Let $v_j$ be a simplification for $v_j(2)$. If there is a difference between the H and L types, we denote $v_{jH}$ for $v_j$ if player $j$ is a type H, and $v_{jL}$ for $v_j$ if player $j$ is a type L. Remember that we make the monotonicity assumption about $1h(.)$ with regard to its off-the-equilibrium-path beliefs and that we only report the case where the responder indexed with a lower subscript is chosen as the coalition partner ex post whenever a proposer is indifferent over which of her two responders to be included in the coalition.

**Proof of Proposition 1**

By assumption, $p_1 = 1$. The posteriors after a rejection of $2\ell(H)$, $1\ell(H)$ and $1h(H)$ are, respectively: $p = (1,1,1)$, $p = (1,1,1/2)$, and $p = (1,1,1/2)$. By Lemma 1-1,

\[ v_1 = v_2 = v_3 = (1/3)(1-(1/3)) + (2/3)(1/2)(1/3) = 1/3. \]  

(A1)

Put in words: after the rejection, each player has a 1/3 chance of becoming the proposer in the second session; once a player becomes the proposer, she adopts the $1h$ proposal with a symmetric tie-breaking.

(A1) implies:

The best of $2\ell(H)$, $1\ell(H)$ and $1h(H)$ are, respectively

\[ (1 - \delta(v_2 + v_3), \delta v_2, \delta v_3) = (1 - (2/3)\delta, (1/3)\delta, (1/3)\delta); \]

\[ (1 - \delta v_2, \delta v_2, 0) = (1 - (1/3)\delta, (1/3)\delta, 0); \]

\[ (1 - v_2, v_2, 0) = (1 - (1/3), 1/3, 0). \]

The above proposals are “best” in the sense that they allocate merely continuation values to the targeted responders and zero to the rest.
The expected payoffs from choosing these best proposals equal, respectively
\[
\begin{align*}
(3/4)[1 - \delta(v_2 + v_3)] + (1/4)v_1 &= 5/6 - (1/2)\delta; \quad \text{(A2-1)} \\
(1/2)[1 - \delta v_2] + (1/2)v_1 &= 2/3 - (1/6)\delta; \quad \text{(A2-2)} \\
1 - v_2 &= 2/3. \quad \text{(A2-3)}
\end{align*}
\]

With the common knowledge that both a type H and L are equally likely a priori, player 1 faces four possible types of responders in combination (i.e. HH, HL, LH and LL) with equal probability. Thus, the probability of failing to receive a majority support is 1/4 if the 2 \( \ell \) (H) proposals are chosen, 1/2 if the 1 \( \ell \) (H) proposals are chosen, and zero if the 1h(H) proposals are chosen. Since player 1 is a type H, her continuation value equals \( v_1 = 1/3 \) if a proposal fails to receive a majority support so that delay occurs.

It is clear from (A2) that, regardless of the value of \( \delta \), the best of 1h(H) strictly dominates the best of 1 \( \ell \) (H). As a result, we only need to make a comparison between 1h(H) and 2 \( \ell \) (H). It can be calculated that the best of 2 \( \ell \) (H) will be chosen over the best of 1h(H) if and only if
\[
\delta \leq 1/3. \quad \text{(A3)}
\]

**Proof of Proposition 2**

By assumption, \( p_1 = 0 \). The posteriors after a rejection of 2 \( \ell \) (L), 1 \( \ell \) (L) and 1h(L) are respectively: \( p = (0,1,1) \), \( p = (0,1,1/2) \), and \( p = (0,1,1/2) \). By Lemma 1-2,
\[
\begin{align*}
\hat{v}_1 &= (1/3)(1-(1/3)) + (2/3)(1/3)\delta = 2/9 + (2/9)\delta; \quad \text{(A4-1)} \\
\hat{v}_2 &= v_3 = (1/3)(1-(1/3)\delta) + (1/3)(1/2)(1/3) = 7/18 - (1/9)\delta. \quad \text{(A4-2)}
\end{align*}
\]

Put in words: after the rejection, each player has a 1/3 chance of becoming the proposer in the second session; once becoming the proposer, player 1 adopts the 1h proposal with a symmetric tie-breaking, while both players 2 and 3 adopt the 1 \( \ell \) proposal and choose player 1 as their coalition partner.

(A4) implies:

The best of 2 \( \ell \) (L), 1 \( \ell \) (L) and 1h(L) are, respectively

1
The expected payoffs from choosing these best proposals equal, respectively

\[(1-\delta(v_2 + v_3), \delta v_2, \delta v_3) = (1-(7/9)\delta+(2/9)\delta^2, (7/18)\delta-(1/9)\delta^2, (7/18)\delta-(1/9)\delta^2);\]

\[(1-\delta v_2, \delta v_2, 0) = (1-(7/18)\delta+(1/9)\delta^2, (7/18)\delta-(1/9)\delta^2, 0);\]

\[(1-v_2, v_2, 0) = (11/18+(1/9)\delta, 7/18-(1/9)\delta, 0).\]

Since player 1 is a type L, her continuation value equals \(\delta v_1 = (2/9)\delta + (2/9)\delta^2\) if a proposal fails to receive a majority support so that delay occurs.

Regardless of the value of \(\delta\), the best of 1\(h\)(L) strictly dominates the best of 1\(h\)(H). As a result, we only need to make a comparison between 1\(h\)(H) and 2\(l\)(H). It can be calculated that the best of 2\(l\)(L) will be chosen over the best of 1\(h\)(L) if and only if

\[\delta \leq \delta_0 = [23-3(41)^{1/2}] / 16 \approx 0.237\]  \(A6\)

Summary

(A3) and (A6) divide \(\delta \in (0,1)\) into three regimes:

- If \(\delta \geq 1/3\), the best of 1\(h\)(H) and 1\(h\)(L) apply;
- If \(\delta \leq \delta_0\), the best of 2\(l\)(H) and 2\(l\)(L) apply;
- If \(\delta_0 \leq \delta \leq 1/3\), the best of 1\(h\)(L) and 2\(l\)(H) apply. \(Q.E.D\)

Proof of Proposition 3

We derive separating and pooling equilibria separately.

Separating equilibria

A separating equilibrium requires that neither a type H nor L have an incentive to masquerade as the other type. We first check if the “first-best” solution of Proposition 1 can be supported as a separating equilibrium (the “no-envy” case).
Step 1. “No-envy” case

We examine three regimes stated in Proposition 1 in turn.

