Chapter 5 - Section A - Mathcad Solutions

5.2 **Let the symbols Q and Work represent rates in kJ/s. Then by Eq. (5.8)**

$$\eta = \frac{|Work|}{|Q_H|} = 1 - \frac{T_C}{T_H}$$

$T_C := 323.15 \cdot K$ $\qquad T_H := 798.15 \cdot K$ $\qquad Q_H := 250 \cdot \dfrac{kJ}{s}$

$Work := \left| Q_H \cdot \left(1 - \dfrac{T_C}{T_H} \right) \right|$ $\qquad |Work| = 148.78 \dfrac{kJ}{s}$

or $|Work| = 148.78\, kW$ **which is the power. Ans.**

By Eq. (5.1), $\qquad Q_C := |Q_H| - |Work|$ $\qquad Q_C = 101.22 \dfrac{kJ}{s}$ **Ans.**

5.3 (a) Let symbols Q and Work represent rates in kJ/s

$T_H := 750 \cdot K$ $\qquad T_C := 300 \cdot K$ $\qquad Work := -95000 \cdot kW$

By Eq. (5.8): $\qquad \eta := 1 - \dfrac{T_C}{T_H}$ $\qquad \eta = 0.6$

But $\qquad \eta = \dfrac{|Work|}{|Q_H|}$ **Whence** $\qquad Q_H := \dfrac{|Work|}{\eta}$

$Q_H = 1.583 \times 10^5\, kW$ **Ans.**

$Q_C := |Q_H| - |Work|$ $\qquad Q_C = 6.333 \times 10^4\, kW$ **Ans.**

(b) $\eta := 0.35$ $\qquad Q_H := \dfrac{|Work|}{\eta}$ $\qquad Q_H = 2.714 \times 10^5\, kW$ **Ans.**

$Q_C := |Q_H| - |Work|$ $\qquad Q_C = 1.764 \times 10^5\, kW$ **Ans.**

5.4 (a) $T_C := 303.15 \cdot K$ $\qquad T_H := 623.15 \cdot K$

$\eta_{Carnot} := 1 - \dfrac{T_C}{T_H}$ $\qquad \eta := 0.55 \cdot \eta_{Carnot}$ $\qquad \eta = 0.282$ **Ans.**

(b) $\eta := 0.35$ $\qquad$ $\eta_{Carnot} := \dfrac{\eta}{0.55}$ $\qquad$ $\eta_{Carnot} = 0.636$

By Eq. (5.8), $\qquad$ $T_H := \dfrac{T_C}{1 - \eta_{Carnot}}$ $\qquad$ $T_H = 833.66\,K$ **Ans.**

5.7 **Let the symbols represent rates where appropriate. Calculate mass rate of LNG evaporation:**

$V := 9000 \cdot \dfrac{m^3}{s}$ $\qquad$ $P := 1.0133 \cdot bar$ $\qquad$ $T := 298.15 \cdot K$

$molwt := 17\dfrac{gm}{mol}$ $\qquad$ $m_{LNG} := \dfrac{P \cdot V}{R \cdot T} \cdot molwt$ $\qquad$ $m_{LNG} = 6254\,\dfrac{kg}{s}$

Maximum power is generated by a Carnot engine, for which

$$\dfrac{|Work|}{|Q_C|} = \dfrac{|Q_H| - |Q_C|}{|Q_C|} = \dfrac{|Q_H|}{|Q_C|} - 1 = \dfrac{T_H}{T_C} - 1$$

$T_H := 303.15 \cdot K$ $\qquad\qquad$ $T_C := 113.7 \cdot K$

$Q_C := 512 \cdot \dfrac{kJ}{kg} \cdot m_{LNG}$ $\qquad$ $Q_C = 3.202 \times 10^6\,kW$

$Work := Q_C \cdot \left(\dfrac{T_H}{T_C} - 1\right)$ $\qquad$ $Work = 5.336 \times 10^6\,kW$ $\qquad$ **Ans.**

$Q_H := Q_C + Work$ $\qquad$ $Q_H = 8.538 \times 10^6\,kW$ $\qquad$ **Ans.**

5.8 **Take the heat capacity of water to be constant at the valu** $C_P := 4.184 \cdot \dfrac{kJ}{kg \cdot K}$

(a) $T_1 := 273.15 \cdot K$ $\qquad$ $T_2 := 373.15 \cdot K$ $\qquad$ $Q := C_P \cdot (T_2 - T_1)$ $\qquad$ $Q = 418.4\,\dfrac{kJ}{kg}$

$\Delta S_{H2O} := C_P \cdot \ln\left(\dfrac{T_2}{T_1}\right)$ $\qquad$ $\Delta S_{H2O} = 1.305\,\dfrac{kJ}{kg \cdot K}$

$\Delta S_{res} := \dfrac{-Q}{T_2}$ $\qquad$ $\Delta S_{res} = -1.121\,\dfrac{kJ}{kg \cdot K}$ $\qquad$ **Ans.**

$$\Delta S_{total} := \Delta S_{H2O} + \Delta S_{res} \qquad \Delta S_{total} = 0.184 \frac{kJ}{kg \cdot K} \quad \text{Ans.}$$

(b) The entropy change of the water is the same as in (a), and the total heat transfer is the same, but divided into two halves.

$$\Delta S_{res} := \frac{-Q}{2} \cdot \left(\frac{1}{323.15 \cdot K} + \frac{1}{373.15 \cdot K} \right) \qquad \Delta S_{res} = -1.208 \frac{kJ}{kg \cdot K}$$

$$\Delta S_{total} := \Delta S_{res} + \Delta S_{H2O} \qquad \Delta S_{total} = 0.097 \frac{kJ}{kg \cdot K} \quad \text{Ans.}$$

(c) The reversible heating of the water requires an infinite number of heat reservoirs covering the range of temperatures from 273.15 to 373.15 K, each one exchanging an infinitesimal quantity of heat with the water and raising its temperature by a differential increment.

