Re-examining Long-Run PPP under an Innovation Regime Switching Framework

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Abstract

This paper avoids the usual dichotomy between unit-root nonstationarity and stationarity in testing long-run PPP and re-examine this hypothesis based on the Innovation Regime-Switching model of Kuan, Huang, and Tsay (2005, *JBES*). This model permits the random shock in each period to be permanent or transitory, depending on a swtiching mechanism, and hence results in distinct dyanmics (unit-root nonstationarity or stationarity) in different periods. Our empirical study on centuried U.S./U.K. real exchange rates shows that there are both temporary and permanent influences on the real exchange rate such that approximately 42% of the shocks in the long run are more likely to have a permanent effect. It is also found that transitory shocks dominate in the fixed-rate regimes, yet permanent shocks play a more important role during the floating regimes. Thus, long-run PPP is rejected due to the presence of a significant amount of permanent shocks, and there are still long periods of time in which the deviations from long-run PPP are only transitory. Moreover, after the distinct effects of shocks are properly accounted for, the half-life of a given transitory shock is considerably shorter than those reported in the literature.

Keywords: Innovation regime-switching model, permanent shock, purchasing power parity, transitory shock.

JEL Classification: C51, F31, F41

1 Introduction

Long-run purchasing power parity (PPP), which asserts that equilibrium exchange rates tend to equalize the ratio of national price levels, is a fundamental equilibrium relationship in international economics. This long-run relationship is a cornerstone of dynamic exchange rate models (e.g., Dornbusch, 1976; Mussa, 1982). It also provides a benchmark exchange rate and hence has some practical appeal to arbitragers and policy makers. See Officer (1976), Froot and Rogoff (1995), MacDonald (1995) and Rogoff (1996) for comprehensive reviews of the PPP theory.

Despite its theoretical importance, long-run PPP received mixed empirical evidences from different studies. In practice, it is common to test long-run PPP by examining whether real exchange rate is a stationarity series (or nominal exchange rate is cointegrated with price levels in a specific form). Since Adler and Lehmann (1983) and Meese and Rogoff (1983), there have been numerous researches on long-run PPP that rely mainly on tests of unit-root, stationarity, and cointegration. Examples include Abuaf and Jorion (1990), Kim (1990), Grilli and Kaminsky (1991), Glen (1992), MacDonald (1993), Cheung and Lai (1998), Culver and Papell (1999), Kuo and Mikkola (1999), Cuddington and Liang (2000), Taylor (2002), Lopez, Murray and Papell (2005), Papell (2006), Papell and Prodan (2006) and Kanas (2006), to name just a few. The conclusions, however, may vary with the tests (also how these tests are implemented) and the sample periods considered in these studies.

Although the test power and data span may be responsible for the contradictory conclusions on long-run PPP, they can not be the only reasons. It should be noted that the rejection of unit root (stationarity) does not necessarily imply that the series must be stationary (unit-root nonstationary). It is thus somewhat simplistic to draw a conclusion about real exchange rate based only on these tests. The dynamic properties of real exchange rates may be more complex than those of a unit-root model or a linear stationary model. This leads researchers to consider different models for real exchange rates, such as the ARFIMA model (Diebold, Husted, and Rush, 1991), stationary models with breaks (Culver and Papell, 1995; Papell and Prodan, 2006), the Markov-swtiching model (Engel and Kim, 1999), and the TAR model (Taylor, 2001). These studies are still restrictive, in the sense that only one model structure is permitted throughout the sample period. Even for the models with structural breaks or Markov switching, it is the parameter, not the model per se, that changes with different regimes.

In this paper we avoid the usual dichotomy between unit-root nonstationarity and stationarity and re-examine long-run PPP based on a flexible model, the Innovation Regime-

Switching (henceforth IRS) model, recently proposed by Kuan, Huang, and Tsay (2005). Intuitively, it is hard to believe that all random shocks exert only one effect (permanent or transitory) on future real exchange rate in a long time span. This intuition underpins the models that allow for breaks and stochastic unit root (e.g., Culver and Papell, 1995; Papell and Prodan, 2005; Kanas, 2006). It is also supported by the opposite conclusions on PPP in different regimes (e.g., Grilli and Kaminsky, 1991; Lothian and Taylor, 1996). As an alternative, the IRS model permits the random shock in each period to be permanent or transitory, depending on a swtiching mechanism, and hence admits distinct dyanmics (unit-root nonstationarity or stationarity) in different periods. Under the IRS framework, standard unit-root models and stationarity models are just two extreme cases. By applying the IRS model to real exchange rate, we circumvent the difficulties arising from unit-root (or stationarity) testing. More importantly, we allow the data to speak for themselves, rather than putting them in the straitjacket of unit-root nonstationarity (or stationarity).

Our empirical study on centuried U.S./U.K. real exchange rates reveals interesting data characteristics. We find the presence of both temporary and permanent influences on the real exchange rate over the entire sample. The simulation-based tests suggest that neither an ARIMA model nor an ARMA model can properly characterize the data. It is also found that transitory shocks dominate in the fixed-rate regimes, yet permanent shocks play a more important role during the floating regimes. Hence, there are both mean-reverting and parity-deviating behaviors in this sample. These findings, while leading to rejection of long-run PPP, indicate that the behavior of real exchange rate is quite different from that asserted in the literature. First, long-run PPP is rejected due to the presence of a significant amount of permanent shocks. Second, there are still long periods of time in which the deviations from long-run PPP are only transitory. These results are compatible with that of Kanas (2006). They are also consistent with the existing results that unit-root nonstationarity is more evident in the floating periods and help to explain why such a conclusion may alter when more pre-float data are included. Moreover, after the effects of shocks are properly accounted for, the half-life of a given transitory shock is considerably shorter than those reported in the literature.

The rest of the paper is organized as the following. In section 2, we briefly discuss the PPP theory and show the danger of relying on unit-root tests to determine PPP. In section 3, we introduce the IRS model and describe model estimation and hypothesis testing. The empirical analysis of U.S./U.K. real exchange rate is presented in section 4. Second 5 concludes the paper. A detailed description of the estimation algorithm is given in Appendix.

2 Long-Run PPP and Unit Root Tests

Long-run PPP states that the "fundamental" or "equilibrium" exchange rate E_t^* is determined by the ratio of domestic price P_t and foreign price Q_t:

$$E_t^* = A\frac{P_t}{Q_t},$$

where A is an arbitrary constant. Let E_t denote the nominal exchange rate; also let e_t, p_t and q_t be the logarithms of E_T, P_t and Q_t, respectively. An empirical representation of the long-run PPP relationship is

$$e_t = \alpha + \beta_1 p_t + \beta_2 q_t + u_t, \tag{1}$$

where $\alpha = \ln A$, and u_t is a disturbance capturing the deviation from the logarithm of the equilibrium exchange rate. A typical approach to testing long-run PPP is to impose the symmetry condition ($\beta_1 = -\beta_2 = \beta^*$) and the proportionality condition ($\beta^* = 1$) in (1) and check the stationarity property of the logarithm of the real exchange rate, $r_t \equiv e_t - p_t + p_t^*$. In the literature, PPP is said to hold in the long run when r_t is driven by transitory shocks, i.e., the deviation from the equilibrium exchange rate is temporary. When r_t has a unit root, the shocks to r_t all have a permanent effect, so that long-run PPP breaks down.

