E C O N O M I C S   B U L L E T I N

Private Provision of Public Goods under Delegated Common Agency

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Abstract

This paper considers a delegated common agent who produces a public good with private information regarding his cost. We show that truthful strategies are not optimal for principals, and that the agent enjoys some rent in equilibrium. It is not always that all principals make contributions: the number of contracts with positive contributions accepted by the agent in equilibrium is non−increasing as the agent becomes less efficient.

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1 Introduction

In the standard voluntary provision of public good models, the allocations are usually inefficient in a Nash equilibrium because of the free-rider problem. However, the result has to be reconsidered when there is a common agent who produces the public good. The difference from the standard voluntary contribution games lies in that the contributions are contingent upon the equilibrium outcomes. Bagnoli & Lipman (1989, 92) consider a special common agency game where the public good is provided in indivisible units and conclude that a subset of Nash equilibria, referred to as undominated perfect equilibria, are always efficient. In a general situation where information is complete, Bernheim and Whinston (1986) show that truthful equilibria can always induce efficient outcomes.

This paper considers a delegated common agent who has private information regarding his cost in producing a public good. In particular, the agent can contract with any subset of principals. In most of the literature that deals with asymmetric information, only "intrinsic common agency," where the agent must either accept or reject *all* contracts, has been considered.[1] A notable exception is Laussel and Le Breton (1998), who analyze the same problem as in this paper but with a crucial difference: the agent considers the *ex ante* participation constraints, that is, he signs the contracts before realizing his type. In a so-called truthful equilibrium, the expected payoff for the agent is zero, and free-riding is never an optimal choice for the contributors. Hence, efficient outcomes are implemented and *all* contracts are accepted, so that the results in Bernheim and Whinstein (1986) are still valid.

It is quite different if the agent learns his type before signing the contracts, i.e. with *ex post* participation constraints. It is no longer optimal for the principals to use truthful strategies so that the equilibrium output is not at the efficient level. Moreover, it is possible that the agent will only contract with a subset of principals in equilibrium. Hence, the results in Bernheim and Whinston (1986) and Laussel and Le Breton (1998) fail to hold. Indeed, we show that players jointly prefer the output to be as large as possible. Since every principal who contributes extracts the information rent from the agent, this part of reduction in output is larger when the less efficient types of the agent accept more contracts. It follows that the less efficient types would like to turn down some undesirable contracts when the reduction in output is too serious. If this is the case, it is optimal for some principals to be free riders because they can enjoy a larger output and pay less. This results in that the number of principals who contribute and whose proposals are accepted by the agent is non-increasing as the agent becomes less efficient. This is obviously different from the previous literature, where all contracts are accepted in a truthful equilibrium.

[1] For example, see Martimort (1992, 1996a, b) and Stole (1991).

2 The Model

There are n consumers (principals) of a global public good. We denote the set of all principals by N. Each principal offers a procurement contract to a common agent who produces the public good. The game proceeds as follows. First, nature draws θ, the cost parameter of the agent. In the second stage, the principals simultaneously offer their (nonlinear) contribution schedules, $T_i(g)$, which are contingent upon the final output g.[2] In the final stage, the agent decides which proposals to accept. If the agent rejects all of them, everyone gets 0 and the game ends, otherwise he chooses the optimal output to maximize his profit and all payoffs are realized.

Principal i has a quasi-linear utility function, $V_i(g) - T_i$, $i \in N$. We assume that $V_i(g)$ is concave in g, $V_i(0) = 0$, and $\lim_{g\to\infty} V_i'(g) = 0$. The total cost of production for the agent is $\theta c(g)$, where $c'(g) > 0, c''(g) > 0$, and $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$. θ is private information to the agent, and it is common knowledge that θ has a distribution function $F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, and a continuous density function $f(\theta)$. We also assume that $\frac{\mathrm{d}}{\mathrm{d}\theta}(\frac{F(\theta)}{f(\theta)}) > 0$ for all θ. Given the set of contribution schedules $\{T_i\}_{i=1}^n$, the profit of the agent is $\Pi(A, \theta) = \sum_{j\in A} T_j - \theta c(g)$, where A is the set of principals who contribute and whose proposals are accepted by the agent. We also assume that $\exists\ g > 0 : \sum_{j=1}^n V_j(g) - [\bar{\theta} + N \cdot \frac{F(\theta)}{f(\theta)}]c(g) > 0$, which means that there exists a positive social surplus to provide the public good for all types. Lastly, we restrict the analysis to equilibria for which $g(\cdot)$ are continuous.