5.9 $P_1 := 1 \cdot bar \qquad T_1 := 500 \cdot K \qquad V := 0.06 \cdot m^3$

$n := \frac{P_1 \cdot V}{R \cdot T_1} \qquad n = 1.443\, mol \qquad C_V := \frac{5}{2} \cdot R \qquad Q := 15000 \cdot J$

(a) Const.-V heating; $\quad \Delta U = Q + W = Q = n \cdot C_V \cdot (T_2 - T_1)$

$$T_2 := T_1 + \frac{Q}{n \cdot C_V} \qquad T_2 = 1 \times 10^3\, K$$

By Eq. (5.18), $\qquad \Delta S = n \cdot \left(C_P \cdot \ln\left(\frac{T_2}{T_1} \right) - R \cdot \ln\left(\frac{P_2}{P_1} \right) \right)$

But $\quad \dfrac{P_2}{P_1} = \dfrac{T_2}{T_1} \quad$ **Whence** $\quad \Delta S := n \cdot C_V \cdot \ln\left(\frac{T_2}{T_1} \right) \qquad \Delta S = 20.794 \frac{J}{K} \quad$ **Ans.**

(b) The entropy change of the gas is the same as in (a). The entropy change of the surroundings is zero. Whence

$$\Delta S_{total} = 10.794 \cdot \frac{J}{K} \quad \text{Ans.}$$

The stirring process is irreversible.

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5.10 (a) The temperature drop of the second stream (B) in either case is the same as the temperature rise of the first stream (A), i.e., 120 degC. The exit temperature of the second stream is therefore 200 degC. In both cases we therefore have:

$$C_P := \frac{7}{2}R$$

$$\Delta S_A := C_P \cdot \ln\left(\frac{463.15}{343.15}\right) \qquad\qquad \Delta S_B := C_P \cdot \ln\left(\frac{473.15}{593.15}\right)$$

$$\Delta S_A = 8.726 \frac{J}{mol \cdot K} \qquad\qquad \Delta S_B = -6.577 \frac{J}{mol \cdot K} \qquad\text{Ans.}$$

(b) For both cases:

$$\Delta S_{total} := \Delta S_A + \Delta S_B \qquad\qquad \Delta S_{total} = 2.149 \frac{J}{mol \cdot K} \qquad\text{Ans.}$$

(c) In this case the final temperature of steam B is 80 degC, i.e., there is a 10-degC driving force for heat transfer throughout the exchanger. Now

$$\Delta S_A := C_P \cdot \ln\left(\frac{463.15}{343.15}\right) \qquad\qquad \Delta S_B := C_P \cdot \ln\left(\frac{353.15}{473.15}\right)$$

$$\Delta S_A = 8.726 \frac{J}{mol \cdot K} \qquad\qquad \Delta S_B = -8.512 \frac{J}{mol \cdot K} \qquad\text{Ans.}$$

$$\Delta S_{total} := \Delta S_A + \Delta S_B \qquad\qquad \Delta S_{total} = 0.214 \frac{J}{mol \cdot K} \qquad\text{Ans.}$$

5.16 By Eq. (5.8),

$$\frac{dW}{dQ} = 1 - \frac{T_\sigma}{T} \qquad\qquad dW = dQ - T_\sigma \cdot \frac{dQ}{T}$$

Since $dQ/T = dS$,

$$dW = dQ - T_\sigma \cdot dS$$

Integration gives the required result.

$$T_1 := 600 \cdot K \qquad\qquad T_2 := 400 \cdot K \qquad\qquad T_\sigma := 300 \cdot K$$

$$Q := C_P \cdot (T_2 - T_1) \qquad\qquad Q = -5.82 \times 10^3 \frac{J}{mol}$$

$$\Delta S := C_P \cdot \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S = -11.799 \frac{J}{mol \cdot K}$$

$$Work := Q - T_\sigma \cdot \Delta S \qquad\qquad Work = -2280 \frac{J}{mol} \qquad \textbf{Ans.}$$

$$Q_\sigma := |Q| - |Work| \qquad\qquad Q_\sigma = 3540 \frac{J}{mol} \qquad \textbf{Ans.}$$

$$\Delta S_{reservoir} := \frac{Q_\sigma}{T_\sigma} \qquad\qquad \Delta S_{reservoir} = 11.8 \frac{J}{mol \cdot K} \qquad \textbf{Ans.}$$

$$\Delta S + \Delta S_{reservoir} = 0 \frac{J}{mol \cdot K} \qquad \textbf{Process is reversible.}$$

5.17 $\quad T_{H1} := 600 \cdot K \qquad T_{C1} := 300 \cdot K \qquad T_{H2} := 300 \cdot K \qquad T_{C2} := 250 \cdot K$

For the Carnot engine, use Eq. (5.8):
$$\frac{|W|}{|Q_{H1}|} = \frac{T_{H1} - T_{C1}}{T_{H1}}$$

The Carnot refrigerator is a reverse Carnot engine.
Combine Eqs. (5.8) & (5.7) to get:
$$\frac{|W|}{|Q_{C2}|} = \frac{T_{H2} - T_{C2}}{T_{C2}}$$

Equate the two work quantities and solve for the required ratio of the heat quantities:

$$r := \frac{T_{C2}}{T_{H1}} \cdot \left(\frac{T_{H1} - T_{C1}}{T_{H2} - T_{C2}}\right) \qquad\qquad r = 2.5 \qquad \textbf{Ans.}$$

5.18 (a) $\quad T_1 := 300K \qquad P_1 := 1.2bar \qquad T_2 := 450K \qquad P_2 := 6bar \qquad C_p := \frac{7}{2}R$

$$\Delta H := C_p \cdot (T_2 - T_1) \qquad\qquad \Delta H = 4.365 \times 10^3 \frac{J}{mol} \qquad \textbf{Ans.}$$

$$\Delta S := C_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right) \qquad\qquad \Delta S = -1.582 \frac{J}{mol \cdot K} \qquad \textbf{Ans.}$$

(b) $\qquad \Delta H = 5.82 \cdot 10^3 \frac{J}{mol} \qquad\qquad \Delta S = 1.484 \frac{J}{mol \cdot K}$

(c) $\Delta H = -3.118 \cdot 10^3 \dfrac{J}{mol}$ $\Delta S = 4.953 \dfrac{J}{mol \cdot K}$

(d) $\Delta H = -3.741 \cdot 10^3 \dfrac{J}{mol}$ $\Delta S = 2.618 \dfrac{J}{mol \cdot K}$

(e) $\Delta H = -6.651 \cdot 10^3 \dfrac{J}{mol}$ $\Delta S = -3.607 \dfrac{J}{mol \cdot K}$

5.19 This cycle is the same as is shown in Fig. 8.10 on p. 305. The equivalent states are A=3, B=4, C=1, and D=2. The efficiency is given by Eq. (A) on p. 305.

