Measuring Business Cycles: A Temporal Disaggregation Model with Regime Switching

Abstract

In this paper, we propose a temporal disaggregation model with regime switches to disaggregate U.S. quarterly GDP into monthly figures. Alternative to the existing literature, our model is able to capture the nonlinear behaviors of both aggregated and disaggregated output series as well as the asymmetric nature of business cycle phases. To demonstrate the applicability of the proposed model, we apply the model with a Markov trend component to U.S. quarterly real GDP. The results suggest that the combination of a temporal disaggregation model with Markov switches leads to a successful representation of the data relative to the existing literature. Also, the inferred probabilities of unobserved states are clearly in close agreement with the NBER reference cycle on a monthly basis, which highlights the importance of nonlinearities in business cycle.

Keywords: business cycle asymmetries; Markov trend; regime-switching model; temporal disaggregation.

JEL Classification: C22. C51, E31

1 Introduction

Real GDP (or GNP) is always considered as one of the most important coincident indicators in measuring business cycles. In practice, however, common indices used to identify business cycle phases do not include real GDP. For example, though the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER) considers U.S. real GDP as the single best measure of aggregate economic activity, the Dating Committee shows particular interests in four other monthly indicators (i.e., real personal income less transfer payments, employment, industrial production and total sales of the manufacturing and wholesale-retail sectors) to determine the months of peaks and troughs. The infrequent use of real GDP is mainly attributed to the fact that real GDP is constructed on a quarterly basis. Without a statistically rigorous method to disaggregate the quarterly data into monthly series, real GDP plays no major role in the prediction of business cycle turning points and the construction of short-term economic indicators.

For better use of real GDP, many researchers have proposed numbers of statistical techniques to circumvent the disaggregation problem. One main approach is to use temporal disaggregation models which have been discussed by a number of authors, one of the first of whom was Friedman (1962). Recent contributions have been made by Harvey and Chung (2000), Santos Silva and Cardoso (2001), Proietti (2004, 2006), Mönch and Uhlig (2005), and Stock and Watson (2010a), to name a few.[1] Though these models can capture some properties of output series, they fail to account for the nonlinear and asymmetric nature of business cycle phases while disaggregating the quarterly real GDP. In fact, the linearity imposed by these models may imply a built-in symmetry in the disaggregated output series, which forces expansions and recessions to have the same magnitude, duration, and amplitude. In addition, when the disaggregated series is aggregated back into quarterly GDP, the linear property is preserved and implies a built-in symmetry for quarterly figures as well. Such a linear and symmetric framework for quarterly GDP is inconsistent with the stylized facts (cf. Sichel, 1993; Milas et al.,

[1] Mönch and Uhlig (2005) propose a unified state-space framework which nests a few of the prominent interpolation methods (e.g., Chow and Lin, 1971; Fernández, 1981) in the literature. Stock and Watson (2010a) use a similar state-space framework to carry out interpolation for components of U.S. nominal GDP.

2006). It is therefore helpful to consider a model that can accommodate the temporal disaggregation of real GDP, business cycle asymmetries and the nonlinear characteristics of both aggregated and disaggregated output dynamics.

In this paper, we propose a nonlinear model to disaggregate real GDP, and demonstrate how such a modeling framework can be applied to analyzing business cycle dynamics. To capture asymmetries in business cycles, we assume that the underlying unobserved monthly GDP follows a Markov switching process considered by Hamilton (1989). Under the restriction that the sum of the three monthly GDPs for each quarter must be equal to the published value for the quarter, the proposed model has a state-space representation with switching parameters, and hence the estimation algorithm derived by Kim (1994) can be used to obtain the expected monthly GDP. One attractive feature of our model is that it is able to draw optimal inferences regarding the monthly unobserved regimes using real GDP, departing from the Markov switching models of Hamilton (1989) and Chauvet and Hamilton (2006) which only provide the information of unobserved regimes at quarterly intervals. Moreover, compared to the models of Wei and Stram (1990), Proietti (2006), and Stock and Watson (2010a) which impose a built-in symmetry in the aggregated and disaggregated output series, the Markovian structure here is preserved under aggregation and entails nonlinear and asymmetric dynamics for both monthly and quarterly GDP series.

To demonstrate the applicability of the proposed model, we apply the proposed model to U.S. seasonally adjusted, quarterly real GDP data for the period of 1952:I–2008:IV (with 228 observations). We use the sample period of 1952:I–2007:IV for the estimation and the remaining data for the out-of-sample forecasts. Our results indicate that our modeling approach reveal important features of output data. We find that the inferred probabilities of the monthly unobserved regimes are strongly correlated with the NBER business cycle dates, suggesting that the proposed model can be used as an alternative approach for dating business cycle turning points.[2] Moreover, we find that the proposed model generates

[2] Our approach to dating business cycle turning points is conceptually related to the "aggregate then date" approach discussed in Stock and Watson (2010b). In addition, unlike the approach of Chauvet and Piger (2003), the proposed model cannot identify business cycle turning points in real time. Only when all relevant data are available, we can identify business cycle phases.

a smooth path of monthly GDP in all in-sample periods and outperforms conventional models in out-of-sample forecasts. Also, we apply a monthly structural vector autoregression (SVAR) framework of Bernanke et al. (1997) to study the effect of oil price shocks and the role of the monetary policy response.

The remainder of this paper is organized as follows. Section 2 introduces the temporal disaggregation model with Markov switches, and discusses the estimation procedure and hypothesis testing. Section 3 applies the proposed model to postwar U.S. data on real GDP. Section 4 presents our conclusions.