As pointed out earlier, opposite conclusions on long-run PPP may result when different tests and/or different data are used. To be sure, we first evaluate the log of U.S./U.K. real exchange rate for the period of 1885:01 to 1995:02 and various subperiods classified according to Grilli and Kaminsky (1991).[1] Note that this data set has been used by Grilli and Kaminsky (1991) and Engel and Kim (1999) and is downloaded from the *Journal of Money, Credit and Banking* Data Archives. We consider 4 tests: the Augmented Dickey-Fuller (ADF) test, the ADF-GLS test of Elliott et al. (1996), the P-P test of Phillips and Perron (1988), and the KPSS test of Kwiatkowski et al. (1992). The testing results are summarized in Table 1. The ADF test is based on the following regression:

$$\Delta r_t = \alpha_0 + \beta_0 r_{t-1} + \sum_{j=1}^{k} \beta_j \Delta r_{t-j} + \epsilon_t, \tag{2}$$

[1]According to Grilli and Kaminsky (1991), the following subperiods are classified as a fixed-rate regime: 1885:01 – 1919:06 (Classical Gold Standard), 1925:05 – 1931:08 (Gold Exchange Standard), and 1949:10 – 1972:05 (Bretton Woods). The floating regime periods are: 1919:04 – 1925:04 (First Inter-War floating), 1931:09 – 1939:08 (Second Inter-War floating), 1973:03 – 1995:02 (Post-Bretton Woods). The rest of periods, 1914:07 – 1919:03 and 1939:09–1949:09, are denoted as Wartime Controls.

Table 1: Unit root tests for the logarithm of the real exchange rate (1885:01 – 1995:02).

Subperiods	ADF	ADF-GLS	P-P	KPSS
Data used in Engel and Kim (1999) (1885:01 $\sim$ 1995:02)	-3.278^{*}	-3.233^{*}	-2.678	0.597^{*}
Data used in Grilli and Kaminsky (1991) (1885:01 $\sim$ 1986:12)	-4.022^{*}	-4.027^{*}	-3.711^{*}	0.492^{*}
Classical Gold Standard (1885:01 $\sim$ 1914:06)	-2.506	-2.329^{*}	-2.582	0.754^{*}
First Inter-War floating (1919:04 $\sim$ 1925:04)	-2.535	-1.025	-2.584	0.445^{*}
Gold Exchange Standard (1925:05 $\sim$ 1931:08)	-2.462	-0.730	-2.170	0.965^{*}
Second Inter-War floating (1931:09 $\sim$ 1939:08)	-1.866	-1.530	-1.383	0.874^{*}
Bretton Woods (1949:10 $\sim$ 1972:05)	-1.620	-2.075^{*}	-1.027	1.555^{*}
Post-Bretton Woods floating (1973:03 $\sim$ 1995:02)	-1.553	-0.752	-1.568	1.128^{*}

Note: The 5% critical values for the ADF, ADF-GLS, P-P, and KPSS statistics are -2.864, -1.942, -2.864, and 0.463, respectively. Statistical significance at the 5% level is indicated by *.

where $\Delta r_t = r_t - r_{t-1}$ denotes the change of r_t and ϵ_t is the disturbance term. The lag length k in (2) was chosen by the Bayesian information criterion with a maximum lag length of 12. The ADF-GLS test is based on the specification (2) but substitutes the GLS detrended data for r_t. The P-P test allows for weakly dependent disturbances without including lagged Δr_t in (2). The KPSS test focuses on the null hypothesis of stationarity around a deterministic mean. The P-P and KPSS tests are computed with a Newey-West heteroskedastic and autocorrelation consistent estimator for the residual spectrum at frequency zero. Details of these tests are omitted to save space but can be found in Stock (1994).

Table 1 shows that there are mixed evidences for long-run PPP, depending on the test and the sample period being considered. For example, for the data used in Engel and Kim (1999), the presence of a unit root is rejected at 5% level by the ADF and ADF-GLS tests but receives support from both the P-P and KPSS tests. As another example, for the shorter sample period used in Grilli and Kaminsky (1991), the null hypothesis of a unit root is rejected by the ADF, ADF-GLS and P-P tests at 5% level, but this hypothesis is unable to rejecte by the KPSS test. In addition, the ADF-GLS test clearly rejects the unit root hypothesis for the Classical Gold Standard and the

Bretton Woods periods at 5% leve, but the other tests lead to an opposite conclusion. Only in the floating-rate regime and the Gold Exchange Standard priod are the testing results more consistent (rejecting long-run PPP). The contradictory results for a given period are usually attributed to the power properties of different tests and/or the length of the sample period, cf. Lopez et al. (2005) and Taylor (2002). On the other hand, the mixed conclusions on PPP across different periods may be an indication that the dynamic patterns of real exchange rate in fact change from time to time and hence can not be fully characterized by a unit-root model or a linear stationary model.

There have been some attempts to characterize real exchange rate using more complex models. For example, Engel and Kim (1999) built a Markov-swtiching model with changing volatility across different regimes. In the context of unit-root testing, Culver and Papell (1995) and Papell and Prodan (2006) based their tests on models that allow for structural breaks, whereas Kanas (2006) considered models with random AR coefficients. In fact, Kanas (2006) found evidence that real exchange rate may exhibit regime-dependent stationarity such that parity deviation and mean reversion are present in different periods.

3 IRS Models

In our study, we employ a variant of the IRS model introduced in Kuan et al. (2005). We shall briefly describe the model specification, estimation and hypothesis testing in the following subsections; more details of this model can be found in Kuan et al. (2005).

