Portfolio Management of Default Risk

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Published by:

KMV
1620 Montgomery Street, Suite 140
San Francisco, CA 94111 U.S.A.
Phone: +1 415-296-9669
FAX: +1 415-296-9458
email: support@kmv.com
website: http: // www.kmv.com

Author(s):

Stephen Kealhofer
Jeffrey R. Bohn

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Introduction

Corporate liabilities have default risk. There is always a chance that a corporate borrower will not meet its obligations to pay principal and interest. For the typical high-grade borrower, this risk is small, perhaps 1/10 of 1% per year. For the typical bank borrower this risk is about 1/2 of 1%.

Although these risks do not seem large, they are in fact highly significant. First, they can increase quickly and with little warning. Second, the margins in corporate lending are very tight, and even small miscalculations of default risks can undermine the profitability of lending. But most importantly, many lenders are themselves borrowers, with high levels of leverage. Unexpected realizations of default risk have destabilized, de-capitalized and destroyed lenders. Banks, finance companies, insurers, investment banks, lessors: none have escaped unscathed.

Default risk cannot be hedged away, or "structured" away. The government cannot insure it away. It is a reflection of the substantial risk in companies' futures. Various schemes exist, and more are coming, which can shift risk, but in the end, someone must bear this risk. It does not "net out" in the aggregate.

Default risk can be reduced and managed through diversification. Default risk, and the rewards for bearing it, will ultimately be owned by those who can diversify it best.

Every lender knows the benefits of diversification. Every lender works to achieve these benefits. However, until recently lenders have been reluctant to, or unable to, implement systems for actually measuring the amount of diversification in a debt portfolio.

Portfolios have "concentrations"; ex post we see them. Ex ante, lenders must look to models and software to quantify concentrations. Until recently, these types of models have not been generally available. Thus it should not come as a surprise that there have been many unexpected default events in lenders' portfolios in the past.

Quantitative methods for portfolio analysis have developed since Markowitz's pioneering work in 1950. These methods have been applied successfully in a variety of areas of finance, notably to equity portfolios. These methods show the amount of risk reduction achievable through diversification. They measure the amount of risk contributed by an asset, or group of assets, to a portfolio. By extension, they also show the amount of diversification provided by a single asset or group of assets. The aim of these methods is to maximize the return to a portfolio while keeping the risk within acceptable bounds. This maximization requires a balancing of return to risk within the portfolio, asset by asset, group of assets by group of assets.

This logic can be illustrated by imagining that it was not the case. If a low-return-to-risk asset is swapped for a high-return-to-risk asset, then the portfolio's return can be improved with no addition to risk. The process is equilibrated by changes in risk. As an asset is swapped out of the portfolio, it changes from being a source of concentration to being a source of diversification, i.e., its risk contribution falls. The reverse applies as an asset is swapped into the portfolio. Thus the return-to-risk increases for the low return asset and decreases for the high return asset, until their return-to-risk ratios are equal. At that point, no further swap can raise return without also raising risk. This then characterizes the optimal portfolio or, equivalently, the optimal set of holdings.

This conceptual model applies to the default risk of debt as surely as it applies to equities. Equity practitioners however have used the last twenty-five years to develop techniques for measuring the asset attributes that are necessary for an actual portfolio management tool.

The same development has not occurred for debt portfolios because of the greater analytical and empirical difficulties. In particular, it is necessary to quantify the level of default risk in a single asset, and to quantify the relationship between the default risks of each pair of assets in the portfolio.

Due to a variety of technical developments in finance, it has become both possible and feasible to make these measurements. KMV has pioneered the development of these methods for the last 12 years in its practice with commercial banks. The fruits of this development effort are several products designed to address the quantification and management of credit risk. KMV estimates an expected default frequency (EDF™) for firms with publicly traded equity and delivers this estimate via a PC-based viewer called Credit Monitor™ or an internet-based viewer called CreditEdge™. Both of these software products cover nearly 30,000 firms globally and come bundled with a variety of analysis tools. For firms without publicly traded equity, KMV offers the Private Firm Model (PFM™) which also produces an EDF credit measure. The PFM EDF values are housed in a software product called the Private Firm Analyst™ that works in tandem with Credit Monitor. KMV's EDF values combined with facility-specific data can be used together with KMV's Global Correlation Model™ and Portfolio Manager™ to analyze and manage portfolios of credit-risky assets. The result is that practical and conceptually sound methods exist for measuring actual diversification, and for determining portfolio holdings to minimize concentrations and maximize return in debt portfolios.

The remainder of this paper provides an introduction to the methods and approaches of quantitative debt portfolio management underlying KMV's products and models, and their implications for bank management.

The Model of Default Risk

A corporation has fixed obligations. These may be no more than its trade obligations, although they could just as well include bank loans and public debt. At one time, there was no legal means to escape the fulfillment of such obligations; a defaulter fled or was jailed. Modern treatment allows the defaulter to escape the obligation but only by relinquishing the corporation's assets to the obligee. In other words, a firm owing a single creditor $75 million fulfills the obligation by either paying the $75 million or by transferring the corporation's assets to the lender.

Which action the borrower will take is an economic decision.

And the economic answer is straightforward: if the corporate assets are worth more than $75 million, the borrower will meet the obligation; if they are worth less, the borrower will default. The critical point is that the action depends on the market value of assets; book or accounting value will not suffice.

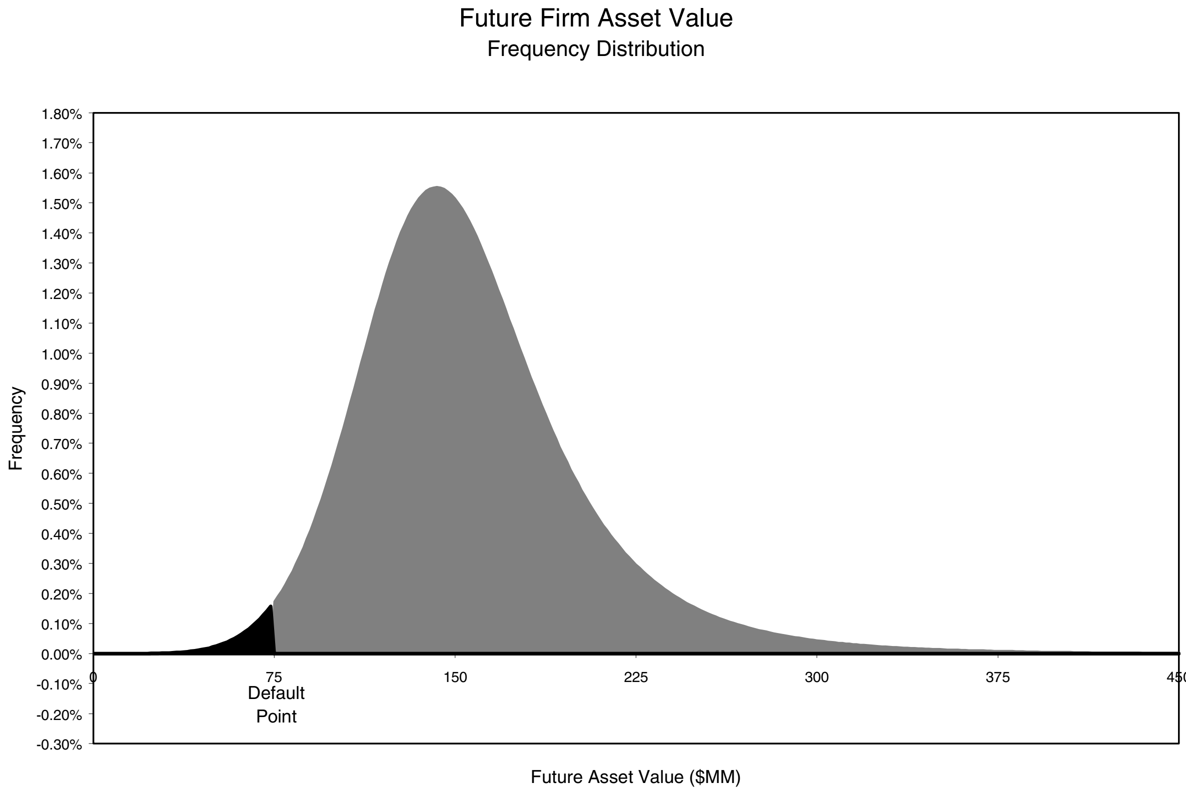
Note that the "option to default" is valuable. Without it, the corporation could be forced to raise additional capital with the benefit accruing not to its owners but instead to its prior lender.

A lender purchasing a corporation's note can be thought of as engaging in two transactions. In the first it is purchasing an "inescapable" debt obligation, i.e. one which cannot be defaulted on. In the second, it is selling a "put" option to the borrower that states that the lender will buy the corporation's assets for the amount of the note at the option of the borrower. In the event the assets turn out to be worth less than the amount of the note, the borrower "puts" the assets to the lender and uses the proceeds to pay the note.

The lender owns a risk-free note and is "short" the default option. The probability of default is the same as the probability of the option being exercised. If the probability of default goes up, the value of the option goes up, and the value of the lender's position (because it is "short" the option) goes down.