2 Temporal Disaggregation with Markov Switches

There is an abundance of empirical evidence to suggest that the time series behaviors of output series may exhibit different patterns over time. Instead of using one model for the output dynamics, it is of interest to employ several models to represent these patterns. A Markov switching model, which is constructed by combining two or more dynamic models via a Markovian switching mechanism, is commonly used to characterize the distinct patterns. In this section, we consider the dynamics of U.S. output by assuming that the underlying unobserved monthly GDP follows a Markov switching process, subject to the constraint that the sum of the three monthly GDPs for each quarter ought to equal the observed GDP for the quarter. We also illustrate several features of the proposed model and briefly discuss the estimation algorithm and hypothesis testing.

2.1 The Proposed Model

Let $\ddot{y}_\tau$ be the seasonally-adjusted, quarterly real GDP at time τ and $\ddot{\boldsymbol{y}}_\tau = (0\ 0\ \ddot{y}_\tau)'$ be a 3×1 vector of observations. We first stack the observations $\ddot{\boldsymbol{y}}_1, \ldots, \ddot{\boldsymbol{y}}_T$ in a one-column vector to obtain $\boldsymbol{y} = (\ddot{\boldsymbol{y}}_1'\ \ddot{\boldsymbol{y}}_2'\ \ldots\ \ddot{\boldsymbol{y}}_T')'$ and denote the t^{th} element of $\boldsymbol{y}$ as y_t for $t = 1, \ldots, 3T$, where T is the number of quarterly observations. Then we assume that the unobserved monthly GDP, y_t^*, satisfies the sum up constraint:

$$y_t = \sum_{i=0}^{2} y_{t-i}^*, \quad t = 3, 6, 9, \ldots, 3T. \tag{1}$$

We further assume that the unobserved monthly GDP may be specified as

$$\begin{aligned} y_t^* &= n_t^* + z_t^*, \\ n_t^* &= n_{t-1}^* + \mu_0(1 - s_t^*) + \mu_1 s_t^*, \\ \Psi(B)\Delta z_t^* &= \alpha + \Phi(B)\varepsilon_t^*, \quad t = 1, 2, \dots, 3T, \end{aligned} \tag{2}$$

where n_t^* is the "Markov trend in level" of Hamilton (1989), $\Delta z_t^* = z_t^* - z_{t-1}^*$ is a stationary autoregressive and moving-average (ARMA) component of the monthly GDP, $s_t^* = \{0, 1\}$ denotes an unobserved monthly state variable whose law of motion is governed by a first-order Markov chain with the transition matrix

$$\begin{bmatrix} \mathbb{P}(s_t^* = 0 \mid s_{t-1}^* = 0) & \mathbb{P}(s_t^* = 1 \mid s_{t-1}^* = 0) \\ \mathbb{P}(s_t^* = 0 \mid s_{t-1}^* = 1) & \mathbb{P}(s_t^* = 1 \mid s_{t-1}^* = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix},$$

ε_t^* is an i.i.d. $N(0, \sigma_{\varepsilon^*}^2)$ sequence that is independent of n_{t+i}^* for all i, and $\Psi(B) = 1 - \psi_1 B - \cdots - \psi_p B^p$ and $\Phi(B) = 1 + \varphi_1 B + \cdots + \varphi_q B^q$ are finite-order polynomials of the back-shift operator B such that they have no common factors and their roots are all outside the unit circle.

The proposed model in equations (1) and (2) has some novel features. First, y_t^* of the proposed model is specified as the sum of a Markov trend component and an ARIMA component. Such a specification is similar to those in Hamilton (1989), Kim and Nelson (1999a), and Chauvet and Hamilton (2006), among many others. A major difference is that the Markovian switching mechanism in our model is applied to the underlying latent monthly GDP. Hence, when the estimation algorithm of the proposed model is derived, it is capable of assessing the likelihoods of the monthly state variables s_t^* using quarterly GDP. Many existing models, on the other hand, postulate that the switching mechanism is applied to the published quarterly GDP. As such, only the likelihoods of quarterly state variables are obtainable; see, e.g., Hamilton (1989), Kim (1994), Kim and Nelson (1999a), and Chauvet and Hamilton (2006).

Second, for the proposed model, the dynamics of unobserved monthly GDP are governed by the Markov state variables s_t^*. When the monthly data are aggregated back into quarterly series, the Markov property is preserved because the sum of current and previous states is also Markovian. To see this, we note that the quarterly GDP can be expressed as $y_t = N_t^* + Z_t^*$, where $N_t^* = \sum_{i=0}^{2} n_{t-i}^*$ and $Z_t^* = \sum_{i=0}^{2} z_{t-i}^*$. By setting $S_t^* = \sum_{i=0}^{2} s_{t-i}^* = \{0, 1, 2, 3\}$, we have

$$\begin{aligned} N_t^* &= N_{t-1}^* + \mu_0 \sum_{i=0}^{2} (1 - s_{t-i}^*) + \mu_1 \sum_{i=0}^{2} s_{t-i}^* \\ &= N_{t-1}^* + \mu_0 (3 - S_t^*) + \mu_1 S_t^*. \end{aligned} \tag{3}$$

As shown in Lam (1990), the term S_t^* in (3) is Markovian, and hence the quarterly component N_t^* is a four-state (at most) Markov trend. In addition, Stram and Wei (1986a) and Marcellino (1999) have shown that the quarterly component Z_t^* is still of the ARIMA type. Consequently, the quarterly GDP consists of a Markov trend component and an ARIMA component; the Markov property is unchanged upon temporal aggregation. This feature enables us to represent many nonlinear business cycle patterns via aggregated quarterly GDP. For example, by using quarterly GDP or GNP, numerous studies have applied Markov switching models to study asymmetries in cyclical expansions and contractions and the differences in the dynamics of business cycle phases.

On the other hand, the traditional temporal disaggregation techniques of Chow and Lin (1971), Stram and Wei (1986b), Wei and Stram (1990), and Proietti (2006) typically consider a linear model to capture the dynamic patterns of unobserved monthly GDP. To illustrate this, let $s_t^* = 0$ with probability one for all t in (2). In this special case, the unobserved monthly GDP becomes $y_t^* = \mu_0 t + z_t^*$ and the proposed model simply reduces to the disaggregation scheme considered by Stram and Wei (1986b) and Wei and Stram (1990). When the monthly series are aggregated to quarterly observations, the dynamic structure of quarterly figures becomes $Y_t = 3\mu_0 t + Z_t^*$ which is still linear and of the ARIMA type; see Marcellino (1999).