3 Nash Equilibrium

We denote a Nash equilibrium by $(\{T_i\}_{i=1}^n, A, g^A)$. In the last stage, given $\{T_i\}_{i=1}^n$, the agent chooses the optimal output g and the set A, such that $g^A(\theta) \in \arg\max_{\tilde{g}} \left[\sum_{j\in A} T_j(\tilde{g}) - \theta c(\tilde{g})\right]$. Hence, in equilibrium, $\sum_{j\in A} T_j'(g^A) = \theta c'(g^A)$.[3]

3.1 The Benchmark Case: Complete Information

When information is complete, given $\{T_j\}_{j\neq i}$, it is a standard result that Principal i's optimal T_i, *if he contributes*, satisfies $T_i'(g^A(\theta)) = V_i'(g^A(\theta))$. That is, T_i is locally truthful.

There exist equilibria where some principals do not contribute and become free riders so that the equilibrium outputs are inefficient. However, if we focus on (globally) truthful equilibria, the efficient allocations are always implemented in equilibrium. The reason for this is as follows. When principals use truthful strategies,

[2]With quasi-linear objective functions, the *Taxation Principle* allows us to consider *simple* nonlinear contribution schedules without loss of generality. See Martimort and Stole (2003).

[3]To consider the out-of-equilibrium behavior, $T_i(g)$ has to be appropriately extended for out-of-equilibrium outputs in order for the optimal solution to be always defined by equation (1). See the discussions in Martimort (1992) and Martimort and Stole (2003).

for another Principal j, $V_j(\tilde{g}^{\tilde{A}}) - \tilde{T}_j(\tilde{g}^{\tilde{A}}) = V_j(g^A) - T_j(g^A)$, where $\tilde{g}^{\tilde{A}}$ is the output when Principal i does not contribute. That is, the contribution of each principal *everywhere* reflects his true net willingness to pay with respect to the equilibrium output. If all principals contribute and the agent accepts all contracts, the payoff of Principal i is $V_i(g^A) - \theta\big[c(g^A) - c(\tilde{g}^{\tilde{A}})\big] + \big[\sum_{j\in N, j\neq i} T_j(g^A) - \sum_{j\in N, j\neq i} T_j(\tilde{g}^{\tilde{A}})\big]$.[4] On the other hand, Principal i obtains $V_i(\tilde{g}^{\tilde{A}})$ when he is a free rider. Since $V_i(g^A) - \theta\big[c(g^A) - c(\tilde{g}^{\tilde{A}})\big] + \big[\sum_{j\in N, j\neq i} V_j(g^A) - \sum_{j\in N, j\neq i} V_j(\tilde{g}^{\tilde{A}})\big] \geq V_i(\tilde{g}^{\tilde{A}})$ is always true because g^A maximizes the social surplus, it is a best response for Principal i to contribute. Therefore, in a truthful equilibrium, $A = N$. Therefore, the results in Bernheim and Whinston (1986) remain valid here. It follows that since $\sum_{i\in N} V_i'(g^A(\theta)) = \theta c'(g^A)$, which is indeed the Samuelson condition, the first-best output is achieved for every θ.

