal can benefit from? If so, how does this added information affect the subsequent renegotiation? If a principal cannot prohibit collusion and renegotiation, to what should he pay attention in order to better his benefits? We seek to shed some light on these questions in this paper.

We restrict our attention to the moral hazard problem; in this case, the most relevant type of collusion among agents is mutual insurance. Since the main issue of the contract design problem is to impose the right amount of risk upon an agent for him to work hard, the agent has an incentive to reduce the amount of risk that he is bearing through, say, side contracting with a third party. To consider the simplest possible type of risk sharing, we assume there is a risk-averse person, called the supervisor, who can observe the agent's effort. While information is symmetric between the agent and the supervisor, the agent's effort is nonetheless unobservable to and noncontractible for the principal. The whole contracting game naturally starts with a grand contract offered by the principal to both the agent and the supervisor and ends with a possible renegotiation contract offered by the principal to the supervisor. In the intervening time, the agent exerts an effort and may collude with the supervisor through side contracting.

Throughout this paper, we assume that effort is not contractible be it in the grand contract, a side contract, or a renegotiation contract. This is a standard assumption in agency models. On the other hand, we assume that the side contract between the agent and the supervisor is observable and can be used by the principal to infer the effort chosen and to structure an optimal renegotiation contract. Put another way, we are interested in environments in which the principal can monitor collusive activities but not the agent's effort.

In the first half of the paper, we study a model in which collusion precedes effort exertion. We find that when renegotiation is forbidden, collusion makes things worse. To implement an effort other than the least costly one, the minimized cost always strictly exceeds the second best cost (as in Varian

1990), which is the cost that implements the effort when effort choice is unobservable by the principal and the three parties can commit to the grand contract (as in Felli and Villas-Boas 2000). However, when both collusion and renegotiation are present, the cost goes down to the second-best cost.

In the second half of the paper, we study a variant in which collusion succeeds effort exertion. We find that when renegotiation is forbidden, collusion enhances efficiency. To implement an effort, the minimized cost equals to the first-best cost, which is the cost that implements the effort when effort choice is verifiable by the principal and the three parties can commit to the grand contract. Interestingly, even if the principal is unable to commit (i.e., renegotiation cannot be forbidden), we find that the cost remains the same (comparable to Ishiguro and Itoh 2001).

To summarize, our first main result is that the order between collusion and effort exertion plays an important role in determining efficiency. Our second main result is that the principal is weakly better off with both collusion and renegotiation than with neither of them; in other words, collusion and renegotiation are weakly countervailing. This result is in sharp contrast with the negative effects of collusion and renegotiation found in most previous papers when collusion and renegotiation are studied separately.¹

Related literature The role of collusion in organizations under adverse selection is highlighted and summarized by Tirole (1986, 1992). The related literature on collusion under the moral hazard focuses on the efficiency implications of different types of collusion technologies. For example, collusion is efficiency worsening if it involves only mutual insurance (as in Varian), but becomes efficiency enhancing if it involves also effort coordination (Itoh 1993). However, most of the studies in this literature do not consider renegotiation, except for, notably, a recent study by Felli and Villas-Boas. Their model resembles the first model in this paper except that in their model, the effort exertion takes place at the very end of the game, after the collusion and the renegotiation contracts are signed. Because of this, the principal's problem in deciding a renegotiation contract is much easier as he is not required to form any beliefs about the effort exerted since it has yet to be exerted. In our model, effort has been determined before the renegotiation is started;

¹According to Hermalin and Katz (1991), the first-best cost can be achieved with renegotiation when the principal can observe the agent's effort. One exception in the literature is found by Ishiguro and Itoh (2001), which we discuss in detail below.

treatment of the belief is more tricky and it is more challenging to solve for the equilibrium.

Similarly, the literature on renegotiation rarely considers collusion. The part of this literature that is closest to our paper focuses on the question whether renegotiation is harmful or beneficial to the principal. There what plays a crucial role is how much information the principal has regarding the agent's effort. Specifically, the principal is always worse off with renegotiation if he does not observe the agent's effort (Fudenberg and Tirole 1990) but can implement at the first-best cost with renegotiation if he does observe it (Hermalin and Katz 1991). Part of our result is obtained by applying a central argument of a recent paper by Ishiguro and Itoh (2001), who extend the literature by allowing multiple agents and show that the first-best cost can be achieved under what they refer to as 'decentralized renegotiation.'^{2,3}

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the game in which collusion precedes effort exertion, while Section 4 studies the game in which collusion succeeds effort exertion. Section 5 concludes.

2 Model

A risk-neutral principal hires an agent and a supervisor. The agent has a utility function, $U(w) - G(e)$, where w is his income and e his effort exerted, and the supervisor, who is not required to exert any effort, has a utility function, $V(s)$, where s is her income. Assume that $U(w)$ and $V(s)$ are both strictly increasing, strictly concave, unbounded functions so that their inverse functions $w(\cdot) \equiv U^{-1}(\cdot)$ and $s(\cdot) \equiv V^{-1}(\cdot)$ are well defined. $G(e)$, the cost function, is increasing.

²In an unpublished paper, Ishiguro and Itoh (1996) study a model similar to our second model. However, they either assume the risk-neutrality of one agent or place other restrictions on the distribution functions or the utility functions.

³There is a relatively large literature on the optimal design of organizational forms. One question raised in this literature is whether there should be a two-tier organization or a three-tier organization. Another is who—the principal or the supervisor—should be responsible for signing the contract with the agent for a three-tier organization (see, for instance, Laffont and Martimort 2000, Macho-Stadler and Perez-Ca Vafai 1998, and Melumad, Mookherjee, and Reichelstein 1995). These issues are not addressed here.