Finally, as the dynamic of y_t^* satisfies the sum up constraint (1), we can follow the approach of Harvey and Pierse (1984) and set up the proposed model in state-space form to estimate the expected monthly GDP. The estimation results may provide an alternative view of the characteristics of monthly real GDP. They can also serve as an important coincident index to measure aggregate economic activity.

The proposed model can be easily extended to include monthly information.

To see this, the unobserved monthly GDP may be specified as

$$
\begin{aligned}
y_t^* &= n_t^* + z_t^*, \\
n_t^* &= n_{t-1}^* + \mu_0(1 - s_t^*) + \mu_1 s_t^*, \\
\Psi(B)\Delta z_t^* &= \alpha + \boldsymbol{x}_t'\boldsymbol{\beta} + \Phi(B)\varepsilon_t^*, \quad t = 1, 2, \ldots, 3T,
\end{aligned} \tag{4}
$$

where the $\boldsymbol{x}_t$ are some GDP-related monthly series. By setting $r = \max(p, q+1)$, the model in (1) and (4) can be expressed as a state-space model with the following measurement and transition equations:

$$
\begin{aligned}
y_t &= \boldsymbol{h}_t'\boldsymbol{\gamma}_t, \\
\boldsymbol{\gamma}_t &= \boldsymbol{\mu}(\boldsymbol{x}_t)_{s_t^*} + \boldsymbol{F}\boldsymbol{\gamma}_{t-1} + \boldsymbol{R}\varepsilon_t^*
\end{aligned} \tag{5}
$$

for $t = 1, 2, \ldots, 3T$, where a $(r+3)$-dimensional vector $\boldsymbol{h}_t = (1\ 0\ \cdots\ 0\ 1\ 2\ 1)'$ if $t = 3, 6, 9, \ldots, 3T$ and $\boldsymbol{h}_t = \boldsymbol{0}$, otherwise;

$$
\boldsymbol{\gamma}_t = \begin{bmatrix} \Delta z_t^* \\ \sum_{i=2}^r \psi_i \Delta z_{t-i+1}^* + \sum_{i=2}^r \varphi_{i-1}\varepsilon_{t-i+2}^* \\ \sum_{i=3}^r \psi_i \Delta z_{t-i+2}^* + \sum_{i=3}^r \varphi_{i-1}\varepsilon_{t-i+3}^* \\ \vdots \\ \psi_r \Delta z_{t-1}^* + \varphi_{r-1}\varepsilon_t^* \\ \Delta n_t^* \\ y_{t-1}^* \\ y_{t-2}^* \end{bmatrix}_{(r+3)\times 1}, \quad \boldsymbol{\mu}(\boldsymbol{x}_t)_{s_t^*} = \begin{bmatrix} \alpha + \boldsymbol{x}_t'\boldsymbol{\beta} \\ 0 \\ 0 \\ \vdots \\ 0 \\ \mu_0(1-s_t^*) + \mu_1 s_t^* \\ 0 \\ 0 \end{bmatrix}_{(r+3)\times 1},
$$

$\psi_i = 0$ for $i > p$ and $\varphi_i = 0$ for $i > q$. The terms $\boldsymbol{F}$ and $\boldsymbol{R}$ are fixed matrices such that

$$
\boldsymbol{F} = \begin{bmatrix} \psi_1 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \psi_2 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \psi_{r-1} & 0 & 0 & \cdots & 1 & 0 & 0 & 0 \\ \psi_r & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 \end{bmatrix}_{(r+3)\times(r+3)}, \quad \boldsymbol{R} = \begin{bmatrix} 1 \\ \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_{r-1} \\ 0 \\ 0 \\ 0 \end{bmatrix}_{(r+3)\times 1}.
$$

Once the model has been put into the state-space form with switching coefficients, the "collapsing" Kalman filter developed in Kim (1994) is applied, and this results in algorithms for filtering and smoothing. In what follows, we will use the extended model (4) to identify business cycle phases.

2.2 Model Estimation and Hypothesis Testing

To start the "collapsing" filter at time $t = 1$, some initial values are needed. These values are supplied either by their limiting unconditional counterparts or by any arbitrary values, as suggested in Kim and Nelson (1999a). Given the initial values, Kim's (1994) algorithm is readily available for inferences on the unobserved state vector $\boldsymbol{\gamma}_t$, which is conditional upon the parameters of the model and the information set. Thus, the expected unobserved monthly GDP (i.e., the last two elements of $\boldsymbol{\gamma}_t$ and the sum of Δz_t^*, Δn_t^*, and y_{t-1}^* in $\boldsymbol{\gamma}_t$) can be extracted from the published quarterly GDP, subject to the sum up constraint. Moreover, the filtering probabilities of monthly state variables, the smoothing probabilities of monthly state variables, and an approximate log-likelihood function can also be obtained as by-products. The approximate maximum likelihood estimates (MLE),

$$\hat{\boldsymbol{\theta}} = (\hat{\mu}_0, \hat{\mu}_1, \hat{\alpha}, \hat{\beta}, \hat{\psi}_1, \ldots, \hat{\psi}_p, \hat{\varphi}_1, \ldots, \hat{\varphi}_q, \hat{\sigma}_{\varepsilon^*}, \hat{p}_{00}, \hat{p}_{11})',$$

can then be found using a numerical-search method. In order to identify the state in the proposed model, $\hat{\mu}_1$ is restricted to be positive in our estimation algorithm. Our program is written in GAUSS which employs the BFGS (Broyden-Fletcher-Goldfarb-Shanno) search algorithm. By plugging $\hat{\boldsymbol{\theta}}$ into the formulae of filtering (smoothing) probabilities, we obtain the estimated filtering (smoothing) probabilities of monthly state variables. These probabilities are used to identify recession and expansion periods in the U.S. A detailed derivation of the estimation algorithm is given in Kim (1994).

