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Coherent control of the group velocity in a dielectric slab doped with duplicated two-level atoms

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Abstract
Coherent control of reflected and transmitted pulses is investigated theoretically through a slab doped with atoms in a duplicated two-level configuration. When a strong control field and a relatively weak probe field are employed, coherent control of the group velocity is achieved via changing the phase shift $\phi$ between control and probe fields. Furthermore, the peak values in the delay time of the reflected and transmitted pulses are also studied by varying the phase shift $\phi$.

Keywords: group velocity, slow and fast light, group delay

(Some figures may appear in colour only in the online journal)

I. Introduction

Optical light pulse propagation in dispersive media has been investigated by many researchers [1–8]. In particular, subluminal (slow light) or superluminal (fast light) phenomena have been revealed in a single atomic system theoretically and experimentally [9–19, 20]. It was observed that the group velocity of a traveling light pulse through the medium can be affected not only by the intensity of the driving fields [11, 18], relative phase of the driving fields [12], and intensity of the incoherent pumping field [13], but also by the spontaneous generated coherence [14, 22]. For example, the experimental work done by Kim et al is prominent [16], and they observed superluminal behavior with a negative group velocity along with induced absorption for a weak coupling power. The behavior of light pulse propagation can be changed from super- to subluminal when the strength of the laser is increased. The sub- and superluminal light pulse propagations have also been observed experimentally using an incoherent pump field, based on electromagnetically induced transparency [20]. Later on, a four-level atomic system was introduced [21], where the effect of incoherent pumping on dispersion and absorption were studied.

In addition to a gaseous atomic medium, studies on the light pulse propagation in solid materials are attractive for their potential applications. For example, sub- and superluminal light propagations have been investigated in photonic crystals (PCs) [23, 24], as well as optical-phase conjugation mirrors [25]. Wang et al investigated sub- and superluminal light pulse propagation in the reflected and transmitted beam for the first time by doping two- or three-level atoms within a slab [26]. Since then, several investigations have been proposed and studied for sub- and superluminal light pulse propagations [27–33]. In these proposals, different atomic configurations are doped within a slab, such as a duplicated two-level atomic configuration [34]. The control of sub- and superluminal light pulse propagation is demonstrated by changing the phase shift $\phi$ between the probe and control fields. Such a duplicated two-level atomic medium has also been used for coherent control of the negative and positive Goos–Hänchen shift [35], and for the relevant optical bistability by turning absorption into amplification without inversion of the multi-photon resonance condition [36].
passing through a slab doped with atoms in a duplicated two-level configuration. The motivation comes from the coherent control of group velocity in a gaseous atomic medium via the phase shift between the control and probe fields. However, the behavior of light pulse propagation through the gaseous medium is definitely different from that in the slab medium. Specifically, the existence of doped atoms changes the dispersive properties of the media. For many, applications of light pulse propagation through a solid-state medium is appreciated. Moreover, one can control the sub- and superluminal pulse propagations via the phase difference \( \phi \), as well as by adjusting the slab thickness.

II. Model for a slab system

We consider a Gaussian-shaped pulse having a temporal width \( \tau_0 \) and central frequency \( \omega_0 \), which is incident on a slab system from vacuum normally, see figure 1. Duplicated two-level atoms are doped with a slab, which is extended in the \( z \)-direction from \( z = 0 \) to \( z = d \). The electric field and its Fourier components of the Gaussian pulse at \( z = 0 \) can be represented as

\[
E_i(0, t) = a_0 e^{-\left(\frac{\tau_0}{2}\right)^2} e^{-i\omega_0 t},
\]

and

\[
E_i(0, \omega_p) = \left(\frac{a_0 \tau_0}{2 \sqrt{\pi}}\right) e^{-i\left(\frac{n(\omega_p) - \omega_0}{2}\right)^2},
\]

where \( a_0 \) and \( \tau_0 \) are the amplitude and temporal width of the incident pulse.

