Bayesian Estimation of Linearized DSGE Models

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1 Posterior distribution

- Chapter 12 of Tsay (2005) provides an elegant introduction to Markov Chain Monte Carlo Methods with applications.

- Greenberg (2008) provides a very good introduction to fundamentals of Bayesian inference and simulation.

- Geweke (2005) provides a more advanced treatment of Bayesian econometrics.

- Bayesian inference combines prior belief (knowledge) with empirical data to form posterior distribution, which is the basis for statistical inference.

- θ : the parameters of a DSGE model
- Y : the empirical data
- $P(\theta)$: prior distribution for the parameters
- The prior distribution $P(\theta)$ incorporates the prior belief and knowledge of the parameters.
- $f(Y|\theta)$: the likelihood function of the data for given parameters
- By the definition of conditional probability:

$$f(\theta|Y) = \frac{f(\theta, Y)}{f(Y)} = \frac{f(Y|\theta)\, P(\theta)}{f(Y)} \tag{1}$$

- The marginal distribution $f(Y)$ is defined as:

$$f(Y) = \int f(Y, \theta)\, d\theta = \int f(Y|\theta)\, P(\theta)\, d\theta$$

- $f(\theta|Y)$ is called the posterior distribution of θ.

- It is the probability density function (PDF) of θ given the observed empirical data Y.

- Omit the scale factor and equation (1) can be expressed as:

$$f\left(\theta|Y\right) \propto f\left(Y|\theta\right) P\left(\theta\right)$$

- Bayes theorem

$$posterior \quad PDF \;\propto\; (likelihood \quad function) \;\times\; (prior \quad PDF)$$

- Expressed in logarithm:

$$\log(posterior \quad PDF) \;\propto\; \log(likelihood \quad function) \;+\; \log(prior \quad PDF)$$

2 Markov Chain Monte Carlo (MCMC) methods

- This section draws from Chapter 7 of Greenberg (2008).

- An advanced textbook is Carlin and Louis (2009), *Bayesian Methods for Data Analysis*, CRC Press.

- The basis of an MCMC algorithm is the construction of a transition kernel, denoted by $p(x, y)$, that has an invariant density equal to the target density.

- Given such a kernel, we can start the process at x_0 to yield a draw x_1 from $p(x_0, x_1)$, x_2 from $p(x_1, x_2)$, x_3 from $p(x_2, x_3)$,..., and x_g from $p\left(x_{g-1}, x_g\right)$.

- The distribution of x_g is approximately equal to the target distribution after a transient period.

- Therefore, MCMC algorithms provide an approximation to the exact posterior distribution of a parameter.

- How to find a kernel that has the target density as its invariant distribution?

- Metropolis-Hasting algorithm provides a general principle to find such kernels.

- Gibbs sampler is a special case of the Metropolis-Hasting algorithm.

2.1 Gibbs algorithm

- The Gibbs algorithm is applicable when it is possible to sample from each conditional distribution.

- Suppose we want to sample from the joint distribution $f(x_1, x_2)$.

- Further suppose that we are able to sample from the two conditional distributions $f(x_1|x_2)$ and $f(x_2|x_1)$.

- Gibbs algorithm

1. Choose $x_2^{(0)}$ (you can also start from $x_1^{(0)}$)

2. The first iteration

$$draw \quad x_1^{(1)} \quad from \quad f\left(x_1 | x_2^{(0)}\right)$$

$$draw \quad x_2^{(1)} \quad from \quad f\left(x_2 | x_1^{(1)}\right)$$

3. The g-th iteration

$$draw \quad x_1^{(g)} \quad from \quad f\left(x_1|x_2^{(g-1)}\right)$$

$$draw \quad x_2^{(g)} \quad from \quad f\left(x_2|x_1^{(g)}\right)$$

4. Draw until the desired number of iterations is obtained.

- We discard some portion of the initial sample.

- This portion is the transient or burn-in sample.