(I) \( \delta \geq 1/3 \)

The IC constraints for this regime require that a type H who chooses the best of \( 1h(H) \) have no incentives to masquerade as a type L who chooses the best of \( 1h(L) \), viz
\[
\frac{2}{3} \geq 1 - v_2; \quad (A7-1-1)
\]
and that a type L who chooses the best of \( 1h(L) \) have no incentives to masquerade as a type H who chooses the best of \( 1h(H) \), viz
\[
1 - v_2 \geq \frac{2}{3}, \quad (A7-1-2)
\]
where \( v_2 \) follows (A4-2) since a rejection of \( 1h(L) \) implies \( p = (0,1,1/2) \).

It can be checked that the IC constraint (A7-1-1) is violated if \( \delta > 1/2 \), while the IC constraint (A7-1-2) is violated if \( \delta < 1/2 \). In other words, a type H has an incentive to masquerade as a type L if \( \delta > 1/2 \), while a type L has an incentive to masquerade as a type H if \( \delta < 1/2 \). The “first-best” solution, the best of \( 1h(H) \) and \( 1h(L) \), cannot be supported as a separating equilibrium in this regime.

(II) \( \delta \leq \delta_0 \approx 0.237 \)

The IC constraints for this regime require that a type H who chooses the best of \( 2\ell(H) \) have no incentives to masquerade as a type L who chooses the best of \( 2\ell(L) \), viz
\[
(3/4)(1-(2/3)\delta) + (1/4)(1/3) \geq (3/4)(1-\delta(v_2+v_3)) + (1/4)V_1; \quad (A7-2-1)
\]
and that a type L who chooses the best of \( 2\ell(L) \) have no incentives to masquerade as a type H who chooses the best of \( 2\ell(H) \), viz
\[
(3/4)(1-\delta(v_2+v_3)) + (1/4)\delta_1 \geq (3/4)(1-(2/3)\delta) + (1/4)(1/3)\delta \quad (A7-2-2)
\]
where \( V_1 = 4/9 \), and \( v_1 \), \( v_2 \) and \( v_3 \) follow (A4) since a rejection of \( 2\ell(L) \) implies \( p = (0,1,1) \).
Note that \( V_1 = 4/9 > \nu_1 = 2/9 + (2/9)\delta \) in this regime. If the best of \( 2\ell (L) \) fails to receive a majority support in the first session, it will give rise to the posterior belief \( p = (0,1,1) \) and hence players 2 and 3 will make the offer \( 1\ell \) to player 1 if either of them becomes the proposer in the second session (Lemma 1-2). However, sequential rationality dictates that player 1 of a type H, who mimics a type L by choosing the best of \( 2\ell (L) \) in the first session, reject the \( 1\ell \) offer in the second session. The rejection moves the game to the third session, which ensures the payoff \( 1/3 \) for player 1 of a type H (her continuation value). This leads to

\[
V_1 = 1/3 \cdot (2/3) + 2/3 \cdot (1/3) = 4/9
\]

in (A7-2-1).

It can be checked that both (A7-2-1) and (A7-2-2) are violated so that both types have an incentives to masquerade as the other type. The “first-best” solution, the best of \( 2\ell (H) \) and \( 2\ell (L) \), cannot be supported as a separating equilibrium in this regime.

\[(III)\] \( \delta_0 \leq \delta \leq 1/3 \)

The IC constraints for this regime require that a type H who chooses the best of \( 2\ell (H) \) have no incentives to masquerade as a type L who chooses the best of \( 1h(L) \), viz

\[
(3/4)(1-(2/3)\delta)+(1/4)(1/3) \geq 1-\nu_2;
\]

(A7-3-1)

and that a type L who chooses the best of \( 1h(L) \) have no incentives to masquerade as a type H who chooses the best of \( 2\ell (H) \), viz

\[
1-\nu_2 \geq (3/4)(1-(2/3)\delta)+(1/4)(1/3)\delta
\]

(A7-3-2)

where \( \nu_2 \) follow (A4-2) since a rejection of \( 1h(L) \) implies \( p = (0,1,1/2) \).

Let \( \delta_i = 5/19 \approx 0.263 \), which satisfies the equality part of (A7-3-2). Then it can be checked that both IC constraints (A7-3-1)-(A7-3-2) are satisfied if \( \delta_i \leq \delta \leq 1/3 \), while the IC constraint (A7-3-2) is violated if \( \delta_0 \leq \delta < \delta_i \). In other words, the “first-best” solution, the best of \( 2\ell (H) \) and \( 1h(L) \), can be supported as a separating equilibrium if \( \delta_i \leq \delta \leq 1/3 \); but it cannot be supported as a separating equilibrium if \( \delta_0 \leq \delta < \delta_i \).
Summary

The “no-envy” separating equilibrium exists if and only if $\delta \leq \frac{1}{3}$ and it consists of the best of $2\ell(H)$ and $1h(L)$. For all other values of $\delta$, some IC constraints are violated, so there exists “envy” between types under “first-best” solutions.

Step 2. “Envy” case

When some IC constraints are violated so that there exists “envy” between types under “first-best” solutions, we seek “second-best” solutions. “Second-best” separating equilibria exist if a type can deviate from her “first-best” solution and find profitable solutions to separate from the type who wants to mimic her. We examine respectively four regimes under which “no-envy” separating equilibria fail to exist in Step 1.