The agent's effort, e , is non-observable to the principal and is chosen from $E = \{e_1, \dots, e_N\}$, where $e_1 < e_2 < \dots < e_N$. Let $Y = \{y_1, \dots, y_K\}$ be the set of possible output levels and $p_k(e)$ be the probability that y_k will result given effort e . We assume that $p_k(e)$ is strictly positive for all k and for all e so that in observing the output level the principal cannot exclude any effort from having been chosen. Alternatively, we may write $p_y(e)$ to represent the probability that outcome $y \in Y$ will result given effort e .

One can easily justify the supervisor's role even though she is not required to exert any effort. Suppose that, in the absence of the supervisor, the agent, say, can abscond to play tennis with friends with impunity (i.e., a very undesirable effort not included in E now becomes feasible); in the presence of the supervisor, however, such overt shirking is not allowed and the agent has to come to the workplace.⁴

The principal can design a grand contract that specifies both the agent's wage and the supervisor's salary contingent on the realization of the output level, which is verifiable. Hence, a grand contract can be written as (\mathbf{w}, \mathbf{s}) where $\mathbf{w} = (w_1, \dots, w_K)$ and $\mathbf{s} = (s_1, \dots, s_K)$ are the agent and the supervisor's wage menus, respectively. Given our assumption regarding utility functions, a grand contract can be represented either in money (\mathbf{w}, \mathbf{s}) or in utilities (\mathbf{u}, \mathbf{v}) , where $u_k = U(w_k)$ and $v_k = V(s_k)$, $k = 1, \dots, K$. Denote \mathfrak{C} as the set of all possible grand contracts in terms of utility.

First-best cost We define the first-best problem to be the one in which the principal can verify and contract upon the effort *while still retaining the supervisor*. Explicitly, the first-best grand contract $(\mathbf{u}^{FB}, \mathbf{v}^{FB})$ that implements effort e solves

$$\min_{\mathbf{u}, \mathbf{v}} \sum_{y \in Y} p_y(e) [w(u_y) + s(v_y)] \quad (1)$$

subject to the following individual rationality (IR) constraints of the agent and supervisor:

$$\sum_{y \in Y} p_y(e) u_y - G(e) \geq U_0 \quad (2)$$

⁴Felli and Villas-Boas give a slightly more active role to the supervisor. They assume that the agent's productivity depends on his type, which is observed by the supervisor. Given that the agent's type is "hard" information (cannot be falsified) and the supervisor always observes the true type (so that she cannot lie about not having observed it), the principal benefits from hiring such a supervisor when her reservation wage is not too high.

and

$$\sum_{y \in Y} p_y(e) v_y \geq V_0. \quad (3)$$

Clearly, the first-best cost of implementing any given effort e is

$$C_{FB}(e) \equiv w(U_0 + G(e)) + s(V_0).$$

where $w(U_0 + G(e))$ is the payment to the agent and $s(V_0)$ is the payment to supervisor, both of which are invariant across output levels.

Second-best cost We define the second best problem to be the one in which the agent's effort is unobservable to the principal while the three parties can commit to a grand contract (so that neither collusion and renegotiation can occur). In this case, the optimal contract to implement effort e , $(\mathbf{u}^{SB}, \mathbf{v}^{SB})$, solves (1) subject to the two IR constraints, (2) and (3), and the incentive compatibility (IC) constraint of the agent, i.e.,

$$\sum_{y \in Y} p_y(e) u_y - G(e) \geq \sum_{y \in Y} p_y(e') u_y - G(e') \text{ for } e' \neq e. \quad (4)$$

We assume that the problem has a solution for each e ; in other words, we assume that every e is implementable under commitment. Hemalin and Katz provide necessary and sufficient conditions for an effort to be implementable under commitment. While there is risk involved in \mathbf{u}^{SB} to induce the agent to exert effort, \mathbf{v}^{SB} simply prescribes a fixed payment of V_0 at every output level. Define the second-best cost of implementing effort e as

$$C_{SB}(e) \equiv \sum_{y \in Y} p_y(e) [w(u_y^{SB}) + s(V_0)].$$

It is clear that unless e is the least costly effort, $C_{SB}(e) > C_{FB}(e)$. We assume that \mathbf{u}^{SB} is unique for each e .

In the first half of the paper, we are interested in the following game.

1. The principal offers a take-it-or-leave-it (henceforth TIOLI) grand contract to the agent and the supervisor.⁵

⁵There are models in which at the outset the principal contracts only with the supervisor while letting the supervisor contract with her subordinate. That approach is examined by, for instance, Macho-Stadler and Perez-Castrillo (1998) in a moral hazard framework and Faure-Grimaud, Laffont, and Martimort (2003) in an adverse selection one.

2. The agent and the supervisor decide simultaneously and independently to accept or reject the grand contract. If both accept the contract, the game goes to the third stage. Otherwise, the game ends and the principal, the agent, and the supervisor receive their respective reservation values, P_0 , U_0 , and V_0 .
3. (a) The agent offers to the supervisor a TIOLI side contract, also known as collusion contract, for her to decide whether or not to accept it. If the collusion contract is accepted, it becomes verifiable between the supervisor and the agent and also observable by the principal. Otherwise, the principal observes only the absence of an accepted collusion contract, but not the content of the rejected contract itself. (b) The agent chooses the effort to exert, which is observable by the supervisor.
4. The principal offers a TIOLI renegotiation contract to the supervisor. If it is accepted, the renegotiation contract replaces the part of the grand contract that *concerns the supervisor*; otherwise, the entire grand contract is in force.
5. Production takes place and payments are made according to all contracts in force.

