

# Appendix

In this appendix we describe the equilibrium conditions of the small open economy employed in the main text. The model follows Galí and Monacelli (2005) and Monacelli (2005), assuming that the law of one price holds. The model consists of a small open economy and the rest of the world. The size of the small open economy is negligible relative to the rest of the world and takes the equilibrium of the latter as given, and the rest of the world is in the limit treated as a closed economy.

Here are some notations. We have express all variables as log deviations from their respective steady-state values.  $c_t$  is domestic consumption, which is a composite of domestic and foreign goods;  $y_t$  is domestic output;  $r_t$  is domestic gross nominal interest rate;  $q_t$  is real exchange rate, namely, the number of domestic goods that a unit of foreign goods can exchange for;  $s_t$  is the terms of trade, defined as the relative price of foreign goods to domestic goods;  $\pi_t$  is the CPI inflation rate;  $\pi_{H,t}$  is the domestic producer inflation, defined as the rate of change in the index of domestic goods prices;  $x_t$  is domestic output gap, defined as the deviation of log domestic output from the equilibrium log level of output in the absence of nominal rigidity;  $z_t$  is total factor productivity; and  $mc_t$  is real marginal cost in terms of domestic goods prices. We use  $\xi_t$  to denotes shocks. Foreign variables are denoted with the superscript "\*".

## The domestic economy

The domestic economy contains 11 endogenous variables:  $c_t$ ,  $y_t$ ,  $r_t$ ,  $q_t$ ,  $s_t$ ,  $\pi_t$ ,  $\pi_{H,t}$ ,  $x_t$ ,  $z_t$ ,  $mc_t$ ,  $\bar{r}\bar{r}_t$ . There are two shocks in the domestic economy: the monetary policy shocks  $\xi_{m,t}$  and the productivity shocks  $\xi_{z,t}$ . The foreign variables  $\pi_t^*$ ,  $r_t^*$ ,  $y_t^*$  are taken as exogenous.

Equations (A1) to (A11) describe the structural relations of the endogenous variables. Equation (A1) is a log-linear approximation to the market clearing condition, where  $\gamma$  is share of foreign goods in the domestic consumption bundle, and  $\eta$  measures the elasticity of substitution between domestic and foreign goods. Equation (A2) relates the terms of trade and the real exchange rate. Equation (A3) is a typical forward-looking Phillips curve, obtained from a log-linear approximation to domestic firms' optimality conditions for price setting and the domestic aggregate price index.  $\beta$  is the discount factor and  $\theta_H$  is the degree of price rigidity according to the standard Calvo-Yun type price setting. Equation (A4) denotes the real marginal cost in terms of domestic goods. The real marginal cost is common across all producers.  $\sigma$  and  $\varphi$  are the inverse elasticities of intertemporal substitution and labour supply respectively. Equation (A5) follows from the assumption of complete markets for nominal state contingent securities. Complete markets imply a link between domestic and foreign consumption levels. Equation (A6) is the uncovered interest rate parity, expressed in terms of real interest rate differential and real exchange rate. Equation (A7) is the monetary policy reaction function. It is assumed

that monetary policy is conducted according to the Taylor-type rule, namely, nominal interest rate responds to the current CPI inflation rate and the output gap. Parameter  $\rho_r$  determines the degree of interest rate smoothing.  $\xi_{m,t}$  denotes the domestic monetary policy shock. Equation (A8) relates the CPI inflation and the domestic producer inflation. Output gap, defined as the deviation of domestic output from the equilibrium level of output that would obtain when prices are fully flexible, is depicted by equation (A9). Equation (A10) defines the small open economy's natural rate of interest. Domestic productivity is assumed to follow a simple stochastic autoregressive process as depicted by equation (A11), where  $\rho$  is a persistence parameter and  $\xi_{z,t}$  is an i.i.d. productivity shock.

$$(1 - \gamma) c_t = y_t - (2 - \gamma) \gamma \eta s_t - \gamma y_t^* \quad (\text{A1})$$

$$q_t = (1 - \gamma) s_t \quad (\text{A2})$$

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \lambda_H \cdot mc_t \quad (\text{A3})$$

$$\lambda_H \equiv \left( \frac{(1 - \theta_H)(1 - \beta \theta_H)}{\theta_H} \right)$$

$$mc_t = \sigma c_t + \varphi y_t + \gamma s_t - (1 + \varphi) z_t \quad (\text{A4})$$

$$c_t = y_t^* + \frac{1}{\sigma} [(1 - \gamma) s_t] \quad (\text{A5})$$

$$(r_t - E_t \{ \pi_{t+1} \}) - (r_t^* - E_t \{ \pi_{t+1}^* \}) = E_t \{ \Delta q_{t+1} \} \quad (\text{A6})$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \cdot (\phi_\pi \pi_t + \phi_x x_t) + z_{m,t} \quad (\text{A7})$$