(I) $\delta \geq \frac{1}{2}$

In this regime a type $H$ has an incentive to masquerade as a type $L$ under the “first-best” solution. Let us fix the “first-best” proposal of a type $H$ (i.e. the best of $1h(H)$), which yields the payoff that a type $H$ can at least guarantee herself in any separating equilibrium. We check if a type $L$ can find profitable “second-best” solutions to separate herself from a type $H$. There are three possibilities:

(i) A type $L$ seeks “second-best” solutions in the set of $1h(L)$ with $x^l(1) = (1 - x^l_2(1), x^l_2(1), 0)$ and $x^l_2(1) \geq 7/18 - (1/9)\delta$ such that it is not profitable for a type $H$ to mimic, viz

$$2/3 \geq 1 - x^l_2(1);$$

but profitable for a type $L$ to separate, viz

$$1 - x^l_2(1) > 2/3.$$  

However, there exist no such “second-best” solutions that satisfy both inequalities above in this regime.
(ii) A type L seeks “second-best” solutions in the set of 1 \( l \) (L) with 
\[
x'(l) = (1 - x^1_2(l), \ x^1_2(l), \ 0) \quad \text{and} \quad x^1_2(l) \geq (7/18)\delta - (1/9)\delta^2 \quad \text{such that it is not}
\]
profitable for a type H to mimic, viz 
\[
2/3 \geq (1/2)(1-x^1_2(l)) + (1/2)V_1;
\]
but profitable for a type L to separate, viz 
\[
(1/2)(1-x^1_2(l)) + (1/2)\delta_1 > 2/3,
\]
where \( V_1=4/9 \), and \( v_i \) follows (A4-1) since a rejection of 1 \( l \) (L) implies 
\( p = (0,1,1/2) \). However, there exist no such “second-best” solutions that satisfy both
inequalities above in this regime.

(iii) A type L seeks “second-best” solutions in the set of 2 \( l \) (L) with 
\[
x'(l) = (1 - x^1_2(l), \ x^1_2(l), \ x^1_3(l)) \quad \text{and} \quad x^1_2(l), \ x^1_3(l) \geq (7/18)\delta - (1/9)\delta^2 \quad \text{such that it is}
\]
not profitable for a type H to mimic, viz 
\[
2/3 \geq (3/4)(1-x^1_2(l)-x^1_3(l)) + (1/4)V_1;
\]
but profitable for a type L to separate, viz 
\[
(3/4)(1-x^1_2(l)-x^1_3(l)) + (1/4)\delta_1 > 2/3,
\]
where \( V_1=4/9 \), and \( v_i \) follows (A4-1) since a rejection of 2 \( l \) (L) implies 
\( p = (0,1,1) \). However, there exist no such “second-best” solutions that satisfy both
inequalities above in this regime.

(II) \( 1/3 \leq \delta \leq 1/2 \)

In this regime a type L has an incentive to masquerade as a type H under
the “first-best” solution. Let us fix the “first-best” proposal of a type L (i.e. the best of 
1h(L)), which yields the payoff that a type L can at least guarantee herself in any
separating equilibrium. We check if a type H can find profitable “second-best”
solutions to separate herself from a type L. There are three possibilities:

(i) A type H seeks “second-best” solutions in the set of 1h(H) with
\[ x^1_l = (1 - x^1_2(l), x^1_2(l), 0) \quad \text{and} \quad x^1_3(l) \geq 1/3 \] such that it is not profitable for a type L to mimic, viz
\[ 11/18 + (1/9)\delta \geq 1 - x^1_2(l); \]
but profitable for a type H to separate, viz
\[ 1 - x^1_2(l) > 11/18 + (1/9)\delta. \]
However, there exist no such “second-best” solutions that satisfy both inequalities above in this regime.

(ii) A type H seeks “second-best” solutions in the set of \( 1 \ell (H) \) with
\[ x^1_l = (1 - x^1_2(l), x^1_2(l), 0) \quad \text{and} \quad x^1_3(l) \geq (1/3)\delta \] such that it is not profitable for a type L to mimic, viz
\[ 11/18 + (1/9)\delta \geq (1/2)(1 - x^1_2(l)) + (1/2)\delta v_1; \]
but profitable for a type H to separate, viz
\[ (1/2)(1 - x^1_2(l)) + (1/2)v_1 > 11/18 + (1/9)\delta, \]
where \( v_1 \) follows (A1) since a rejection of \( 1 \ell (H) \) implies \( p = (1,1,1,1/2) \). However, there exist no such “second-best” solutions that satisfy both inequalities above in this regime.

(iii) A type H seeks “second-best” solutions in the set of \( 2 \ell (H) \) with
\[ x^1_l = (1 - x^1_2(l), x^1_2(l), x^1_3(l)) \quad \text{and} \quad x^1_2(l), x^1_3(l) \geq (1/3)\delta \] such that it is not profitable for a type L to mimic, viz
\[ 11/18 + (1/9)\delta \geq (3/4)(1 - x^1_2(l) - x^1_3(l)) + (1/4)\delta v_1; \]
but profitable for a type H to separate, viz
\[ (3/4)(1 - x^1_2(l) - x^1_3(l)) + (1/4)v_1 > 11/18 + (1/9)\delta, \]
where \( v_1 \) follows (A1) since a rejection of \( 2 \ell (H) \) implies \( p = (1,1,1,1) \). However, there exist no such “second-best” solutions that satisfy both inequalities above in this regime.
(III) $\delta \leq \delta_0$

In this regime both types have an incentive to masquerade as the other type under the “first-best” solution. Let us fix the “first-best” proposal of a type L (i.e. the best of $2\ell(L)$), which yields the payoff that a type L can at least guarantee herself in any separating equilibrium. A rejection of $2\ell(L)$ implies $p = (0,1,1)$, which in turn implies that in the second session both players 2 and 3 will choose $1\ell$ with player 1 as their coalition partner according to Lemma 1-2. A type H could choose $2\ell(L)$ and pretend to be a type L to ensure a coalitional offer in the second session on the equilibrium path and then reject the offer. The rejection implies that the game would move to the third session in equilibrium, which in turn implies that it would ensure a type H the payoff $1/3$. This would give a type H a higher payoff than choosing $2\ell(H)$. In other words, a type H would mimic a type L rather than find solutions to separate herself from a type L.