3.1 Model Specification

The IRS model is an unobserved component model consisting of a unit-root component and a stationarity component such that there is a switching mechanism determining the prevailing component at each time. Specifically, the logarithm of the real exchange rate, r_t, is expressed as $r_t = r_{1,t} + r_{0,t}$, with

$$\begin{aligned} \Gamma(B)\Delta r_{1,t} &= s_t \upsilon_t, \\ \Psi(B) r_{0,t} &= (1 - s_t)\upsilon_t, \end{aligned} \tag{3}$$

where $\Gamma(B) = 1 - \gamma_1 B - \cdots - \gamma_n B^n$ and $\Psi(B) = 1 - \psi_1 B - \cdots - \psi_m B^m$ are the polynomials of the backshift operator B such that they have no common factors and their roots are all outside the unit circle, s_t is a unobserved state variable taking the value of one or zero, and υ_{s_t} are uncorrelated random variables with mean zero and variances depending on s_t: $\sigma^2_{s_t}$. This model will be referred to as an IRS$(n, 1; m)$ model, signifying one component ($r_{1,t}$) has an ARIMA$(n, 1, 0)$ structure and the other ($r_{0,t}$) has a stationary AR(m) structure. Compared with the IRS model originally considered by Kuan et al. (2005), the model (3) allows for more general short-run dynamics in the first component and accommodates potential asymmetry in volatility across different regimes by permitting switching variances in the random shocks.

A feature of model (3) is that only one component is activated in a given time period, depending on the realization of s_t. When $s_t = 1$, the first component $r_{1,t}$ is excited by the random shock, while $r_{0,t}$ keeps evolving according to AR dynamics without the new shock. As long as $s_t = 1$, the corresponding random shock has a permanent effect on future r_{t+j} $(j > 0)$ and generates unit-root type dynamics. When $s_t = 0$, the random shock activates $r_{0,t}$ while leaving $r_{1,t}$ intact. The random shock thus has a transitory effect on future r_{t+j} and results in stationary AR dynamics. This model specification permits the effect of a random shock to alternate from time to time and exhibits both nonstationary and stationary behaviors. In particular, when $s_t = 1$ ($s_t = 0$) with probability one for all t, the model (3) simply reduces to a conventional unit-root or a stationary AR model.

The reduced form of the IRS model (3) is

$$r_t = \Delta^{-1}\Gamma^*(B)s_t\upsilon_t + \Psi^*(B)(1 - s_t)\upsilon_t,$$

where $\Delta = (1 - B)$, $\Gamma^*(B) = \Gamma^{-1}(B)$ and $\Psi^*(B) = \Psi^{-1}(B)$. The first term on the right-hand side is understood as a flexible stochastic trend whose behavior depends on the number of $s_t = 1$, whereas the second term is a stationarity component. Moreover, assuming that the initial variables $\upsilon_i = 0$ for $i \leq 0$, the decomposition of Beveridge and Nelson (1981) yields

$$\begin{aligned} r_t &= \Gamma^*(1)\Delta^{-1}s_t\upsilon_t + \Delta^{-1}\big[\Gamma^*(B) - \Gamma^*(1)\big]s_t\upsilon_t + \Psi^*(B)(1 - s_t)\upsilon_t \\ &= \Gamma^*(1)\sum_{i=1}^{t} s_i\upsilon_i + \sum_{i=1}^{t}\tilde{\gamma}_{t-i}s_i\upsilon_i + \sum_{i=1}^{t}\psi^*_{t-i}(1 - s_i)\upsilon_i, \end{aligned} \tag{4}$$

where $\tilde{\gamma}_i$ and ψ^*_i are the coefficients of $\Delta^{-1}\big[\Gamma^*(B) - \Gamma^*(1)\big]$ and $\Psi^*(B)$, respectively. The decomposition of (4) provides an interesting interpretation of r_t. As $s_i\upsilon_i$ and $(1 - s_i)\upsilon_i$ are both present in the stationarity compoent (the second and third terms on the right-hand side), each υ_i must have a transitory effect regardless of the value of corresponding s_i (though the magnitude of such effect depend on s_i), but υ_i may also have a permanent effect when $s_i = 1$. As such, transitory deviations from PPP in the IRS model are "standard," while permanent deviations from PPP are "exceptional" and take place only when $s_i = 1$.

The IRS model is in contrast with the model that includes both permanent and transitory shocks at each time, e.g., Engel and Kim (1999). As these shocks together must have a permanent effect, such a model already implies the failure of long-run PPP. The IRS model also differs from the model considered by Kanas (2006): $r_t = \beta_{s_t} r_{t-1} + u_t$, where $\beta_{s_t} = 1$ when $s_t = 1$ and $\beta_{s_t} = \beta^*$ with $|\beta^*| < 1$ when $s_t = 0$. Under this random AR coefficient framework, it can be seen that

$$r_t = \sum_{i=1}^{t-1} \left(\prod_{j=0}^{i-1} \beta_{s_{t-j}} \right) u_{t-i}.$$

Then provided that $s_i = 0$ infinitely often, $\prod_{j=0}^{i-1} \beta_{s_{t-j}}$ would be small, and *all* random shocks eventually have a transitory effect. Such a model appears to be in favor of long-run PPP because there will be no permanent deviation from PPP. For more comparisions between the IRS model and other switching models see Kuan et al. (2005).

3.2 Model Estimation and Hypothesis Testing

In what follows, we postulate that the switching variable s_t follows a first-order Markov chain with the transition matrix

$$\begin{bmatrix} \mathbb{P}(s_t = 0 \mid s_{t-1} = 0) & \mathbb{P}(s_t = 1 \mid s_{t-1} = 0) \\ \mathbb{P}(s_t = 0 \mid s_{t-1} = 1) & \mathbb{P}(s_t = 1 \mid s_{t-1} = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix},$$

as in Hamilton (1989). The parameters of the resulting IRS$(n, 1; m)$ model are:

$$\boldsymbol{\theta} = (\gamma_1, \ldots, \gamma_n, \psi_1, \ldots, \psi_m, \sigma_0^2, \sigma_1^2, p_{00}, p_{11})',$$

which may be estimated by the approximate quasi-maximum likelihood method or the Markov chain Monte Carlo method. In this study, we adopt the former approach; the estimation algorithm is described in Appendix. This algorithm is initialized by a broad range of random initial values. The covariance matrix of the quasi-maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ is $-\boldsymbol{H}(\hat{\boldsymbol{\theta}}_T)^{-1}$, the Hessian matrix of the log-likelihood function evaluated at $\hat{\boldsymbol{\theta}}_T$.

Based on the estimation result, we can compute the estimated smoothing probabilities $\mathbb{P}(s_t = 0 \mid \boldsymbol{\Omega}^T; \hat{\boldsymbol{\theta}}_T)$, where $\boldsymbol{\Omega}^t = \{\Delta r_1, \ldots, \Delta r_t\}$ is the collection of all the observed variables up to time t. The estimated smoothing probabilities will be used to determine whether a shock is more likely to be permanent or transitory. We may also compute the ergodic probability of $s_t = 1$ according to:

$$\mathbb{P}(s_t = 1) \equiv \lim_{T \to \infty} \mathbb{E}\left[\frac{1}{T} \sum_{t=1}^{T-1} \mathbf{1}_{\{s_t = 1\}} \right] = \frac{1 - p_{00}}{2 - p_{00} - p_{11}},$$

where $\mathbf{1}_{\{s_t=1\}}$ is the indicator function of $s_t = 1$. The ergodic probability is understood as the likelihood of $s_t = 1$ in the long run.