Temperature T_4 is not given and must be calaculated. The following equations are used to derive and expression for T_4.

For adiabatic steps 1 to 2 and 3 to 4:

$T_1 \cdot V_1^{\gamma-1} = T_2 \cdot V_2^{\gamma-1}$ $T_3 \cdot V_3^{\gamma-1} = T_4 \cdot V_4^{\gamma-1}$

For constant-volume step 4 to 1: $V_1 = V_4$

For isobaric step 2 to 3: $\dfrac{P_2}{T_2} = \dfrac{P_3}{T_3}$

Solving these 4 equations for T_4 yields: $T_4 = T_1 \cdot \left(\dfrac{T_2}{T_3}\right)^{-\gamma}$

$C_p := \dfrac{7}{2}R$ $C_v := \dfrac{5}{2}R$ $\gamma := \dfrac{C_p}{C_v}$ $\gamma = 1.4$

$T_1 := (200 + 273.15)K$ $T_2 := (1000 + 273.15)K$ $T_3 := (1700 + 273.15)K$

$T_4 := T_1 \cdot \left(\dfrac{T_2}{T_3}\right)^{-\gamma}$ $T_4 = 873.759\,K$

Eq. (A) p. 306 $\eta := 1 - \dfrac{1}{\gamma}\cdot\left(\dfrac{T_4 - T_1}{T_3 - T_2}\right)$ $\eta = 0.591$ **Ans.**

5.21 $C_V := C_P - R$ $P_1 := 2 \cdot bar$ $P_2 := 7 \cdot bar$ $T_1 := 298.15 \cdot K$

$\gamma := \dfrac{C_P}{C_V}$ $\gamma = 1.4$

With the reversible work given by Eq. (3.34), we get for the actual W:

$$Work := 1.35 \cdot \frac{R \cdot T_1}{\gamma - 1} \cdot \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$ $Work = 3.6 \times 10^3 \dfrac{J}{mol}$

But Q = 0, and $W = \Delta U = C_V \cdot \left(T_2 - T_1 \right)$ **Whence** $T_2 := T_1 + \dfrac{Work}{C_V}$

$T_2 = 471.374 \, K$

$$\Delta S := C_P \cdot \ln\left(\frac{T_2}{T_1} \right) - R \cdot \ln\left(\frac{P_2}{P_1} \right)$$ $\Delta S = 2.914 \dfrac{J}{mol \cdot K}$ **Ans.**

5.25 $P := 4$ $T := 800$

Step 1-2: Volume decreases at constant P.
Heat flows out of the system. Work is done on the system.

$$W_{12} = -\left[P \cdot \left(V_2 - V_1 \right) \right] = -\left[R \cdot \left(T_2 - T_1 \right) \right]$$

Step 2-3: Isothermal compression. Work is done on the system. Heat flows out of the system.

$$W_{23} = R \cdot T_2 \cdot \ln\left(\frac{P_3}{P_2} \right) = R \cdot T_2 \cdot \ln\left(\frac{P_3}{P_1} \right)$$

Step 3-1: Expansion process that produces work. Heat flows into the system. Since the PT product is constant,

$P \cdot dT + T \cdot dP = 0$ $T \cdot \dfrac{dP}{P} = -dT$ **(A)**

$P \cdot V = R \cdot T$ $P \cdot dV + V \cdot dP = R \cdot dT$

$P \cdot dV = R \cdot dT - V \cdot dP = R \cdot dT - R \cdot T \cdot \dfrac{dP}{P}$

In combination with (A) this becomes

$$P \cdot dV = R \cdot dT + R \cdot dT = 2 \cdot R \cdot dT$$

Moreover,
$$P_3 = P_1 \cdot \frac{T_1}{T_3} = P_1 \cdot \frac{T_1}{T_2}$$

$$W_{31} = -\int_{V_3}^{V_1} P \, dV = -2 \cdot R \cdot \left(\right) T_1 - T_3 \ = -2 \cdot R \cdot \left(\right) T_1 - T_2$$

$$Q_{31} = \Delta U_{31} - W_{31} = C_V \cdot \left(\right) T_1 - T_3 \ + 2 \cdot R \cdot \left(\right) T_1 - T_3$$

$$Q_{31} = \left(\right) C_V + 2 \cdot R \ \cdot \left(\right) T_1 - T_3 \ = \left(\right) C_P + R \ \cdot \left(\right) T_1 - T_2$$

$$\eta = \frac{|W_{net}|}{Q_{in}} = \frac{|W_{12} + W_{23} + W_{31}|}{Q_{31}}$$

$$C_P := \frac{7}{2} \cdot R \qquad T_1 := 700 \cdot K \qquad T_2 := 350 \cdot K$$

$$P_1 := 1.5 \cdot bar \qquad P_3 := P_1 \cdot \frac{T_1}{T_2}$$

$$W_{12} := -\left[R \cdot \left(\right) T_2 - T_1 \ \right] \qquad W_{12} = 2.91 \times 10^3 \ \frac{J}{mol}$$

$$W_{23} := R \cdot T_2 \cdot \ln\left(\frac{P_3}{P_1}\right) \qquad W_{23} = 2.017 \times 10^3 \ \frac{J}{mol}$$

$$W_{31} := -2 \cdot R \cdot \left(\right) T_1 - T_2 \qquad W_{31} = -5.82 \times 10^3 \ \frac{J}{mol}$$

$$Q_{31} := \left(\right) C_P + R \ \cdot \left(\right) T_1 - T_2 \qquad Q_{31} = 1.309 \times 10^4 \ \frac{J}{mol}$$

$$\eta := \frac{|W_{12} + W_{23} + W_{31}|}{Q_{31}} \qquad \eta = 0.068 \qquad \textbf{Ans.}$$

5.26 $T := 403.15 \cdot K$ $P_1 := 2.5 \cdot bar$ $P_2 := 6.5 \cdot bar$ $T_{res} := 298.15 \cdot K$

By Eq. (5.18), $\Delta S := -R \cdot \ln\left(\dfrac{P_2}{P_1}\right)$ $\Delta S = -7.944 \dfrac{J}{mol \cdot K}$ **Ans.**

With the reversible work given by Eq. (3.27), we get for the actual W:

$Work := 1.3 \cdot R \cdot T \cdot \ln\left(\dfrac{P_2}{P_1}\right)$ **(Isothermal compresion)** $Work = 4.163 \times 10^3 \dfrac{J}{mol}$