To justify whether the Markov switching model is appropriate, it is natural to consider the following hypotheses: (1) the switching parameters (μ_0 and μ_1) are in fact the same; (2) the state variables s_t^* are independent. Rejecting the first hypothesis suggests that switching does occur in y_t^*. Failure to reject the second hypothesis is evidence against the Markovian structure, yet rejecting this hypothesis provides only a partial support for Markov switching. Note that under the null hypothesis of $\mu_0 = \mu_1$, the likelihood function of model (4) is nonquadratic and flat with respect to the nuisance parameters at the optimum and the scores are identically zero. Hence, conventional statistics do not have an asymptotic standard χ^2-distribution under the null hypothesis. To test this hypothesis, Hansen (1992), Garcia (1998), Carrasco et al. (2005), and recently Cho and White (2007) have proposed several solutions. However, their solutions cannot be applied to our case because constructing these test statistics requires information on the monthly GDP which is not observable here. We therefore follow Di Sanzo (2007) and use Monte Carlo analysis to examine the significance of the Markov switching form of y_t^*.

More specifically, we note that the monthly GDP $y_t^* = \mu t + z_t^*$ in (4) is a nonstationary process under the first hypothesis $\mu_0 = \mu_1 = \mu$. This implies that the quarterly GDP $y_t = 3\mu t + \mathcal{Z}_t^*$ is also nonstationary, where $\mathcal{Z}_t^*$ is an ARIMA process with some GDP-related quarterly variables. Thus, given the quarterly GDP and these related series, we first estimate an array of ARIMA models and choose an appropriate specification based on an information criterion; e.g., Akaike information criterion (AIC) or Schwartz information criterion (SIC). We then save the standardized residuals of the selected ARIMA model. We also estimate an array of proposed models and select a suitable specification. Based on these two estimation results, we can obtain the likelihood ratio statistic which is denoted as $\hat{LR}$. The selected ARIMA model is then taken as the data generating process to generate simulated samples by bootstrapping the standardized residuals. For each simulated sample, we re-estimate these two selected models and construct the likelihood ratio statistics. Replicating this procedure many times yields a finite-sample reference distribution of the likelihood ratio statistic on which we can compute the p-value of $\hat{LR}$. We reject the null hypothesis if the p-value of $\hat{LR}$ is small, say, less than 5%. As shown in Di Sanzo (2007), this bootstrap-based test works well and outperforms the Hansen (1992) test and the Carrasco et al. (2005) test. Note that this test does not solve all the problems when there are unidentified nuisance parameters under the null, but it is used to provide some justification of the proposed model.[3] A better testing procedure would be highly desirable but is

[3]As mentioned above, the asymptotic distributions of conventional statistics are very difficult to obtain under the null hypothesis. The bootstrap method discussed here only provides an approximation of the asymptotic distributions.

beyond the scope of this paper.

3 Empirical Study

3.1 Monthly Business Cycle Measurement

To demonstrate the applicability of the proposed model, we apply the model in (5) with a Markov trend component to U.S. quarterly real GDP. The aim of this paper is to disaggregate the quarterly data into monthly figures and to extract the likelihoods of monthly state variables from the quarterly GDP. The prevailing approaches for the temporal disaggregation of published quarterly GDP include (1) a method that involves the use of observed related series at the desired higher frequency, and (2) a method that only relies on pure time series dynamic models. The former approach, discussed in Chow and Lin (1971), Litterman (1983) Mönch and Uhlig (2005), and Stock and Watson (2010a), employs the index of industrial production as the related series to disaggregate GDP. The second approach, explored by Stram and Wei (1986b) and Wei and Stram (1990), depends on the ARIMA dynamic structure of the series to be disaggregated. These temporal disaggregation techniques do not, however, take account of the business cycle asymmetries and nonlinear business cycle dynamics. In the literature, leading models for exploring business cycle dynamics include the Markov switching models of Hamilton (1989), the current depth of the recession models of Beaudry and Koop (1993), and the plucking models of Friedman (1993). By using the quarterly GDP as an indicator, these existing models can only provide quarterly information on business cycles. As the proposed model provides more in-depth information regarding economic states at the monthly level and bridges the gap between temporal disaggregation techniques and the regime-switching models, it would be interesting to know if it is capable of accounting for the monthly fluctuations in U.S. real GDP.

The data set is taken from the Bureau of Economic Analysis. We take real GDP and the monthly industrial production index as $\ddot{y}_\tau$ and $\boldsymbol{x}_t$ and estimate an array of models in (5) with $0 \leq p, q \leq 4$. The parameters are estimated using the algorithm described in Kim (1994). This algorithm is initialized by a broad range of random initial values. The covariance matrix of $\hat{\boldsymbol{\theta}}$ is $-\boldsymbol{H}(\hat{\boldsymbol{\theta}})^{-1}$, where $\boldsymbol{H}(\hat{\boldsymbol{\theta}})$ is the

Table 1: Approximate maximum likelihood estimates of the proposed state-space model.

Parameter	Estimate	Standard error	t-statistic
z_t^* component:			
$\hat{\alpha}$	0.00338	0.00186	1.81720*
$\hat{\beta}$	0.06431	0.03198	2.01094*
$\hat{\psi}_1$	0.96932	0.51220	1.89246*
$\hat{\varphi}_1$	-0.29591	0.13869	-2.13359*
$\hat{\varphi}_2$	-0.83200	0.64360	-1.29273
$\hat{\varphi}_3$	-0.29159	0.15300	-1.90582*
$\hat{\sigma}_{\varepsilon^*}$	13.07249	1.52805	8.55501*
Markov trend component:			
$\hat{\mu}_0$	-6.18484	1.90661	-3.24389*
$\hat{\mu}_1$	7.91320	1.16461	6.79472*
$\hat{p}_{00}$	0.88018	0.05151	
$\hat{p}_{11}$	0.97894	0.01067	
Log-Likelihood=-1134.83		SIC=2341.11	AIC=2291.67

Note: t-statistics with one asterisk are significant at the 5% level.