Some explanations are in order here. Firstly, we assume that the collusion contract is made contingent on output, but not on the amount of effort chosen to exert. This is in line with the assumption that effort is not verifiable and hence no contract can be made contingent on the effort. Secondly, we assume that the collusion contract is observable by the principal. This is consistent with the assumption that the collusion contract is enforceable by the court. Thirdly, though similar to one formulation studied by Felli and Villas-Boas,⁶ our game differs in the timing of the effort exertion. In their work, it takes place at the very end, after the renegotiation stage.

In the second half of this paper, we study the same game except that we reverse stage 3a and stage 3b. In other words, the two games to be studied differ only in the timing of the collusion contracting and effort exertion. Depending on whether the effort exertion occurs before or after the collusion contracting, the efficiency implications turn out to differ greatly, contrasting

⁶Felli and Villas-Boas also study the environment in which collusion contracts are non-observable. The possibility is not pursued here.

the insights between Felli and Villas-Boas on one hand and Ishiguro and Itoh (2001) on the other.

As our main goal is to understand the impact of renegotiation on collusion, we will analyze the case with only collusion as well as the case with both collusion and renegotiation. The proper equilibrium concepts of these two cases are the subgame perfect equilibrium (SPE) and the perfect Bayesian equilibrium (PBE), respectively, for the following reason. For the former case, in the game tree no players have an information set containing multiple decision nodes; for the latter case, the principal needs to renegotiate without knowing the effort of the agent that has been exerted.

3 Collusion Precedes Effort Choice

3.1 Collusion only

We now consider the agent's problem when he needs to propose a collusion contract to the supervisor before choosing effort. We assume that each collusion contract specifies the payment transferred from the agent to the supervisor so that no money is wasted. Instead of creating notation for the collusion contract itself, we use $(\mathbf{u}^C, \mathbf{v}^C)$ to represent the *grand-cum-collusion* contract so that the supervisor's income is $s(v_y^C)$ and the agent's income $w(u_y^C) \equiv w(u_y) + s(v_y) - s(v_y^C)$ for each y , where (\mathbf{u}, \mathbf{v}) is the grand contract. Note that, with this interpretation, no collusion contract signed means $(\mathbf{u}^C, \mathbf{v}^C) = (\mathbf{u}, \mathbf{v})$. Provided that no confusion is made, we will refer to $(\mathbf{u}^C, \mathbf{v}^C)$ simply as a collusion contract, rather than the more cumbersome name of the grand-cum-collusion contract. The set of all such possible contracts is simply \mathfrak{C} .

Given any grand contract (\mathbf{u}, \mathbf{v}) , the agent's program, denoted by $P^1(\mathbf{u}, \mathbf{v})$, is as follows:

$$\max_{e, \mathbf{u}^C, \mathbf{v}^C} \sum_{y \in Y} p_y(e) U(w(u_y) + s(v_y) - s(v_y^C)) - G(e) \quad (5)$$

subject to the agent's IC constraint,

$$\begin{aligned} & \sum_{y \in Y} p_y(e) U(w(u_y) + s(v_y) - s(v_y^C)) - G(e) \\ & \geq \sum_{y \in Y} p_y(e') U(w(u_y) + s(v_y) - s(v_y^C)) - G(e') \text{ for } e' \neq e \end{aligned} \quad (6)$$

and the supervisor's IR constraint,

$$\sum_{y \in Y} p_y(e) v_y^C \geq V(\mathbf{u}, \mathbf{v}), \quad (7)$$

where $V(\mathbf{u}, \mathbf{v})$ is the supervisor's expected utility in the absence of a collusion contract, and should equal V_0 when (\mathbf{u}, \mathbf{v}) is optimally chosen by the principal. Equation (7) states that by correctly foreseeing the agent's effort, the supervisor finds the collusion contract acceptable. Equation (6) states that given the collusion contract, the effort, e , that the supervisor anticipates is indeed optimal for the agent; finally, according to (5), we require that the 3-tuple $(e, \mathbf{u}^C, \mathbf{v}^C)$ indeed maximizes the agent's expected utility among all $(e, \mathbf{u}^C, \mathbf{v}^C)$'s that satisfy the agent's IC and the supervisor's IR constraints.

Remark 1 *The choice of \mathbf{u}^C does not appear explicitly in the objective function in (5) because it is determined from (\mathbf{u}, \mathbf{v}) and \mathbf{v}^C by $u_y^C = U(w(u_y) + s(v_y) - s(v_y^C))$ for each y .*

Let the set of maximizers to the program be denoted by $B^1(\mathbf{u}, \mathbf{v}) \subset E \times \mathfrak{C}$, where, as we recall, E is the set of feasible efforts and \mathfrak{C} is the set of all possible (grand-cum-)collusion contracts.

We now move back to the very beginning when the principal chooses the grand contract with which to implement effort e , anticipating subsequent optimal collusion and effort choice. His problem is

$$\min_{\mathbf{u}^G, \mathbf{v}^G} \sum p_y(e)(w(u_y) + s(u_y))$$

subject to $(e, \mathbf{u}^C, \mathbf{v}^C) \in B^1(\mathbf{u}, \mathbf{v})$, $\sum_y p_y(e) u_y^C \geq U_0 + G(e)$, and $\sum_{y \in Y} p_y(e) v_y^C \geq V_0$.

The first constraint ensures that e and $(\mathbf{u}^C, \mathbf{v}^C)$ will indeed be chosen in equilibrium. The last two constraints are versions of the agent's and the supervisor's IR constraints (2) and (3). They state that, by anticipating effort e and collusion contract $(\mathbf{u}^C, \mathbf{v}^C)$, both the agent and the supervisor find the grand contract acceptable. Since the supervisor does not have bargaining power in collusion, in order for her to accept the grand contract as well as for the grand contract to be optimal to the principal, it must be that $V(\mathbf{u}, \mathbf{v}) = V_0$.