The probability of exercising the default option can be determined by application of option valuation methods. Assume for a moment that the market value of the corporation's assets is known, as well as the volatility of that value. The volatility measures the propensity of the asset value to change within a given time period. This information determines the probability of default, given the corporation's obligations. For instance, if the current asset market value is $150 million and the corporation's debt is $75 million and is due in one year, then default will occur if the asset value turns out to be less than $75 million in one year.

If the firm's asset volatility is 17% per year, then a fall in value from $150 million to $75 million is a three standard deviation event with a probability of 0.3%. Thus the firm has a default probability of 0.3%. [17% of 150 is 25. This is the amount of a one standard deviation move. The probability calculation assumes that the assets have a lognormal distribution.]



The market value of the firm's assets in one year is unknown. Based on firm characteristics including past performance, the expected asset value is determined to be $150 million, with a standard deviation of $25 million. This information makes it possible to represent the range of possible asset values and their frequencies in the diagram above.

The firm has obligations due in one year of $75 million. If the market asset value turns out to be less than $75 million, then the owners of the firm will prefer to default. If the asset value is greater than $75 million, then the owners will prefer to pay the debt, since they will retain the residual value.

The probability of default is thus represented by the shaded area. It represents the frequency of outcomes where the asset value is less than $75 million.

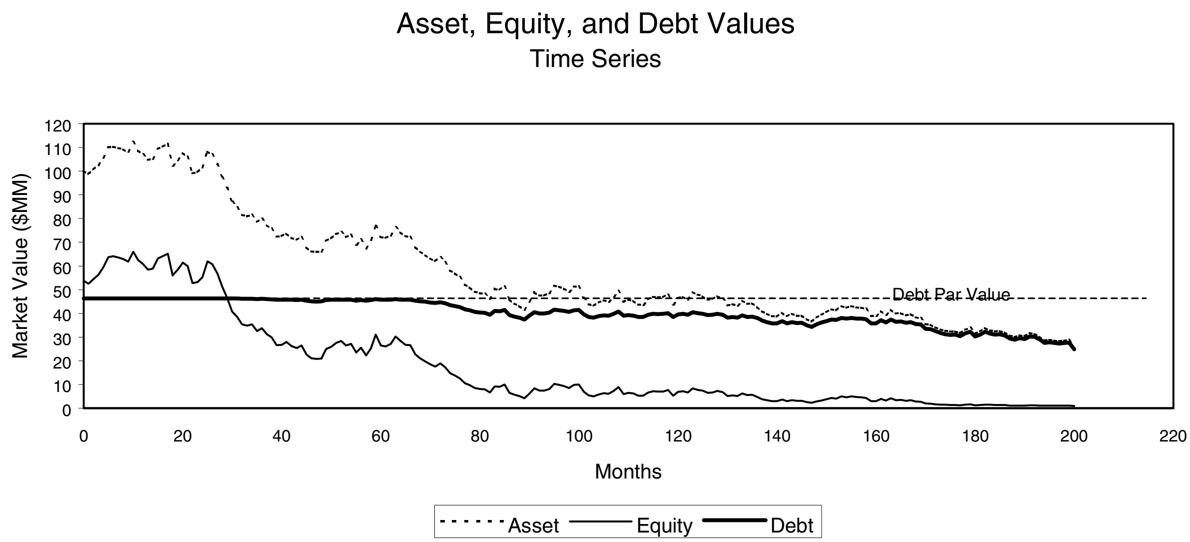
The shape of the frequency distribution is often simply assumed, given the expected value and standard deviation. For many purposes this is satisfactory but practical experience with default rates shows that this shape must be measured, rather than assumed, to obtain sufficiently precise estimated default rates.

Asset Market Value and Volatility

Just as the firm's default risk can be derived from the behavior of the firm's asset value and the level of its obligations, the firm's equity behavior can be similarly derived. The shareholders of the firm can be viewed as having a call option on the firm's asset value, where the exercise price is equal to the firm's obligations. If the market asset value exceeds the obligation amount at the maturity date, then the shareholders will exercise their option by paying off the obligation amount. If the asset value is less, they will prefer to default on the obligation and relinquish the remaining asset value to the lenders.

Using this framework, the equity value and volatility can be determined from the asset value, asset volatility, and the amount and maturity of obligations. What is actually more important is that the converse is also true: the asset value and volatility can be inferred from the equity value, equity volatility, and the amount and maturity of obligations. This process enables us to determine the market characteristics of a firm's assets from directly observable equity characteristics.

Knowing the market value and volatility of the borrower's assets is critical, as we have seen, to the determination of the probability of default. With it we can also determine the correlation of two firms' asset values. These correlations play an important role in the measurement of portfolio diversification.



The market value of assets changes unpredictably through time. The volatility in the historical time series is measured by the asset standard deviation, which was used in the previous box to describe the range of possible future asset values.

The liabilities of the firm including equity represent a complete set of claims on all the cashflows produced by the assets. Thus the market value of the assets exactly equals the market value of the liabilities including equity. As the market value of assets changes, the market value of liabilities changes, but the changes are not evenly apportioned across the liabilities due to differences in seniority.

The equity value changes close to dollar-for-dollar with the asset value. The vertical distance between the asset and equity values in Figure 2 is the market value of obligations senior to the equity ("debt"). The difference, i.e. the debt value, is shown below the axis. If the asset value falls enough, the probability of default on the debt increases and the market value of the debt also falls. A $1 fall in the asset value leads to perhaps a $0.10 fall in the debt value and a $0.90 fall in the equity value.

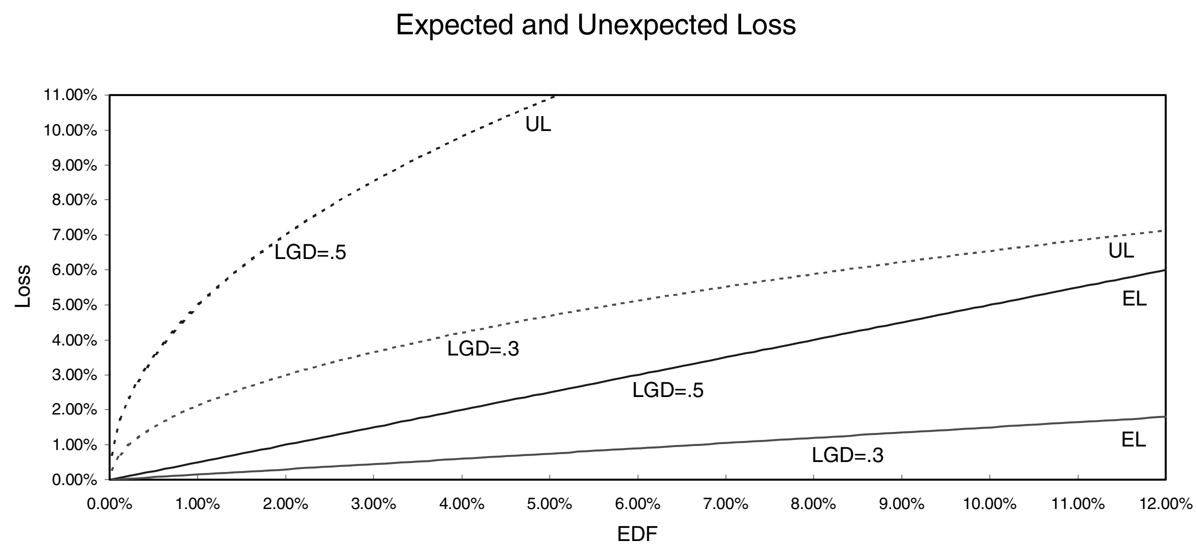
In percentage terms, the changes in the equity value are always larger than the changes in the asset value, because the equity value is a fraction of the total asset value. As the asset value, and thus equity value, falls, the equity volatility increases dramatically. The relationship between the asset and equity value is described by option theory. This theory makes it possible to infer the asset value and volatility by knowing the level of fixed obligations and the equity value and volatility.

Measurement of Portfolio Diversification

Defaults translate into losses. The loss associated with a single default depends on the amount recovered. For the purposes of this exposition, we will assume that the recovery in the event of default is known, and that this recovery is net of the expenses of collection including the time value of the recovery process. Thinking of the recovery as a percent of the face value of the loan, we can also specify the "loss given default" as one minus the expected recovery.

Using this structure, the expected loss for a single borrowing is the probability of default times the loss given default. Interestingly, the unexpected loss depends on the same variables as the expected loss. [It equals the loss given default times the square root of the product of the probability of default times one minus the probability of default.] The unexpected loss represents the volatility, or standard deviation of loss. This approach raises the question of how to deal with instruments of different maturities. The analysis here uses a single time horizon for measuring risk. Establishing one horizon for analysis forms the basis of a framework for comparing the attractiveness of different types of credit exposures on the same scale. The risk at the horizon has two parts: the risk due to possible default, and the risk of loss of value due to credit deterioration. Instruments of the same borrower with different maturities (as long as the maturity is at or beyond the horizon) have the same default risk at the horizon, but the value risk (i.e. uncertainty around the value of the instrument at horizon) depends upon the remaining time to maturity. The longer the remaining time, the greater the variation in value due to credit quality changes.