The transfer matrix of a TE plane wave for the electric and magnetic components of a monochromatic wave of frequency \( \omega \) through the slab can be written as [26, 27]

\[
\begin{pmatrix}
\cos(kd) & i \frac{1}{n(\omega_p)} \sin(kd) \\
\sin(\omega_p) \sin(kd) & \cos(kd)
\end{pmatrix},
\]

where \( n(\omega_p) = \sqrt{\varepsilon_p} \) is the refractive index of the slab, which is doped with duplicated two-level atoms, whereas \( \varepsilon_p \) is the permittivity of the doped slab and can be expressed as

\[
\varepsilon_p = \varepsilon_b + \chi(\omega_p),
\]

where \( \varepsilon_b \) is the background permittivity of the slab and \( \chi(\omega_p) \) is the optical susceptibility of duplicated two-level atomic system. To calculate the reflection and transmission coefficients of monochromatic light, we use the transfer matrix method as [26, 27]

\[
r(\omega_p) = \frac{\cos(kd) - \frac{i}{2} \left[ \frac{1}{n(\omega_p)} - n(\omega_p) \right] \sin(kd)}{\cos(kd) - \frac{i}{2} \left[ \frac{1}{n(\omega_p)} + n(\omega_p) \right] \sin(kd)},
\]

\[
t(\omega_p) = \frac{1}{\cos(kd) - \frac{i}{2} \left[ \frac{1}{n(\omega_p)} + n(\omega_p) \right] \sin(kd)},
\]

where \( d \) is the thickness of the slab. The thickness of the slab for the resonance and off-resonance conditions can be expressed \( d = 2m \left( \frac{\lambda_0}{2\sqrt{\varepsilon_b}} \right) \) and \( d = 2(m + 1) \left( \frac{\lambda_0}{2\sqrt{\varepsilon_b}} \right) \), respectively, where \( m \) and \( \lambda_0 \) are the integer and central wavelength of the pulse, respectively.

III. Atom–field interaction

We consider a duplicated two-level atomic configuration doped with a slab, as presented in figure 1(b). In this atomic configuration each individual system is excited by two fields having orthogonal polarizations, i.e. \( \pi \)-polarized and \( \sigma \)-polarized. The proposed atomic system was first suggested by Hashmi and Bouchene for coherent control of the effective susceptibility [34]. They considered the excitation of the transition \( ^2S_{1/2} F = 1/2 \rightarrow ^2P_{1/2} F = 1/2 \) of \(^6\) Li at 671 nm by two co-propagating fields. Here, the control field \( E_c \) couples the transitions with identical \( m_F \) i.e. \( |a \rangle \leftrightarrow |c \rangle \) and \( |b \rangle \leftrightarrow |d \rangle \).
the probe field $E_p$ couples the energy levels with different $m_F$, i.e. $|a⟩\leftrightarrow |d⟩$ and $|b⟩\leftrightarrow |c⟩$. In the proposed atomic system, the atomic transition forms a closed loop i.e. the parallel and non-parallel states coupled via the phase shift $\phi$. The manipulation of the phase shift leads to coherent control of the optical susceptibility of the doped atoms.

The effective Hamiltonian for the atom–field system under the rotating wave approximation can be written as

$$\mathcal{H} = \hbar(-\Omega_\sigma |c⟩\langle a| + \Omega_\pi |d⟩\langle b| - \Omega_\sigma e^{-i\phi} |c⟩\langle b| - \Omega_\pi e^{-i\phi} |d⟩\langle a| + c.c) + \hbar\Delta |c⟩\langle c| + |d⟩\langle d|),$$

(6)

where $\Omega_\sigma$ and $\Omega_\pi$ are the Rabi frequencies corresponding to the pump and probe field, respectively. The detuning parameter is defined as $\Delta = \omega - \omega_0$. Using the density matrix approach, the resulting rate equations are shown in [34, 35].