(IV) $\delta_0 \leq \delta \leq \delta_1$

In this regime a type L has an incentive to masquerade as a type H under the “first-best” solution. Let us fix the “first-best” proposal of a type L (i.e. the best of $1h(L)$), which yields the payoff that a type L can at least guarantee herself in any separating equilibrium. We check if a type H can find profitable “second-best” solutions to separate herself from a type L. There are three possibilities:

(i) A type H seeks “second-best” solutions in the set of $1h(H)$ with $x^1(l) = (1 - x^1_2(l), x^1_2(l), 0)$ and $x^1_3(l) \geq 1/3$ such that it is not profitable for a type L to mimic, viz

$$11/18 + (1/9)\delta \geq 1 - x^1_2(l);$$

but profitable for a type H to separate, viz

$$1 - x^1_2(l) > 11/18 + (1/9)\delta.$$

However, there exist no such “second-best” solutions that satisfy both inequalities above in this regime.
(ii) A type $H$ seeks “second-best” solutions in the set of $1\ell (H)$ with
\[ x^1(1) = (1 - x^1_2(1), x^1_2(1), 0) \] and $x^1_2(1) \geq (1/3)\delta$ such that it is not profitable for a
type $L$ to mimic, viz
\[ 11/18 + (1/9)\delta \geq (1/2)(1 - x^1_2(1)) + (1/2)\delta v^1_1; \]
but profitable for a type $H$ to separate, viz
\[ (1/2)(1 - x^1_2(1)) + (1/2) v^1_1 > 11/18 + (1/9)\delta, \]
where $v^1_1$ follows (A1) since a rejection of $1\ell (H)$ implies $p = (1,1,1/2)$. However,
there exist no such “second-best” solutions that satisfy both inequalities above in this
regime.

(iii) A type $H$ seeks “second-best” solutions in the set of $2\ell (H)$ with
\[ x^1(1) = (1 - x^1_2(1), x^1_2(1), x^1_3(1)) \] and $x^1_2(1), x^1_3(1) \geq (1/3)\delta$ such that it is not
profitable for a type $L$ to mimic, viz
\[ 11/18 + (1/9)\delta \geq (3/4)(1 - x^1_2(1) - x^1_3(1)) + (1/4)\delta v^1_1; \]
but profitable for a type $H$ to separate, viz
\[ (3/4)(1 - x^1_2(1) - x^1_3(1)) + (1/4) v^1_1 > 11/18 + (1/9)\delta, \]
where $v^1_1$ follows (A1) since a rejection of $2\ell (H)$ implies $p = (1,1,1)$.

The above two inequalities imply
\[ 5/27 - (1/27)\delta \leq x^1_2(1) + x^1_3(1) < 8/27 - (4/27)\delta \] (A8)

The $2\ell (H)$ proposals that meet both $x^1_2(1), x^1_3(1) \geq (1/3)\delta$ and (A8) do exist in this
regime. Thus, separating equilibria are characterized by the feature that a type $L$
proposes the best of $1h(L)$, while a type $H$ proposes the $2\ell (H)$ proposals that satisfy
(A8) with $x^1_2(1), x^1_3(1) \geq (1/3)\delta$.

Ascribing zero weight to the mimicking type in the set of “second-best”
separating equilibria allows the mimicked type to choose the separating equilibrium
she prefers most. This leads to the “least-cost” separating equilibrium in which a
type $L$ proposes the best of $1h(L)$, while a type $H$ makes a proposal merely satisfying
the first equality part of (A8), that is,
\[ x^1(1) = (22/27 + (1/27)\delta, \ 5/54 - (1/54)\delta, \ 5/54 - (1/54)\delta). \]  \hspace{1cm} (A8-1)

Summary

“Second-best” separating equilibria exist if and only if \( \delta_0 \leq \delta \leq \delta_1 \) and they consist of the best of \( 1h(L) \) and “second-best” deviations from the best of \( 2\ell(H) \). The “least-cost” separating equilibrium is characterized by the best of \( 1h(L) \) and the least “second-best” deviation from the best of \( 2\ell(H) \) that enables a type \( H \) to separate from a type \( L \), that is, (A8-1).

Step 3. Off-the-equilibrium-path beliefs

We now specify players’ off-the-equilibrium-path beliefs that support the derived separating equilibria.

(I) “No-envy” separating equilibria

There are many such beliefs. Let us consider the following:

\[ p_1 = 1 \text{ if } x^1(1) \geq 1 - (2/3)\delta, \text{ and } p_1 = 0 \text{ otherwise;} \]
\[ p_2 = p_3 = 1 \text{ after a rejection of } x^1(1). \]

Then the posteriors in the second session will be

\[ p = (0,1,1) \text{ if } x^1(1) < 1 - (2/3)\delta, \text{ and } p = (1,1,1) \text{ if } x^1(1) \geq 1 - (2/3)\delta. \]

When \( p = (0,1,1) \), the best choice for a type \( L \) is the best of \( 1h(L) \) and a type \( H \) would rather choose the best of \( 2\ell(H) \) than deviate to masquerade as a type \( L \). When \( p = (1,1,1) \), the best choice for a type \( H \) is the best of \( 2\ell(H) \) and a type \( L \) would rather choose the best of \( 1h(L) \) than deviate to masquerade as a type \( H \). Thus, neither type would like to deviate from the “no-envy” separating equilibrium under the above off-the-equilibrium-path belief.

(II) “Least-cost” separating equilibria
Consider the following off-the-equilibrium-path beliefs:

\[ p_1 = 1 \text{ if any proposal } x'(1) \text{ yields player 1 of a type } L \text{ less than } \frac{11}{18} + \frac{(1/9)}{\delta}; \text{ and } p_1 = 0 \text{ otherwise.} \]

If a proposal yields player 1 of L type the payoff less than \( \frac{11}{18} + \frac{(1/9)}{\delta} \), such a proposal is considered making by a type H. Obviously player 1 if she is indeed a type L has no incentive to deviate to it. However, given this belief, the type H will choose the least-cost separating equilibrium proposal, \( 2 \ell (H) \) which is the best among all the choices. On the other hand, if a proposal yields player 1 of L type more than \( \frac{11}{18} + \frac{(1/9)}{\delta} \), player 1 is considered as a type L. Given this belief, player 2 (3) will vote for any proposal if and only if \( x'_j(1) \geq \frac{7}{18} - \frac{(1/9)}{\delta} \) if she is a type H and \( x'_j(1) \geq \frac{7}{18} - \frac{(1/9)}{\delta^2} \) if she is a type L. Thus, player 1 if she is a type H has no incentive to deviate away from the equilibrium proposal since a deviation that can make her better off will be rejected; and the type L will still choose equilibrium proposal since it is the best one among all the choices. Therefore, the least-cost separating equilibrium can be supported. Q.E.D.

**Pooling equilibria**

The posteriors after a rejection of \( 2 \ell (HL), \ 1 \ell (HL) \) and \( 1h(HL) \) are, respectively: \( p = (1/2,1,1) \), \( p = (1/2,1,1/2) \), and \( p = (1/2,1,1/2) \). We consider three ranges of \( \delta \)'s separately according to Lemma 1-3, that is, \( \delta \geq 1/3 \), \( 1/5 \leq \delta \leq 1/3 \), and \( \delta \leq 1/5 \).