For simplicity of presentation, the following analysis will assume that the maturity of all instruments is the same as the horizon. While this will eliminate maturity as an aspect of credit risk, it will not change the qualitative nature of any of the results. Although maturity effects are important in practice, they are generally of lesser importance than the risk due to default.



$$EDF \equiv \text{ probability of default}$$
$$LGD \equiv \text{ loss given default, (\% of face)}$$
$$EL \equiv \text{ Expected loss} = EDF \times LGD$$
$$UL \equiv \text{ Unexpected loss} = LGD\sqrt{EDF\left(1 - EDF\right)}$$

Measuring the diversification of a portfolio means specifying the range and likelihood of possible losses associated with the portfolio. All else equal, a well-diversified portfolio is one that has a small likelihood of generating large losses.

The average expected loss for a portfolio is the average of the expected losses of the assets in the portfolio. It is not a simple average but rather a weighted average, with the weights equal to each exposure amount as a percent of the total portfolio exposure.

It would be convenient if the volatility, or unexpected loss, of the portfolio were simply the weighted average of the unexpected losses of the individual assets, but it is not. The reason is that portfolio losses depend also on the relationship (correlation) between possible defaults.

A simple example illustrates this point. Consider an island on which it always rains on one side or the other in a given year, but never on both, with each side equally likely to receive rain. Consider two farms, one on each side of the island, each with debt on which they will default if it doesn't rain. A portfolio holding both loans in equal amounts, and nothing else, will have an expected default rate of 50%. Each borrowing will have an unexpected default rate of 50% (= $\sqrt{0.5 \times (1 - 0.5)}$. In other words, each of the portfolio assets is quite risky. But the portfolio as a whole has an unexpected loss rate of zero. The actual default rate and the expected default rate for the portfolio are identical. In each year, one and only one borrower will default, though which one is uncertain. There is a perfect negative correlation.

The alternative extreme is that it only rains half of the years on average, but when it rains, it always rains on both sides. This is perfect positive correlation; the farms will default in exactly the same years. Holding one loan is equivalent to holding both loans: there is no risk reduction from diversification.

An intermediate case is that raining on one farm makes it no more likely or less likely that it will rain on the other. The events of rainfall are independent. In this case, in 1/4 of the years both loans will default, in 1/4 neither will default, and 1/2 the time only one will default. There is now substantial diversification versus the perfect positive correlation case, since the likelihood of both defaulting is reduced by half.

Now let us extend this notion of diversification to the more general case of a portfolio with multiple risky securities. The portfolio loss measures can be calculated as follows:

$X_i \equiv$ face value of security i

$P_i \equiv$ price of security i (per $1 of face value)

$V_P \equiv$ portfolio value = $P_1X_1 + P_2X_2 + \ldots + P_nX_n$

$w_i \equiv$ value proportion of security i in portfolio ("weight") = P_iX_i / V_P

$\rho_{ij} \equiv$ loss correlation between security i and security j

Note that $w_1 + w_2 + \ldots + w_n = 1$

$EL_i \equiv$ expected loss for security i

$EL_P \equiv$ portfolio expected loss = $w_1EL_1 + w_2EL_2 + \ldots w_nEL_n$

$UL_i \equiv$ unexpected loss for security i

$UL_p \equiv$ unexpected loss for portfolio

$$= \sqrt{\begin{array}{l} w_1w_1UL_1UL_1\rho_{11} + w_1w_2UL_1UL_2\rho_{12} + \ldots + w_1w_nUL_1UL_n\rho_{1n} + w_2w_1UL_2UL_1\rho_{21} + w_2w_2UL_2UL_2\rho_{22} \\ + \ldots + w_2w_nUL_2UL_n\rho_{2n} + \ldots + w_nw_1UL_nUL_1\rho_{n1} + w_nw_2UL_nUL_2\rho_{n2} + \ldots + w_nw_nUL_nUL_n\rho_{nn} \end{array}}$$

Note that $\rho_{ij} = 1$ if $i = j$, and $\rho_{ij} = \rho_{ji}$

The portfolio expected loss is the weighted average of the expected losses of the individual securities, where the weights are the value proportions. On the other hand, the portfolio's unexpected loss is a more complex function of the ULs of the individual securities, the portfolio weights, and the pairwise loss correlations between securities.

In practice, actual defaults are positively but not perfectly positively correlated. Diversification, while not perfect, conveys significant benefits. Unfortunately negative default correlations are rare to non-existent.

Calculating portfolio diversification means determining the portfolio's unexpected loss. To do this, default correlations and, ultimately, correlation in instrument values are required.

Model of Default Correlation

Default correlation measures the strength of the default relationship between two borrowers. If there is no relationship, then the defaults are independent and the correlation is zero. In such a case, the probability of both borrowers being in default at the same time is the product of their individual probabilities of default.

When two borrowers are correlated, this means that the probability of both defaulting at the same time is heightened, i.e. it is larger than it would be if they were completely independent. In fact, the correlation is just proportional to this difference. Thus, holding their individual default probabilities fixed, it is equivalent to say either that two borrowers are highly correlated or that they have a relatively high probability of defaulting in the same time period.

The basic default model says that the firm will default when its market asset value falls below the face value of obligations (the "default point"). This means that the joint probability of default is the likelihood of both firms' market asset values being below their respective default points in the same time period.

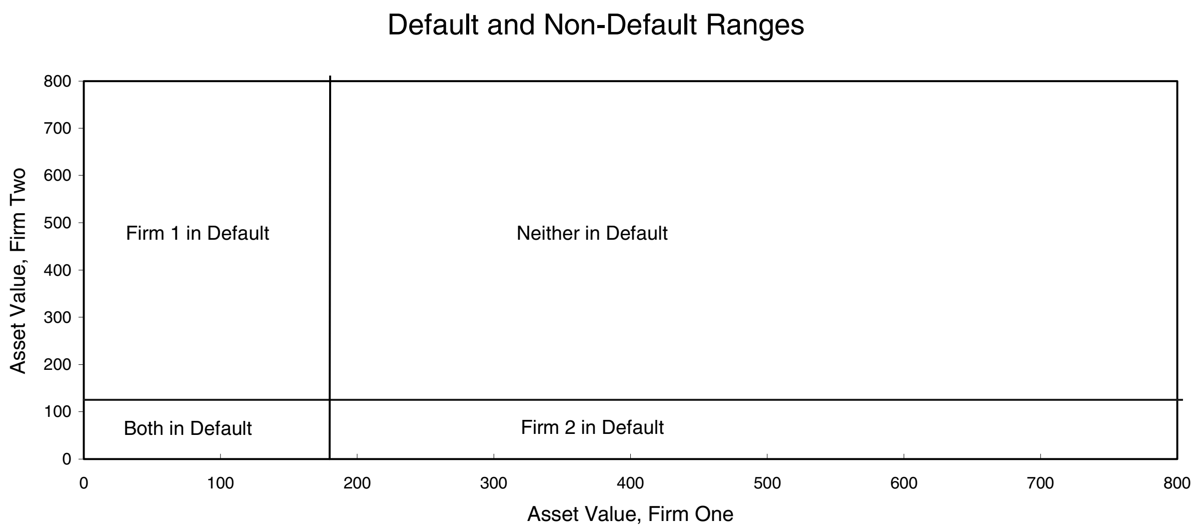
This probability can be determined quite readily from knowing (i) the firms' current market asset values; (ii) their asset volatilities; and (iii) the correlation between the two firms' market asset values. In other words, the derivatives framework enables us to use the firms' asset correlation to obtain their default correlation.

This may not seem to be all that helpful, but in fact it is critically important to the empirical determination of default correlations. The correlation, for example, between equity returns, can be directly calculated because the histories of firms' stock returns are easily observable. Default correlations cannot be successfully measured from default experience.

The historically observed joint frequency of default between two companies is usually zero. Exxon and Chevron have some chance of jointly defaulting, but nothing in their default history enables us to estimate the probability since neither has ever defaulted. Grouping firms enables us to estimate an average default correlation in the group using historical data, but the estimates so obtained are highly inaccurate.

No satisfactory procedure exists for directly estimating default correlations. Not surprisingly, this has been a major stumbling block to portfolio management of default risk.

The derivatives approach enables us to measure the default correlation between two firms, using their asset correlation and their individual probabilities of default. The correlation between the two firms' asset values can be empirically measured from their equity values, as was described in the previous section.



The figure above illustrates the ranges of possible future asset values for two different firms. The two intersecting lines represent the default points for the two firms. For instance, if firm one's asset value ends up being below $180 million (the point represented by the vertical line), then firm one will default.

The intersecting lines divide the range of possibilities into four regions. The upper right region represents those asset values for which neither firm one nor firm two will default. The lower left region represents those asset values for which both will default.

The probabilities of all these regions taken together must equal one. If the asset values of the two firms were independent, then the probabilities of the regions could be determined simply by multiplying the individual probabilities of default and non-default for the two firms. For instance, suppose that firm one's default probability is 0.6% and firm two's is 0.3%. Then the probability of both defaulting, if they are independent, is the product of the default probabilities, or 0.0018%.