The expression for the optical susceptibility of the doped atomic system can be derived by observing the modification in the probe field using the coherence $\rho_{ab} = \rho_{ba} + \rho_{da}$ [34]

$$\chi(\omega_p) = \frac{-2\alpha\Gamma_d}{k(\Delta + i\Gamma_d)} \omega^{2\alpha},$$

(7)

with

$$\alpha = ND^2\omega_0^2/2\hbar\epsilon_0\Gamma_d,$$

(8)

where $D$ is the dipole matrix element, $\phi$ is the phase shift between the control and the probe fields and $N$ is the atomic density.

**IV. Results and discussion**

We consider a dielectric slab doped with duplicated two-level atoms. The expression for the optical susceptibility ($\chi(\omega_p)$) of the doped atoms shows that the response of the medium is independent of the control field intensity. The atomic medium can be manipulated by the phase shift between the probe and control fields, which leads to control of the group velocity of the medium. The optical susceptibility of the duplicated two-level atomic medium has previously been studied in a gaseous medium [34]. It has been shown that the medium behaves as an absorber with superluminal characteristics when the phase shift $\phi = 0$; however, the medium turns into an amplifier for the phase shift $\phi = \pi/2$ and exhibits subluminal characteristics. Here, we dope the duplicated two-level atomic medium with a slab and investigate the control of group velocity in the reflected and transmitted pulses. The control of the group velocity is the phase shift $\phi$ between the probe and control fields. In the following discussion, we concentrate on the suband superluminal light pulse propagation in the reflected and transmitted pulses. We follow Agarwal’s reciprocity theorem [37], which states that the peak in the curve of reflected and transmitted pulses corresponds to subluminal pulse propagation. However, the dip in the curve of the reflected and transmitted light pulses corresponds to superluminal pulse propagation.

First, we consider an off-resonant condition of the thickness slab and plot the reflectivity $R = |r|^2$ and transmittivity $T = |t|^2$ curves versus probe field detuning $\Delta$ for two different cases i.e. $\phi = 0$ and $\phi = \pi/2$. Previously, sub- and superluminal pulse propagation in a gaseous atomic medium has been observed for $\phi = \pi/2$ and $\phi = 0$, respectively [34]. In figure 2 we plot the reflectivity and transmittivity versus probe field detuning by considering the phase shift $\phi = 0$. The plot shows that both the reflectivity and transmittivity curves have dips, which means that superluminal light propagates through the medium, simultaneously. In figure 2 one can note that the sum of reflection and transmission is less than unity, which shows that our proposed system acts like an absorber when we consider the phase shift $\phi = 0$ between control and probe fields. Similar behavior has also been investigated in a gaseous atomic medium consisting of duplicated two-level atoms [34]. Further, we change the phase from $\phi = 0$ to $\phi = \pi/2$ and again plot the reflectivity and transmittivity versus probe field detuning. The behavior of reflected and transmitted pulse propagation through the medium changes dramatically. In this time we investigate peak curves of the reflectivity and transmittivity simultaneously, see figure 3. The peak curves of the reflectivity and transmittivity lead to subluminal light propagation through the medium. It is noted that the sum of reflection and transmission is greater than unity, which reflects that the medium becomes an amplifier for the phase $\phi = \pi/2$. For the off-resonant condition of the slab system it is noted that the superluminal light is reflected and transmitted from the slab simultaneously when the phase $\phi = 0$ is considered, whereas subluminal light is reflected and transmitted from a slab simultaneously by considering $\phi = \pi/2$ between the control and probe fields.

Figure 2. (a) Reflectivity and (b) transmittivity versus probe field detuning with the parameters $\gamma = 1$ MHz, $\Gamma = \gamma, \alpha = 0.015, \phi = 0$, $d = (2m + 1)\left(\frac{\lambda_0}{4\epsilon_0 \epsilon}\right)$, $m = 100$ and $\epsilon_0 = 4$. 

