Determination of Forward and Futures Prices

Chapter 3

Consumption vs Investment Assets

- Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)
- Consumption assets are assets held primarily for consumption (Examples: copper, oil)

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Short Selling **(Page 41-42)**

- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way

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Short Selling

(continued)

- At some stage you must buy the securities back so they can be replaced in the account of the client
- You must pay dividends and other benefits the owner of the securities receives

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Measuring Interest Rates

- The compounding frequency used for an interest rate is the unit of measurement
- The difference between quarterly and annual compounding is analogous to the difference between miles and kilometers

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Continuous Compounding

(Page 43)

- In the limit as we compound more and more frequently we obtain continuously compounded interest rates
- \$100 grows to $\$100e^{RT}$ when invested at a continuously compounded rate R for time T
- \$100 received at time T discounts to $\$100e^{-RT}$ at time zero when the continuously compounded discount rate is R

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Conversion Formulas

(Page 44)

Define

R_c : continuously compounded rate

R_m: same rate with compounding m times per year

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c/m} - 1\right)$$

Notation

S_0:	Spot price today
F_0:	Futures or forward price today
T:	Time until delivery date
r:	Risk-free interest rate for maturity T

Gold Example (From Chapter 1)

- For gold

$$F_0 = S_0(1 + r)^T$$

(assuming no storage costs)

- If r is compounded continuously instead of annually

$$F_0 = S_0e^{rT}$$

Extension of the Gold Example

(Page 46, equation 3.5)

- For any investment asset that provides no income and has no storage costs

$$F_0 = S_0e^{rT}$$

When an Investment Asset Provides a Known Dollar Income (page 48, equation 3.6)

$$F_0 = (S_0 - I)e^{rT}$$

where I is the present value of the income

When an Investment Asset Provides a Known Yield

(Page 49, equation 3.7)

$$F_0 = S_0\, e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

Valuing a Forward Contract

Page 50

- Suppose that

  K is delivery price in a forward contract

  F_0 is forward price that would apply to the contract today
- The value of a long forward contract, f, is

  $$f = (F_0 - K)e^{-rT}$$
- Similarly, the value of a short forward contract is

  $$(K - F_0)e^{-rT}$$

Forward vs Futures Prices

- Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different:
- A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
- A strong negative correlation implies the reverse

Stock Index (Page 52)

- Can be viewed as an investment asset paying a dividend yield
- The futures price and spot price relationship is therefore

$$F_0 = S_0 \, e^{(r-q)T}$$

where q is the dividend yield on the portfolio represented by the index

Stock Index

(continued)

- For the formula to be true it is important that the index represent an investment asset
- In other words, changes in the index must correspond to changes in the value of a tradable portfolio
- The Nikkei index viewed as a dollar number does not represent an investment asset

Index Arbitrage

- When $F_0 > S_0 e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures
- When $F_0 < S_0 e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index

Index Arbitrage

(continued)

- Index arbitrage involves simultaneous trades in futures and many different stocks
- Very often a computer is used to generate the trades
- Occasionally (e.g., on Black Monday) simultaneous trades are not possible and the theoretical no-arbitrage relationship between F_0 and S_0 does not hold

3.19

Futures and Forwards on Currencies (Page 55-58)

- A foreign currency is analogous to a security providing a dividend yield
- The continuous dividend yield is the foreign risk-free interest rate
- It follows that if r_f is the foreign risk-free interest rate

$$F_0 = S_0 e^{(r - r_f)T}$$

3.20

Futures on Consumption Assets (Page 59)

$$F_0 \leq S_0 \, e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.

Alternatively,

$$F_0 \leq (S_0 + U)e^{rT}$$

where U is the present value of the storage costs.

3.21

The Cost of Carry (Page 60)

- The cost of carry, c, is the storage cost plus the interest costs less the income earned
- For an investment $F_0 = S_0 e^{cT}$
- For a consumption asset $F_0 \leq S_0 e^{cT}$
- The convenience yield on the consumption asset, y, is defined so that

$$F_0 = S_0 \, e^{(c-y)T}$$

3.22

Futures Prices & Expected Future Spot Prices (Page 61)

- Suppose k is the expected return required by investors on an asset
- We can invest $F_0 e^{-rT}$ now to get S_T back at maturity of the futures contract
- This shows that

$$F_0 = E\,(S_T)e^{(r-k)T}$$

3.23

Futures Prices & Future Spot Prices (continued)

- If the *asset* has
  - no systematic risk, then $k = r$ and F_0 is an unbiased estimate of S_T
  - positive systematic risk, then $k > r$ and $F_0 < E\,(S_T)$
  - negative systematic risk, then $k < r$ and $F_0 > E\,(S_T)$