Lemma 1 (*The collusion-proofness principle.*) Suppose collusion precedes the effort choice and no renegotiation is allowed. Let (\mathbf{u}, \mathbf{v}) be any optimal grand contract under this environment that implements e . Let $(\mathbf{u}^C, \mathbf{v}^C)$ be the corresponding collusion contract along the equilibrium path. We argue that there exists a collusion-proof grand contract $(\mathbf{u}', \mathbf{v}')$ that is outcome-equivalent to (\mathbf{u}, \mathbf{v}) (i.e., the effort e is implementable and all players have the same expected utilities under the new grand contract as under the old one.) In particular, we argue that $(\mathbf{u}', \mathbf{v}') = (\mathbf{u}^C, \mathbf{v}^C)$.

Proof. We first note that the principal's total payment in each output level, y , under the new grand contract is the same as under the old one, i.e., $w(u'_y) + s(v'_y) = w(u_y^C) + s(v_y^C) = w(u_y) + s(v_y)$. Making use of this equality in (5), (6) and (7), we establish that the two programs for the agent, namely $P^1(\mathbf{u}', \mathbf{v}')$ and $P^1(\mathbf{u}, \mathbf{v})$, are identical and so are their solutions, $B^1(\mathbf{u}', \mathbf{v}') = B^1(\mathbf{u}, \mathbf{v})$. Therefore, given the new grand contract $(\mathbf{u}', \mathbf{v}')$, it is an equilibrium play that the agent proposes no collusion contracts and chooses e subsequently. In this case, all parties receive the same expected utilities as in the equilibrium that implements e under the old grand contract. Since the old grand contract is optimal, this new collusion-proof contract must also be optimal. ■

This collusion proofness principle implies that, without loss of generality, we can focus on the set of collusion-proof grand contracts when looking for the optimal grand contract.

Suppose the principal wants to implement effort e and (\mathbf{u}, \mathbf{v}) is an optimal collusion-proof grand contract that implements it. Then (\mathbf{u}, \mathbf{v}) solves the principal's program (1) subject to the agent's IC constraint (4) and the supervisor's IR constraint (3), *as well as the following collusion-proofness constraint*:

$$\frac{s'(v_y)}{w'(u_y)} \left\{ 1 + \sum_{\{l|e_l \neq e\}} \lambda_l \left[1 - \frac{p_y(e_l)}{p_y(e)} \right] \right\} = \gamma \text{ for all } y, \quad (8)$$

where γ and λ_l 's are the multipliers for the supervisor's IR constraint (7) and the agent's IC constraint (6), respectively (the agent's IR constraint is automatically satisfied). Compared with the second-best problem, now that there is exactly one more constraint to satisfy, the minimized cost that implements e must not be smaller. This result was first proposed by Varian (1990) and summarized in the following proposition.

Proposition 1 *Suppose collusion precedes effort exertion. The cost of implementing any given effort $e > e_1$ under the collusion-only environment is strictly higher than the corresponding second-best cost.*

3.2 Collusion and Renegotiation

3.2.1 Renegotiation

In the last subsection, we found that the cost of implementing any given effort (except for e_1) under collusion is strictly greater than the corresponding second-best cost. Here, we argue that introduction of renegotiation can mitigate the problem so much that the cost is brought down to the second-best cost.

After observing the collusion contract $(\mathbf{u}^C, \mathbf{v}^C)$ and effort exertion, the principal forms his beliefs about the actual effort chosen. His beliefs are captured by a belief function $\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C)$, which is a probability distribution over the set of feasible efforts, E . The formulation here allows for a mixed strategy used by the agent, even though in the equilibrium that we are going to examine no mixed strategy will be used on and off the equilibrium path. The optimal renegotiation contract is one that maximizes the principal's expected payoff given his beliefs; clearly, the renegotiation contract is deterministic, making a fixed payment to the supervisor in every output level. We use $R[\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C), e]$ to denote the supervisor's expected payoff from accepting the principal's optimal renegotiation contract when the principal holds belief function $\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C)$ and the actual effort is e .

3.2.2 Collusion

Consider an effort, e , which is implementable under a collusion-only environment and let (\mathbf{u}, \mathbf{v}) be the corresponding collusion-proof optimal contract. This means that $(e, \mathbf{u}, \mathbf{v})$ is the solution to the agent's problem, $P^1(\mathbf{u}, \mathbf{v})$, which is (5) subject to the agent's IC constraint (6) and the supervisor's IR constraint (7), which can be rewritten as:

$$\sum_{y \in Y} p_y(e) v_y^C \geq \sum_{y \in Y} p_y(e) v_y = V_0. \quad (9)$$

Now we consider the problem of implementing the same e under both collusion and renegotiation and consider the same grand contract, (\mathbf{u}, \mathbf{v}) .