As the postulated model admits both stationarity and unit-root nonstationarity, it is important to test whether the log of real exchange rate is in fact a stationary series. This amounts to testing $p_{00} = 1$. Under this null hypothesis, the permanent component does not enter the model so that the parameters in $\Gamma(B)$ and p_{11} are not identified. In this case, standard likelihood-based tests are not applicable, as discussed in Davies (1977, 1987) and Hansen (1996).[2] We thus follow Kuan et al. (2005) and adopt a simulation-based test. We first estimate an array of ARMA(p, q) models for r_t and choose an appropriate specification based on an information criterion (AIC or SIC). Denote the selected model as ARMA$(\tilde{p}, \tilde{q})$. We also estimate an array of IRS$(n, 1; m)$ models for r_t and choose the best model based on AIC or SIC; the selected model is denoted as IRS$(n^*, 1; m^*)$ with the estimated transition probability $\hat{p}^*_{00}$. The selected ARMA$(\tilde{p}, \tilde{q})$ model is then taken as the data generating process to generate simulated samples. For each simulated sample, we re-estimate the IRS$(n^*, 1; m^*)$ model and obtain an estimate of p_{00}, denoted as $\tilde{p}_{00}$. Replicating this procedure many times yields an empirical distribution of $\tilde{p}_{00}$. We then compare $\hat{p}^*_{00}$ with the quantiles of this empirical distribution. The null hypothesis that r_t is a stationary process would be rejected if the empirical p-value of $\hat{p}^*_{00}$ is less than, say, 5%.

Similarly, one may also be interested in knowing whether r_t is a pure unit-root process and would like to test $p_{11} = 1$. Following the idea of the simulation-based test described above, we now select an appropriate ARIMA$(\tilde{p}, 1, \tilde{q})$ model to generate the simulated samples. We then obtain $\hat{p}^*_{11}$ from the IRS$(n^*, 1; m^*)$ model and compute a finite-sample reference distribution of $\tilde{p}_{11}$. The null hypothesis that the series follows the ARIMA$(\tilde{p}, 1, \tilde{q})$ model is rejected if the empirical p-value of $\hat{p}^*_{11}$ is small. When the simulation-based tests reject their respective null hypothesis, we have evidence that r_t can be characterized by an IRS$(n, 1; m)$ model.

4 Empirical Analysis

4.1 Model Estimation Results

Given the sample data from 1885:01 through 1995:11, we estimate the IRS$(n, 1; m)$ model with n and m no greater than 4. The best model selected by SIC is the IRS$(1, 1; 3)$ model;

[2]Hansen (1992), Garcia (1998) and Carrasco et al. (2004) proposed solutions to test parameter constancy in the Markov-switching model. Their tests cannot be directly applied to our problem because the primary concern here is to check $p_{00} = 1$, not parameter constancy.

Table 2: Quasi-maximum likelihood estimates of the proposed IRS model.

Estimator	Estimate	Standard error	t-statistic
$\hat{\gamma}_1$	0.9621	0.0054	177.0523*
$\hat{\psi}_1$	0.0990	0.0279	3.5382*
$\hat{\psi}_2$	−0.0931	0.0166	−5.6066*
$\hat{\psi}_3$	−0.1338	0.0346	−3.8610*
$\hat{\sigma}_0$	0.0095	0.0022	4.3181*
$\hat{\sigma}_1$	0.0330	0.0035	9.4285*
$\hat{p}^*_{00}$	0.9432	0.0104	
$\hat{p}^*_{11}$	0.9211	0.0136	
Log-Likelihood=−3482.6558	SIC=−6907.8005		

Note: t-statistics with an asterisk are significant at the 5% level.

the estimation results are summarized in Table 2. In particular, the estimated transition probabilities are $\hat{p}^*_{00} \approx 0.9432$ and $\hat{p}^*_{11} \approx 0.9211$. The diagnostic tests of the model residuals, including the Q test of Ljung-Box (1978) on serial correlations and the LM test of Engle (1982) on the ARCH effect, have statistics: $Q(12) = 20.247$, $Q(24) = 29.115$ and ARCH$(4) = 2.170$, which are all insignificant at 5% level, under the $\chi^2(12)$, $\chi^2(24)$ and $\chi^2(4)$ distributions, respectively. Hence, there appears no serial correlation and conditional heteroskedasticity in these residuals. Following Engel and Hamilton (1990), we also test whether the state variables are independent over time, i.e., $p_{00} + p_{11} = 1$. The resulting Wald statistic is 2584.5868 and rejects the null at 1% level under the $\chi^2(1)$ distribution. This provides a support of the Markovian specification.

We then apply the simulation-based test discussed in the preceding section to check whether the log of U.S./U.K. real exchange rate is actually a pure ARMA or ARIMA process. We estimate an array of ARMA(p, q) model for r_t with p and q no greater than 4; the best model based on SIC is the following ARMA$(2, 0)$ model:

$$r_t = 1.0498 + 1.2579 r_{t-1} - 0.2713 r_{t-2} + \dot{e}_t, \tag{5}$$

with $\sigma_{\dot{e}} = 0.0207$. Simulated data are then generated using the estimated parameters in (5). By estimating an IRS$(1, 1; 3)$ model based on the simulated data, we obtain a new estimate $\tilde{p}_{00}$. With 3000 replications, we get an empirical distribution of $\tilde{p}_{00}$. The empirical p-value of $\hat{p}^*_{00} = 0.9432$ is 0.0279, rejecting the hypothesis that the data are generated from (5) at 5% level. This test result is consistent with the KPSS testing result in Table 1. To examine whether r_t is actually an ARIMA process, we estimate an array of ARIMA$(p,1,q)$ model with p and q no greater than 4; the best model based on SIC is the ARIMA$(1,1,0)$ model:

$$\Delta r_t = 0.0002 + 0.263\Delta r_{t-1} + \ddot{e}_t, \tag{6}$$

with $\sigma_{\ddot{e}} = 0.0208$. For each simulated sample generated according to equation (6), we re-estimate the IRS$(1,1;3)$ to get an estimate of p_{11}. With 3000 replications we obtain a simulated distribution of $\tilde{p}_{11}$. The p-value of $\hat{p}_{11}^* = 0.9211$ is 0.0233, resulting a rejection of the hypothesis that the data are generated from (6) at 5% level. This is consistent with the ADF and ADF-GLS testing results in Table 1. These testing results suggest that this exchange rate series is neither a pure ARMA process nor a pure ARIMA process.