$Q := -Work$ **Q here is with respect to the system.**

So for the heat reservoir, we have

$\Delta S_{res} := \dfrac{-Q}{T_{res}}$ $\Delta S_{res} = 13.96 \dfrac{J}{mol \cdot K}$ **Ans.**

$\Delta S_{total} := \Delta S + \Delta S_{res}$ $\Delta S_{total} = 6.02 \dfrac{J}{mol \cdot K}$ **Ans.**

5.27 (a) By Eq. (5.14) with P = const. and Eq. (5.15), we get for the entropy change of 10 moles

$n := 10 \cdot mol$

$\Delta S := n \cdot R \cdot ICPS\left(473.15K, 1373.15K, 5.699, 0.640 \cdot 10^{-3}, 0.0, -1.015 \cdot 10^5\right)$

$\Delta S = 536.1 \dfrac{J}{K}$ **Ans.**

(b) By Eq. (5.14) with P = const. and Eq. (5.15), we get for the entropy change of 12 moles

$n := 12 \cdot mol$

$\Delta S := n \cdot R \cdot ICPS\left(523.15K, 1473.15K, 1.213, 28.785 \cdot 10^{-3}, -8.824 \cdot 10^{-6}, 0.0\right)$

$\Delta S = 2018.7 \dfrac{J}{K}$ **Ans.**

5.28 (a) The final temperature for this process was found in Pb. 4.2a to be 1374.5 K. The entropy change for 10 moles is then found as follows

$n := 10 \cdot mol$

$\Delta S := n \cdot R \cdot ICPS\left(473.15K, 1374.5K, 1.424, 14.394 \cdot 10^{-3}, -4.392 \cdot 10^{-6}, 0.0\right)$

$\Delta S = 900.86 \dfrac{J}{K}$ **Ans.**

(b) The final temperature for this process was found in Pb. 4.2b to be 1413.8 K. The entropy change for 15 moles is then found as follows:

$n := 15 \cdot mol$

$\Delta S := n \cdot R \cdot ICPS\left(533.15K, 1413.8K, 1.967, 31.630 \cdot 10^{-3}, -9.873 \cdot 10^{-6}, 0.0\right)$

$\Delta S = 2657.5 \dfrac{J}{K}$ **Ans.**

(c) The final temperature for this process was found in Pb. 4.2c to be 1202.9 K. The entropy change for 18.14 kg moles is then found as follows

$n := 18140 \cdot mol$

$\Delta S := n \cdot R \cdot ICPS\left(533.15K, 1202.9K, 1.424, 14.394 \cdot 10^{-3}, -4.392 \cdot 10^{-6}, 0.0\right)$

$\Delta S = 1.2436 \times 10^{6} \dfrac{J}{K}$ **Ans.**

5.29 The relative amounts of the two streams are determined by an energy balance. Since $Q = W = 0$, the enthalpy changes of the two streams must cancel. Take a basis of 1 mole of air entering, and let x = moles of chilled air. Then 1 - x = the moles of warm air.

$T_0 := 298.15 \cdot K$ **Temperature of entering air**

$T_1 := 248.15 \cdot K$ **Temperature of chilled air**

$T_2 := 348.15 \cdot K$ **Temperature of warm air**

$x \cdot C_P \cdot \left(T_1 - T_0\right) + (1 - x) \cdot C_P \cdot \left(T_2 - T_0\right) = 0$

$x := 0.3$ **(guess)**

Given $\quad \dfrac{x}{1-x} = -\left(\dfrac{T_2 - T_0}{T_1 - T_0}\right)$ $\qquad$ x := Find(x) $\qquad$ x = 0.5

Thus x = 0.5, and the process produces equal amounts of chilled and warmed air. The only remaining question is whether the process violates the second law. On the basis of 1 mole of entering air, the total entropy change is as follows.

$C_P := \dfrac{7}{2}\cdot R$ $\qquad$ $P_0 := 5\cdot bar$ $\qquad$ $P := 1\cdot bar$

$$\Delta S_{total} := x\cdot C_P\cdot \ln\left(\dfrac{T_1}{T_0}\right) + (1-x)\cdot C_P\cdot \ln\left(\dfrac{T_2}{T_0}\right) - R\cdot \ln\left(\dfrac{P}{P_0}\right)$$

$\Delta S_{total} = 12.97\,\dfrac{J}{mol\cdot K}$ $\qquad$ **Ans.**

Since this is positive, there is no violation of the second law.

5.30 $\quad$ $T_1 := 523.15\cdot K$ $\qquad$ $T_2 := 353.15\cdot K$ $\qquad$ $P_1 := 3\cdot bar$ $\quad$ $P_2 := 1\cdot bar$

$T_{res} := 303.15\cdot K$ $\qquad$ $Work := -1800\cdot \dfrac{J}{mol}$ $\qquad$ $C_P := \dfrac{7}{2}\cdot R$

$C_V := C_P - R$ $\qquad$ $Q = \Delta U - Work$ $\qquad$ $Q := C_V\cdot\left(T_2 - T_1\right) - Work$

$\Delta S_{res} := \dfrac{-Q}{T_{res}}$ $\qquad$ $\Delta S_{res} = 5.718\,\dfrac{J}{mol\cdot K}$ $\qquad$ $Q = -1.733\times 10^3\,\dfrac{J}{mol}$

$\Delta S := C_P\cdot \ln\left(\dfrac{T_2}{T_1}\right) - R\cdot \ln\left(\dfrac{P_2}{P_1}\right)$ $\qquad$ $\Delta S = -2.301\,\dfrac{J}{mol\cdot K}$

$\Delta S_{total} := \Delta S + \Delta S_{res}$ $\qquad$ $\Delta S_{total} = 3.42\,\dfrac{J}{mol\cdot K}$ $\quad$ **PROCESS IS POSSIBLE.**

5.33 For the process of cooling the brine:

$$C_P := 3.5 \cdot \frac{kJ}{kg \cdot K} \qquad \Delta T := -40 \cdot K \qquad mdot := 20 \cdot \frac{kg}{sec} \qquad \eta_t := 0.27$$

$T_1 := (273.15 + 25) \cdot K \qquad T_1 = 298.15\,K$

$T_2 := (273.15 - 15) \cdot K \qquad T_2 = 258.15\,K$

$T_\sigma := (273.15 + 30) \cdot K \qquad T_\sigma = 303.15\,K$

$\Delta H := C_P \cdot \Delta T \qquad \Delta H = -140\,\dfrac{kJ}{kg}$

$\Delta S := C_P \cdot \ln\left(\dfrac{T_2}{T_1}\right) \qquad \Delta S = -0.504\,\dfrac{kJ}{kg \cdot K}$