**I** \( \delta \geq 1/3 \)

According to Lemmas 1-1 and 1-3, all players will adopt the \( 1h \) proposal in the second session. Thus,

\[ v_1 = v_2 = v_3 = (1/3)(1-(1/3)) + (2/3)(1/2)(1/3) = 1/3 \]

which implies:

The best of \( 2 \ell (HL), \ 1 \ell (HL) \) and \( 1h(HL) \) are, respectively
There are two sub-regimes to consider.

\((\text{I-1})\)  \(\delta \geq 1/2\)

The mimicking player is a type H in this regime. A type H can reveal herself and guarantee to obtain the payoff \(2/3\) by proposing the best of \(1h(H)\). Thus, to prevent a deviation to the best of \(1h(H)\), any pooling equilibrium must give a type H at least the payoff \(2/3\). For a type H, the best of \(2\ell(\text{HL})\) yields the expected payoff \((3/4)(1-(2/3)\delta) + (1/4)(1/3) = 5/6 - (1/2)\delta\), while the best of \(1\ell(\text{HL})\) yields the expected payoff \((1/2)(1-(1/3)\delta) + (1/2)(1/3) = 2/3 - (1/6)\delta\). Both are strictly less than \(2/3\) with \(\delta \geq 1/2\). This leads to the conclusion that the best of \(1h(\text{HL})\), which yields player 1 the payoff \(2/3\), is the only potential candidate for the pooling equilibrium.

We check if the best of \(1h(\text{HL})\) survives after applying the refinement of the intuitive criterion. This refinement requires us to examine if there exists a proposal \(x^1(I)\) deviating from the best of \(1h(\text{HL})\) such that if the deviation is regarded as from a type L would be worse off for a type H to mimic but better off for a type L to separate. There are three possibilities:

(i) Deviations are from the set of \(1h(L)\) with \(x^1(I) = (1-x^1_2(I), x^1_2(I), 0)\) and \(x^1_2(I) \geq 7/18 - (1/9)\delta\) such that it is worse off for a type H, viz \(2/3 > 1-x^1_2(I)\);

but better off for a type L, viz \(1-x^1_2(I) > 2/3\).

However, there exist no such proposals that satisfy both inequalities above in this regime.
(ii) Deviations are from the set of $1 \ell (L)$ with $x^1(1) = (1-x^1_1(1), x^1_2(1), 0)$ and

$$x^2(1) \geq (7/18)\delta - (1/9)\delta^2$$

such that it is worse off for a type H, viz

$$2/3 > (1/2)(1-x^1_1(1)) + (1/2)V_1;$$

but better off for a type L, viz

$$(1/2)(1-x^1_1(1)) + (1/2)\delta_1 > 2/3,$$

where $V_1=4/9$, and $\delta_1$ follows (A4-1) since a rejection of $1 \ell (L)$ implies $p = (0,1,1/2)$. However, there exist no such proposals that satisfy both inequalities above in this regime.

(iii) Deviations are from the set of $2 \ell (L)$ with $x^1(1) = (1-x^1_2(1), x^1_1(1), x^1_3(1))$ and

$$x^2(1), x^3(1) \geq (7/18)\delta - (1/9)\delta^2$$

such that it is worse off for a type H, viz

$$2/3 \geq (3/4)(1-x^1_2(1) - x^3_1(1)) + (1/4)V_1;$$

but better off for a type L, viz

$$(3/4)(1-x^1_2(1) - x^3_1(1)) + (1/4)\delta_1 > 2/3,$$

where $V_1=4/9$, and $\delta_1$ follows (A4-1) since a rejection of $2 \ell (L)$ implies $p = (0,1,1)$. However, there exist no such proposals that satisfy both inequalities above in this regime.

(I-2) $1/3 \leq \delta \leq 1/2$

The mimicking player is a type L in this regime. A type L can reveal herself and guarantee to obtain the payoff $11/18 + (1/9)\delta$ by proposing the best of $1h(L)$. Thus, to prevent a deviation to the best of $1h(L)$, any pooling equilibrium must give a type L at least the payoff $11/18 + (1/9)\delta$. For a type L, the best of $2 \ell (HL)$ yields the expected payoff $(3/4)(1-(2/3)\delta) + (1/4)(1/3)\delta = 3/4 - (5/6)\delta$, while the best of $1 \ell (HL)$ yields the expected payoff $(1/2)(1-(1/3)\delta) + (1/2)(1/3)\delta = 1/2$. Both are strictly less than $11/18 + (1/9)\delta$ with $1/3 \leq \delta \leq 1/2$. This leads to the conclusion that the potential candidates for the pooling equilibrium are those of
$1h(HL)$ with $1/3 \leq x_2^1(1) \leq 7/18 - (1/9)\delta$.

We check if these $1h(HL)$ proposals survive after the refinement of the intuitive criterion. This refinement requires us to examine if there exists a proposal $x^1(1)$ deviating from these $1h(HL)$ such that if the proposal is regarded as from a type H would be worse off for a type L to mimic but better off for a type H to separate. There are three possibilities:

(i) Deviations are from the set of $1h(H)$ with $x^1(1) = (1 - x_2^1(1), x_2^1(1), 0)$ and $x_2^1(1) \geq 1/3$ such that it is worse off for a type L, viz

$$11/18 + (1/9)\delta > 1 - x_2^1(1);$$

but better off for a type H, viz

$$1 - x_2^1(1) > 11/18 + (1/9)\delta.$$

However, there exist no such proposals that satisfy both inequalities above in this regime.

(ii) Deviations are from the set of $1\ell(H)$ with $x^1(1) = (1 - x_2^1(1), x_2^1(1), 0)$ and $x_2^1(1) \geq (1/3)\delta$ such that it is worse off for a type L, viz

$$11/18 + (1/9)\delta > (1/2)(1 - x_2^1(1)) + (1/2)\delta v_1;$$

but better off for a type H, viz

$$(1/2)(1 - x_2^1(1)) + (1/2)\delta v_1 > 11/18 + (1/9)\delta,$$

where $v_1$ follows (A1) since a rejection of $1\ell(H)$ implies $p = (1, 1, 1/2)$. However, there exist no such proposals that satisfy both inequalities above in this regime.