4.2 The Effects of Random Shocks

In Figure 1, we plot $\mathbb{P}(s_t = 0 \mid \boldsymbol{\Omega}^T; \hat{\boldsymbol{\theta}}_T)$, the smoothing probabilities of $s_t = 0$ evaluated at $\hat{\boldsymbol{\theta}}_T$, and the log of real exchange rate. The shaded areas denote the first and the second Wartime Controls periods classified by Grilli and Kaminsky (1991). It turns out that there are 831 periods (about 63% of the sample) with $\mathbb{P}(s_t = 0 \mid \boldsymbol{\Omega}^T; \hat{\boldsymbol{\theta}}_T) > 0.5$. This shows that stationarity is more likely to prevail in about 63 percent of the sample periods, yet unit-root non-stationarity appears in the remaining periods. This is compatible with the result of Kanas (2006). Since there are both stationary and non-stationary behaviors in the sample, it is now not surprising to see the conflicting unit-root testing results for different subsamples in Table 1.

The estimated ergodic probability is

$$\mathbb{P}(s_t = 1) \approx \frac{1 - \hat{p}_{00}^*}{2 - \hat{p}_{00}^* - \hat{p}_{11}^*} = 0.4185, \tag{7}$$

suggesting that approximately 42% (58%) of the shocks may have a permanent (transitory) effect in the long-run. This result supports that not all shocks in r_t have a permanent effect, contrary to the findings of Engel and Kim (1999) and Cuddington and Lian (2000). Also, not all shocks are transitory, cf. Grilli and Kaminsky (1991), Lothian and Taylor (1996) and Taylor (2002). In view of the estimated ergodic probability and the estimated σ_1, we can see that permanent shocks play a relatively small but significant role in the long run. We therefore conclude that PPP is invalid in the long run for the U.S./U.K. real exchange rate considered.

For comparison, we also plot the estimated smoothing probabilities of $s_t = 0$ for some subperiods in Figure 2. The smoothing probabilities under the fixed-rate regime are in the left panels, and those under the floating regime are in the right panels. For

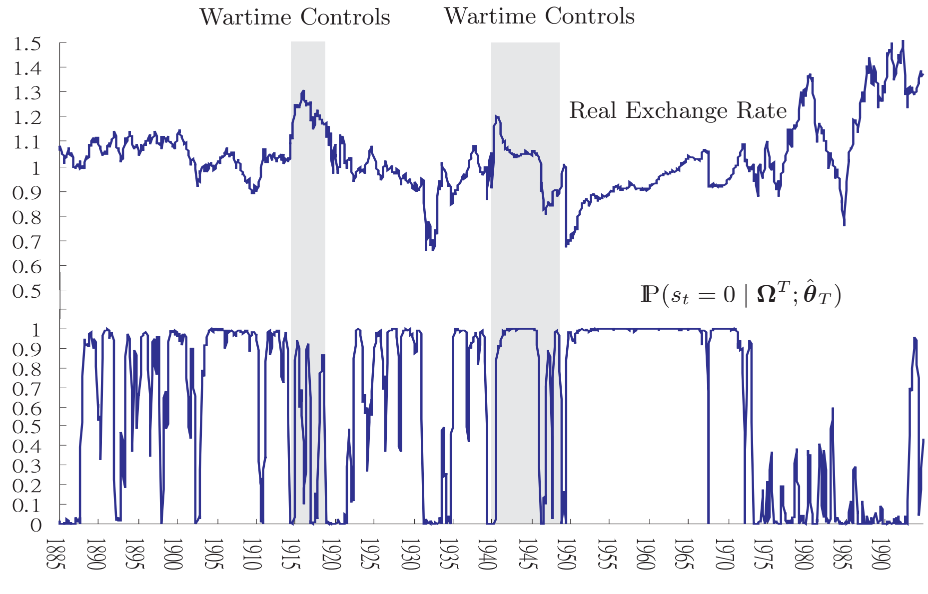

Figure 1: The logarithm of U.S./U.K. real exchange rate and the estimated smoothing probabilities of $s_t = 0$.

each panel, the numbers in parentheses are the ratios of the number of observation with $\mathbb{P}(s_t = 0 \mid \mathbf{\Omega}^T; \hat{\boldsymbol{\theta}}_T) > 0.5$ to all observations in each subperiod. It can be seen that in Gold Exchange Standard and Bretton Woods periods, most of the shocks are more likely to be transitory shocks; in the Classical Gold standard period, there are still more than 75% of the shocks appear to be transitory. That is, transitory shocks dominate in the fixed-rate regime. Observe also that most of the shocks in the Post-Bretton Woods periods are more likely to be permanent shocks. Yet, slightly more than half of the shocks are permanent in two inter-war floating periods, and the permanent shocks concentrate mainly in the beginning of these two periods. Thus, the presence of permanent shocks does not always agree with the floating regime; they dominate only in the Post-Bretton Woods floating period. The latter is consistent with the consensus view of parity-deviating behavior of r_t during the recent floating period. It also explains why unit-root tests tend to reject the null when more pre-float data (those with transitory shocks) are included in the sample.

It can also be seen that $\hat{\sigma}_1$ is about 3.5 times of $\hat{\sigma}_0$ in Table 2. This indicates that the series tends to be more volatile when permanent shocks are present, and it is consistent with the finding that the floating regime is usually more volatile than fixed-rate regime; see, e.g., Baxter and Stockman (1989) and Lothian and Taylor (1996). Moreover, as the estimated coefficients of the stationary AR component r_{0t} are $\hat{\psi}_1 = 0.099$, $\hat{\psi}_2 = -0.0931$