- Let n be the number of total iterations and m be the number of burn-in sample.

- The point estimate of x_1 (similarly for x_2) and its variance are:

$$\hat{x}_1 = \frac{1}{n-m} \sum_{j=m+1}^{n} x_1^{(j)}$$

$$\hat{\sigma}_1^2 = \frac{1}{n-m-1} \sum_{j=m+1}^{n} \left(x_1^{(j)} - \hat{x}_1 \right)^2$$

- The invariant distribution of the Gibbs kernel is the target distribution.

- Proof:

- $x = (x_1, x_2)$

- $y = (y_1, y_2)$

- $p(x, y) : x \rightarrow y \Leftrightarrow x_1 \rightarrow y_1; \quad x_2 \rightarrow y_2$

- The Gibbs kernel is:

$$p\left(x,y\right)=f\left(y_1|x_2\right)\cdot f\left(y_2|y_1\right)$$

$$f\left(y_1|x_2\right)\quad:\quad draw\quad x_1^{(g)}\quad from\quad f\left(x_1|x_2^{(g-1)}\right)$$

$$f\left(y_2|y_1\right)\quad:\quad draw\quad x_2^{(g)}\quad from\quad f\left(x_2|x_1^{(g)}\right)$$

- It follows that:

$$
\begin{aligned}
\int p(x,y)\, f(x)\, dx &= \int f(y_1|x_2) \cdot f(y_2|y_1)\, f(x_1,x_2)\, dx_1 dx_2 \\
&= f(y_2|y_1) \int f(y_1|x_2)\, f(x_1,x_2)\, dx_1 dx_2 \\
&= f(y_2|y_1) \int f(y_1|x_2)\, f(x_2)\, dx_2 \\
&= f(y_2|y_1)\, f(y_1) = f(y_1,y_2) = f(y)
\end{aligned}
$$

- This proves that $f(y)$ is the invariant distribution for the Gibbs kernel $p(x,y)$.

- The invariant distribution of the Gibbs kernel is the target distribution is a necessary, but not a sufficient condition for the kernel to converge to the target distribution.

- Please refer to Tierney (1994) for a further discussion of such conditions.

- The Gibbs sampler can be easily extended to more than two blocks.

- In practice, convergence of Gibbs sampler is an important issue.

- I will use Brooks and Gelman (1998)'s method for convergence check.

2.2 Metropolis-Hasting algorithm

- Metropolis-Hasting algorithm is more general than the Gibbs sampler because it does not require the availability of the full set of conditional distribution for sampling.

- Suppose that we want to draw a random sample from the distribution $f(X)$.

- The distribution $f(X)$ contains a complicated normalization constant so that a direct draw is either too time-consuming or infeasible.

- However, there exists an approximate distribution (jumping distribution, proposal distribution) for which random draws are easy to obtain.

- The Metropolis-Hasting algorithm generates a sequence of random draws from the approximate distribution whose distributions converge to $f(X)$.

- MH algorithm

1. Given x, draw Y from $q\left(x,y\right)$.
2. Draw U from $U\left(0,1\right)$.
3. Return Y if:

$$U \leq \alpha\left(x,Y\right) = \min\left\{\frac{f\left(Y\right)q\left(Y,x\right)}{f\left(x\right)q\left(x,Y\right)},1\right\}$$

4. Otherwise, return x and go to step 1.

5. Draw until the desired number of iterations is obtained.

- $q(x, y)$ is the proposal distribution.

- The normalization constant of $f(X)$ is not needed because only a ratio is used in the computation.

- How to choose the proposal density $q(x, y)$?

- The proposal density should generate proposals that have a reasonably good probability of acceptance.

- The sampling should be able to explore a large part of the support.

- Two well-known proposal kernels are the random walk kernel and the independent kernel.