(iii) Deviations are from the set of $2\ell(H)$ with $x^1(1) = (1 - x_2^1(1), x_2^1(1), x_3^1(1))$ and $x_2^1(1), x_3^1(1) \geq (1/3)\delta$ such that it is worse off for a type L, viz

$$11/18 + (1/9)\delta \geq (3/4)(1 - x_2^1(1) - x_3^1(1)) + (1/4)\delta v_1;$$

but better off for a type H, viz
\[(3/4)(1-x_2^1(l)-x_3^1(l))+(1/4)v_1 > 11/18+(1/9)\delta,\]

where \(v_1\) follows (A1) since a rejection of \(2\ell\) (H) implies \(p = (1,1,1)\). However, there exist no such proposals that satisfy both inequalities above in this regime.

**Summary**

The best of \(1h(HL)\) survives the refinement of the intuitive criterion and is the unique pooling equilibrium if \(\delta \geq 1/2\). Those of \(1h(HL)\) with \(1/3 \leq x_2^1(l) \leq 7/18-(1/9)\delta\) survive the refinement of the intuitive criterion and are the pooling equilibria if \(1/3 \leq \delta \leq 1/2\).

**(II) 1/5 \leq \delta \leq 1/3**

When \(p = (1/2,1,1)\) (from a rejection of \(2\ell\) (HL)), all players will choose \(1h\) in the second session according to Lemma 1-1. This implies that \(v_1 = v_2 = v_3 = 1/3\). Thus the best of \(2\ell\) (HL) is 

\((1-(2/3)\delta, (1/3)\delta, (1/3)\delta)\).

When \(p = (1/2,1,1/2)\) (from a rejection of \(1h(HL)\) or \(1\ell\) (HL)), player \(j\) with \(p_j = 1/2\) will choose the \(1h\) proposal in the second session, while player 2 with \(p_2 = 1\) if she is a type H will choose \(2\ell\), but if she is a type L will choose \(1h\). This result is on the basis of Lemma 1-3. Our derivation of the best of \(1h(HL)\) and \(1\ell\) (HL) proceeds differently according to whether player 2 is a type H or L.

(i) Player 2 is a type L

All players will adopt \(1h\) in the second session. Thus, \(v_1 = v_2 = v_3 = 1/3\). The best of \(1h(HL)\) and \(1\ell\) (HL) remain the same as those with \(\delta \geq 1/3\).

(ii) Player 2 is a type H

\[
v_{2h} = (1/3)(1-(1/3)) + (1/3)(1/2)(1/3) + (1/3)[(1/2)(1/3)\delta + (1/2)(1/3)] = 1/3 + (1/18)\delta, j \neq 2;
\]

\[
v_{2l} = 1/3, j \neq 2;
\]
\[ v_{2h} = (1/3)[(3/4)(1 - (2/3)\delta) + (1/4)(1/3)] + (2/3)(1/2)(1/3) = 7/18 - (1/6)\delta. \]

Put in words: after the rejection of 1h(HL) or 1\( \ell \) (HL), each player has a 1/3 chance of becoming the proposer in the second session; once becoming the proposer, both players 1 and 3 adopt 1h with a symmetric tie-breaking, while player 2 adopts 2\( \ell \). Note that when player 2 is the proposer, player \( j \neq 2 \) if she is a type H will face a 1/2 probability that 2\( \ell \) fails to receive a majority support; but if she is a type L, 2\( \ell \) will always receive a majority support. The best of 1h(HL) and 1\( \ell \) (HL) are, respectively
\[(11/18 + (1/6)\delta, 7/18 - (1/6)\delta, 0)\;
(1 - (1/3)\delta, (1/3)\delta, 0).

There are three sub-regimes to consider.

**III-1** \( \delta_i \leq \delta \leq 1/3 \)

In this regime there exists the so-called “no-envy” separating equilibrium. We show that all pooling equilibria are dominated by the “no-envy” separating equilibrium.

Consider the best of 1h(HL). It yields player 1 the payoff \( 11/18 + (1/6)\delta \). However, this payoff is less than the payoff \( 5/6 - (1/2)\delta \) of proposing the best of 2\( \ell \) (H) for a type H. This means that the best of 1h(HL) is dominated by the best of 2\( \ell \) (H) for player 1 if she is a type H.

Consider the best of 2\( \ell \) (HL). It yields player 1 the payoff \( 3/4 - (5/12)\delta \) if she is type L. However, this payoff is less than the payoff of proposing the best of 1h (L) for a type L. This means that the best of 2\( \ell \) (HL) is dominated by the best of 1h (L) for player 1 if she is a type L.

Similarly, the best of 1\( \ell \) (HL) yields player 1 the payoff 1/2 if she is type L. This is less than the payoff of proposing the best of 1h (L) for a type L, which means that 1\( \ell \) (HL) is dominated by the best of 1h (L) for player 1 if she is a type L.
(II-2) $\delta_0 \leq \delta \leq \delta_1$

In this regime there exists the so-called “least-cost” separating equilibrium. We show that all pooling equilibria which are not dominated by the “least-cost” separating equilibrium survive after the refinement of the intuitive criterion.

The mimicking player is a type L in this regime. A type L can reveal herself and guarantee to obtain the payoff $11/18 + (1/9)\delta$ by proposing the best of $1 h(L)$. Thus, to prevent a deviation to the best of $1 h(L)$, any pooling equilibrium must give a type L at least the payoff $11/18 + (1/9)\delta$. $1 \ell (HL)$ will not be supportable as a pooling equilibrium, since the best of $1 \ell (HL)$ in this regime yields player 1 of a type L only $1/2$, which is less than that from the “least-cost” separating equilibrium, $11/18 + (1/9)\delta$. Likewise, the best of $1 h(HL)$ yields player 1 the expected payoff $11/18 + (1/6)\delta$, which is less than the payoff from the “least-cost” separating equilibrium for a type H. Thus, any pooling equilibrium in the form of $1 h(HL)$ will be broken by a type H’s deviation to the “least-cost” separating equilibrium.