Hessian matrix of the log-likelihood function evaluated at the approximate MLE $\hat{\boldsymbol{\theta}}$. Among all the models considered, both AIC and SIC select the ARMA$(1,3)$ model for Δz_t^*. The estimation results are summarized in Table 1. As the table shows, all parameter estimates are statistically significant at the 5% level except for $\hat{\varphi}_2$.

To check the model's adequacy, we first apply the Monte Carlo analysis in Section 2.2 to the published quarterly GDP. We estimate an array of ARIMA(m,n) models with m and n being no greater than 4; the SIC selects the ARIMA(1,1,1) model:

$$\Delta\ddot{y}_\tau = 946.2422 + 0.9683\Delta\ddot{y}_{\tau-1} - 0.7869u_{\tau-1} + 12.6495X_t + u_\tau \tag{6}$$

with $\sigma_u = 29.5421$, where X_t denotes the quarterly industrial production index. The log-likelihood value of the selected ARIMA(1,1,1) model in (6) is -1160.73 and the resulting likelihood ratio statistic is $\hat{LR} = 51.8$. We generate the simulated data by bootstrapping standardized residuals of equation (6). We then re-estimate the proposed model using the simulated data and obtain the likelihood ratio statistic. Using 3,000 replications we obtain a finite-sample reference distribution of $\hat{LR}$. The p-value of $\hat{LR}$ based on this simulated distribution is about 0.0027 and hence we reject the model in equation (6) at the 5% significance level. In addition, we also test whether the state variables s_t^* are independent over time. Following Engel and Hamilton (1990), this amounts to testing whether $p_{00}+p_{11}=1$. The resulting Wald statistic is 10.499 and the null hypothesis of $p_{00}+p_{11}=1$ is rejected at the 5% level under the $\chi^2(1)$ distribution. The rejection of the null hypothesis justifies our Markovian specification of the state variable.

Since $\hat{\mu}_0 < \hat{\mu}_1$ in Table 1, it suggests that $s_t^* = 0$ ($s_t^* = 1$) corresponds to a "recessionary" ("expansionary") state. In addition, the estimated transition probabilities ($\hat{p}_{00} \approx 0.87$, $\hat{p}_{11} \approx 0.97$) suggest that expansions are more persistent than recessions, much like the NBER reference cycle. Indeed, the expected durations of expansions and recessions can be calculated from the transition probabilities: $1/(1-0.97894) = 47.483$ months for an expansion and $1/(1-0.88018) = 8.345$ months for a recession. According to NBER dating, the average durations for expansions and recessions are, respectively, 57.00 and 11.33 months. Compared with NBER dating, our results indicate shorter expected durations for both states. Similar results have also been found in Hamilton (1989) and Kim (1994) which apply the Markov switching model to the quarterly real GDP from 1952:II to 1984:IV.

In Figure 1, we plot the published quarterly GDP and the estimated monthly series in the left and right figures, respectively. The shaded areas denote the recession periods identified by NBER. To evaluate the performance of the proposed model and other temporal disaggregation techniques such as Chow and Lin (1971), Litterman (1983), Wei and Stram (1990), Santos Silva and Cardoso (2001), and Mönch and Uhlig (2005), we compare these disaggregated monthly GDPs with the Macroeconomic Advisers' index of monthly GDP.[4] We follow Chow and Lin (1971) and select the index of industrial production as the observed related monthly indicator for these temporal disaggregated techniques. Table 2 presents the correlation

[4]The Macroeconomic Advisers' index of monthly GDP is an indicator of real aggregate output that is conceptually consistent with real GDP in NIPA. The data are extracted from the web site: www.macroadvisers.com.

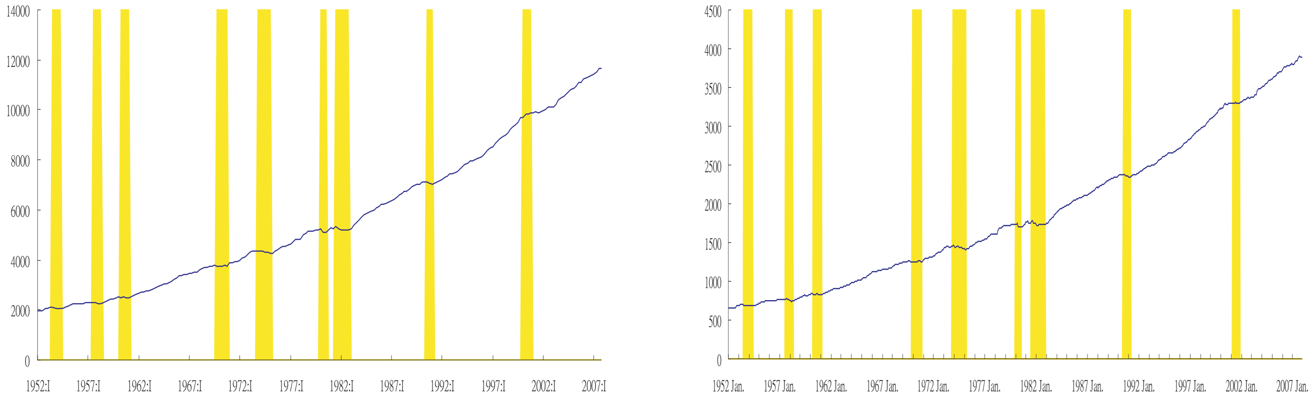

Figure 1: Published quarterly GDP (left) and estimated monthly GDP (right).

coefficients between the monthly GDP index of Macroeconomic Advisers and these disaggregated series from 1959:1 to 2007:12. In this table we can see that, when the levels of the series are considered, the correlation coefficient is higher than 0.999 for all the methods. However, when the growth rates of the series are compared, we find that the Macroeconomic Advisers' index of monthly GDP has a higher degree of correlation with our monthly GDP. These findings suggest that the estimated monthly series could be treated as a monthly time series of real GDP.