A. Random walk kernel:

$$y = x + u$$

- $h(u) = h(-u) \rightarrow q(x, y) = q(y, x) \rightarrow \alpha(x, y) = \frac{f(y)}{f(x)}$

B. Independent kernel:

$$q(x, y) = q(y)$$

- $q(x, y) = q(y) \rightarrow \alpha(x, y) = \frac{f(y)/q(y)}{f(x)/q(x)}$

2.2.1 Metropolis algorithm

- The algorithm uses a symmetric proposal function, namely $q\left(Y,x\right)=q\left(x,Y\right)$.

- Metropolis algorithm

1. Given x, draw Y from $q\left(x,y\right)$.

2. Draw U from $U\left(0,1\right)$.

3. Return Y if:

$$U \leq \alpha(x, Y) = \min \left\{ \frac{f(Y)}{f(x)}, 1 \right\}$$

4. Otherwise, return x and go to step 1.

5. Draw until the desired number of iterations is obtained.

2.2.2 Properties of MH algorithm

- This part draws from Chib and Greenberg (1995), "Understanding the Metropolis-Hasting Algorithm," *The American Statistician*, 49 (4), pp. 327-335.

- A kernel $q(x, y)$ is reversible if:

$$f(x)\, q(x, y) = f(y)\, q(y, x)$$

- It can be shown that f is the invariant distribution for the reversible kernel q defined above.

- Now we begin with a kernel that is not reversible:

$$f(x)\, q(x, y) > f(y)\, q(y, x)$$

- We make the irreversible kernel into a reversible kernel by multiplying both sides by a function α.

$$f(x)\underbrace{\alpha(x,y)\,q(x,y)} = f(y)\underbrace{\alpha(y,x)\,q(y,x)}$$

- $p(x,y) \equiv \alpha(x,y)\,q(x,y)$

$$f(x)\,p(x,y) = f(y)\,p(y,x)$$

- This turns the irreversible kernel $q(x, y)$ into the reversible kernel $p(x, y)$.

- Now set $\alpha(y, x) = 1$.

$$f(x)\,\alpha(x, y)\,q(x, y) = f(y)\,q(y, x)$$

$$\alpha(x, y) = \frac{f(y)\,q(y, x)}{f(x)\,q(x, y)} < 1$$

- By letting $\alpha(x, y) < \alpha(y, x)$, we equalize the probability that the kernel goes from x to y with the probability that the kernel goes from y to x.

- Similar consideration for the general case implies that:

$$\alpha(x, y) = \min\left\{\frac{f(y)\, q(y, x)}{f(x)\, q(x, y)}, 1\right\}$$

2.2.3 Metropolis-Hasting algorithm with two blocks

- Suppose we are at the $(g-1)$-th iteration $x = (x_1, x_2)$ and want to move to the g-th iteration $y = (y_1, y_2)$.

- MH algorithm

1. Draw Z_1 from $q_1(x_1, Z| \quad x_2)$.

2. Draw U_1 from $U(0, 1)$.

3. Return $y_1 = Z_1$ if:

$$U_1 \leq \alpha(x_1, Z_1| \quad x_2) = \frac{f(Z_1, x_2)\, q_1(Z_1, x_1| \quad x_2)}{f(x_1, x_2)\, q_1(x_1, Z_1| \quad x_2)}$$

4. Otherwise, return $y_1 = x_1$.

5. Draw Z_2 from $q_2(x_2, Z| \quad y_1)$.

6. Draw U_2 from $U(0, 1)$.

7. Return $y_2 = Z_2$ if:

$$U_2 \leq \alpha(x_2, Z_2| \quad y_1) = \frac{f(y_1, Z_2)\, q(Z_2, x_2| \quad y_1)}{f(y_1, x_2)\, q(x_2, Z_2| \quad y_1)}$$

8. Otherwise, return $y_2 = x_2$.

3 Estimation algorithm

- Karagedikli et al. (2010) provide an overview of the Bayesian estimation of a simple RBC model.

- This paper provides internet linkages of several sources of useful computation code.