Eq. (5.26): $\quad Wdot_{ideal} := mdot \cdot \left(\Delta H - T_\sigma \cdot \Delta S\right) \quad Wdot_{ideal} = 256.938\,kW$

By Eq. (5.28): $\quad Wdot := \dfrac{Wdot_{ideal}}{\eta_t} \qquad\qquad Wdot = 951.6\,kW \qquad$ **Ans.**

5.34 $\quad E := 110 \cdot volt \qquad\qquad i := 9.7 \cdot amp \qquad T_\sigma := 300 \cdot K$

$Wdot_{mech} := -1.25 \cdot hp \qquad Wdot_{elect} := i \cdot E \qquad Wdot_{elect} = 1.067 \times 10^3\,W$

At steady state: $\quad Qdot + Wdot_{elect} + Wdot_{mech} = \dfrac{d}{dt}U^t = 0$

$$\dfrac{Qdot}{T_\sigma} + Sdot_G = \dfrac{d}{dt}S^t = 0$$

$Qdot := -Wdot_{elect} - Wdot_{mech} \qquad Qdot = -134.875\,W$

$Sdot_G := \dfrac{-Qdot}{T_\sigma} \qquad\qquad Sdot_G = 0.45\,\dfrac{W}{K} \qquad$ **Ans.**

5.35 $\Omega := 25 \cdot ohm$ $i := 10 \cdot amp$ $T_\sigma := 300 \cdot K$

$Wdot_{elect} := i^2 \cdot \Omega$ $\boxed{Wdot_{elect} = 2.5 \times 10^3\, W}$

At steady state: $Qdot + Wdot_{elect} = \dfrac{d}{dt} U^t = 0$ $Qdot := -Wdot_{elect}$

$\dfrac{Qdot}{T_\sigma} + Sdot_G = \dfrac{d}{dt} S^t = 0$ $Sdot_G := \dfrac{-Qdot}{T_\sigma}$

$\boxed{Qdot = -2.5 \times 10^3\, watt}$ $\boxed{Sdot_G = 8.333 \dfrac{watt}{K}}$ **Ans.**

5.38 $\boxed{mdot := 10 \dfrac{kmol}{hr}}$ $T_1 := (25 + 273.15)K$ $\boxed{P_1 := 10bar}$ $\boxed{P_2 := 1.2bar}$

$C_p := \dfrac{7}{2}R$ $C_v := C_p - R$ $\gamma := \dfrac{C_p}{C_v}$ $\boxed{\gamma = \dfrac{7}{5}}$

(a) Assuming an isenthalpic process: $T_2 := T_1$ $\boxed{T_2 = 298.15\,K}$ **Ans.**

(b) $\dfrac{\Delta S}{R} = \displaystyle\int_{T_1}^{T_2} \dfrac{C_p}{R}\dfrac{1}{T}\, dT - \ln\!\left(\dfrac{P_2}{P_1}\right)$ **Eq. (5.14)**

$\Delta S := \dfrac{7}{2}R \cdot \ln\!\left(\dfrac{T_2}{T_1}\right) - R \cdot \ln\!\left(\dfrac{P_2}{P_1}\right)$ $\boxed{\Delta S = 17.628 \dfrac{J}{mol \cdot K}}$ **Ans.**

(c) $Sdot_G := mdot \cdot \Delta S$ $\boxed{Sdot_G = 48.966 \dfrac{W}{K}}$ **Ans.**

(d) $\boxed{T_\sigma := (20 + 273.15)K}$ $W_{lost} := T_\sigma \cdot \Delta S$ $\boxed{W_{lost} = 5.168 \times 10^3 \dfrac{J}{mol}}$ **Ans.**

5.39 (a) $\boxed{T_1 := 500K}$ $\boxed{P_1 := 6bar}$ $\boxed{T_2 := 371K}$ $\boxed{P_2 := 1.2bar}$ $\boxed{C_p := \dfrac{7}{2}R}$

$\boxed{T_\sigma := 300K}$ **Basis: 1 mol** $\boxed{n := 1mol}$

$\Delta H := n \cdot C_p \cdot \left(\right)T_2 - T_1$ $W_s := \Delta H$ $\boxed{W_s = -3753.8\, J}$ **Ans.**

$$\Delta S := n \cdot \left(C_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right) \right)$$

$$\Delta S = 4.698 \frac{J}{K}$$

Eq. (5.27) $\quad W_{ideal} := \left(\right) \Delta H - T_\sigma \cdot \Delta S$ $\qquad W_{ideal} = -5163\,J$ **Ans.**

Eq. (5.30) $\quad W_{lost} := \left| W_{ideal} - W_s \right|$ $\qquad W_{lost} = 1409.3\,J$ **Ans.**

Eq. (5.39) $\quad S_G := \dfrac{W_{lost}}{T_\sigma}$ $\qquad S_G = 4.698 \dfrac{J}{K}$ **Ans.**

	W_s	W_{ideal}	W_{lost}	S_G
(a)	$-3753.8J$	$-5163J$	$1409.3J$	$4.698\frac{J}{K}$
(b)	$-2460.9J$	$-2953.9J$	$493J$	$1.643\frac{J}{K}$
(c)	$-3063.7J$	$-4193.7J$	$1130J$	$3.767\frac{J}{K}$
(d)	$-3853.5J$	$-4952.4J$	$1098.8J$	$3.663\frac{J}{K}$
(e)	$-3055.4J$	$-4119.2J$	$1063.8J$	$3.546\frac{J}{K}$

5.41 $\quad P_1 := 2500\,kPa \qquad P_2 := 150\,kPa \qquad T_\sigma := 300K \qquad mdot := 20\dfrac{mol}{sec}$

$$\Delta S := -R \cdot \ln\left(\frac{P_2}{P_1}\right) \qquad \Delta S = 0.023 \frac{kJ}{mol \cdot K}$$

$$Sdot_G := mdot \cdot \Delta S \qquad Sdot_G = 0.468 \frac{kJ}{sec \cdot K} \qquad \textbf{Ans.}$$

$$Wdot_{lost} := T_\sigma \cdot Sdot_G \qquad Wdot_{lost} = 140.344\,kW \qquad \textbf{Ans.}$$

5.42 $\quad Q_H := 1kJ \qquad W := 0.45kJ \qquad T_H := (250 + 273.15)K \qquad T_H = 523.15\,K$

$$T_C := (25 + 273.15)K \qquad T_C = 298.15\,K$$

$$\eta_{actual} := \frac{|W|}{|Q_H|} \qquad \eta_{actual} = 0.45$$

$$\eta_{max} := 1 - \frac{T_C}{T_H} \qquad \eta_{max} = 0.43$$

Since $\eta_{actual} > \eta_{max}$, the process is impossible.