If the two firms' asset values are positively correlated, then the probability of both asset values being high or low at the same time is higher than if they were independent, and the probability of one being high and the other low is lower. For instance, using the previous default probabilities, the probability of both defaulting might now be 0.01%, if their asset values are positively correlated.

By knowing the individual firms' default probabilities, and knowing the correlation of their asset values, the likelihood of both defaulting at the same time can be calculated. As noted in a previous box, the time series of a firm's asset values can be determined from its equity values. The correlation between two firms' asset values can be calculated from their respective time series.

We can calculate default correlation as follows:

$JDF \equiv$ joint default frequency of firm 1 and firm 2 i.e. actual probability of both firms defaulting together

$\rho_D \equiv$ default correlation for firm 1 and firm 2 = $\dfrac{JDF - EDF_1 EDF_2}{\sqrt{EDF_1(1-EDF_1)EDF_2(1-EDF_2)}}$

The numerator of this formula represents the difference of the actual probability of both firms defaulting and the probability of both defaulting if they were *independent*. Note that if the asset values are independent, then the default correlation is zero.

In practice, we extend this model to consider the correlation in the value of claims such as loans or bonds within a portfolio. The default state corresponds with a particularly low value realization for the loan or bond issued by the defaulted firm. This extension requires estimation of the joint value distribution between each pair of credit risky assets in the portfolio. KMV's Portfolio Manager™ incorporates this richer approach to determine the value correlation among all securities in a portfolio. In this way, the correlated credit migration over time can be captured to determine a more accurate measure of possible losses in the future.

Model of Value Correlation

An important strength of the structural model of default presented here and implemented in KMV technology is the ability to generalize relationships in a way to create a comprehensive credit portfolio model. In addition to the EDF values for each firm, the joint default frequency (JDF) must be calculated to determine a value correlation. In the context of the structural model explained above, the JDF can be calculated by focusing on the relationship between a firm's market asset value and its respective default point. EDF values embed this information on an individual firm level. The remaining piece of the puzzle is the correlation between each firm's market asset value.

Mathematically we can write down the following function for the JDF:

$N_2(\ ) \equiv$ bivariate normal distribution function

$N^{-1}(\ ) \equiv$ inverse normal distribution function

$\rho_A \equiv$ correlation between firm 1's asset return and firm 2's asset return

$$JDF = N_2\left(N^{-1}\left(EDF_1\right), N^{-1}\left(EDF_2\right), \rho_A\right)$$

Estimating pair-wise asset correlations for publicly traded firms can be done in a number of ways. One method would be to calculate a time series of asset values for each firm and then calculate a sample correlation between each pair of asset value series. While this method may seem reasonable in theory, in practice it is the least effective way to calculate correlations for credit portfolio modeling. We are most interested in the systematic co-movement and work to estimate efficiently this co-movement over a subsequent time horizon. Because the movement in a typical firm's asset value is mostly driven by factors idiosyncratic to that firm, sample correlations will reflect co-movement that is unique to that sample period—not very useful for predicting ex ante correlation over a subsequent time horizon. Given the weakness in this approach (not to mention the problems associated with insufficient observations needed to even calculate sample correlations), we turn to factor modeling to calculate correlations.

A factor model relates the systematic or non-diversifiable components of the economy that drive changes in asset value. For example, the entire economy may follow a business cycle which affects most companies' prospects. The impact may differ from company to company, but they are affected nonetheless. Determining the sensitivity of changes in asset values to changes in a particular economic factor provides the basis for estimating asset correlation.

Changes in a firm's asset value constitutes an asset value return or firm return. We can decompose this return as follows:

$$\begin{bmatrix} Firm \\ Return \end{bmatrix} = \begin{bmatrix} Composite \\ Factor \\ Return \end{bmatrix} + \begin{bmatrix} Firm \\ Specific \\ Effects \end{bmatrix}$$

The composite factor return proxies for the systematic risk factors in the economy. We can further decompose this composite factor return as follows:

$$\begin{bmatrix} Composite \\ Factor \\ Return \end{bmatrix} = \begin{bmatrix} Country \\ Factor \\ Returns \end{bmatrix} + \begin{bmatrix} Industry \\ Factor \\ Returns \end{bmatrix}$$

$$\begin{bmatrix} Country \\ Factor \\ Return \end{bmatrix} = \begin{bmatrix} Global \\ Economic \\ Effect \end{bmatrix} + \begin{bmatrix} Regional \\ Factor \\ Effect \end{bmatrix} + \begin{bmatrix} Sector \\ Factor \\ Effect \end{bmatrix} + \begin{bmatrix} Country \\ Specific \\ Effect \end{bmatrix}$$

$$\begin{bmatrix} Industry \\ Factor \\ Return \end{bmatrix} = \begin{bmatrix} Global \\ Economic \\ Effect \end{bmatrix} + \begin{bmatrix} Regional \\ Factor \\ Effect \end{bmatrix} + \begin{bmatrix} Sector \\ Factor \\ Effect \end{bmatrix} + \begin{bmatrix} Industry \\ Specific \\ Effect \end{bmatrix}$$

Firm asset correlation can then be calculated from each firm's systematic or composite factor return. In this way, we relate the systematic component of changes in asset value which produces a better estimate of future co-movements in asset values. In KMV's Global Correlation Model, industry and country indices are produced from a global database of market asset values (estimated from the traded equity prices together with each firm's liability information) for nearly 30,000 publicly traded firms. These indices are used to create a composite factor index for each firm depending on its country and industry classifications.

Mathematically, the following relationship is constructed:

$w_{kc} \equiv$ weight of firm k in country c

$w_{ki} \equiv$ weight of firm k in industry i

note that $\sum_{c=1}^{\bar{c}} w_{kc} = \sum_{i=1}^{\bar{i}} w_{ki} = 1$ where $\bar{c}$ is currently 45 countries and $\bar{i}$ is 61 industries for KMV's Global Correlation Model (as more data become available this coverage increases.)

$r_c \equiv$ return index for country c (estimated from publicly traded firms.)

$r_i \equiv$ return index for industry I (estimated from publicly traded firms.)

$\phi_k \equiv$ composite (custom) market factor index for firm k

$$\phi_k = \sum_{c=1}^{\bar{c}} w_{kc} r_c + \sum_{i=1}^{\bar{i}} w_{ki} r_i$$

Once the custom index is calculated for a particular firm, the sensitivity (i.e. beta) to the market factors reflected in this index can be estimated. The relationship used for this estimation is written as follows:

$r_k \equiv$ return for firm k

$\beta_k \equiv$ beta for firm k

$\varepsilon_k \equiv$ firm-specific component of return for firm k

$$r_k = \beta_k \phi_k + \varepsilon_k$$

We can similarly estimate the sensitivity or beta ($\beta_{Country,Common\ Factor}$ and $\beta_{Industry,Common\ Factor}$) of countries and industries on factors we specify. In KMV's model, we have chosen two global factors, five regional factors, and seven sectoral factors. Since we may have effects unique to industries and countries (i.e. not linked through the 14 common factors), we also have country (45 countries) and industry (61 industries) specific factors. An example of calculating the sensitivity of firm k to a global factor is written below:

$$\beta_{kG} = \beta_k \left(\sum_{c=1}^{45} w_{kc} \beta_{cG} + \sum_{i=1}^{61} w_{ki} \beta_{iG} \right)$$

These calculations produce the parameters necessary to estimate the firm asset value correlation. We construct this calculation as follows:

$\sigma(j,k) \equiv$ covariance between firm j and firm k

$\rho_{jk} \equiv$ correlation between firm j's and firm k's asset value returns

$\sigma_k \equiv$ standard deviation of firm k's asset value return

$$\sigma(j,k) = \sum_{G=1}^{2} \beta_{jG}\beta_{kG}\sigma_G^2 + \sum_{R=1}^{5} \beta_{jR}\beta_{kR}\sigma_R^2 + \sum_{S=1}^{7} \beta_{jS}\beta_{kS}\sigma_S^2 + \sum_{i=1}^{61} \beta_{ji}\beta_{ki}\varepsilon_i^2 + \sum_{c=1}^{45} \beta_{jc}\beta_{kc}\varepsilon_c^2$$

$$\begin{bmatrix} Return \\ Covariance \\ j \text{ and } k \end{bmatrix} = \begin{bmatrix} Global(G) \\ Economic \\ Factors \end{bmatrix} + \begin{bmatrix} Regional(R) \\ Economic \\ Factors \end{bmatrix} + \begin{bmatrix} Industrial(S) \\ Sector \\ Factors \end{bmatrix} + \begin{bmatrix} Industry(i) \\ Specific \\ Factors \end{bmatrix} + \begin{bmatrix} Country(c) \\ Specific \\ Factors \end{bmatrix}$$

$$\begin{bmatrix} Return \\ Correlation \\ j \text{ and } k \end{bmatrix} = \frac{\begin{bmatrix} Return \\ Covariance \\ j \text{ and } k \end{bmatrix}}{\begin{bmatrix} Return \\ Volatility\ j \end{bmatrix}\begin{bmatrix} Return \\ Volatility\ k \end{bmatrix}}$$

$$\rho_{jk} = \frac{\sigma(j,k)}{\sigma_j \sigma_k}$$

The covariance calculation depends on the sensitivities or betas ($\beta_{Company,Factor}$) for each firm combined with the factor variances (σ_{Factor}^2). To arrive at the correlation we must scale the covariance by the standard deviation of the returns as shown in the final equation of the above group.