In Table 3, we report the 1-step to 12-step ahead out-of-sample forecasts for the monthly real GDP using the approaches of Chow and Lin (1971), Litterman (1983), Wei and Stram (1990), Santos Silva and Cardoso (2001), Mönch and Uhlig (2005), and the proposed model. The associated quarterly forecasts and the published quarterly real GDP are also given. For comparison, we also report the 1-step to 4-step ahead out-of sample forecasts of the Markov switching model for quarterly GDP.[5] In this table, the forecasts of monthly GDP from 2008:01 to 2008:12 for the proposed model are 3908.04 billion, 3914.25 billion, ..., 3976.68 billion, and 3983.59 billion, respectively. These results show that the predicted quarterly GDP for the periods 2008:I – 2008:IV are 11743.48, 11805.22, 11867.69 and 11930.01, while the associated mean square error (MSE) is 51516.8. Compared with the MSEs from other approaches, the proposed model possesses the smallest MSE

[5] We apply the Markov switching model of Hamilton (1989) to the log of real GDP for the period 1985:I to 2007:IV. A detailed result is available upon request.

Table 2: Correlations between the monthly index and the disaggregated series.

	M.A.	Proposed Model	Chow & Lin	Litterman	Wei & Stram	Santos Silva & Cardoso	Mönch & Uhlig
Levels:	1	0.99972	0.99966	0.99972	0.99970	0.99972	0.99972
Growth Rates:	1	0.55938	0.37284	0.42272	0.42095	0.45789	0.37569

Note: M.A. denotes the Macroeconomic Advisers' index of monthly GDP.

which is about 1/2 of the MSE of the simple Markov switching model. In addition, the MSE of the proposed model is only about 1/13 of the MSEs of Chow and Lin (1971), Litterman (1983) and Santos Silva and Cardoso (2001). Note also that the proposed nonlinear model forecasts better than the linear model used in Wei and Stram (1990). This forecasting result apparently shows that the proposed model achieves better prediction power and thereby proves to serve as a better temporal disaggregation model.

In Figure 2 we plot the estimated filtering and smoothing probabilities of $s_t^* = 0$ (the recessionary state) in the panel on the left and in that on the right, respectively. In this figure, we can observe that the monthly filtering and smoothing probabilities are clearly in close agreement with the NBER reference cycle on a monthly basis. For seven of the nine NBER recessions in the sample, both filtering and smoothing probabilities spike up by more than 50% after the business cycle peak date established by the NBER.[6] It is also worth mentioning that the proposed model successfully identifies the recession period that started in March 2001 which is at the end of the sample. These results are consistent with the view that conventional regime-switching models may provide reasonable inferences on the probabilities of a recession (or an expansion). Clearly, an advantage of the proposed model is that it may extract the likelihoods of monthly state variables from the published quarterly GDP. Many existing models, on the other hand, provide only valuable information regarding the quarterly state variables. When compared with dating business cycles at the quarterly level, our empirical results are thus likely to provide more precise information as to the exact turning points.

[6] The 1953 and 1960 recessions are the exceptions. For these recessions both probabilities move up during the NBER recession dates, but remain below 50%.

Table 3: The out-of-sample forecasts of U.S. real GDP.

Units of Measure of GDP: Billians of Dollars

Date	Published GDP	Markov Switching	Proposed Model	Chow & Lin	Litterman	Wei & Stram	Santos Silva & Cardoso	Mönch & Uhlig
2008:01			3908.04	3738.02	3738.02	3925.33	3737.94	3756.71
2008:02			3914.25	3738.02	3723.93	3934.32	3723.84	3750.21
2008:03			3921.19	3726.09	3726.10	3943.32	3726.02	3747.77
2008:I	**11646.0**	**11774.51**	**11743.48**	**11202.13**	**11188.05**	**11802.97**	**11187.80**	**11254.69**
2008:04			3928.13	3699.01	3699.01	3952.35	3698.93	3724.30
2008:05			3935.07	3691.04	3691.05	3961.39	3690.97	3718.04
2008:06			3942.02	3709.31	3709.32	3970.45	3709.23	3735.39
2008:II	**11727.4**	**11873.44**	**11805.22**	**11099.36**	**11099.38**	**11884.19**	**11099.13**	**11177.73**
2008:07			3948.96	3663.49	3663.49	3979.53	3663.41	3695.06
2008:08			3955.91	3622.10	3622.10	3988.62	3622.02	3658.71
2008:09			3962.82	3475.37	3475.37	3997.74	3475.31	3527.57
2008:III	**11712.4**	**11973.05**	**11867.69**	**10760.96**	**10760.96**	**11965.89**	**10760.74**	**10881.34**
2008:10			3969.75	3524.36	3524.36	4006.87	3613.42	3572.57
2008:11			3976.68	3478.62	3478.63	4016.03	3578.21	3532.33
2008:12			3983.59	3391.50	3391.51	4025.20	3509.90	3454.84
2008:IV	**11522.1**	**12073.60**	**11930.01**	**10394.48**	**10394.50**	**12048.10**	**10394.28**	**10559.74**
MSE		102,481.6	51,516.8	692,049.6	695,204.1	97,539.5	695,570.7	518,014.6

Note: MSE stands for mean square error. The predicted value of, e.g., 2008:I (2008:II), is the sum of the forecasts from 2008:01 to 2008:03 (2008:04 to 2008:06).