- The program appendix includes a whole set of DYNARE programs to estimate, simulate the simple RBC model by using the U.S. output data, as well as to diagnose the convergence of MCMC.

- The references include a rich and most updated literature on estimation of DSGE models.

- However, the paper is confined to Bayesian estimation, and it is not useful for researchers who want to understand the computational details and to build their own programs.

- An and Schorfheide (2007) review Bayesian estimation and evaluation techniques that have been developed in recent years for empirical work with DSGE models.

- Why using Bayesian method to estimate DSGE models?

- Bayesian estimation of DSGE models has 3 characteristics (An and Schorfheide, 2007).

- First, compared to GMM estimation, Bayesian estimation is system-based. (This is also true for maximum likelihood estimation)

- Second, the estimation is based on likelihood function generated by the DSGE model, rather than the discrepancy between model-implied impulse responses and VAR impulse responses.

- Third, prior distributions can be used to incorporate additional information into the parameter estimation.

- Counter-argument:

- Fukač and Pagan (2010) show that DSGE models should be not estimated and evaluated only with full information methods.

- If the assumption that the complete system of equations is specified properly seems dubious, limited information estimation, which focuses on specific equations, can provide useful complementary information about the adequacy of the model equations in matching the data.

3.1 Draw from the posterior by Random Walk Metropolis algorithm

- Remember that posterior is proportional to likelihood function times prior.

$$f(\theta|Y) \propto f(Y|\theta)\, P(\theta)$$

- How to compute posterior moments?

- Random Walk Metropolis (RWM) algorithm allows us to draw from the posterior $f(\theta|Y)$.

- RWM algorithm belongs to the more general class of Metropolis-Hasting algorithm.

- RWM algorithm

1. Initialize the algorithm with an arbitrary value θ_0 and set $j = 1$.

2. Draw θ_j^* from $\theta_j^* = \theta_{j-1} + \varepsilon \sim \mathcal{N}\left(\theta_{j-1}, \mathbf{\Sigma}_\varepsilon\right)$.

3. Draw u from $U(0,1)$.

4. Return $\theta_j = \theta_j^*$ if

$$u \leq \alpha\left(\theta_{j-1}, \theta_{j-1}^{*}\right) = \min\left\{\frac{f\left(Y|\theta_{j}^{*}\right) P\left(\theta_{j}^{*}\right)}{f\left(Y|\theta_{j-1}\right) P\left(\theta_{j-1}\right)}, 1\right\}$$

5. Otherwise, return $\theta_j = \theta_{j-1}$.

6. If $j \leq N$ then $j \rightarrow j+1$ and go to step 2.

- Kalman filter is used to evaluate the above likelihood values $f\left(Y|\theta_{j}^{*}\right)$ and $f\left(Y|\theta_{j-1}\right)$.

3.2 Computational algorithm

- Schorfheide (2000) and An and Schorfheide (2007).

1. Use a numerical optimization routine to maximize $\log f(Y|\theta) + \log P(\theta)$.

2. Denote the posterior model by $\tilde{\theta}$.

3. Denote by $\tilde{\Sigma}$ the inverse of the Hessian computed at the posterior mode $\tilde{\theta}$.

4. Specify an initial value for θ_0, or draw θ_0 from $\mathcal{N}\left(\tilde{\theta}, c_0^2 \cdot \tilde{\Sigma}\right)$.

5. Set $j = 1$ and set the number of MCMC N.

6. Evaluate $f(Y|\theta_0)$ and $P(\theta_0)$

A evaluate $P(\theta_0)$ for given θ_0

B use Paul Klein's method to solve the model for given θ_0

C use Kalman filter to evaluate $f(Y|\theta_0)$

7. Draw θ_j^* from $\theta_j^* = \theta_{j-1} + \varepsilon \sim \mathcal{N}\left(\theta_{j-1}, c^2 \cdot \tilde{\boldsymbol{\Sigma}}\right)$.