We argue that there exists an equilibrium under which, given that this grand contract is offered and accepted, no collusion contract will be signed, the same e will be adopted, and the supervisor will accept from the principal a risk-free renegotiation contract that always ensures the supervisor a constant payment of $s(V_0)$. Note that given that no collusion contract is signed, e is indeed optimal for the agent. Moreover, given that the principal correctly believes that e has been chosen, the free risk renegotiation contract — paying $\sum_{y \in Y} p_y(e) v_y = V_0$ in each state — offered by the principal is indeed optimal for him and will be accepted by the supervisor. Hence, to show the existence of the equilibrium, it suffices to show that $(e, \mathbf{u}, \mathbf{v})$ is a solution to the agent's program given (\mathbf{u}, \mathbf{v}) , which is to solve (5) subject to the agent's IC constraint (6) and the following modified IR constraint of the supervisor,

$$\max \left\{ \sum_{y \in Y} p_y(e) v_y^C, R[\pi(\mathbf{u}^C, \mathbf{v}^C), e] \right\} \geq V_0. \quad (10)$$

where the left-hand side (right-hand side) is the supervisor's expected utility when a collusion contract is signed (is not signed). In case a collusion contract $(\mathbf{u}^C, \mathbf{v}^C) \neq (\mathbf{u}, \mathbf{v})$ is signed, her expected utility equals $\sum_{y \in Y} p_y(e) v_y^C$ if no renegotiation contract is accepted, and otherwise equals $R[\pi(\mathbf{u}^C, \mathbf{v}^C), e]$. In case no collusion contract is signed, her expected utility equals V_0 whether or not the optimal renegotiation contract is accepted.

Consider the following off-equilibrium belief by the principal: if any collusion contract $(\mathbf{u}^C, \mathbf{v}^C) \neq (\mathbf{u}, \mathbf{v})$ is signed, the effort chosen is:

$$\arg \min_{e'} \sum_{y \in Y} p_y(e') v_y^C.$$

Given his belief, the principal's optimal renegotiation contract is a risk-free contract that ensures the supervisor with her reservation utility, or the same amount of $\min_{e'} \sum_{y \in Y} p_y(e') v_y^C$ in each state. This implies that $R[\pi(\mathbf{u}^C, \mathbf{v}^C), e] \leq \sum_{y \in Y} p_y(e) v_y^C$, and that the supervisor's modified IR constraint (10) collapses back to (9). Furthermore, $(e, \mathbf{u}, \mathbf{v})$ — the solution to the agent's problem under the collusion only environment given the grand contract (\mathbf{u}, \mathbf{v}) — will continue to be a solution to this agent's problem under both collusion and renegotiation given the grand contract (\mathbf{u}, \mathbf{v}) .

Finally, we note that foreseeing the equilibrium play after the grand contract is accepted, both the agent and the supervisor find this grand contract acceptable because it ensures them with their reservation utilities.

Lemma 2 *Suppose collusion precedes effort exertion. Let (\mathbf{u}, \mathbf{v}) be the optimal grand contract that implements e under the collusion only environment, which is collusion-proof. Then, under the collusion and renegotiation environment, given (\mathbf{u}, \mathbf{v}) is offered, there exists an equilibrium under which the grand contract is accepted, no collusion contract is signed, e is chosen, and the supervisor accepts from the principal a renegotiation contract that makes a fixed payment of $s(V_0)$.*

The next thing to note is that since the renegotiation contract is risk-free for the supervisor, the principal is strictly better off under the collusion and renegotiation environment than under the collusion-only environment when (\mathbf{u}, \mathbf{v}) is used.

Proposition 2 *Suppose collusion precedes effort exertion. Suppose that $e > e_1$ is an effort implementable under the collusion-only environment. Then, the effort is also implementable under the collusion-cum-renegotiation environment. In addition, the cost is strictly lower than that under the collusion-only environment but is bound below by the second-best cost.*

To make a stronger point, consider the grand contract $(\mathbf{u}, \mathbf{v}) = (\mathbf{u}^{SB}, \mathbf{v})$ where \mathbf{u}^{SB} is the contract that implements e under the second-best cost and \mathbf{v} solves the collusion-proofness condition (8) and the supervisor's IR condition. Because of this grand contract, no collusion contract will be signed ((8) is satisfied) and e is optimal (e solves (8)). Then, e is chosen and the supervisor in fact receives a risk-free contract after renegotiation, the total cost incurred is equivalent to the cost implied by $(\mathbf{u}^{SB}, (V_0))$ when neither collusion nor renegotiation is allowed, which is simply the second-best cost. Lastly, we remark that no effort $e > e_1$ can be implemented at a cost lower than second

best cost because the agent's IC and both the agent and the supervisor's IR constraints have to be met. We summarize our result as follows.

Proposition 3 *Suppose that collusion precedes effort exertion. Suppose that $e > e_1$ is an effort implementable under the collusion-only environment. Then, the cost of implementing it under the collusion-cum-renegotiation environment equals the second-best cost.*

The role of collusion and the joint role of collusion and renegotiation found in this Section resemble those found in Felli and Villas-Boas. However,

our model is different in several aspects. Most importantly, in their model effort exertion takes place at the very end of the game, after collusion and renegotiation contracts are signed.⁷ Because of this, the principal's problem of choosing a renegotiation contract is much easier since he is not required to form any beliefs about the effort, which has yet to be determined. In our model, effort is chosen prior to renegotiation; treatment of the belief is more tricky and it is generally more difficult to solve for the equilibrium. Nonetheless, our results illustrate that the results on the effect of collusion and the joint effect of both collusion and renegotiation are robust in the timing of the effort choice so long as it takes place subsequent to the collusion contracting.

4 Collusion Succeeds Effort Choice

If collusion succeeds effort exertion, the effort which the calculation of the supervisor's expected payoff is based on will be the same whether or not the collusion contract is accepted. When the collusion takes the form of a TIOLI contract from the agent, the supervisor's reservation price in the collusion stage depends on the actual effort (instead of a constant). The collusion can help provide the right incentive to the agent if the grand contract is designed so that a constant-sum game is entailed between the agent and the supervisor and that the supervisor can obtain more if an undesirable effort is chosen.