Note that, unlike the approach of Chauvet and Piger (2003), our model cannot identify business cycle turning points in real time. Only when all relevant data are available and the early data revisions are completed, we can use the historical data and identify business cycle phases based on estimated filtering and smoothing probabilities.

3.2 The Effect of Oil Price Shocks

Other than identifying business cycle turning points, we can also use the estimated monthly real GDP to investigate economic questions. For example, recent papers by Bernanke et al. (1997; 2004) suggest that monetary policy could be used to eliminate any recessionary consequences of an oil price shock. Using the estimated monthly GDP, we thus apply a monthly SVAR model of Bernanke et al. (1997) to study the effect of oil price shocks and the role of the monetary policy response. Note that previous studies investigate the effects of shocks at quarterly intervals;

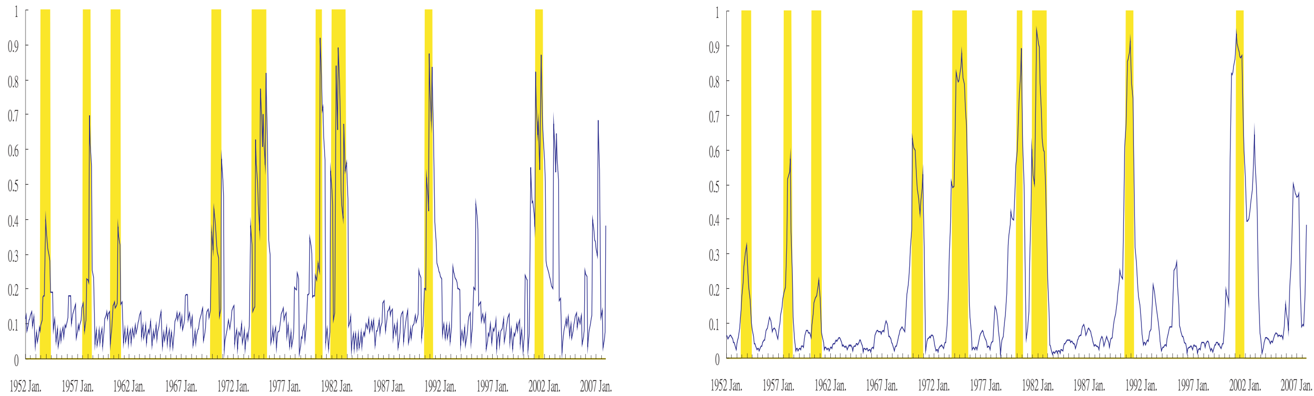

Figure 2: Filtering (left) and smoothing (right) probabilities of the recessionary state.

see, e.g., Bernanke et al. (2004) and Herrera and Pesavento (2009).

Our SVAR model is specified by using seven variables over the period 1960:1-2007:12. These variables include the rate of growth of estimated monthly real GDP ($\tilde{y}_{GDP,t}$), the log of the GDP deflator ($\tilde{y}_{P,t}$), the log of the commodity price index ($\tilde{y}_{COM,t}$), Hamilton's (1996) net oil price increase measure ($\tilde{y}_{OIL,t}$), the Fed funds rate ($\tilde{y}_{FED,t}$), the 3-month Treasury bill rate ($\tilde{y}_{TB3,t}$), and the 10-year Treasury bond rate ($\tilde{y}_{TB10,t}$).[7] The SVAR model is specified as

$$\boldsymbol{A}_0\tilde{\boldsymbol{y}}_t = \boldsymbol{c}_0 + \boldsymbol{A}_1\tilde{\boldsymbol{y}}_{t-1} + \cdots + \boldsymbol{A}_{\tilde{p}}\tilde{\boldsymbol{y}}_{t-\tilde{p}} + \boldsymbol{v}_t \tag{7}$$

with $\tilde{\boldsymbol{y}}_t = \{\tilde{y}_{GDP,t}, \tilde{y}_{P,t}, \tilde{y}_{COM,t}, \tilde{y}_{OIL,t}, \tilde{y}_{FED,t}, \tilde{y}_{TB3,t}, \tilde{y}_{TB10,t}\}$, where $\boldsymbol{A}_0$ is a lower triangular matrix with ones along the principal diagonal, and the lagged Fed funds rate are assumed to exert its macroeconomic effects only through the short-term and long-term interest rates (so the row i, column 5 element of $\boldsymbol{A}_j$ is zero for $i = 1, 2, 3, 4$ and $j = 1, 2, \ldots, \tilde{p}$). That is, the policy instrument, the Fed funds rate, has no independent effect on the economy. This SVAR model is estimated by OLS, equation by equation. The lag length $\tilde{p}$ is set to 12 in accordance with previous studies; see, e.g., Bernanke et al. (2004), Hamilton and Herrera (2004), and Herrera and Pesavento (2009).

[7] We use the same interpolation process discussed in Bernanke et al. (1997) to obtain the monthly GDP deflator.

To check on the reasonableness of the estimated system in (7), we calculate the impulse response functions to determine the effect of a 10% increase in the net oil price on the value of each element of $\tilde{\boldsymbol{y}}_t$. The resulting impulse response functions are plotted as the solid lines in Figure 3. As can be seen in this figure, the estimated results are reasonable, with all variables exhibiting their expected qualitative behaviors. In particular, a 10% increase in oil prices would result in 0.32% slower real GDP growth and 0.2% higher prices after 6 quarters, with the Fed funds rate rising 50 basis points within the first year. These findings are consistent with the results in Bernanke et al. (1997) who consider the case in which the monetary policy is allowed to respond to the oil shock. Following Bernanke et al. (2004), we also consider a counterfactual scenario in which the response of monetary policy is "shut off" for 12 months. The dashed lines in Figure 3 plot the response of the economy to a 10% oil price shock when we shut down the monetary policy response for one year. We find that the oil shock reduces output and raises the price level, as occurs in the previous case. However, shutting off the monetary policy response for a year reduces the depressing effect of the oil shock on output by about 20% on average (based on the point estimates), which is smaller than those reported by Bernanke et al. (1997, 2004). This result is consistent with Hamilton and Herrera's (2004) and Herrera and Pesavento's (2009) findings that the adverse effect of the oil shock on output is moderate when the endogenous response of the Fed funds rate is shut off.