Simply said, the factor model focuses attention on the components driving co-movements. These components can be separated into the effects listed above; however, the important aspect of this process is identifying the total systematic component. To the extent that is correctly estimated, the subsequent decomposition into constituent effects is only necessary for gaining intuition behind the source of correlation between any two firms. This approach relies on the embedded systematic components reflected in the data on publicly traded firms around the world.

Returning to the JDF calculation, we combine this asset value correlation with the individual firm EDF values to arrive at a default correlation. Default correlation is sufficient if we do not model possible credit migration over our horizon of analysis. If we plan to consider the possibility of credit migration at horizon, we need to calculate the joint distribution of values among the loans made to the firms being analyzed. Explicitly calculating this relationship requires calculation of a double integral (over all possible firm asset values) for each pair of firms in the portfolio. For most sizable credit portfolios, this approach is cost prohibitive from a computational perspective.

Instead we can make use of the factor structure explained above to construct a Monte Carlo simulation which draws the individual factors over and over again to determine the possible portfolio value realizations. Each of these portfolio value realizations embeds the loan value correlations since each loan value is calculated based on the relationship feeding back to each firm's asset value. These asset values derive from the sensitivity to each of the risk factors.

A simple example will make this process clearer: Assume we are analyzing a portfolio of three loans to three different companies. We determine that the asset values of company A and company B increase (decrease) whenever interest rates decline (rise). Company C is unaffected by changes in interest rates. In this economy, we have only one factor—interest rate movement. We then simulate this one factor. Whenever this interest rate factor is high, A's and B's asset values are small. These low asset values result in the loans to A and B being valued at a discount; C's loan value is unchanged, since C is not affected by the interest-rate factor. If the interest rate factor is low, A's and B's loans will be valued at a premium. The key to the correlation arises from the similar behavior in loan value whenever a particular factor level is drawn.

Clearly, the movement in the value of A's and B's loans are correlated while C's loans are uncorrelated with the rest of the portfolio. The process of simulating different factor realizations generates a variety of portfolio value realizations. These value realizations can then be transformed into a loss distribution. The extent to which loan values move together link back to the sensitivities to the different risk factors in the factor model.

The Likelihood of Large Losses

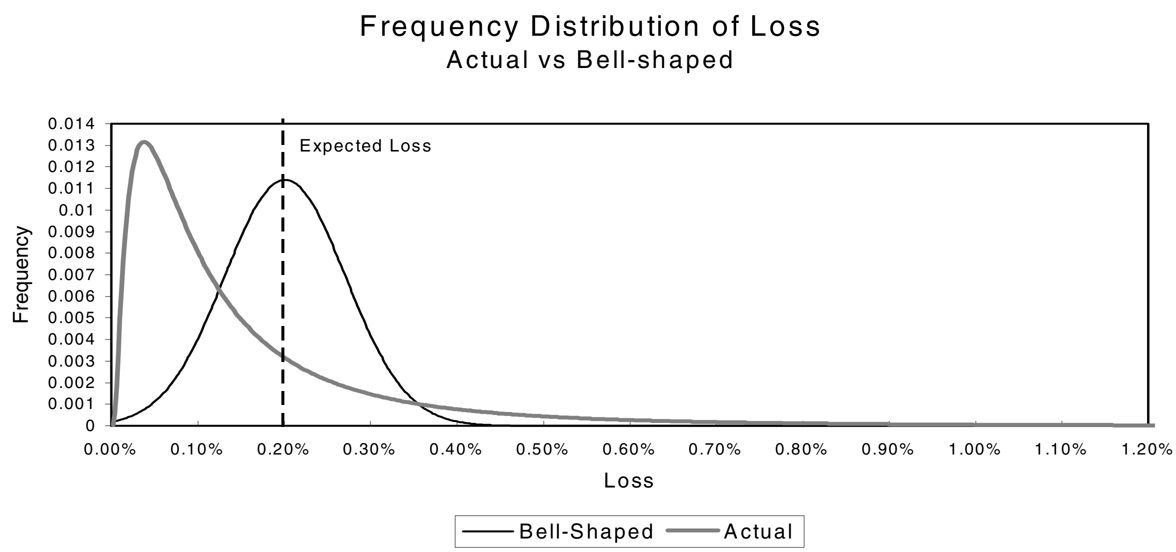
We are all familiar with the "bell-shaped" or Normal distribution. If portfolio losses had such a bell shaped distribution, we could accurately specify the likelihood of large losses simply by knowing the expected and unexpected loss for the portfolio. The problem is that individual debt assets have very "skewed" loss probabilities. Most of the time the borrower does not default and the loss is zero. However, when default occurs, the loss is usually substantial.

Given the positive correlation between defaults, this unevenness of loss never fully "smoothes out", even in very large portfolios. There is always a large probability of relatively small losses, and a small probability of rather large losses.

This "skewness" leads to an unintuitive result: a very high percentage of the time (around 80%), the actual losses will be less than the average loss. The reason is that the average is pulled upwards by the potential for large losses. There is a great danger of being "lulled" by a string of low losses into believing that the portfolio is much better diversified than in fact it is.

Fortunately, the frequency distribution of portfolio losses can be determined using the information we have already discussed. Knowing this distribution for a given portfolio gives an alternative characterization of diversification:

> Portfolio A is better diversified than portfolio B if the probability of loss exceeding a given percent is smaller for A than for B, and both portfolios have the same expected loss.



The graph above contrasts the loan loss distribution for a portfolio with a bell shaped loss distribution having the same expected loss and unexpected loss. There are two striking differences. The most obvious is that the actual loan loss distribution is asymmetric. There is a small probability of large losses and large probability of small losses.

If losses were determined as per the bell shaped distribution, then losses would exceed the expected loss about half the time, and the other half of the time they would be less than the expected loss. For the actual loss distribution, realized losses will be less than the expected loss approximately 75% of the time. There is a significant likelihood that even a risky portfolio will generate consecutive years of low realized losses.

The second major difference is that the probability of very large losses approaches zero much more quickly for the bell shaped distribution than for the skewed distribution. In fact, for a portfolio with a skewed loss distribution there is an economically significant chance of realized losses that are six to eight standard deviations in excess of the expected loss. For the bell shaped distribution, there is virtually no chance of a four standard deviation event occurring.

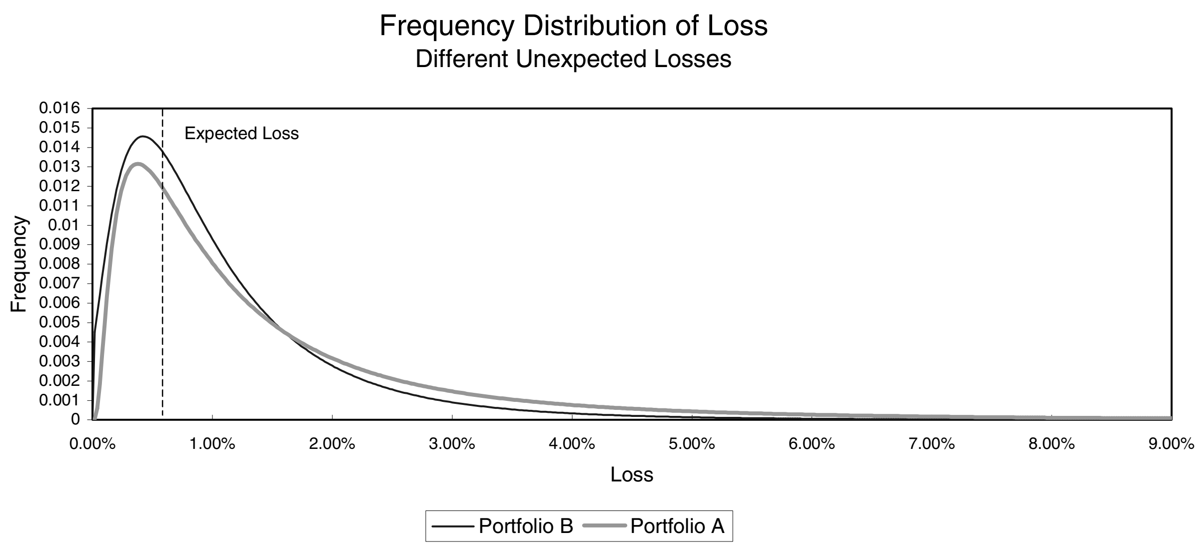


Figure 6 contrasts two loan loss distributions for different portfolios. The two portfolios have the same level of expected loss, but portfolio A has a higher unexpected loss. There is a significantly higher chance of incurring a large loss in portfolio A than in portfolio B. These probabilities can be seen by looking at the areas under the respective curves. For instance, the probability of a 4% loss in portfolio A is 0.1%, but the probability of a 4% loss in portfolio B is only 0.05%. The implication of this difference for the two portfolios in debt rating terms is the difference between a single B rating and a single A rating.