5.43 $Q_H := -150 \cdot kJ \qquad Q_1 := 50 \cdot kJ \qquad Q_2 := 100 \cdot kJ$

$T_H := 550 \cdot K \qquad T_1 := 350 \cdot K \qquad T_2 := 250 \cdot K \qquad T_\sigma := 300 \cdot K$

(a) $S_G := \dfrac{Q_H}{T_H} + \dfrac{Q_1}{T_1} + \dfrac{Q_2}{T_2} \qquad S_G = 0.27 \dfrac{kJ}{K} \qquad$ **Ans.**

(b) $W_{lost} := T_\sigma \cdot S_G \qquad\qquad W_{lost} = 81.039 \, kJ \qquad$ **Ans.**

5.44 $Wdot := -750 \cdot MW \qquad T_H := (315 + 273.15) \cdot K \qquad T_C := (20 + 273.15) \cdot K$

$$T_H = 588.15 \, K \qquad\qquad T_C = 293.15 \, K$$

(a) $\eta_{max} := 1 - \dfrac{T_C}{T_H} \qquad \eta_{max} = 0.502 \qquad$ **Ans.**

$$Qdot_H := \frac{|Wdot|}{\eta_{max}} \qquad\qquad Qdot_C := Qdot_H - |Wdot|$$

$$Qdot_C = 745.297 \, MW \qquad \textbf{(minimum value)}$$

(b) $\eta := 0.6 \cdot \eta_{max} \qquad Qdot_H := \dfrac{|Wdot|}{\eta} \qquad\qquad Qdot_H = 2.492 \times 10^9 \, W$

$$Qdot_C := Qdot_H - |Wdot| \qquad Qdot_C = 1.742 \times 10^3 \, MW \qquad \textbf{(actual value)}$$

River temperature rise: $Vdot := 165 \cdot \dfrac{m^3}{s} \qquad \rho := 1 \cdot \dfrac{gm}{cm^3}$

$$Cp := 1 \cdot \frac{cal}{gm \cdot K} \qquad \Delta T := \frac{Qdot_C}{Vdot \cdot \rho \cdot Cp} \qquad \Delta T = 2.522 \, K \qquad \textbf{Ans.}$$

(a) With $dQ_C = C_C^t dT_C$, Eq. (B) becomes:

$$\frac{dQ_H}{C_C^t dT_C} = -\frac{T_H}{T_C} \qquad \text{or} \qquad dQ_H = -C_C^t \frac{T_H}{T_C} dT_C$$

Substitute for dQ_H and dQ_C in Eq. (A):

$$dW = -C_C^t T_H \frac{dT_C}{T_C} + C_C^t dT_C$$

Integrate from T_{C_0} to T_C:

$$W = -C_C^t T_H \ln \frac{T_C}{T_{C_0}} + C_C^t (T_C - T_{C_0}) \qquad \text{or} \qquad \boxed{W = C_C^t \left(T_H \ln \frac{T_{C_0}}{T_C} + T_C - T_{C_0} \right)}$$

(b) For infinite time, $T_C = T_H$, and the boxed equation above becomes:

$$\boxed{W = C_C^t \left(T_H \ln \frac{T_{C_0}}{T_H} + T_H - T_{C_0} \right)}$$

5.15 Write Eqs. (5.8) and (5.1) in rate form and combine to eliminate $|\dot{Q}_H|$:

$$\frac{|\dot{W}|}{|\dot{W}| + |\dot{Q}_C|} = 1 - \frac{T_C}{T_H} = 1 - r \qquad \text{or} \qquad \frac{|\dot{W}|}{1 - r} = |\dot{W}| + |\dot{Q}| \qquad \text{where} \qquad r \equiv \frac{T_C}{T_H}$$

With $|\dot{Q}_C| = kA(T_C)^4 = kA(rT_H)^4$, this becomes:

$$|\dot{W}| \left(\frac{1}{1-r} - 1 \right) = |\dot{W}| \left(\frac{r}{1-r} \right) = kAr^4(T_H)^4 \qquad \text{or} \qquad A = \left[\frac{|\dot{W}|}{k(T_H)^4} \right] \frac{1}{(1-r)r^3}$$

Differentiate, noting that the quantity in square brackets is constant:

$$\frac{dA}{dr} = \left[\frac{|\dot{W}|}{k(T_H)^4} \right] \left[\frac{-3}{(1-r)r^4} + \frac{1}{(1-r)^2 r^3} \right] = \left[\frac{|\dot{W}|}{k(T_H)^4} \right] \left[\frac{4r-3}{(1-r)^2 r^4} \right]$$

Equating this equation to zero, leads immediately to: $\quad 4r = 3 \quad$ or $\quad \boxed{r = 0.75}$

5.20 Because $W = 0$, Eq. (2.3) here becomes:

$$Q = \Delta U^t = mC_V \Delta T$$

A necessary condition for ΔT to be zero when Q is non-zero is that $m = \infty$. This is the reason that natural bodies (air and water) that serve as heat reservoirs must be massive (oceans) or continually renewed (rivers).

5.22 An appropriate energy balance here is: $\quad Q = \Delta H^t = 0$

Applied to the process described, with T as the final temperature, this becomes:

$$m_1 C_P (T - T_1) + m_2 C_P (T - T_2) = 0 \qquad \text{whence} \qquad T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} \qquad\qquad (1)$$

$$\text{If } m_1 = m_2, \quad T = (T_1 + T_2)/2$$

The total entropy change as a result of temperature changes of the two masses of water:

$$\Delta S^t = m_1 C_P \ln \frac{T}{T_1} + m_2 C_P \ln \frac{T}{T_2} \tag{2}$$

Equations (1) and (2) represent the general case. If $m_1 = m_2 = m$,

$$\Delta S^t = m C_P \ln \frac{T^2}{T_1 T_2} \qquad \text{or} \qquad \boxed{\Delta S^t = 2m C_P \ln \frac{T}{\sqrt{T_1 T_2}}}$$

Because $T = (T_1 + T_2)/2 > \sqrt{T_1 T_2}$, ΔS^t is positive.