3.2 Asymmetric Information

Under asymmetric information, Principal i's maximization problem is:

$$\max_{\{g(\cdot),\Pi(\cdot)\}} \; E_\Theta\Big[V_i\big(g(\theta)\big) + \sum_{j\in A, j\neq i} T_j\big(g(\theta)\big) - \theta c\big(g(\theta)\big) - \Pi(A,\theta)\Big]$$

$$s.t. \;\; \frac{\mathrm{d}\Pi(A,\theta)}{\mathrm{d}\theta} = -c\big(g(\theta)\big),$$

$$\Pi(A,\theta) \geq \max\big[0, \Pi(K,\theta)\big] \;\; \forall\, K \subseteq N - \{i\} \text{ and } \forall\, \theta \in [\underline{\theta}, \bar{\theta}].$$

The incentive constraint is obtained from the envelope theorem. The individual rationality constraint means that, in equilibrium, the agent cannot be better off by accepting any other subset K or by rejecting them all. Then in equilibrium, if T_i is positive, we have[5]

$$T_i'\big(g^A(\theta)\big) = V_i'\big(g^A(\theta)\big) - \frac{F(\theta)}{f(\theta)}\, c'\big(g^A(\theta)\big), \tag{1}$$

$$\sum_{j\in A} V_j'(g^A(\theta)) = \big[\theta + |A| \cdot \frac{F(\theta)}{f(\theta)}\big]\; c'(g^A(\theta)), \tag{2}$$

where $|A|$ is the number of principals in the set $A(\theta)$. Equation (1) shows that truthful strategies are no longer optimal for the principals. Moreover, there is a positive profit in equilibrium for the agent with $\theta < \bar{\theta}$, i.e. $\Pi(A,\theta) = \int_{x=\theta}^{\bar{\theta}} c\big(g^A(x)\big)\mathrm{d}x > 0$. This is also different from the result under complete information.

We can show our first result:

[4] This is so because the agent's profit is always 0 in equilibrium and $T_k = 0$ if $k \in N - \tilde{A}$.

[5] To ensure that there exists a Nash equilibrium, we assume that $\sum_{j\in A} T_j$ is sufficiently concave and twice differentiable.

Proposition 1. *In a Nash equilibrium, for any $\tilde{A} \subseteq N$:*
$E_\Theta\big[V_i\big(g^A(\theta)\big) + \sum_{j\in A, j\neq i} T_j\big(g^A(\theta)\big) - \theta c\big(g^A(\theta)\big)\big] \geq E_\Theta\big[V_i\big(\tilde{g}^{\tilde{A}}(\theta)\big) + \sum_{j\in \tilde{A}, j\neq i} T_j\ \big(\tilde{g}^{\tilde{A}}(\theta)\big) - \theta c\big(\tilde{g}^{\tilde{A}}(\theta)\big)\big]$.

Proof. Suppose not. Principal i may offer $\tilde{T}_i$ (in which output $\tilde{g}^{\tilde{A}}$ is induced) such that $E_\Theta\big[V_i\big(g^A(\theta)\big) + \sum_{j\in A, j\neq i} T_j\big(g^A(\theta)\big) - \theta c\big(g^A(\theta)\big)\big] < E_\Theta\big[V_i\big(\tilde{g}^{\tilde{A}}(\theta)\big) + \sum_{j\in \tilde{A}, j\neq i} T_j\big(\tilde{g}^{\tilde{A}}(\theta)\big) - \theta c\big(\tilde{g}^{\tilde{A}}(\theta)\big)\big]$. However, since $\Pi(A,\theta) \geq \Pi(\tilde{A},\theta)$ for any θ, it follows that $E_\Theta\big[V_i\big(g^A(\theta)\big) - T_i\big(g^A(\theta)\big)\big] < E_\Theta\big[V_i\big(\tilde{g}^{\tilde{A}}(\theta)\big) - \tilde{T}_i\big(\tilde{g}^{\tilde{A}}(\theta)\big)\big]$. That is, there exists a profitable deviation for Principal i, which contradicts the fact that the $\{T_i\}$ are equilibrium schedules. □

This generalizes the result in Laussel and Le Breton (1998), where each principal offers a contribution schedule such that the output maximizes the sum of her expected utility and the expected profit of the agent. Contrary to them, however, with *ex post* participation constraints, the agent obtains a positive profit in equilibrium.