This view of diversification has an immediate concrete implication for capital adequacy. Given the frequency distribution of loss, we can determine the likelihood of losses which exceed the amount of capital held against the portfolio. This probability can be set to the desired level by varying the amount of capital.

To illustrate how this can be done in practice, it is necessary to consider the market value, rather than the book value, of the portfolio. To do that, we need to be able to determine the market value of a security. Ideally, we want to use data from a deep and liquid market in the security we are modeling. In the case of credit risky securities, the markets are typically neither deep nor liquid. Consequently, we must rely on models to determine a mark-to-market value. Let us consider the case of determining the market value of a loan.

Valuation

The market value of a loan is simply the price for which it can be bought or sold. Although there is a loan sales market, loans by and large are not actively transacted for extended periods. The result is that current market prices do not exist for most loans.

The objective of valuation is to determine what a loan should sell for, were it to trade. The value cannot be determined in the abstract or in some absolute sense, but only by comparison to the market prices of financial instruments that are traded. Valuation consists of extrapolating actual market prices to non-traded assets, based on the relationship between their characteristics. In the case of loans, the process involves consideration of the borrower's traded equity, and traded debt should it exist.

Bank assets have a variety of complexities: default risk, utilization, covenants, priority, and so forth. Many of these complexities can be viewed, and valued, as options belonging to the borrower or lender. Some of these complexities are addressed subsequently in this paper.

For this exposition, one of the simplest cases will suffice, namely a fixed term, fixed rate corporate loan. If this loan were not subject to default risk, then it could be valued readily by comparison with the pricing on similar term AAA rated notes. The level of default risk in AAA-rated notes is very small, making them a good market benchmark. On the other hand, Treasury notes, which literally have no default risk, are not as good a benchmark for corporate liabilities, due to persistent and unpredictable differences between corporate and government issues.

The so-called "pricing" on a loan is the set of fees and spreads which determines the promised cashflows between borrower and lender. This is the equivalent of the coupon rate on a bond. The value of the loan is obtained by discounting the loan cashflows by an appropriate set of discount rates. The discount rates, in the absence of default risk, would simply differ by increments of term, according to the current term structure.

In the presence of default risk, the discount rates must contain two additional elements. The first is the expected loss premium. This reflects an adjustment to the discount rate to account for the actuarial expectation of loss. It is based on the probability of default and the loss given default. The second is the risk premium. This is compensation for the non-diversifiable loss risk in the loan.

If the loan did not contain a risk premium, then on average it would only return the risk-free base rate. The key point is the qualifier: "on average." In years when default did not occur, the loan would return a little more due to the expected loss premium. However, in the event of default, it would return much less.

Since an investor could obtain the risk free base rate not just "on average", but all the time by buying the risk free asset, the risky asset must provide additional compensatory return. This would not be the case if default risk was completely diversifiable, but (as we have discussed) it is not. The market will provide compensation for unavoidable risk bearing, i.e. the portion of the loan's loss risk that cannot be eliminated through diversification.

The amount of non-diversifiable risk can be determined from knowing the borrower's probability of default and the risk characteristics of the borrower's assets. The market price for risk bearing can be determined from the equity and fixed income markets. This approach, based on option valuation methods, can be used to construct discount rates, specific to the borrower, which correctly account for both the time value of money and the expected and unexpected loss characteristics of the particular borrower.

There are only two possible outcomes for a loan. Either it is repaid, or the borrower defaults. The loss distribution for a single loan is simply:

Event	**Probability**
Default	EDF
No default	1-EDF

In the event of default, we expected to lose a percentage of the face value of the loan equal to LGD. If the yield on the loan is Y and the risk-free base rate is R_f, then the return distribution can be characterized as follows:

Event	**Probability**	**Return**
Default	EDF	$R_f - LGD$
No default	1-EDF	Y

The expected return is the probability weighted average of the returns.

$$E(R) = EDF\left(R_f - LGD\right) + (1 - EDF)Y$$

The required compensation for the actuarial risk of default is equal to $(LGD \times EDF)/(1 - EDF)$. This is called the expected loss premium. If the loan yield equaled the risk-free base rate plus the expected loss premium, then

$$Y = R_f + \frac{LGD \times EDF}{(1 - EDF)}, \text{ and}$$

$$E(R) = EDF\left(R_f - LGD\right) + (1 - EDF)\left(R_f + \frac{LGD \times EDF}{(1 - EDF)}\right)$$

$$E(R) = R_f.$$

The expected loss premium provides just enough additional return when the borrower does not default to compensate for the expected loss when the borrower does default.

However, this is not the end of the story. What the calculation above shows is that if the only additional compensation were the expected loss premium, then the lender on average would receive only the risk-free base rate. It would be much better for the lender to just lend at the risk-free base, since it would get the same average return and would incur no default risk. There must be additional compensation for the fact that the realized return is risky even for a large, well-diversified portfolio of loans. That additional compensation is called the risk premium.

The required pricing on a loan is thus the risk-free base rate plus the expected loss premium plus the risk premium.

$$Y = R_f + \text{EL Premium} + \text{Risk Premium}.$$

The required risk premium in the market can be determined by taking the credit spread on debt securities and subtracting the appropriate EL premium. The remainder is the market risk premium.

If we think of the yield on a loan as being an average of these various discount rates (as "yield-to-maturity" is for a bond), then the value of the loan is simply its promised cashflows discounted at its yield. If the yield exceeds the loan rate, then the loan will be at a discount. An increase in the probability of default will push up the yield required in the market, and push down the price of the loan. Other factors remaining the same, loan value moves inversely to changes in default probability.

Economic Capital and Fund Management

Consider now a bank. Think of the bank as being divided into two parts. One part is the actual portfolio of assets; the second is an amalgam of all other bank functions. Let us call the part containing the portfolio "the fund," and think of the fund as containing a portfolio management function but no other bank functions.

The fund is leveraged. It borrows from the rest of the bank at the appropriate market rate; we may think of it also as borrowing directly in the bond or money markets. Equity supporting the fund is owned by the rest of the bank, although in principle some or all could be owned outside of the bank. In essence, the fund is an odd sort of leveraged money market fund. The fund's assets have a market value, either because the individual assets have actual market prices, or because we can value them as was discussed in the previous section.

The fund has fixed obligations, i.e. its borrowings. These borrowings also have determinable market values. The value of the fund's equity is exactly equal to the excess of the market value of its assets over the market value of its obligations. The economic capital of a bank is closely related to the market value of its equity. Rather than being the excess of the market value of assets over the *market* value of liabilities, economic capital is the excess of the market value of assets over the market value of liabilities, *assuming the liabilities had no default risk*. For a bank with low default risk, these values are virtually identical. However, for a distressed bank, economic capital can be zero or negative, whereas market equity is always positive.

The "economic capital" fluctuates with the market value of assets. The fund can raise more equity or more debt and invest it in additional assets, or it can make payouts to debt or equity, reducing its assets.

The objective of fund management is to maximize the value of the fund's equity. In a hypothetical world of frictionless markets and common information, i.e. a world without institutional constraints, this would be achieved by:

- purchasing assets at or below market; selling assets at or above market.

Regardless of circumstance, this is a desirable policy, and its implementation requires rigorous measurement of default risk, and pricing which by market standards at least compensates for the default risk. However, institutional constraints do exist, and markets are not frictionless nor information symmetrically dispersed. In fact, it is the existence of these market "imperfections" which makes intermediation a valuable service. There is no need for banks or mutual funds to exist in a world of perfect capital markets.

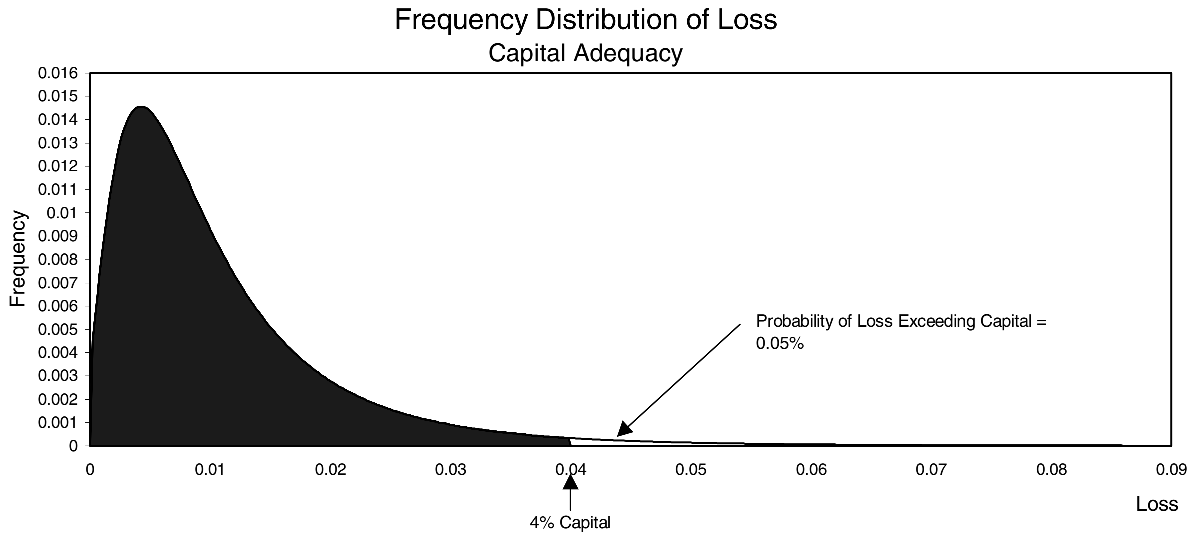
In practice, equity funding is "expensive." This may be because equity returns are taxed at the fund level, or because the "opaqueness" of bank balance sheets imposes an additional "risk" cost (agency cost). The result is that banks feel constrained to use the minimal amount of capital consistent with maintaining their ability to freely access debt markets. For wholesale banks, that access is permitted only to banks with extremely low default probabilities (.10% or less per year).