8. Draw u from $U(0,1)$.

9. Evaluate $f\left(Y|\theta_j^*\right)$ and $P\left(\theta_j^*\right)$

A evaluate $P\left(\theta_j^*\right)$ for given θ_j^*

B use Paul Klein's method to solve the model for given θ_j^*

C use Kalman filter to evaluate $f\left(Y|\theta_j^*\right)$

10. Return $\theta_j = \theta_j^*$ if

$$u \leq \alpha\left(\theta_{j-1}, \theta^{*}_{j-1}\right) = \min\left\{\frac{f\left(Y|\theta^{*}_{j}\right) P\left(\theta^{*}_{j}\right)}{f\left(Y|\theta_{j-1}\right) P\left(\theta_{j-1}\right)}, 1\right\}$$

11. Otherwise, return $\theta_j = \theta_{j-1}$.

12. If $j \leq N$ then $j \rightarrow j+1$ and go to step 7.

13. Approximate the posterior expected value of a function $h\left(\theta\right)$ by:

$$E\left[h\left(\theta\right)|Y\right] = \frac{1}{N_{sim}} \sum_{j=1}^{N_{sim}} h\left(\theta_j\right)$$

- $N_{sim} = N - N_{burn-in}$

- It is recommended to adjust the scale factor c so that the acceptance rate is roughly 25 percent in WRM algorithm.

4 An example: business cycle accounting

- This example illustrates Bayesian estimation of the wedges process in Chari, Kehoe and McGrattan (2007)'s business cycle accounting.

- The wedges process is $s_t = \left(\hat{A}_t, \hat{\tau}_{lt}, \hat{\tau}_{xt}, \hat{g}_t\right)$.

$$s_{t+1} = Ps_t + Q\varepsilon_{t+1}$$

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ q_{21} & q_{22} & 0 & 0 \\ q_{31} & q_{32} & q_{33} & 0 \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix}$$

$$\varepsilon_{t+1} \sim \mathcal{N}\left(0_{4\times 1}, I_{4\times 4}\right)$$

- We estimate the lower triangular matrix Q to ensure that the estimate of $V = QQ'$ is positive semidefinite.

- The matrix Q has no structural interpretation.

- Given the wedges, which are functionally similar to shocks, the next step is to solve the log-linearized model.

- We use Paul Klein's MATLAB code to solve the log-linearized model.

- The state variables of the model are: $\left(\hat{k}_t, s_t\right) = \left(\hat{k}_t, \hat{A}_t, \hat{\tau}_{lt}, \hat{\tau}_{xt}, \hat{g}_t\right)$

- The control variables of the model are: $\left(\hat{c}_t, \hat{x}_t, \hat{y}_t, \hat{l}_t\right)$

- The observed variables of the model are: $\left(\hat{y}_t, \hat{x}_t, \hat{l}_t, \hat{g}_t\right)$

- Here again the log-linearized model:

$$\frac{\tilde{c}}{\tilde{y}}\hat{c}_t + \frac{\tilde{x}}{\tilde{y}}\hat{x}_t + \frac{\tilde{g}}{\tilde{y}}\hat{g}_t = \hat{y}_t \tag{1.c}$$

$$\hat{y}_t = \hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\,\hat{l}_t \tag{2.c}$$

$$\hat{c}_t = \hat{A}_t + \alpha \hat{k}_t - \left[\alpha + \frac{l}{(1-l)}\right] \hat{l}_t - \frac{1}{(1-\tau_l)} \hat{\tau}_{lt} \tag{3.c}$$

$$\begin{aligned} & \hat{\tau}_{xt} \frac{(1+\gamma)}{\beta} + (1+\tau_x) \frac{(1+\gamma)}{\beta} E_t \hat{c}_{t+1} - (1+\tau_x) \frac{(1+\gamma)}{\beta} \hat{c}_t \\ = \;& E_t \left[\alpha \frac{\tilde{y}}{\tilde{k}} \left(\hat{y}_{t+1} - \hat{k}_{t+1}\right) + (1-\delta) \hat{\tau}_{xt+1}\right] \end{aligned} \tag{4.c}$$

$$(1+\gamma_n)(1+\gamma) \hat{k}_{t+1} = (1-\delta) \hat{k}_t + \frac{\tilde{x}}{\tilde{k}} \hat{x}_t \tag{5.c}$$

- In Lecture 4, I have shown the maximum likelihood estimation of the wedges process.