The next proposition claims that, in equilibrium, players jointly prefer a set A such that the output is maximal.

Proposition 2. *In a Nash equilibrium, for any $\tilde{A} \subseteq N$, $g^A(\theta) \geq \tilde{g}^{\tilde{A}}(\theta)$ for any θ.*

Proof. For the most inefficient type, $\bar{\theta}$, his rent is always 0, and so the principals can offer $\{T_i\}$ such that $g^A(\bar{\theta}) \geq \tilde{g}^{\tilde{A}}(\bar{\theta})$, and obtain the highest utility at $\bar{\theta}$.

Suppose that the claim is not true, and there exists an $\tilde{A}(\theta_k)$ such that $\tilde{g}^{\tilde{A}}(\theta_k) > g^A(\theta_k)$ for any $\theta_k \in [\theta', \theta'') = \Theta_\epsilon$, where $|\Theta_\epsilon| = \epsilon$; and $g^A(\theta) \geq \tilde{g}^{\tilde{A}}(\theta)$ for any $\theta \in [\theta'', \bar{\theta}]$. The types $\theta \geq \theta'$ have no incentives to deviate as long as the rent is higher when they make contracts with the principals in $A(\theta)$; that is, $\Pi(A,\theta) = \int_{x=\theta}^{\bar{\theta}} c\big(g^A(x)\big)\mathrm{d}x \geq \Pi(\tilde{A},\theta) = \int_{x=\theta}^{\bar{\theta}} c\big(\tilde{g}^{\tilde{A}}(x)\big)\mathrm{d}x$ holds.

However, even so, there can exist a profitable deviation for the principals for the following reason. Consider the following schedules that induce a higher output $\hat{g} > g^A$: Principal $i \in A$ offers $\{\tilde{T}_i\}$ for θ_k, where $\sum_{j\in A} \tilde{T}'_j(\hat{g}) = \theta c'(\hat{g})$ and $\tilde{T}_i(\hat{g}) = T_i(\hat{g}) + \varepsilon$; and the same $\{T_i\}$ for any other $\theta \geq \theta''$. For an arbitrarily small ϵ, every type $\theta \geq \theta'$ will accept these new schedules since the agent can obtain at least the same rent as before, because now $\{\tilde{T}_i\}$ is higher or at least the same as $\{T_i\}$.[6] On the other hand, every principal will enjoy a higher output at θ_k. According to equation (1), the equilibrium schedules must make the net utility $V_i - T_i$ increase in g (since $V'_i - T'_i$ is positive), which implies that $E_{\Theta_\epsilon}\big[V_i\big(\hat{g}(\theta)\big) - \tilde{T}_i\big(\hat{g}(\theta)\big)\big] \geq E_{\Theta_\epsilon}\big[V_i\big(g^A(\theta)\big) -$

[6] To make this possible, consider a two-part tariff, $\{t_i(\theta), s_i(\theta)\}$, such that $t_i g + s_i = T_i$. To induce the agent to produce a higher $\hat{g}$ in Θ_ϵ, the principals can let $\sum_{j\in A} \tilde{t}_j = \theta c'(\hat{g})$, and $\tilde{T}_i(\hat{g}) = \tilde{t}_i\hat{g} + \tilde{s}_i$ for θ_k. By carefully choosing $\tilde{s}_i$, the principals can control the rent such that it is increased only by an infinitesimal amount compared with the original $\{T_i\}$ such that the agent will still accept them.

$T_i\big(g^A(\theta)\big)\big]$ if $i \in A$, and $E_{\Theta_\epsilon} V_i\big(\hat{g}(\theta)\big) \geq E_{\Theta_\epsilon} V_i\big(g^A(\theta)\big)$ if $i \notin A$. Hence, for an arbitrary ϵ, all the principals and the agent can be better off, which contradicts the assumption that the $\{T_i\}$ are equilibrium schedules. Hence, $g^A(\theta) \geq \tilde{g}^{\tilde{A}}(\theta)$ holds for any $\theta \geq \theta'$.