Finally, the fund is an investment vehicle. In a world where transactions are costly, one of the fund's functions is to minimize those costs for final investors. It does this by providing competitive return for its risk. Failure to do this makes it a secondary rather than primary investment vehicle; in other words, if it is less than fully diversified, another layer of investment vehicles, and another round of transactions costs, is required to provide diversification to the investor.

Both of these considerations add two additional objectives:

- obtain maximal diversification;
- determine and maintain capital adequacy.

As previously discussed, capital adequacy can be determined by considering the frequency distribution of portfolio losses. Maintaining capital adequacy means that the desired leverage must be determined for each new asset as it is added to the portfolio. This leverage must be such as to leave the fund's overall default risk unchanged. Assets which, net of diversification, add more than average risk to the portfolio, must be financed (proportionately) with more equity and less debt than the existing portfolio.



Capital adequacy means using enough equity funding that the fund's default risk is acceptably low. A conventional measure is the actual or implied debt rating of the fund; the debt rating can be interpreted as corresponding to a probability of default. For instance, an AA-rated fund typically has a default probability less than 0.05%.

The fund will default if it suffers losses that are large enough to eliminate the equity. The graph above shows the loss distribution of the fund's portfolio. For any given level of equity funding, it is possible to determine the probability of losses that would eliminate the equity. For instance, if this portfolio were 4% equity funded, then the probability of depleting the equity is 0.10%. This is equivalent to a single A debt rating.

Maximal diversification means the lowest level of portfolio unexpected loss, conditional on a given level of expected return. Note that this is different than minimizing risk without regard to return. The latter can be accomplished by holding U.S. Treasuries.

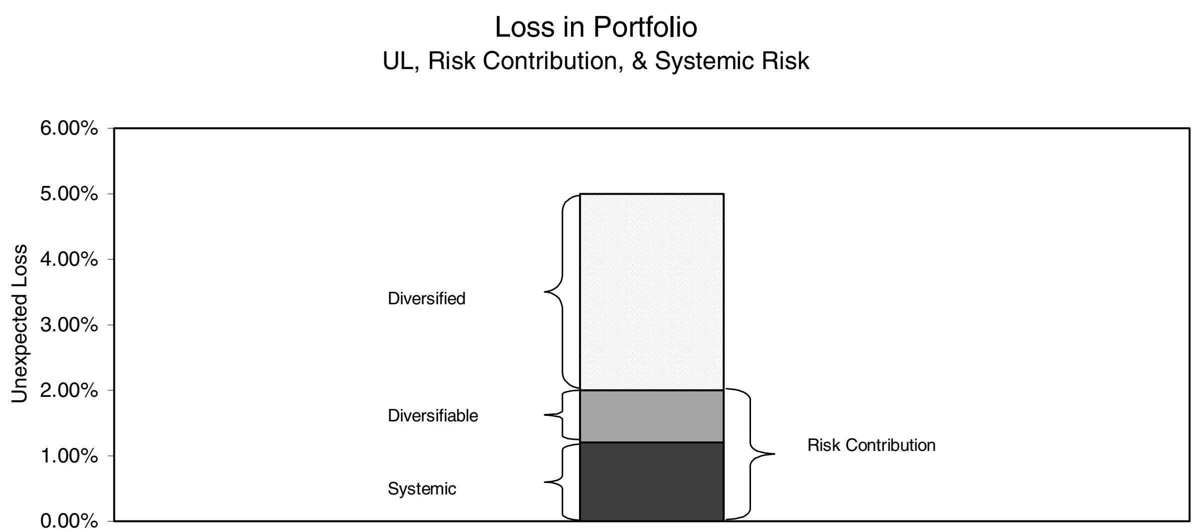
For each level of return, and for a given set of possible assets, there is a unique set of holdings that gives the minimum unexpected loss. When we depict the expected return and unexpected loss associated with each of these portfolios, the resulting graph is called the "efficient frontier."

The process for determining how much "economic capital" (equity) to use in financing an asset, and the process for maximizing diversification both require measuring how much risk an individual asset contributes to the portfolio, net of risk reduction due to diversification.

Risk Contribution and Optimal Diversification

Diversification means that the risk in the portfolio is less than the average of each asset's stand-alone risk. Some part of each asset's stand-alone risk is diversified away in the portfolio. Thinking of it in this way, we can divide the stand-alone risk of an asset into the part which is diversified away and the part which remains. This latter part is the risk contribution of the asset to the portfolio, and the risk of the portfolio is the holdings-weighted average of these risk contributions.

The residual risk contribution of an asset changes as the composition of the portfolio varies. In particular, as the holding of the asset increases, its risk contribution increases. The percentage of its stand-alone risk that is not being diversified away increases at the same time as the value weight of the asset in the portfolio increases.



The figure above shows the loss risk of a single asset. The total height of the bar represents the unexpected loss of the asset.

The bottom segment of the bar represents the portion of the unexpected loss that could not be eliminated through diversification even in the broadest possible portfolio. This is called the non-diversifiable, or systemic, risk of the asset. When one speaks of the "beta" of an asset, one is referring to this portion of an asset's risk.

In the context of an actual portfolio, diversification will generally be less than optimal, and some portion of its risk that could be diversified away has not been. This portion is represented by the second segment of the bar.

The sum of the bottom two segments is the risk contribution of the asset to the portfolio. It represents the risk that has not been diversified away in the portfolio. Some has not been diversified away because it cannot be (the systemic portion); some has not been diversified away because the portfolio is less than optimally diversified.

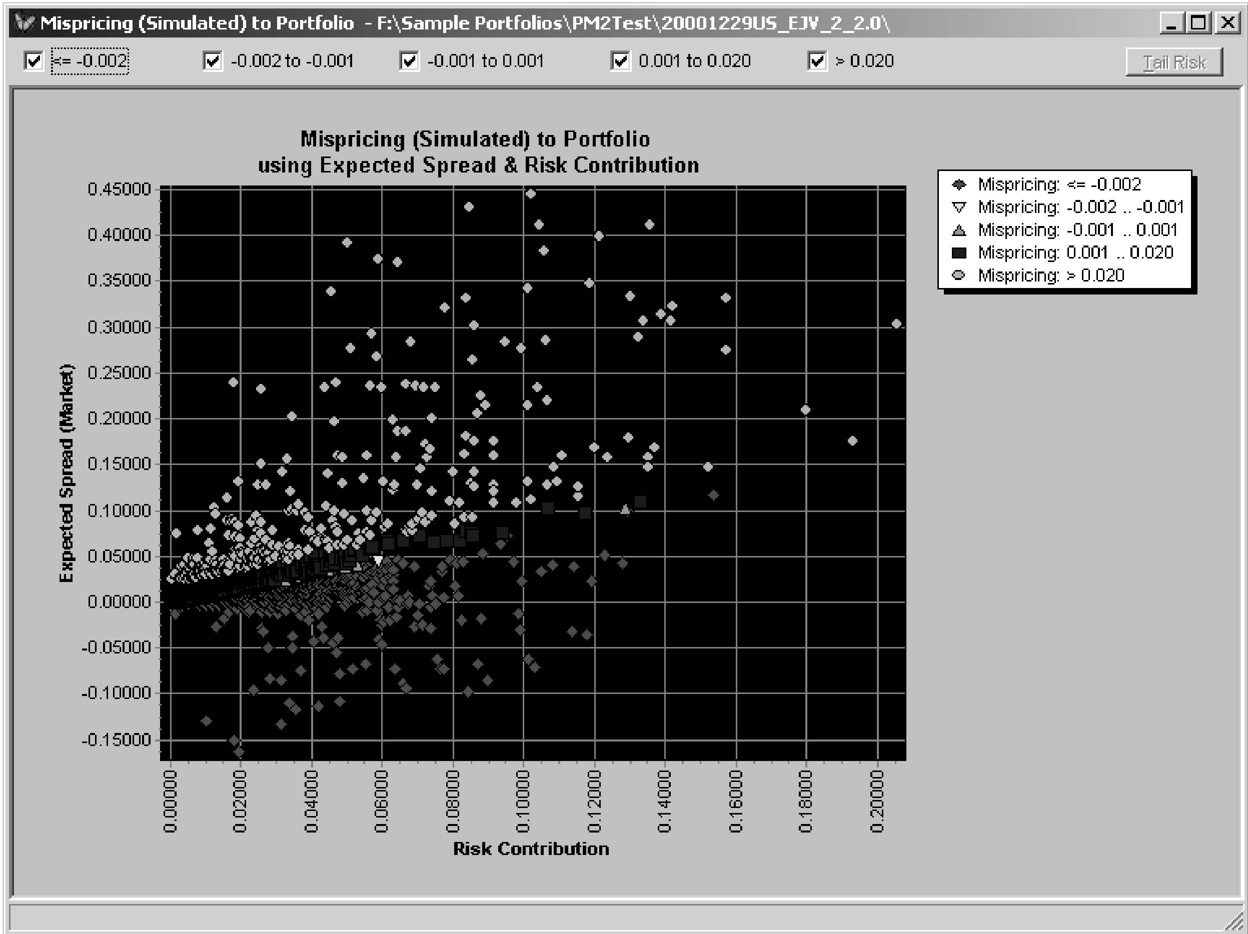
The portfolio's unexpected loss is simply the holdings-weighted average of the risk contributions of the individual assets. Risk contribution is the appropriate measure of the risk of an asset in a portfolio because it is net of the portion of risk that has been diversified away.