- Going from MLE to Bayesian estimation is straightforward.

- The first step is to set the priors.

- The choice of the priors follows Saijo, Hikaru (2008), "The Japanese Depression in the Interwar Period: A General Equilibrium Analysis," *B. E. Journal of Macroeconomics*.

- The prior for diagonal terms of matrix P is assumed to follow a beta distribution with mean 0.7 and standard deviation 0.2.

- The prior for non-diagonal terms of matrix P is assumed to follow a normal distribution with mean 0 and standard deviation 0.3.

- The prior for diagonal terms of matrix Q is assumed to follow an uniform distribution between 0 and 0.5.

- The prior for non-diagonal terms of matrix Q is assumed to follow an uniform distribution between -0.5 and 0.5.

- The table below summarizes the priors.

Name	Domain	Density	Prior Parameter 1	Parameter 2
$p_{11}, p_{22}, p_{33}, p_{44}$	$[0, 1)$	Beta	0.035	0.015
p_{12}, p_{13}, p_{14}	$\mathbb{R}$	Normal	0	0.3
p_{21}, p_{23}, p_{24}	$\mathbb{R}$	Normal	0	0.3
p_{31}, p_{32}, p_{34}	$\mathbb{R}$	Normal	0	0.3
p_{41}, p_{42}, p_{43}	$\mathbb{R}$	Normal	0	0.3
$q_{11}, q_{22}, q_{33}, q_{44}$	$[0, 0.5]$	Uniform	0	0.5
$q_{21}, q_{31}, q_{32}, q_{41}, q_{42}, q_{43}$	$[-0.5, 0.5]$	Uniform	-0.5	0.5

- If the sequence obtained from MCMC were i.i.d., we could use a central limit theorem to derive the standard error (known as the numerical standard error) associate with the estimate.

- Koop, Gary, Dale J. Poirier, and Justin L. Tobias (2007), Bayesian Econometric Methods, page 119.

$$E\left[\theta|y\right] = \hat{\theta} = \frac{1}{R}\sum_{r=1}^{R}\theta^{(r)}$$

$$Var\left(\theta|y\right) = \hat{\sigma}^2 = \frac{1}{R}\sum_{r=1}^{R}\left(\theta^{(r)} - \hat{\theta}\right)^2$$

$$\sqrt{R}\left(\hat{\theta} - \theta\right) \sim \mathcal{N}\left(0, \hat{\sigma}^2\right)$$

$$\hat{\theta} \overset{approx}{\sim} \mathcal{N}\left(\theta, \frac{\hat{\sigma}^2}{R}\right)$$

$$NSE = \frac{\hat{\sigma}}{\sqrt{R}}$$

- However, serial correlation is inherent in MCMC.

- Several methods have been proposed to deal with this problem.

1. the approach of Newey and West (1987)

2. estimate the spectral density at frequency zero

3. batch means

- See Greenberg (2008) page 103 and Geweke (2005) page 149 for discussions.

- Newey and West (1987) estimator:

$$Var\left(\theta|y\right)=\hat{\sigma}^{2}=\hat{\gamma}_{0}+2\sum_{j=1}^{M_{T}}w_{j,T}\cdot\hat{\gamma}_{j}$$

$$w_{j,T} = 1 - \frac{j}{M_T + 1}$$

$$M_T = \mathsf{integer}\left[4 \cdot \left(\frac{T}{100}\right)^{\frac{2}{9}}\right]$$

$$NSE = \frac{\hat{\sigma}}{\sqrt{R}}$$

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