The best of $2 \ell (HL)$ yields player 1 the expected payoff $(3/4)(1 - (2/3)\delta) + (1/4)(1/3) = 5/6 - (1/2)\delta$ if she is a type H, and the expected payoff $(3/4)(1 - (2/3)\delta) + (1/4)(1/3)\delta = 3/4 - (5/12)\delta$ if she is a type L. Each type’s payoff is higher than her payoff in the “least-cost” separating equilibrium. This leads to the conclusion that the potential candidates for the pooling equilibrium are those of $2 \ell (HL)$ with $(1/3)\delta \leq x^1_2(1) = x^1_3(1) \leq 5/54 - (1/54)\delta$.

We check if these $2 \ell (HL)$ proposals survive after the refinement of the intuitive criterion. This refinement requires us to examine if there exists a proposal $x^1(l)$ deviating from these $2 \ell (HL)$ such that if the deviation is regarded as from a type H would be worse off for a type L to mimic but better off for a type H to separate. There are three possibilities:

(i) Deviations are from the set of $1 h(H)$ with $x^1(l) = (1 - x^1_2(1), x^1_2(1), 0)$ and $x^1_2(1) \geq 1/3$ such that it is worse off for a type L, viz
\((3/4)(22/27 + (1/27)\delta) + (1/4)(1/3)\delta > 1 - x_1^1(l)\);

but better off for a type H, viz
\[1 - x_2^1(l) > (3/4)(22/27 + (1/27)\delta) + (1/4)(1/3).\]

However, there exist no such proposals that satisfy both inequalities above in this regime.

(ii) Deviations are from the set of \(1 \ell (H)\) with \(x^1(l) = (1 - x_2^1(l), x_2^1(l), 0)\) and \(x_2^1(l) \geq (1/3)\delta\) such that it is worse off for a type L, viz
\[(3/4)(22/27 + (1/27)\delta) + (1/4)(1/3)\delta > (1/2)(1 - x_2^1(l)) + (1/2)\delta v_1;\]

but better off for a type H, viz
\[(1/2)(1 - x_2^1(l)) + (1/2)v_1 > (3/4)(22/27 + (1/27)\delta) + (1/4)(1/3),\]

where \(v_1\) follows (A1) since a rejection of \(1 \ell (H)\) implies \(p = (1,1,1/2).\) However, there exist no such proposals that satisfy both inequalities above in this regime.

(iii) Deviations are from the set of \(2 \ell (H)\) with \(x^1(l) = (1 - x_2^1(l), x_2^1(l), x_3^1(l))\) and \(x_2^1(l), x_3^1(l) \geq (1/3)\delta\) such that it is worse off for a type L, viz
\[(3/4)(22/27 + (1/27)\delta) + (1/4)(1/3)\delta \geq (3/4)(1 - x_2^1(l) - x_3^1(l)) + (1/4)\delta v_1;\]

but better off for a type H, viz
\[(3/4)(1 - x_2^1(l) - x_3^1(l)) + (1/4)v_1 > (3/4)(22/27 + (1/27)\delta) + (1/4)(1/3),\]

where \(v_1\) follows (A1) since a rejection of \(2 \ell (H)\) implies \(p = (1,1,1).\) However, there exist no such proposals that satisfy both inequalities above in this regime.

(II-3) \(1/5 \leq \delta \leq \delta_0\)

In this regime there exist no separating equilibria. We show that those of \(2 \ell (HL)\) with \((1/3)\delta \leq x_2^1(l) = x_3^1(l) \leq (7/18)\delta - (1/9)\delta^2\) survive the refinement of the intuitive criterion.

In this regime a type L can always reveal herself and guarantee to obtain the
payoff \( (3/4)[1-2((7/18)\delta-(1/9)\delta^2)] + (1/4)[(2/9)\delta+(1/9)\delta^2] \) by proposing the best of \( 2 \ell (L) \). Thus, to prevent a deviation to the best of \( 2 \ell (L) \), any pooling equilibrium must give a type L at least the payoff \( 3/4-(19/36)\delta+(2/9)\delta^2 \). As in the previous case, \( 1 \ell (HL) \) will not be supported as a pooling equilibrium. The best of \( 1h(HL) \) yields player 1 the expected payoff \( 11/18+(1/6)\delta \), which is less than the payoff from the best of \( 2 \ell (L) \). Thus, any pooling equilibrium in the form of \( 1h(HL) \) will be broken by a type L’s deviation to the best of \( 2 \ell (L) \).

The best of \( 2 \ell (HL) \) yields player 1 of L type the expected payoff \( (3/4)(1-(2/3)\delta)+(1/4)(1/3)\delta = 3/4-(5/12)\delta \), which is higher than the payoff from the best of \( 2 \ell (L) \) with \( 1/5 \leq \delta \leq \delta_0 \). This leads to the conclusion that the potential candidates for the pooling equilibrium are those of \( 2 \ell (HL) \) with \( (1/3)\delta \leq x^1_2(1) = x^1_3(1) \leq (7/18)\delta-(1/9)\delta^2 \).

We check if these \( 2 \ell (HL) \) proposals survive after the refinement of the intuitive criterion. This refinement requires us to examine if there exists a proposal \( x^i(I) \) deviating from these of \( 2 \ell (HL) \) such that if the deviation is regarded as from a type H is worse off for a type L to mimic but better off for a type H to separate. There are three possibilities:

(i) Deviations are from the set of \( 1h(H) \) with \( x^i(I) = (1-x^1_2(I), x^1_2(I), 0) \) and \( x^1_2(I) \geq 1/3 \) such that it is worse off for a type L, viz

\[
(3/4)(1-(7/18)\delta+(1/9)\delta^2 )+(1/4)(1/3)\delta > 1-x^i_2(1);
\]

but better off for a type H, viz

\[
1-x^i_2(1) > (3/4)(1-(7/18)\delta+(1/9)\delta^2 )+(1/4)(1/3).
\]

However, there exist no such proposals that satisfy both inequalities above in this regime.