4 Conclusion

In this paper, we propose a temporal disaggregation technique with regime switches to disaggregate quarterly U.S. GDP. The model has several interesting features. First, it allows researchers to specify different dynamic patterns in unobserved monthly GDP by using a Markovian switching mechanism. Second, it can capture the asymmetric nature business cycle phases and describe nonlinear characteristics of output dynamics at both monthly and quarterly levels. Third, it can disaggregate the quarterly data into monthly figures and obtain optimal inferences of unobserved economic states on a monthly basis. Thus, the proposed model bridges the gap between temporal disaggregation techniques and regime-switching models

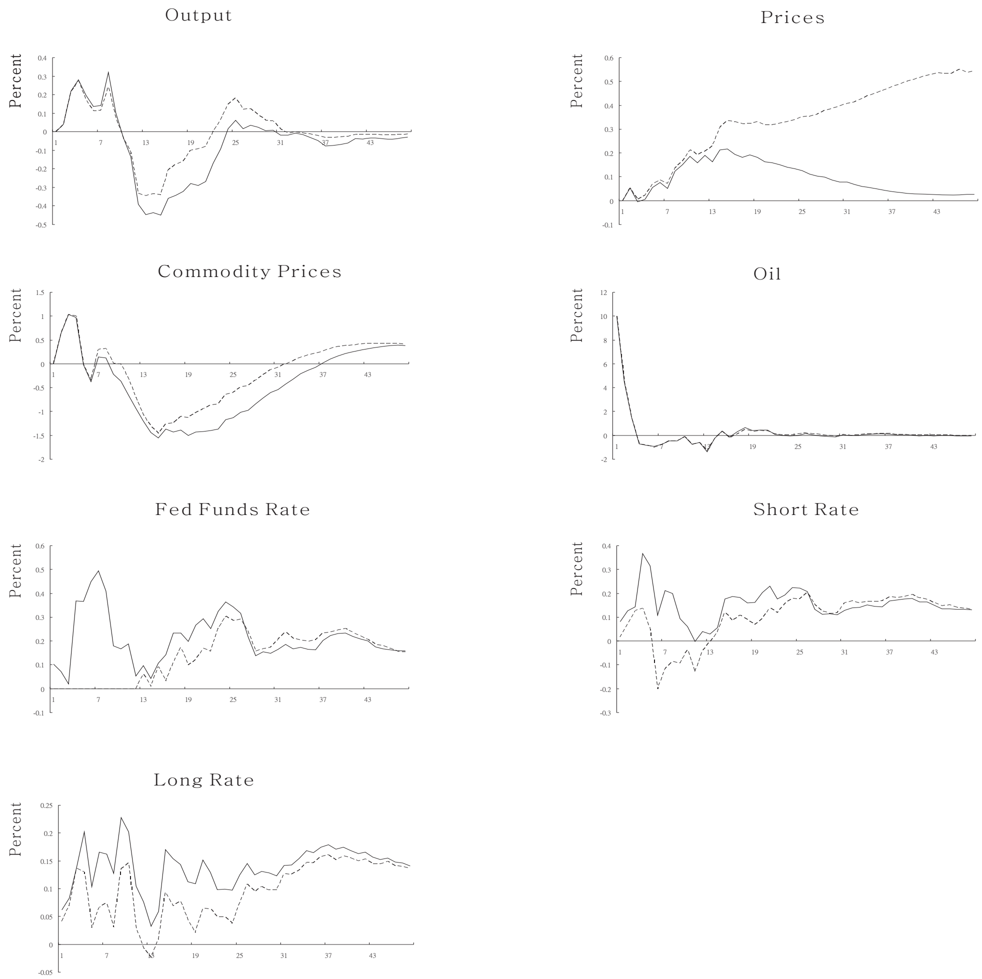

Figure 3: Impulse response functions for effect of 10% oil price increase.

and is able to accommodate underlying asymmetries in business cycles.

The application of the proposed model to U.S. quarterly real GDP suggests that our model provides a useful analytical tool in describing the data characteristics. In particular, it shows that the optimally inferred dates of business cycle turning points exhibit a strong correlation with the NBER dating of business cycles on a monthly basis. This result differs from those of Hamilton's (1989) models in that dating business cycles at the monthly level is available based on the published quarterly GDP. Our empirical results also show that the estimated monthly GDP and published quarterly GDP share a very similar dynamic smoothing pattern during the period of analysis, suggesting that the estimates of monthly GDP may serve as an alternative coincident index to measure economic activity and represent business cycle asymmetries. In addition, the proposed model provides more accurate out-of-sample forecasts than those obtained from other selected models. Finally, the estimated monthly real GDP can be used in studying various economic problems. One example is to study the effects of oil shocks, which has been reported in previous section. As another example, recent work finds evidence that the volatility of the U.S. economy fell dramatically in the mid-1980s (e.g., Kim and Nelson, 1999b; McConnell and Quiros, 2000). According to our estimated monthly GDP, the standard deviation of U.S. real GDP growth during the 1984:01–2002:12 period was 60.29% smaller than that during the 1960:01–1983:12 period.[8] A number of papers call this phenomenon the "Great Moderation". Using our estimated monthly GDP, we may ask whether the reaction of monetary policy to a specific shock (e.g., oil prices) contributed to the "Great Moderation", which can be one direction of our future research.

[8] Stock and Watson (2002) report that the standard deviation of U.S. quarterly real GDP growth during the period of 1984:I–2002:IV was 61% smaller than that during the 1960:I–1983:IV period.

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