4.1 Collusion only

Suppose the principal wants to implement effort $\bar{e} > e_1$. Let $C_{FB}(\bar{e})$ be the first-best cost that implements \bar{e} . Consider a grand contract (\mathbf{u}, \mathbf{v}) so that $w(u_y) + s(v_y) = C_{FB}(\bar{e})$ for all y ; that is, while the supervisor or the agent may face individual risk, there is no aggregate risk when their incomes are pooled together. Given such a grand contract, the agent's problem is to choose e and a collusion contract $(\mathbf{u}^C, \mathbf{v}^C)$ to maximize his expected utility

$$\max_{e, \mathbf{u}^C, \mathbf{v}^C} \sum_{y \in Y} p_y(e) U(C_{FB}(\bar{e}) - s(v_y^C)) - G(e) \quad (11)$$

⁷In addition, the effort space is continuous in their paper, while it is finite in this paper; the output space is binary in their paper, while it is finite but can consist of many levels here.

subject to the supervisor's IR constraint

$$\sum_{y \in Y} p_y(e) v_y^C \geq \sum_{y \in Y} p_y(e) v_y, \quad (12)$$

where the left-hand side (right-hand side) is supervisor's expected utility after the collusion contract is accepted (rejected). Note that the efforts on both sides are the same because the agent's effort has been exerted before any collusion contract is proposed. It is clear that the optimal solution is such that (12) is binding and that the optimal \mathbf{v}^C (and hence the optimal \mathbf{u}^C) is a constant contract. Call this program $P^2(\mathbf{u}, \mathbf{v})$ and let the set of maximizers to the program be denoted by $B^2(\mathbf{u}, \mathbf{v}) \subset E \times \mathfrak{C}$.

We now argue that, in order to implement \bar{e} , there does exist a grand contract (\mathbf{u}, \mathbf{v}) where $w(u_y) + s(v_y) = C_{FB}(\bar{e})$ for all y , so that there exists a subgame perfect equilibrium with the following two properties. (i) Given that the grand contract is accepted, \bar{e} is indeed chosen, i.e., there exists a collusion contract, denoted by $(\bar{\mathbf{u}}^C, \bar{\mathbf{v}}^C)$, so that $(\bar{e}, \bar{\mathbf{u}}^C, \bar{\mathbf{v}}^C) \in B^2(\mathbf{u}, \mathbf{v})$. (ii) The grand contract is accepted by both the agent and the supervisor at the outset, i.e., $\sum_{y \in Y} p_y(e) v_y \geq V_0$ and $\sum_y p_y(e) u_y^C \geq U^0 + G(e)$.

In particular, we argue that the grand contract takes the following form

$$v_y = -f_y + g + V_0 \quad (13)$$

where $g = \sum_y p_y(\bar{e}) f_y$ and

$$u_y = U(w_y) \text{ where } w_y = C_{FB}(\bar{e}) - s(v_y). \quad (14)$$

Later, we show how f_y can be found. This grand contract has an interesting property: The supervisor's expected utility from the grand contract is simply V^0 if \bar{e} is indeed chosen, and in case a smaller effort is chosen ($e < \bar{e}$) it is strictly greater, so much so that the agent decides to stick to \bar{e} rather than the smaller effort.

Proposition 4 *Suppose that collusion follows effort exertion. Consider any effort \bar{e} that is implementable without collusion and renegotiation. Then, the effort is also implementable under the collusion-only environment, and the cost of implementation is the first-best cost. In particular, the grand contract that is offered and accepted in equilibrium is (\mathbf{u}, \mathbf{v}) , which satisfies (13) and (14).*

Proof. Suppose the grand contract is proposed and accepted. When effort \bar{e} is chosen, followed by optimal collusion, the agent and the supervisor will receive a fixed payment of $C_{FB}(\bar{e}) - s(V_0)$ and $s(V_0)$ and utility U_0 and V_0 , respectively. When effort $e' \neq \bar{e}$ is chosen, the supervisor will receive an expected utility of $\sum_k p_y(e') v_y$ from the grand contract (note that v_y in general is different across y). The optimal collusion contract made by the agent that is acceptable to the supervisor is a fixed payment, $s(\sum_k p_y(e') v_y)$, regardless of the output level. Hence, the agent's payment from the (grand-cum-)collusion contract is $C_{FB}(\bar{e}) - s(\sum_k p_y(e') v_y)$ regardless of the output level. In order for the agent to have the incentive to choose \bar{e} over any $e' \neq \bar{e}$, we require that, for all $e' \neq \bar{e}$,

$$U(C_{FB}(\bar{e}) - s(V_0)) - G(\bar{e}) \geq U\left(C_{FB}(\bar{e}) - s\left(\sum_y p_y(e') v_y\right)\right) - G(e'),$$

which, after writing the LHS as U_0 , is equivalent to

$$C_{FB}(\bar{e}) - s\left(\sum_y p_y(e') \left(-f_y + \sum_y p_y(\bar{e}) f_y + V_0\right)\right) \leq w(U_0 + G(e')), \quad (15)$$

where the LHS can be further simplified to

$$C_{FB}(\bar{e}) - s\left(\sum_y (p_y(\bar{e}) - p_y(e')) f_y + V_0\right).$$

Hence, (15) is rewritten as

$$w(U_0 + G(e')) \geq C_{FB}(\bar{e}) - s\left(\sum_y (p_y(\bar{e}) - p_y(e')) f_y + V_0\right)$$

or

$$\sum_y (p_y(e') - p_y(\bar{e})) f_y \leq V_0 - V(C_{FB}(\bar{e}) - w(U_0 + G(e'))) \quad (16)$$