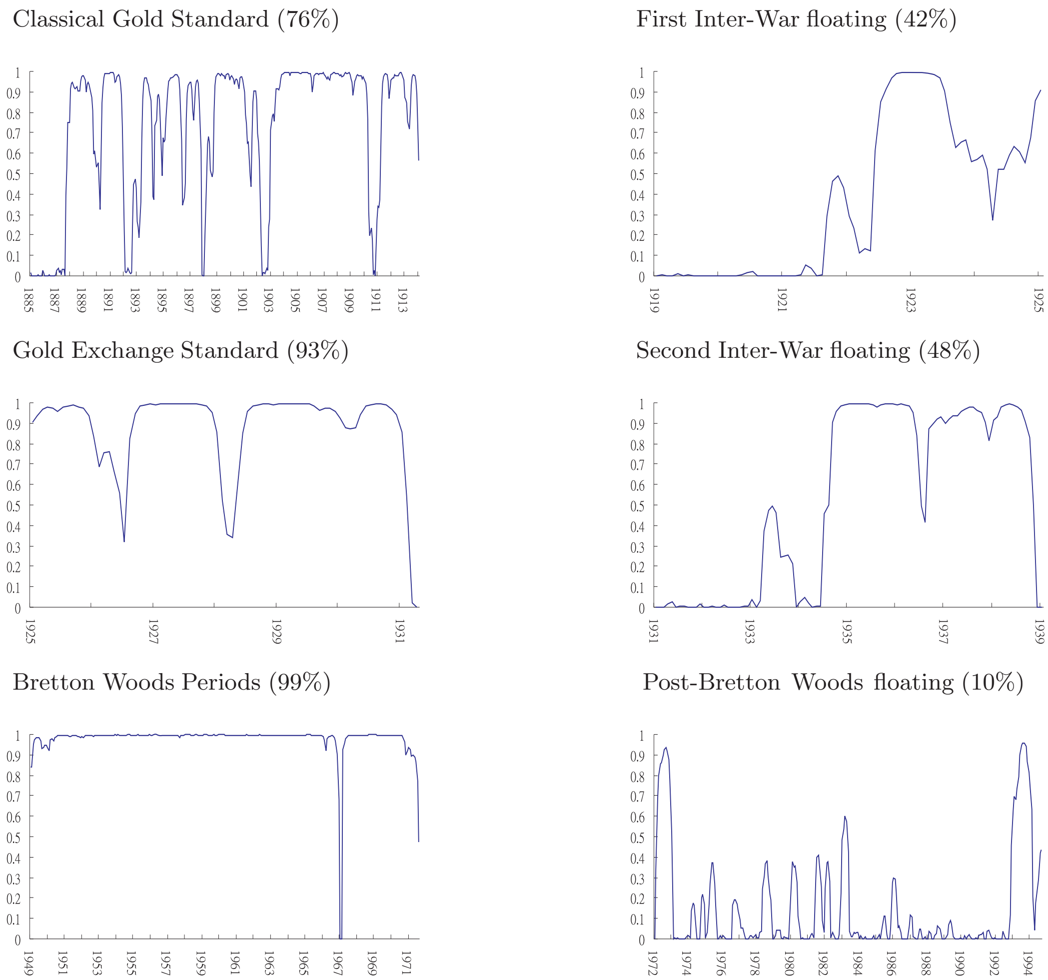

Figure 2: Estimated smoothing probabilities $\mathbb{P}(s_t = 0 \mid \mathbf{\Omega}^T; \hat{\boldsymbol{\theta}}_T)$; the values in parentheses are the ratios of $\mathbb{P}(s_t = 0 \mid \mathbf{\Omega}^T; \hat{\boldsymbol{\theta}}_T) > 0.5$ for each subperiod.

and $\hat{\psi}_3 = -0.1338$, we obtain the estimated half-life of a given transitory shock is less than one month. This estimate is considerably shorter than those usually reported in the literature, e.g., 3 years in Rogoff (1996) and 55 months in Engel and Kim (1999). This result suggests that the long half-life estimates obtained in other empirical studies may be due to the fact that permanent shocks in the sample were not properly accounted for.

5 Concluding Remarks

In this paper we re-examine long-run PPP hypothesis based on an IRS model that permits both stationarity and unit-root nonstationarity. It is found that approximately 37 percent of the shocks in the sample (42 percent in the long run) are more likely to have permanent effects. As such, long-run PPP is rejected in this data set. Despite the rejection of long-run PPP, our results show that mean reversion still occurs in the fixed-rate periods, due to the fact that transitory shocks prevail in these periods. Yet, there seems no tendency of mean reversion during the Post-Bretton Woods period, in which permanent shocks dominate. We thus conclude that the centuried real exchange rate exhibits different characteristics over different periods. Traditional modeling approach that only allows for one dynamic pattern is unable to characterize such behavior and hence may yield misleading conclusion on long-run PPP.

It is worth emphasizing that our examination is built upon the usual definition of long-run PPP which, under the IRS framework, amounts to requiring *all* random shocks to be transitory, i.e., $\mathbb{P}(s_t = 0) = 1$ (or $\mathbb{P}(s_t = 1) = 0$) for all t. This seems to be too strong a requirement for long-run PPP. Note that when $\sum_{t=1}^{\infty} \mathbb{P}(s_t = 1)$ converges, the well known Borel-Cantelli lemma ensures that $\mathbb{P}(s_t = 1 \text{ infinitely often}) = 0$. That is, permanent shocks can only be present for finitely many t. In this case, there will be transitory shocks for all but finitely many t, so that PPP would hold *eventually*. Similarly, when $\mathbb{P}(s_t = 0 \text{ infinitely often}) = 0$, there will be at most finitely many transitory shocks, so that PPP would fail eventually. These two conditions appear to be more appropriate for determining the validity of long-run PPP. How to construct tests for these two conditions is an interesting research topic and currently being investigated.

Appendix

It is easy to show that the IRS$(n, 1; m)$ model (3) has an ARIMA representation with random MA coefficients:

$$\Gamma(B)\Psi(B)\Delta r_t = \sum_{i=1}^{\kappa+1} \xi_{i,s_{t-i}} \upsilon_{t-i} + \upsilon_t, \tag{8}$$

where $\kappa = \max\{m, n\}$,

$$\xi_{1,s_{t-1}} = \begin{cases} -\psi_1, & \text{if } s_{t-1} = 1, \\ -1 - \gamma_1, & \text{otherwise,} \end{cases} \qquad \xi_{i,s_{t-i}} = \begin{cases} -\psi_i, & \text{if } s_{t-i} = 1, \\ \gamma_{i-1} - \gamma_i, & \text{otherwise,} \end{cases}$$

for $i = 2, \ldots, \kappa$, and the last coefficient is

$$\xi_{\kappa+1,s_{t-\kappa-1}} = \begin{cases} 0, & \text{if } s_{t-\kappa-1} = 1, \\ \gamma_\kappa, & \text{otherwise;} \end{cases}$$

$\psi_i = 0$ for $i > m$ and $\gamma_i = 0$ for $i > n$. From equation (8) we see that the past $\kappa + 1$ state variables affect Δr_t. Following Hamilton (1994), we define the new state variable $s^*_{t-1} = 1, 2, \ldots, 2^{\kappa+1}$ such that each of these values represents a particular combination of the realizations of $(s_{t-1}, \ldots, s_{t-\kappa-1})$. It is easy to show that s^*_t also forms a first-order Markov chain with the transition matrix $\boldsymbol{P}^*$. This transition matrix can be expressed as

$$\boldsymbol{P}^* = \begin{bmatrix} \boldsymbol{P}_{00} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{10} \\ \boldsymbol{P}_{01} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{11} \end{bmatrix},$$

with $\boldsymbol{P}_{ji}$ $(j, i = 0, 1)$ being a $2^{\kappa-1} \times 2^\kappa$ block diagonal matrix given by

$$\boldsymbol{P}_{ji} = \begin{bmatrix} p_{ji} & p_{ji} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & p_{ji} & p_{ji} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & p_{ji} & p_{ji} \end{bmatrix}.$$