Since the same logic can be applied to another smaller θ_k having $\tilde{g}^{\tilde{A}}(\theta_k) > g^A(\theta_k)$, by induction, $g^A(\theta) \geq \tilde{g}^{\tilde{A}}(\theta)$ must be true for *every* θ. □

This result suggests that all the principals and the agent prefer the output produced at its maximal level. In the case of the agent, his rent is higher when the output is larger. For each principal, according to equation (1), the marginal contribution is less than the marginal benefit due to the information rent. The equilibrium level of the public good is thus *under-provided* compared to the first-best level, where $V_i'(g) = T_i'(g)$, and so the principal indeed has the incentive to expand the output. Thus, the principals will make contributions such that the output determined in equation (2) is the largest through the selection of A, in which case they cannot give up more rent to produce a higher output and also benefit themselves.

Another important feature in a truthful Nash equilibrium with complete information or *ex ante* participation is that all principals make contributions, i.e. $A = N$ for every type. We show in the following result that this is not always the case under asymmetric information:

Proposition 3. *In equilibrium,* $A(\theta'') \subseteq A(\theta')$ *if* $\theta'' > \theta'$.

Proof. First of all, the incentive constraint implies that $g^A(\cdot)$ is non-increasing in Θ, that is, $g^A(\theta') \geq g^A(\theta'')$ if and only if $\theta' < \theta''$. Second, if $i \notin A(\theta')$, then $T_i(g^A(\theta')) = 0$. We show as follows that $T_i(g^A(\theta)) \leq 0$ for all $\theta > \theta'$, so that $i \notin A(\theta'')$. When $T_i(g^A(\theta')) = 0$, Principal i obtains $V_i(g^A(\theta'))$. If we suppose that $T_i(g^A(\theta'')) > 0$, she then obtains $V_i(g^A(\theta'')) - T_i(g^A(\theta''))$ if $i \in A(\theta'')$, and $V_i(\tilde{g})$ if $i \notin A(\theta'')$, where $\tilde{g} \in \arg\max_{g^K} \Pi(K, \theta'')$ for all $K \subseteq N - \{i\}$. From equation (2), $\tilde{g}$ is close to $g^A(\theta')$ if θ'' is close to θ', since the agent can at least select $A(\theta'') = A(\theta')$. However, since $g^A(\theta'') \leq g^A(\theta')$, $V_i(g^A(\theta'')) - T_i(g^A(\theta'')) \leq V_i(\tilde{g})$. Therefore, $T_i(g^A(\theta'')) \leq 0$. We can infer that $T_i \leq 0$ for all $\theta > \theta'$, and hence $i \notin A(\theta'')$. This implies that $A(\theta'') \subseteq A(\theta')$. □

Under asymmetric information, since every principal who contributes extracts the information rent, this part of the reduction in output is larger when the less efficient types of agent accept more contracts. It follows that the less efficient types will be better off by turning down some undesirable contracts when the reduction in output is too serious. If this is the case, it will be optimal for some principals to become free riders because they will then be able to enjoy a larger output and pay less. In particular, fewer principals will want to contribute when the agent becomes less efficient.

According to Proposition 2, since the agent selects an $A(\theta)$ such that g^A is maximal for every θ, the following result is immediate:

Proposition 4. *Suppose in equilibrium, $g^A(\theta) > 0$ for every θ. Then:*
(i) *If $A < N$, then $V_j'(g^A) > V_k'(g^A)$ for any $j \in A$ and $k \notin A$.*
(ii) *If $V_j(\cdot) = V(\cdot)$ for all j, then $A = N$.*

When not all principals contribute in equilibrium, it is because they have enough heterogeneous preferences. In particular, the agent wants to contract with those who have the highest valuations. On the other hand, if the principals have homogeneous preferences, the agent contracts with all of the principals. This result is intriguing because it implies that the equilibrium allocation under delegated common agency can be very different from the one under intrinsic common agency, especially when the principals' preferences are not homogeneous, unless *every* principal in the society obtains a sufficiently high marginal benefit from the public good compared to its cost. Hence, it is reasonable to view intrinsic common agency as a special case since it rarely happens in an equilibrium outcome when the number of contracts that the agent can select is flexible.