Because \bar{e} is implementable without collusion and renegotiation, there is a vector of coefficients, $\hat{\mathbf{f}} \geq 0$, that satisfies the system of inequalities

$$\sum_y p_y(\bar{e}) \hat{f}_y - G(\bar{e}) \geq \sum_y p_y(e') \hat{f}_y - G(e') \text{ for all } e' \neq \bar{e}. \quad (17)$$

By an argument similar to that of Ishiguro and Itoh, which makes use of an earlier result in Hermalin and Katz (Proposition 2), we guarantee that the set of coefficients, \widehat{f}_y , for all y in (17) also satisfy (16). This shows that \bar{e} is implementable at the first-best cost with collusion. ■

The difference between the two supervisor's IR constraints, (7) and (12), is the key reason why the order between collusion and effort exertion matters so much in connection to the efficiency of the grand contract. The reservation value of the supervisor in the collusion stage is exogenous when collusion precedes effort exertion, but it depends on the equilibrium effort when collusion follows effort exertion. In the latter case, the grand contract affects the gain from collusion and how it is divided between the agent and the supervisor. Because the supervisor can observe the effort before signing a collusion contract, the way the gain from collusion is divided is contingent on the effort. This serves as a very effective way of sharing incentives – the first-best cost can be achieved because the grand contract stipulates a risk free transfer between the agent and the supervisor that ensures the former the highest expected utility. The same idea is also behind the first-best results of Hermalin and Katz and Ishiguro and Itoh (2001). The difference between these two studies is that the person who is making the risk free payment with the agent is the principal himself in the former or another agent in the latter.⁸

Ishiguro and Itoh (2001) attribute their first-best result to ‘decentralized renegotiation’ in a setting similar to ours. However, our result in the proposition suggests that renegotiation involving the principal is not required at all: the true driving force of the first-best result is that the agent and the supervisor are allowed to create a side contract after effort exertion.

4.2 Collusion and Renegotiation

Since the first-best outcome can be achieved with collusion, adding renegotiation will never help. In fact, the best the principal can do is to commit to never renegotiating. The goal of this subsection is to show that a lack of such commitment will not compromise the first-best result obtained in the pre-

⁸Kofman and Lawarree (1996) also argue that sometimes it is optimal to allow collusion rather than forbidding it. But the framework they study is an adverse selection one and they do not obtain a first-best result. Ma (1988) achieves the first-best in a multiple agent environment through a message game in which the agents' payoffs depend also on the messages sent out. In our paper, contracts are simple and cannot be made dependent on exchanged messages.

vious subsection: there always exists a perfect Bayesian equilibrium so that the effort is still implementable at the first-best cost under the collusion-cum-renegotiation environment.

Suppose that the principal wants to implement effort $\bar{e} > 0$ and let (\mathbf{u}, \mathbf{v}) be the optimal grand contract implementing \bar{e} that we solved for the problem under the collusion-only environment. We argue that the same grand contract can implement \bar{e} at the first-best cost under the collusion-and-renegotiation environment.

4.2.1 Renegotiation

As in Section 3, the principal's beliefs about the actual effort chosen after observing the collusion contract, $(\mathbf{u}^C, \mathbf{v}^C)$, are captured by a belief function, $\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C)$, which is a probability distribution over E . The optimal renegotiation contract is one that maximizes the principal's expected payoff given his beliefs; clearly, the renegotiation contract is deterministic, making a fixed payment in every level of y . We use $R[\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C), e]$ to denote the supervisor's expected payoff from accepting the principal's optimal renegotiation contract given his belief function $\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C)$ and chosen effort e .

4.2.2 Collusion

At the collusion stage, given such a grand contract, the agent's problem is to choose e and a collusion contract $(\mathbf{u}^C, \mathbf{v}^C)$ to solve (11) subject to the following supervisor's IR constraint

$$\max \left\{ \sum_{y \in Y} p_y(e) v_y^C, R[\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C), e] \right\} \geq \max \left\{ \sum_{y \in Y} p_y(e) v_y, R[\pi(\cdot | \mathbf{u}, \mathbf{v}), e] \right\}, \quad (18)$$

where the left-hand side (right-hand side) is her expected utility when a collusion contract is signed (no collusion contract is signed). In case collusion contract $(\mathbf{u}^C, \mathbf{v}^C)$ is signed, by accepting the renegotiation contract, she obtains expected utility $R[\pi(\cdot | \mathbf{u}^C, \mathbf{v}^C), e]$; by not accepting it, she simply obtains $\sum_{y \in Y} p_y(e) v_y^C$. In case no collusion contract is signed, by accepting the renegotiation contract, she obtains expected utility $R[\pi(\cdot | \mathbf{u}, \mathbf{v}), e]$; by not accepting it, she obtains $\sum_{y \in Y} p_y(e) v_y$.

Note that (18) is in general different from (12). However, we argue that there always exist beliefs on the part of the supervisor, along with the optimal

renegotiation contract given these beliefs, so that this new IR constraint reduces back to the old one and the first-best cost is achievable.

Recall that $(\bar{\mathbf{u}}^C, \bar{\mathbf{v}}^C)$ is the collusion contract that will result under the collusion-only environment. Consider the following belief by the principal. (i) If $(\bar{\mathbf{u}}^C, \bar{\mathbf{v}}^C)$ is observed, then \bar{e} must have been chosen; (ii) if some different collusion has been signed, then the chosen effort is:

$$\arg \min_{e'} \sum_{y \in Y} p_y(e') v_y^C;$$

and (iii) if no collusion contract has been signed, then the chosen effort is:

$$\arg \min_{e'} \sum_{y \in Y} p_y(e') v_y.$$

In case (i), since $\bar{\mathbf{v}}^C$ already gives the supervisor a constant payment over all states, the optimal renegotiation contract offered by the principal is a null contract, i.e., there will not be renegotiation. In case (ii) and case (iii), the renegotiation contracts are risk-free contracts that pay the same in each state, and the payments are $\min_{e'} \sum_{y \in Y} p_y(e') v_y^C$ and $\min_{e'} \sum_{y \in Y} p_y(e') v_y$, respectively. Note that the effort used in each of these terms reflects what the principal believes, not necessarily what the agent has chosen.