(ii) Deviations are from the set of \( 1 \ell (H) \) with \( x^i(I) = (1-x^1_2(I), x^1_2(I), 0) \) and
\( x_2'(1) \geq (1/3)\delta \) such that it is worse off for a type L, viz

\[
(3/4)(1 - (7/18)\delta + (1/9)\delta^2) + (1/4)(1/3)\delta > (1/2)(1 - x_2'(1)) + (1/2)\delta \nu_1;
\]

but better off for a type H, viz

\[
(1/2)(1 - x_2'(1)) + (1/2)\nu_1 > (3/4)(1 - (7/18)\delta + (1/9)\delta^2) + (1/4)(1/3),
\]

where \( \nu_1 \) follows (A1) since a rejection of \( 1H \) implies \( \rho = (1, 1, 1/2) \). However, there exist no such proposals that satisfy both inequalities above in this regime.

(iii) Deviations are from the set of \( 2L(H) \) with \( x'(1) = (1 - x_2'(1), x_2'(1), x_3'(1)) \) and \( x_2'(1), x_3'(1) \geq (1/3)\delta \) such that it is worse off for a type L, viz

\[
(3/4)(1 - (7/18)\delta + (1/9)\delta^2) + (1/4)(1/3)\delta \geq (3/4)(1 - x_2'(1) - x_3'(1)) + (1/4)\delta \nu_1;
\]

but better off for a type H, viz

\[
(3/4)(1 - x_2'(1) - x_3'(1)) + (1/4)\nu_1 > (3/4)(1 - (7/18)\delta + (1/9)\delta^2) + (1/4)(1/3),
\]

where \( \nu_1 \) follows (A1) since a rejection of \( 2H \) implies \( \rho = (1, 1, 1) \). However, there exist no such proposals that satisfy both inequalities above in this regime.

Summary

There exist no pooling equilibria if \( \delta_0 \leq \delta \leq 1/3 \). Those of \( 2L \) (HL) with \( (1/3)\delta \leq x_2'(1) = x_3'(1) \leq 5/54 - (1/54)\delta \) survive the refinement of the intuitive criterion and are the pooling equilibria if \( \delta_0 \leq \delta \leq \delta_1 \). Those of \( 2L \) (HL) with \( (1/3)\delta \leq x_2'(1) = x_3'(1) \leq (7/18)\delta - (1/9)\delta^2 \) survive the refinement of the intuitive criterion and are the pooling equilibria if \( 1/5 \leq \delta \leq \delta_0 \).

(III) \( \delta \leq 1/5 \)

When \( \rho = (1/2, 1, 1/2) \) (from a rejection of \( 1h(HL) \) or \( 1l \) (HL)), player \( j \) with \( \rho_j = 1/2 \) will choose the \( 1h \) proposal in the second session, while player 2 with
\( p_2 = 1 \) will choose 2 \( \ell \), no matter what type she is. The continuation values following a rejection to 1h(HL) and 1 \( \ell \) (HL) are:

\[ v_{jH} = 1/3 + (1/18)\delta, \; j \neq 2; \]

\[ v_{2H} = 7/18 - (1/6)\delta; \]

\[ v_{jL} = (1/3)(1 - (1/3)) + (1/3)(1/2)(1/3) + (1/3)(1/3)\delta = 5/18 + (1/9)\delta, \; j \neq 2; \]

\[ v_{2L} = (1/3)[(3/4)(1 - (2/3)\delta) + (1/4)(1/3)\delta] + (2/3)(1/2)(1/3) = 13/36 - (5/36)\delta. \]

The best of 1 \( \ell \) (HL) is then \( 1 - (13/36)\delta + (5/36)\delta^2, \; (13/36)\delta - (5/36)\delta^2, \; 0 \), which yields player 1

\[ (1/2)(1 - (13/36)\delta + (5/36)\delta^2) + (1/2)(1/3 + (1/18)\delta) = 2/3 - (11/72)\delta + (5/72)\delta^2 \]

if she is a type H; and

\[ (1/2)(1 - (13/36)\delta + (5/36)\delta^2) + (1/2)((5/18)\delta + (1/9)\delta^2) = 1/2 - (1/24)\delta + (1/8)\delta^2 \]

if she is a type L. However, 1 \( \ell \) (HL) is inferior to 2 \( \ell \) (L), from which player 1 of L type obtains \( 3/4 - (19/36)\delta + (2/9)\delta^2 \); and to 2 \( \ell \) (H), from which player 1 of a type H obtains \( 5/6 - (1/2)\delta \). Thus, 1 \( \ell \) (HL) will not be supportable as a pooling equilibrium.

The rest of the argument is the same as that in (II-3), so the result derived there can be directly applied to this regime.

**Summary**

Those of 2 \( \ell \) (HL) with \( (1/3)\delta \leq x_2^1(1) = x_3^1(1) \leq (7/18)\delta - (1/9)\delta^2 \) survive the refinement of the intuitive criterion and are the pooling equilibria if \( \delta \leq 1/5 \).

**Off-the-equilibrium-path beliefs**

Finally, we specify players’ off-the-equilibrium-path beliefs that support the derived pooling equilibria.

**(I)** \( \delta \geq 1/2 \)

\( p_1 = 0 \) if any proposal \( x^1(1) \) yields player 1 of a type H less than \( 2/3 \); and \( p_1 = 1 \) otherwise.
If player 1 offers a proposal other than \((2/3, 1/3, 0)\), whose payoff is less than 2/3, this offer is considered from a type L. Player 2 will accept it; however, such a deviation is not profitable for either type. If player 1 deviates to a proposal whose payoff is more than 2/3, she is considered a type H. However, this deviation will be rejected by players 2 and 3, because they each demand \(x^j_1(1) \geq 1/3\) for a type H and \(x^j_1(1) \geq (1/3)\delta\) for a type L. Thus, the pooling equilibria can be supported.

**(II)** \(\delta < 1/2\)

\[p_1 = 1\] if any proposal \(x^j_1(1)\) yields player 1 of a type L less than \(11/18 + (1/9)\delta\); and \(p_1 = 0\) otherwise.

If player 1 offers a proposal other than the equilibrium one, whose payoff is less than \(11/18 + (1/9)\delta\), she is considered as a type H. Player 2 will accept it; however, such a deviation is not profitable for neither type. If player 1 offers a proposal whose payoff is more than \(11/18 + (1/9)\delta\), she is considered a type L. However, this deviation will be rejected by players 2 and 3, since they each demand \(x^j_1(1) \geq 7/18 - (1/9)\delta\) if they are type H and \(x^j_1(1) \geq (7/18)\delta - (1/9)\delta^2\) if they are type L. Thus, the pooling equilibria can be supported. Q.E.D.