5.23 Isentropic processes are **not** necessarily reversible and adiabatic. The term *isentropic* denotes a process for which the *system* does not change in entropy. There are two causes for entropy changes in a system: The process may be internally irreversible, causing the entropy to increase; heat may be transferred between system amd surroundings, causing the entropy of the system to increase or decrease. For processes that are internally irreversible, it is possible for heat to be transferred **out** of the system in an amount such that the entropy changes from the two causes exactly compensate each other. One can imagine irreversible processes for which the state of the system is the same at the end as at the beginning of the process. The process is then necessarily isentropic, but neither reversible nor adiabatic. More generally, the system conditions may change in such a way that entropy changes resulting from temperature and pressure changes compensate each other. Such a process is isentropic, but not necessarily reversible. Expansion of gas in a piston/cylinder arrangement is a case in point. It may be reversible and adiabatic, and hence isentropic. But the same change of state may be irreversible with heat transfer to the surroundings. The process is still isentropic, but neither reversible nor adiabatic. An isentropic process must be either reversible *and* adiabatic or irreversible *and* non-adiabatic.

5.24 By definition,
$$\langle C_P \rangle_H = \frac{\int_{T_0}^{T} C_P dT}{T - T_0} = \frac{\int_{T}^{T_0} C_P dT}{T_0 - T}$$

By inspection, one sees that for both $T > T_0$ and $T_0 > T$ the numerators and denominators of the above fractions have the same sign. Thus, for both cases $\langle C_P \rangle_H$ is positive.

Similarly,
$$\langle C_P \rangle_S = \frac{\int_{T_0}^{T} C_P \frac{dT}{T}}{\ln(T/T_0)} = \frac{\int_{T}^{T_0} C_P \frac{dT}{T}}{\ln(T_0/T)}$$

By inspection, one sees that for both $T > T_0$ and $T_0 > T$ the numerators and denominators of the above fractions have the same sign. Thus, for both cases $\langle C_P \rangle_S$ is positive.

When $T = T_0$, both the numerators and denominators of the above fractions become zero, and the fractions are indeterminate. Application of l'Hôpital's rule leads to the result: $\langle C_P \rangle_H = \langle C_P \rangle_S = C_P$.

5.31 The process involves three heat reservoirs: the house, a heat sink; the furnace, a heat source; and the surroundings, a heat source. Notation is as follows:

$|Q|$ Heat transfer **to** the house at temperature T
$|Q_F|$ Heat transfer **from** the furnace at T_F
$|Q_\sigma|$ Heat transfer **from** the surroundings at T_σ

The first and second laws provide the two equations:

$$|Q| = |Q_F| + |Q_\sigma| \qquad \text{and} \qquad \frac{|Q|}{T} - \frac{|Q_F|}{T_F} - \frac{|Q_\sigma|}{T_\sigma} = 0$$

Combine these equations to eliminate $|Q_\sigma|$, and solve for $|Q_F|$:

$$|Q_F| = |Q|\left(\frac{T - T_\sigma}{T_F - T_\sigma}\right)\frac{T_F}{T}$$

With $\qquad T = 295 \text{ K} \qquad T_F = 810 \text{ K} \qquad T_\sigma = 265 \text{ K} \qquad$ and $|Q| = 1000 \text{ kJ}$

The result is: $\qquad\qquad\qquad\qquad |Q_F| = 151.14 \text{ kJ}$

Shown to the right is a scheme designed to accomplish this result. A Carnot heat engine operates with the furnace as heat source and the house as heat sink. The work produced by the engine drives a Carnot refrigerator (reverse Carnot engine) which extracts heat from the surroundings and discharges heat to the house. Thus the heat rejected by the Carnot engine ($|Q_1|$) and by the Carnot refrigerator ($|Q_2|$) together provide the heat $|Q|$ for the house. The energy balances for the engine and refrigerator are:

$$|W|_{\text{engine}} = |Q_F| - |Q_1|$$

$$|W|_{\text{refrig}} = |Q_2| - |Q_\sigma|$$

Equation (5.7) may be applied to both the engine and the refrigerator:

$$\frac{|Q_F|}{|Q_1|} = \frac{T_F}{T} \qquad\qquad \frac{|Q_\sigma|}{|Q_2|} = \frac{T_\sigma}{T}$$

Combine the two pairs of equations:

$$|W|_{\text{engine}} = |Q_1|\left(\frac{T_F}{T} - 1\right) = |Q_1|\frac{T_F - T}{T} \qquad\qquad |W|_{\text{refrig}} = |Q_2|\left(1 - \frac{T_\sigma}{T}\right) = |Q_2|\frac{T - T_\sigma}{T}$$

Since these two quantities are equal,

$$|Q_1|\frac{T_F - T}{T} = |Q_2|\frac{T - T_\sigma}{T} \qquad \text{or} \qquad |Q_2| = |Q_1|\frac{T_F - T}{T - T_\sigma}$$

Because the total heat transferred to the house is $|Q| = |Q_1| + |Q_2|$,

$$|Q| = |Q_1| + |Q_1|\frac{T_F - T}{T - T_\sigma} = |Q_1|\left(1 + \frac{T_F - T}{T - T_\sigma}\right) = |Q_1|\frac{T_F - T_\sigma}{T - T_\sigma}$$

But $\qquad |Q_1| = |Q_F|\dfrac{T}{T_F} \qquad\qquad$ whence $\qquad\qquad |Q| = |Q_F|\dfrac{T}{T_F}\left(\dfrac{T_F - T_\sigma}{T - T_\sigma}\right)$

Solution for $|Q_F|$ yields the same equation obtained more easily by direct application of the two laws of thermodynamics to the overall result of the process.