3.3 An Example: $n = 2$.

Suppose that there are two principals, $i, j \in \{1, 2\}$. We denote in equilibrium $\Theta^{**} = \{\theta : A(\theta) = \{1, 2\}\}$, and $\Theta^* = \{\theta : A(\theta) = \{i\}\}$. We also denote the equilibrium outputs by g^{**} and g_i^*, which satisfy (2) respectively. Then according to Proposition 3, there are only two possible outcomes in equilibrium: (i) $\Theta^{**} = [\underline{\theta}, \bar{\theta}]$ and $\Theta^* = \phi$; (ii) $\Theta^{**} = [\underline{\theta}, \theta_0)$ and $\Theta^* = [\theta_0, \bar{\theta}]$, where $g^{**}(\theta) = g_i^*(\theta)$ at θ_0.

If the reduction in the output is large enough for types in $[\theta_0, \bar{\theta}]$, so that $g^{**} < g_i^*$, the agent has the incentive to reject one contract, and it is also optimal for Principal j to offer $T_j = 0$ for $\theta > \theta_0$, because he can save money and also enjoy a larger output, and thus obtain a higher expected utility.

For example, suppose that $V_i(g) = k_i \cdot g^{\frac{1}{2}}$, $i = 1, 2$. Without loss of generality, suppose that $k_1 \geq k_2$. In addition, $c(g) = g$, and θ is distributed uniformly on $[1, 2]$. It can be shown that $\theta_0 = \frac{k_1 - k_2}{k_1 - 2k_2}$ if and only if $k_2 < \frac{1}{3}k_1$, where $T_2 = 0$ in equilibrium for all $\theta \geq \theta_0$. That is, when the preferences of the two principals are heterogeneous enough, in equilibrium, the types in $[\theta_0, 2]$ contract with only one principal.

Suppose we double the number of both kinds of principals. We can show that $\theta_0 = \frac{k_1 - k_2}{k_1 - 1.5k_2}$, if and only if $k_2 < \frac{1}{2}k_1$, where $A = \{1, 1, 2, 2\}$ in $[1, \theta_0)$, and $A = \{1, 1\}$ in $[\theta_0, 2]$. Note that $\tilde{\theta} = \frac{k_1 - k_2}{k_1 - 1.5k_2} < \frac{k_1 - k_2}{k_1 - 2k_2}$, that is, the set where all contracts are accepted becomes smaller. We can expect that, in a large economy, it becomes rare for the output under intrinsic common agency to be implemented in equilibrium.

4 Concluding Remarks

Under full information, the efficient output can be implemented in a truthful equilibrium through a common agent. When information is asymmetric, in order to induce the agent to reveal his true type, truthful strategies are not optimal for principals so that the equilibrium output is inefficient. The distortion in output could be significantly large when less efficient types of the agent contract with all principals. Since the agent can choose any subset of proposals, he is better off to not accept all contracts. In perceiving this, it is optimal for some principals to stop contributing because they can enjoy a larger output while paying less.

It is often suggested that intrinsic and delegated common agency have no allocative differences when contracting activities are complements. This is true in cases where there are no direct contractual externalities (Stole, 1991). However, this paper shows that, when there are direct contractual externalities and asymmetric information between principals and the agent, there is a substantial impact on allocations under delegated common agency. This suggests that intrinsic common agency can only be viewed as a benchmark. Further research in providing the general profile of equilibria under delegated common agency (when contracting activities can be either complementary or substitutive) with asymmetric information would definitely be of value.

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