Also let $\boldsymbol{\upsilon}_{t-1} = (\upsilon_{t-1}, \ldots, \upsilon_{t-\kappa-1})'$ and for $s^*_{t-1} = \ell$, $\ell = 1, 2, \ldots, 2^{\kappa+1}$, let

$$\boldsymbol{\xi}_{t-1,\ell} = (\xi_{1,s_{t-1}}, \xi_{2,s_{t-2}}, \ldots, \xi_{\kappa+1,s_{t-\kappa-1}})',$$

where the realizations of $s_{t-1}, \ldots, s_{t-\kappa-1}$ are such that $s^*_{t-1} = \ell$. Then,

$$\boldsymbol{\xi}'_{t-1,\ell}\boldsymbol{\upsilon}_{t-1} = \sum_{j=1}^{\kappa+1} \xi_{j,s_{t-j}} \upsilon_{t-j}$$

in equation (8).

To derive the estimation algorithm, we first discuss the optimal forecasts of the state variable s_t based on the information up to time t. Under the normality assumption, the density of Δr_t conditional on $s^*_{t-1} = \ell$ and $\mathbf{\Omega}^{t-1}$ is

$$\begin{aligned} f\big(\Delta r_t \mid s^*_{t-1} = \ell, \mathbf{\Omega}^{t-1}; \boldsymbol{\theta}\big) \\ = \frac{1}{\sqrt{2\pi\sigma^2_{t-1}}} \exp\left\{ \frac{-\big[\Gamma(B)\Psi(B)\Delta r_t - \boldsymbol{\xi}'_{t-1,\ell}\boldsymbol{\upsilon}_{t-1}\big]^2}{2\sigma^2_{t-1}} \right\}, \end{aligned} \tag{9}$$

where $\ell = 1, 2, \cdots, 2^{\kappa+1}$ and $\sigma^2_{t-1} = \sigma^2_0(1 - s_{t-1}) + \sigma^2_1 s_{t-1}$. Although the innovations υ_t depend on s^*_{t-1} ($t = m+1, \ldots, T$), we follow Gray (1996) and compute υ_t ($t = m+1, \ldots, T$) as

$$\begin{aligned} \upsilon_t &= \Delta r_t - \mathbb{E}(\Delta r_t \mid \mathbf{\Omega}^{t-1}) \\ &= \Gamma(B)\Psi(B)\Delta r_t - \sum_{\ell=1}^{2^{\kappa+1}} \mathbb{P}\big(s^*_{t-1} = \ell \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta}\big)\, \boldsymbol{\xi}'_{t-1,\ell}\boldsymbol{\upsilon}_{t-1}, \end{aligned} \tag{10}$$

with the initial values $\upsilon_m, \ldots, \upsilon_1$ being zero, where $\mathbb{P}\big(s^*_{t-1} = \ell \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta}\big)$ is the probability of $s^*_{t-1} = \ell$ based on the information up to time $t-1$.

Given $\mathbb{P}(s^*_{t-1} = \ell \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta})$, the density of Δr_t conditional on $\mathbf{\Omega}^{t-1}$ alone can be obtained via (9) as

$$f(\Delta r_t \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta}) = \sum_{\ell=1}^{2^{\kappa+1}} \mathbb{P}(s^*_{t-1} = \ell \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta})\, f(\Delta r_t \mid s^*_{t-1} = \ell, \mathbf{\Omega}^{t-1}; \boldsymbol{\theta}). \tag{11}$$

To compute $\mathbb{P}(s^*_t = \ell \mid \mathbf{\Omega}^t; \boldsymbol{\theta})$, note that

$$\mathbb{P}(s^*_{t-1} = \ell \mid \mathbf{\Omega}^t; \boldsymbol{\theta}) = \frac{\mathbb{P}(s^*_{t-1} = \ell \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta})\, f(\Delta r_t \mid s^*_{t-1} = \ell, \mathbf{\Omega}^{t-1}; \boldsymbol{\theta})}{f(\Delta r_t \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta})}. \tag{12}$$

As in Hamilton (1989), we also assume that the (j,i)th element of $\boldsymbol{P}^*$ is such that $p^*_{ji} = \mathbb{P}(s^*_t = i \mid s^*_{t-1} = j, \mathbf{\Omega}^t)$. These in turn yield

$$\begin{aligned} \mathbb{P}(s^*_t = \ell \mid \mathbf{\Omega}^t; \boldsymbol{\theta}) &= \sum_{j=1}^{2^{\kappa+1}} \mathbb{P}(s^*_{t-1} = j \mid \mathbf{\Omega}^t; \boldsymbol{\theta})\ \mathbb{P}(s^*_t = \ell \mid s^*_{t-1} = j, \mathbf{\Omega}^t; \boldsymbol{\theta}) \\ &= \sum_{j=1}^{2^{\kappa+1}} p^*_{j\ell}\ \mathbb{P}(s^*_{t-1} = j \mid \mathbf{\Omega}^t; \boldsymbol{\theta}). \end{aligned} \tag{13}$$

Thus, with the initial value $\mathbb{P}(s_m^* \mid \mathbf{\Omega}^m; \boldsymbol{\theta})$ being its limiting unconditional counterpart, we can iterate the equations (9)–(13) to obtain $\mathbb{P}(s_t^* = \ell \mid \mathbf{\Omega}^t; \boldsymbol{\theta})$ for $t = m+1, \ldots, T$. Then for each t, the filtering probability is

$$\mathbb{P}(s_t = 1 \mid \mathbf{\Omega}^t; \boldsymbol{\theta}) = \sum \mathbb{P}(s_t^* = \ell \mid \mathbf{\Omega}^t; \boldsymbol{\theta}),$$

and $\mathbb{P}(s_t = 0 \mid \mathbf{\Omega}^t; \boldsymbol{\theta}) = 1 - \mathbb{P}(s_t = 1 \mid \mathbf{\Omega}^t; \boldsymbol{\theta})$, where the summation is taken over all ℓ that associated with $s_t = 1$. To calculate the desired smoothing probabilities $\mathbb{P}(s_t \mid \mathbf{\Omega}^T; \boldsymbol{\theta})$, we just follow the approach of Kim (1994).

From the recursions above we also obtain the quasi-log-likelihood function:

$$\ln \mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ln f(\Delta r_t \mid \mathbf{\Omega}^{t-1}; \boldsymbol{\theta}),$$

from which the quasi-maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ can be found via a numerical-search algorithm. In this paper, the estimation program for our simulation and empirical study is written in GAUSS which employs the BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm.

Reference

Abuaf, N. and P. Jorion (1990). Purchasing power parity in the long run, *Journal of Finance*, **45**, 157–174.