Ziauddin et al
Further, we consider a resonant condition of the slab i.e. the slab thickness \(d = 2(m)\left(\frac{\lambda_0}{\sqrt{n}}\right)\) and study the light pulse propagation in the reflectivity and transmittivity for two different cases. We plot the reflectivity and transmittivity versus probe field detuning and consider the phase shift between the control and probe fields \(\phi = 0\). We investigate a peak in the reflectivity and a dip in the transmittivity, simultaneously, see figure 4. Therefore, for the resonance condition of the slab, subluminal light is reflected from the slab, while superluminal light is transmitted through the slab when \(\phi = 0\) is considered and the medium behaves like an absorber. Previously, for the off-resonant condition, superluminal behavior was investigated in the reflected pulse, see figure 2(a). It is noted that by changing the slab’s thickness there is a switching from superluminal to subluminal in the reflected light, compare figures 2(a) and 4(a). Next, we change the phase shift from \(\phi = 0\) to \(\phi = \pi/2\) and plot the reflectivity and transmittivity versus probe field detuning, see figure 5. The plots show that there are peak curves in the reflectivity and transmittivity, simultaneously. It is also noticed that the sum of reflection and transmission becomes greater than unity, which clearly shows that the medium becomes an amplifier for transmittivity.

In the above discussion, we studied sub- and superluminal light propagation in the reflected and transmitted pulses for two different conditions i.e. off-resonant and resonant. In the following, we consider a Gaussian-shaped pulse that has a narrow spectral limit in the time domain. The reflected and transmitted pulses can therefore be written as

\[
E_i(0,t) = \int E_i(0, \omega_p) r(\omega_p) e^{-i\omega_p t} d\omega_p
\]

(9)

and

\[
E_i(d,t) = \int E_i(d, \omega_p) t(\omega_p) e^{-i\omega_p t} d\omega_p
\]

(10)

The temporal width of the Gaussian pulse that we have considered is 1 \(\mu s\). we study the peak delay time of the reflected and transmitted pulses, so first we define the phase delay time i.e.

\[
\tau_{\Phi} = \frac{\partial \Phi}{\partial \omega},
\]

(11)

where, \(\Phi_{\text{r,t}}\) are the phases of the reflection and transmission coefficients. The phase time delay is equivalent to the peak time delay, which can be represented by \(T_{\text{peak}}^r\) of the reflected and transmitted pulses. The peak time delay can be measured from the shapes of the reflected and transmitted pulses. The peak of the reflected pulse \(T_{\text{r,peak}}\) corresponds to superluminal reflection of the pulse; the peak of the transmitted pulse \(T_{\text{t,peak}} < 0\) corresponds to superluminal transmission of the pulse. In figure 6, we plot the normalized intensity profile of the incident \((I_0)\), reflected \((I_r)\) and transmitted \((I_t)\) pulses in time domain for the off-resonant condition of the slab. The incident pulse \((I_0)\) shows the reference pulse, i.e. the pulse that travels in free space with a distance \(d\). For the phase \(\phi = 0\), the propagation of reflected and transmitted pulses through the slab system is simultaneously superluminal (see figure 2). In the case of simultaneous superluminal reflection and transmission, the peak times for the reflected and transmitted pulses are \(-33\)
and $-39 \text{ ns}$, respectively, see figure 6(a). Next, we change the phase from $\phi = 0$ to $\phi = \pi/2$ and plot the intensity profile of the incident, reflected and transmitted pulses in the time domain, see figure 6(b). In the case of simultaneous subluminal reflection and transmission, the peak times of the reflected and transmitted pulses shift to the positive time domain. We calculate the peak times of the reflected and transmitted pulses which are $+43 \text{ ns}$ and $+37 \text{ ns}$, respectively.