5.32 The process involves three heat reservoirs: the house, a heat source; the tank, a heat source; and the surroundings, a heat sink. Notation is as follows:
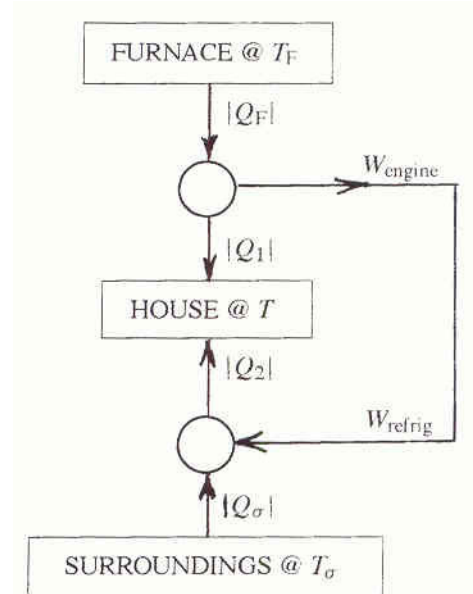
$|Q|$ Heat transfer **from** the tank at temperature T

$|Q'|$ Heat transfer **from** the house at T'

$|Q_\sigma|$ Heat transfer **to** the surroundings at T_σ

The first and second laws provide the two equations:

$$|Q| + |Q'| = |Q_\sigma| \qquad \text{and} \qquad \frac{|Q_\sigma|}{T_\sigma} - \frac{|Q|}{T} - \frac{|Q'|}{T'} = 0$$

Combine these equations to eliminate $|Q_\sigma|$, and solve for $|Q|$:

$$\boxed{|Q| = |Q'|\left(\frac{T_\sigma - T'}{T - T_\sigma}\right)\frac{T}{T'}}$$

With $T = 448.15\ \text{K}$ $T' = 297.15\ \text{K}$ $T_\sigma = 306.15\ \text{K}$ and $|Q'| = 1500\ \text{kJ}$

The result is: $\boxed{|Q| = 143.38\ \text{kJ}}$

Shown to the right is a scheme designed to accomplish this result. A Carnot heat engine operates with the tank as heat source and the surroundings as heat sink. The work produced by the engine drives a Carnot refrigerator (reverse Carnot engine) which extracts heat $|Q'|$ from the house and discharges heat to the surroundings. The energy balances for the engine and refrigerator are:

$$|W|_{\text{engine}} = |Q| - |Q_{\sigma_1}|$$

$$|W|_{\text{refrig}} = |Q_{\sigma_2}| - |Q'|$$

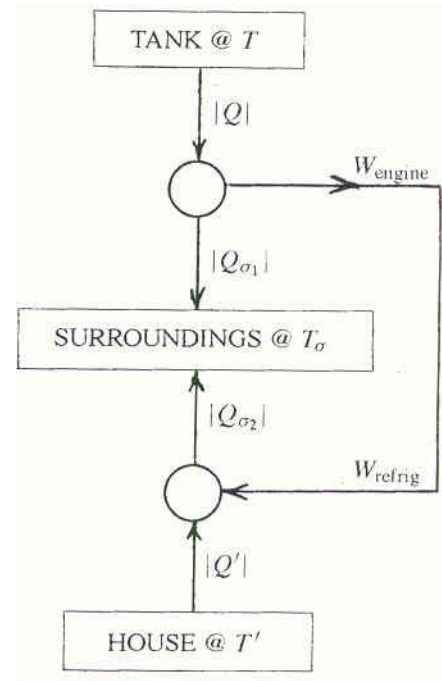Equation (5.7) may be applied to both the engine and the refrigerator:

$$\frac{|Q_{\sigma_1}|}{|Q|} = \frac{T_\sigma}{T} \qquad\qquad \frac{|Q_{\sigma_2}|}{|Q'|} = \frac{T_\sigma}{T'}$$

Combine the two pairs of equations:

$$|W|_{\text{engine}} = |Q|\left(1 - \frac{T_\sigma}{T}\right) = |Q|\frac{T - T_\sigma}{T} \qquad\qquad |W|_{\text{refrig}} = |Q'|\left(\frac{T_\sigma}{T'}\right) = |Q'|\frac{T_\sigma - t'}{T'}$$

Since these two quantities are equal,

$$|Q|\frac{T - T_\sigma}{T} = |Q'|\frac{T_\sigma - T'}{T'} \qquad \text{or} \qquad \boxed{|Q| = |Q'|\left(\frac{T_\sigma - T'}{T - T_\sigma}\right)\frac{T}{T'}}$$

5.36 For a closed system the first term of Eq. (5.21) is zero, and it becomes:

$$\frac{d(mS)_{\text{cv}}}{dt} + \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}} = \dot{S}_G \geq 0$$

where $\dot{Q}_j$ is here redefined to refer to the system rather than to the surroundings. Nevertheless, the second term accounts for the entropy changes of the surroundings, and can be written simply as dS^t_{surr}/dt:

$$\frac{d(mS)_{cv}}{dt} - \frac{dS^t_{surr}}{dt} = \dot{S}_G \geq 0 \qquad \text{or} \qquad \frac{dS^t_{cv}}{dt} - \frac{dS^T_{surr}}{dt} = \dot{S}_G \geq 0$$

Multiplication by dt and integration over finite time yields:

$$\Delta S^t_{cv} + \Delta S^t_{surr} \geq 0 \qquad \text{or} \qquad \boxed{\Delta S_{total} \geq 0}$$

5.37 The general equation applicable here is Eq. (5.22):

$$\boxed{\Delta(S\dot{m})_{fs} - \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}} = \dot{S}_G \geq 0}$$

(a) For a single stream flowing within the pipe and with a single heat source in the surroundings, this becomes:

$$(\Delta S)\dot{m} - \frac{\dot{Q}}{T_\sigma} = \dot{S}_G \geq 0$$

(b) The equation is here written for two streams (I and II) flowing in two pipes. Heat transfer is internal, between the two streams, making $\dot{Q} = 0$. Thus,

$$(\Delta S)_I \dot{m}_I + (\Delta S)_{II} \dot{m}_{II} = \dot{S}_G \geq 0$$

(c) For a pump operatiing on a single stream and with the assumption of negligible heat transfer to the surroundings:

$$(\Delta S)\dot{m} = \dot{S}_G \geq 0$$

(d) For an adiabatic gas compressor the result is the same as for Part (c).

(e) For an adiabatic turbine the result is the same as for Part (c).

(f) For an adiabatic throttle valve the result is the same as for Part (c).

(g) For an adiabatic nozzle the result is the same as for Part (c).

5.40 The figure on the left below indicates the direct, irreversible transfer of heat $|Q|$ from a reservoir at T_1 to a reservoir at T_2. The figure on the right depicts a completely reversible process to accomplish the *same changes in the heat reservoirs* at T_1 and T_2.