Adler, M. and B. Lehmann (1983). Deviations from purchasing power parity in the long run, *Journal of Finance*, **38**, 1471–1487.

Baxter, M. and A.C. Stockman (1989). Business cycles and the exchange rate system, *Journal of Monetary Economics*, **23**, 377–400.

Beveridge, S. and C.R. Nelson (1981). A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the "business cycle", *Journal of Monetary Economics*, **7**, 151–174.

Carrasco, M., L. Hu and W. Ploberger (2004). Optimal test for Markov switching, *Working paper*.

Cheung, Y.W. and K.S. Lai (1998). Parity reversion in real exchange rates during the post-Bretton Woods period, *Journal of International Economics*, **17**, 597–641.

Cuddington, J.T. and H. Liang (2000). Purchasing power parity over two centuries?, *Journal of International Money and Finance*, **19**, 753–757.

Culver, S.E. and D.H. Papell (1995). Real exchange rates under the gold standard: can they be explained by the trend break model? *Journal of International Economics*, **14**, 539–548.

Culver, S.E. and D.H. Papell (1999). Long-run purchasing power parity with shortrun data: evidence with a null hypothesis of stationarity, *Journal of International Economics*, **18**, 751–768.

Davies, R. B. (1977). Hypothesis testing when a nuisance parameter is present only under the alternative, *Biometrika*, **64**, 247–254.

Davies, R. B. (1987). Hypothesis testing when a nuisance parameter is present only under the alternative, *Biometrika*, **74**, 33–43.

Diebold, F.X., S. Husted and M. Rush (1991). Real exchange rates under the gold standard, *Journal of Political Economy*, **99**, 1252–1271.

Dornbusch, R (1976). Expectations and exchange rate dynamics, *Journal of Political Economy*, **84**, 1161–1176.

Elliott, G., T. Rothenberg, and J.H. Stock (1996). Efficient tests for an autoregressive unit root, *Econometrica*, **64**, 813–836.

Engel, C. and J.D. Hamilton (1990). Long swings in the dollar: Are they in the data and do markets know it?, *American Economic Review*, **80**, 689–713.

Engel, C. and C.J. Kim (1999). The long-run U.S./U.K. real exchange rate, *Journal of Money, Credit, and Banking*, **31**, 335–356.

Engle, R.F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, *Econometrica*, **50**, 987–1008.

Froot, K. and K. Rogoff (1995). Perspectives on PPP and long-run real exchange rates, *The Handbook of International Economics*, Grossman G. and Rogoff K. ed., **3**, 1647–1688, Amsterdam: North Holland.

Garcia, R. (1998). Asymptotic null distribution of the likelihood ratio test in Markov switching models, *International Economic Review*, **39**, 763–788.

Glen, J.D. (1992). Real exchange rates in the short, medium, and long run, *Journal of International Economics*, **33**, 147–166.

Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime-switching process, *Journal of Financial Economics*, **42**, 27–62.

Grilli, V. and G. Kaminsky (1991). Nominal exchange rate regimes and the real exchange rate: Evidence from the United States and Great Britain, 1885–1986, *Journal of Monetary Economics*, **27**, 191–212.

Hamilton, J.D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica*, **57**, 357–384.

Hamilton, J.D. (1994). *Time Series Analysis*, Princeton, NJ: Princeton University Press.

Hansen, B. E. (1992). The likelihood ratio test under non-standard conditions: Testing the Markov switching model of GNP, *Journal of Applied Econometrics*, **7**, S61–S82.

Hansen, B. E. (1996). Inference when a nuisance parameter is not identified under the null hypothesis, *Econometrica*, **64**, 413–430.

Kanas, A. (2006). Purchasing Power Parity and Markov regime switching, *Journal of Money, Credit, and Banking*, forthcoming.

Kim, C.-J. (1994). Dynamic linear models with Markov-switching, *Journal of Econometrics*, **60**, 1–22.

Kim, Y. (1990). Purchasing power parity in the long run: A cointegration approach, *Journal of Money, Credit, and Banking*, **22**, 491–503.

Kuan, C.-M., Y.-L. Huang and R.-S. Tsay (2005). An unobserved component model with switching permanent and transitory innovations, *Journal of Business and Economic Statistics*, **23**, 443–454.

Kuo, B.S. and A. Mikkolab (1999). Re-examining long-run purchasing power parity, *Journal of International Money and Finance*, **18**, 251–266.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992). Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics*, **54**, 159–178.

Ljung, G.M. and G.E.P. Box (1978). On a measure of lack of fit in time series models, *Biometrika*, **65**, 297–303.

Lopez, C., C.J. Murray and D.H. Papell (2005). State of the art unit root tests and purchasing power parity *Journal of Money, Credit, and Banking*, **37**, 361–369.

Lothian, J. and M.P. Taylor (1996). Real exchange rate behavior: The recent float from the perspective of the past two centuries, *Journal of Political Economy*, **104**, 488–509.

MacDonald, R. (1993). Long-run purchasing power parity: Is it for real? *The Review of Economics and Statistics*, **75**, 690–695.

MacDonald, R. (1995). Long-run exchange rate modeling: A survey of the recent evidence. *International Monetary Fund Staff Papers*, **42**, 437–489.

Meese, R. and K. Rogoff (1983). Empirical exchange rate models of the seventies: Do they fit out-of-sample? *Journal of International Economics*, **14**, 3–24.

Mussa, M. L. (1982). A model of exchange rate dynamics, *Journal of Political Economy* **90**, 74–104.

Officer, L.H. (1976). The purchasing power parity theory of exchange rates: A review article, *International Monetary Fund Staff Papers*, **23**, 1–59.

Papell, D.H. (2006). The panel purchasing power parity puzzle, *Journal of Money, Credit, and Banking*, **38**, 447–467.

Papell, D.H. and R. Prodan (2006). Additional evidence of long run purchasing power parity with restricted structural change, *Journal of Money, Credit, and Banking*, forthcoming.

Phillips, P.C.B. and P. Perron (1988). Testing for a unit root in time series regression, *Biometrika*, **73**, 335–346.

Rogoff, K. (1996). The purchasing power parity puzzle, *Journal of Economic Literature*, **34**, 647–668.

Stock, J.H. (1994). Unit roots, structure breaks and trends, in Engle, R. and D. McFadden, *Handbook of Econometrics IV*, 2739V841. Amsterdam: North Holland.

Taylor, A.M. (2001). Potential pitfalls for the purchasing-power-parity puzzle? Sampling and specification biases in mean-reversion tests of the law of one price. *Econometrica*, **69**, 473–498.

Taylor, A.M. (2002). A century of purchasing power parity, *The Review of Economics and Statistics*, **84**, 139–150.