To study the peak time delay for the resonance condition of the slab system, i.e. $d = 2n \left( \frac{\omega_0}{c} \right)$, we again plot the intensity profiles for incident, reflected and transmitted pulses in the time domain for $\phi = 0$ and $\phi = \pi/2$. For simultaneous subluminal reflection and superluminal transmission for $\phi = 0$, we plot the incident, reflected and transmitted pulses in the time domain, see figure 7(a). We calculate the peak times of the reflected and transmitted pulses which are $+638 \text{ ns}$ and $-63 \text{ ns}$, respectively. Then we switch the phase from $\phi = 0$ to $\phi = \pi/2$ and plot the intensity profile of the incident, reflected and transmitted pulses in the time domain. In the case of simultaneous subluminal reflection and transmission we calculate the peak times $+784 \text{ ns}$ and $+82 \text{ ns}$ of the reflected and transmitted pulses.

Further, based on equation (11), consider the derivative of the phases in the reflected and transmitted light with probe frequency, which is consistent with Agarwal’s reciprocity theorem. For the off-resonant condition of the slab, we study the group delays in the reflected and transmitted light versus
probe field detuning, as shown in figure 8 for $\phi = 0$ and $\pi/2$, respectively. For $\phi = 0$, we have the simultaneous negative group delays both in the reflected and transmitted light fields, see figures 8(a) and (b). By comparing figures 8(a) and (b) and figures 2(a) and (b), it is obvious that we have similar results for slow and fast light propagation. By switching the phase $\phi$ from 0 to $\pi/2$, a switch from superluminal light propagation to subluminal is demonstrated both for the reflected and transmitted light, see figures 8(c) and (d). By comparing figures 8(c) and (d) with figures 3(a) and (b), again, we have similar behavior for light pulse propagation.

For the resonant condition of the slab, the plots in figures 9(a) and (b) show that the group delays are positive and negative for reflected and transmitted light beams for $\phi = 0$, respectively. This implies that the light pulse propagation in the reflected and transmitted beams are sub- and superluminal, respectively. By switching the phase $\phi$ from 0 to $\pi/2$, we have a positive group delay time in both the reflected and transmitted light pulses (subluminal behavior), as shown in figures 9(c) and (d). Based on the results shown in figures 8 and 9, we conclude that the behavior in light pulse propagation using equation (11) is similar to what we previously obtained using Agarwal’s reciprocity theorem.

V. Conclusion

We doped duplicated two-level atoms with a slab and investigated the propagation of a light pulse incident into the slab. We considered the off-resonant and resonant conditions of the slab system and investigated sub- and superluminal light pulse propagation of the reflected and transmitted light in each case. For the off-resonant condition, we investigated superluminal

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**Figure 8.** Group delay times in the reflected and transmitted light fields for off-resonant condition of the slab ((a), (b)) $\phi = 0$ and ((c), (d)) $\phi = \pi/2$; the other parameters remain the same as those in figure 2.

**Figure 9.** Group delay times in the reflected and transmitted light fields for resonant condition of the slab ((a), (b)) $\phi = 0$ and ((c), (d)) $\phi = \pi/2$; the other parameters remain the same as those in figure 2.
pulse propagation of the reflected and transmitted light simultaneously through the slab by considering the phase $\phi = 0$. By switching the phase from $\phi = 0$ to $\phi = \pi/2$, the behavior of the light propagation of the reflected and transmitted pulses through the slab is changed, and subluminal light propagation of the reflected and transmitted pulses is investigated. For the resonant condition of the slab system, we investigated the sub- and superluminal pulse propagation of the reflected and transmitted pulses simultaneously, respectively for $\phi = 0$. For $\phi = \pi/2$, subluminal light propagation is investigated in both reflected and transmitted pulses simultaneously. Moreover, the peak times of the Gaussian pulse during reflection and transmission through the slab system are calculated. Such a doped slab may be a tool to control the group velocity in quantum information applications. It may also used to control the dispersion and transmission in different fields of applied photonics.

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