Suppose $\mathbf{v}^C = \bar{\mathbf{v}}^C = (V_0)$ and consider any chosen effort, e . There will not be any renegotiation upon accepting the collusion contract and the supervisor's expected utility from accepting the collusion contract – the left-hand side of (18) – is simply V_0 . By rejecting the collusion contract, she expects to obtain $\sum_{y \in Y} p_y(e) v_y$ if she turns down the renegotiation contract and $\min_{e'} \sum_{y \in Y} p_y(e') v_y$ (which is weakly dominated) otherwise. Hence, her IR constraint (18) becomes

$$V^0 \geq \sum_{y \in Y} p_y(e) v_y \quad \text{if } \mathbf{v}^C = \bar{\mathbf{v}}^C. \quad (19)$$

Suppose $\mathbf{v}^C \neq \bar{\mathbf{v}}^C$ and consider any chosen effort e . Consider her expected utility upon accepting the collusion contract. If she does not accept the subsequent renegotiation contract, it simply equals $\sum_{y \in Y} p_y(e) v_y^C$; otherwise, it equals $\min_{e'} \sum_{y \in Y} p_y(e') v_y^C$ (which is weakly dominated). Now consider her expected utility upon rejection of the collusion contract. If she does not accept the subsequent renegotiation contract, it simply equals

$\sum_{y \in Y} p_y(e) v_y$; otherwise, it equals $\min_{e'} \sum_{y \in Y} p_y(e') v_y$ (which is weakly dominated). Hence, her IR constraint (18) becomes

$$\sum_{y \in Y} p_y(e) v_y^C \geq \sum_{y \in Y} p_y(e) v_y \quad \text{if } \mathbf{v}^C \neq \bar{\mathbf{v}}^C. \quad (20)$$

Hence, given the grand contract (\mathbf{u}, \mathbf{v}) , the agent's problem becomes one solving (11) subject to this two-part constraint (19) and (20). We first note that $(\bar{e}, \bar{\mathbf{u}}^C, \bar{\mathbf{v}}^C)$ is feasible for this program; the supervisor's IR is satisfied because her expected utility from the collusion contract is just V_0 and there will be no renegotiation. Foreseeing this outcome, both the agent and supervisor will find the grand contract acceptable.

We next argue that there does not exist another 3-tuple $(\tilde{e}, \tilde{\mathbf{u}}^C, \tilde{\mathbf{v}}^C)$ that is both feasible for the program and makes the agent strictly better off. Suppose on the contrary it existed and $\tilde{\mathbf{v}}^C \neq \bar{\mathbf{v}}^C$. Then the supervisor's IR was exactly the same as her IR constraint under the collusion-only environment, and hence $(\tilde{e}, \tilde{\mathbf{u}}^C, \tilde{\mathbf{v}}^C)$ satisfied the old program too and would give the agent a strictly higher payoff. This is contradictory to the claim that $(\bar{e}, \bar{\mathbf{u}}^C, \bar{\mathbf{v}}^C)$ is a solution to the program under the collusion only environment. Suppose $\tilde{\mathbf{v}}^C = \bar{\mathbf{v}}^C$ and $\tilde{e} \neq \bar{e}$. Then, this 3-tuple also satisfies the supervisor's IR constraint under the collusion-only environment, and the agent is made strictly better off there too. Again this is a contradiction. Therefore, the same effort, \bar{e} , is implementable by the grand contract (\mathbf{u}, \mathbf{v}) under collusion and renegotiation as it is under collusion only. It is straightforward to see that the belief along the equilibrium path is indeed correct.

Proposition 5 *Suppose collusion follows effort exertion. Consider any effort e that is implementable under the collusion-only environment. Then the effort is also implementable under the collusion and renegotiation environment at the first-best cost. In particular, the grand contract that is offered and accepted in equilibrium is the same as the one used in the collusion-only environment. There is collusion but no renegotiation in equilibrium.*

A few comments are in order here. Firstly, the belief systems used in this model and the model in the previous section are similar; they can be called pessimistic because they construe the effort chosen to be most unfavorable whenever unexpected collusion contracts are seen. Secondly, the two seeming unrelated results of Felli and Villas-Boas and Ishiguro and Itoh (2001) are actually special cases of a unifying framework, and the essential difference between them is merely the order between collusion and effort exertion.

Thirdly, while our model with both collusion and renegotiation is able to implement the first-best cost, it differs from our earlier model under the collusion only environment and the model in Ishiguro and Itoh (2001). Given that the first-best cost is already achieved when renegotiation is forbidden, it appears to be more difficult to sustain the first-best cost in our model. Nonetheless, we were able to show that it is still feasible, and interestingly renegotiation never occurs in equilibrium.

5 Concluding Remarks

This paper has aimed at analyzing the interactions between collusion and renegotiation. We found that they jointly play a non-negative role, and that the order of collusion and effort exertion was identified as an important factor determining efficiency. Our results encompass several important existing results as special cases, and we view the synthesis as a main contribution of our paper. As both collusion and renegotiation are common in practice, our analysis suggests that they should be studied jointly, instead of separately as in most of the existing literature.

We have looked at the role for collusion in terms of risk sharing. One can think of other roles of collusion, such as effort coordination, joint utility maximization, and report manipulation. We hope to pursue these different roles in future research.

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