$$\pi_t = \pi_{H,t} + \gamma \Delta s_t \quad (\text{A8})$$

$$x_t = E_t \{ x_{t+1} \} - \frac{\varpi_s}{\sigma} (r_t - E_t \{ \pi_{H,t+1} \} - \bar{r} r_t)$$

$$x_t = y_t - \left( \frac{\varpi_s (1 + \varphi)}{\sigma + \varphi \varpi_s} \right) z_t - \left( \frac{\sigma (1 - \varpi_s)}{\sigma + \varphi \varpi_s} \right) y_t^* \quad (\text{A9})$$

$$\bar{r} r_t \equiv \sigma \left( \frac{\varphi (\varpi_s - 1)}{\sigma + \varphi \varpi_s} \right) E_t \{ \Delta y_{t+1}^* \} - \left( \frac{\sigma (1 - \rho) (1 + \varphi)}{\sigma + \varphi \varpi_s} \right) z_t$$

$$\varpi_s \equiv 1 + \gamma(2 - \gamma)(\sigma\eta - 1) > 0$$

$$z_t = \rho z_{t-1} + \xi_{z,t} \quad (\text{A10})$$

$$\xi_{z,t} \sim \mathcal{N}(0, \sigma_z^2)$$

$$z_{m,t} = \rho_m z_{m,t-1} + \xi_{m,t} \quad (\text{A11})$$

$$\xi_{m,t} \sim \mathcal{N}(0, \sigma_m^2)$$

## The foreign economy

The foreign economy shares the same preferences, technology, and market structure with the domestic economy. The foreign economy contains 7 endogenous variables:  $y_t^*$ ,  $\pi_t^*$ ,  $r_t^*$ ,  $x_t^*$ ,  $mc_t^*$ ,  $\bar{r}r_t^*$ ,  $z_t^*$ . Like the domestic economy, the foreign economy also faces monetary policy shocks  $\xi_{m^*,t}$  and productivity shocks  $\xi_{z^*,t}$ .

The equilibrium conditions of the foreign economy are given by equations (A12) to (A18). Equation (A12) is the aggregate demand function of the foreign economy. Equation (A13) is the aggregate supply curve. Equation (A14) defines the real marginal cost function. Equation (A15) is the monetary policy reaction function. It is assumed that monetary policy of the foreign economy is conducted according to the Taylor-type rule, too. Equation (A16) is the dynamic equation for foreign output gap. The natural rate of interest and the productivity level is depicted by equation (A17) and (A18), respectively. The foreign economy has the form of the small-scale sticky price model that has become the benchmark of monetary policy analysis. The equilibrium conditions can be reduced to a three-equation dynamic system for inflation and output gap, consisting of a new Keynesian Phillips curve, a forward-looking IS curve, and a monetary policy rule.

$$y_t^* = E_t \{y_{t+1}^*\} - \frac{1}{\sigma} (r_t^* - E_t \{\pi_{t+1}^*\}) \quad (\text{A12})$$

$$\pi_t^* = \beta E_t \{\pi_{t+1}^*\} + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} mc_t^* \quad (\text{A13})$$

$$mc_t^* = (\sigma + \varphi) y_t^* - (1 + \varphi) z_t^* \quad (\text{A14})$$

$$r_t^* = \rho_{r^*} \cdot r_{t-1}^* + (1 - \rho_{r^*}) \cdot (\phi_{\pi^*} \pi_t^* + \phi_{x^*} x_t^*) + z_{m,t}^* \quad (\text{A15})$$

$$x_t^* = y_t^* - \left( \frac{1 + \varphi}{\sigma + \varphi} \right) z_t^* \quad (\text{A16})$$

$$x_t^* = E_t \{ x_{t+1}^* \} - \frac{1}{\sigma} (r_t^* - E_t \{ \pi_{t+1}^* \} - \overline{rr}_t^*)$$

$$\overline{rr}_t^* \equiv - \frac{\sigma (1 - \rho) (1 + \varphi)}{\sigma + \varphi} z_t^*$$

$$z_t^* = \rho^* z_{t-1}^* + \xi_{z^*,t} \quad (\text{A17})$$

$$\xi_{z^*,t} \sim \mathcal{N}(0, \sigma_{z^*}^2)$$

$$z_t^* = \rho z_{t-1}^* + \xi_{z^*,t}$$

$$z_{m,t}^* = \rho_{m^*} z_{m,t-1}^* + \xi_{m^*,t} \quad (\text{A18})$$

$$\xi_{m^*,t} \sim \mathcal{N}(0, \sigma_{m^*}^2)$$

## Fixed exchange rates

So far the model assumes flexible exchange rates. The model can be easily adjusted to fixed exchange rates. Under fixed exchange rates, equation (A6) becomes

$$\pi_t - \pi_t^* = -\Delta q_t$$

and equation (A7) becomes

$$r = r^*$$

Finally, domestic monetary policy becomes endogenous so that equation (A11) that describes monetary shocks is dropped out. The other equilibrium conditions of the model remain unchanged.