As the holdings change, the risk contributions change. For instance, if the proportionate holding of this asset were increased in the portfolio, less of its risk would be diversified away, and the risk contribution would go up.

Systemic risk is measured relative to the whole market of risky assets. Risk contribution is specific to a particular portfolio: the particular set of assets and the particular proportions in which the assets are held. In a typical portfolio, there are assets whose returns are large relative to the amount of risk they contribute; there are also assets whose returns are small relative to the amount of risk they contribute to the portfolio. These assets are mispriced relative to the portfolio in which they are being held.

In some cases, this "mispricing to portfolio" simply reflects that the assets are mispriced in the market, and it is ultimately fixed as the market price adjusts. More often, however, it reflects that the portfolio has too much or too little of the particular assets. If an asset that has too little return for its risk is partially swapped for an asset that is generously compensated for its risk, two things happen. First, the portfolio improves: without any increase in risk, the portfolio return improves. Second, as the holding of the former asset decreases, its risk contribution goes down; similarly, the risk contribution of the latter asset increases. As the risk contributions change, the return-to-risk ratios change for each asset. The former asset is no longer so under-rewarded, the latter is no longer so over-rewarded. Continuing the swap will continue to improve the portfolio until the return-to-risk ratios for each of the assets are brought into alignment with the overall portfolio.

This process, applied to all assets in the portfolio, leads to the maximization of diversification for any given level of return. Thus, a key part of the portfolio management process is to measure the risk contribution of each asset, and its return relative to that risk. The optimized portfolio will not contain the same amount of all assets; the holdings will be based upon the risk contribution of each asset relative to its return. In fact, an optimized portfolio is one where all assets have the same return-to-risk ratio. Any deviation would imply the existence of a swap that could improve the overall portfolio.



A portfolio is optimized by swapping low return-to-risk assets for high return-to-risk assets. To do this requires identifying which are the high and low return-to-risk assets.

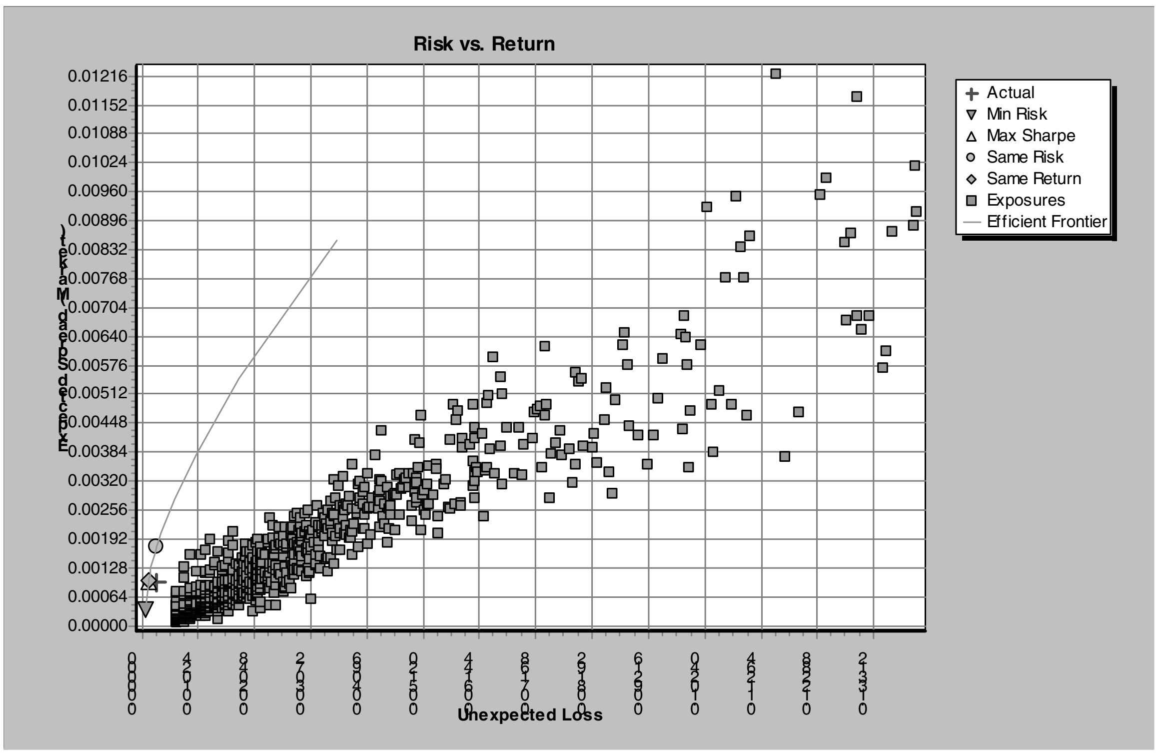
The above graph is taken from KMV's Portfolio Manager software. It illustrates for a bond portfolio the return-to-risk characteristics of all the assets in the portfolio. The return to each asset is measured by its spread adjusted for expected loss. The risk is measured by the risk contribution to the portfolio.

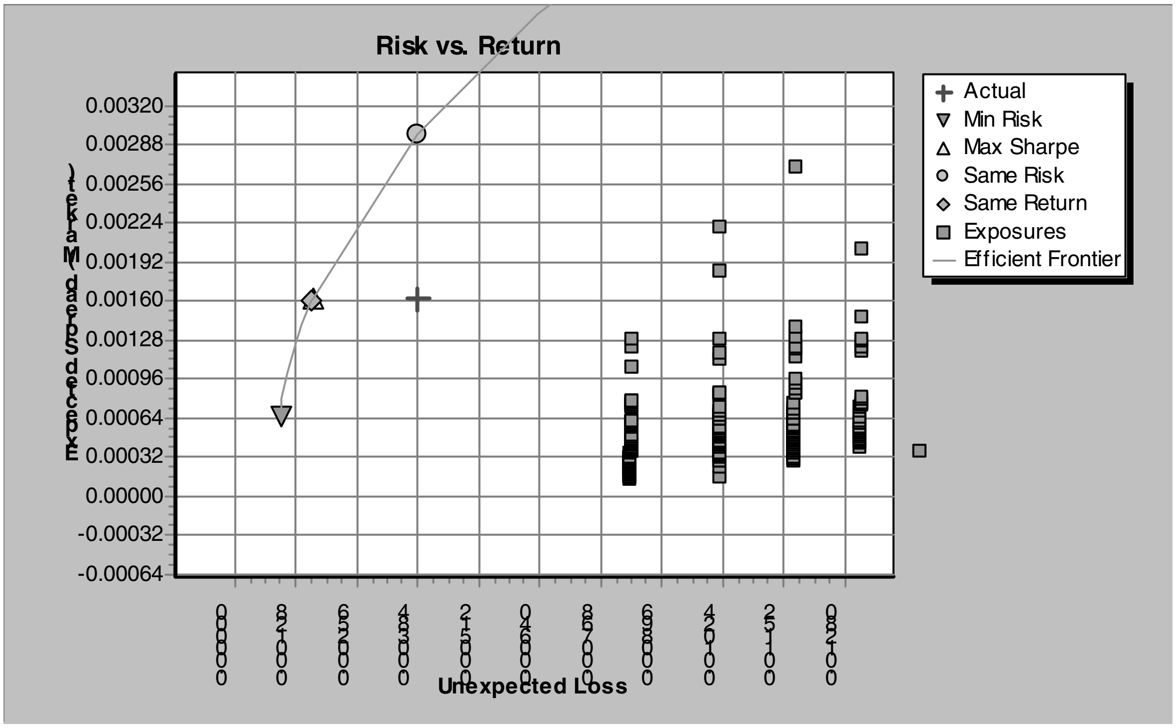
The assets represented by the (non-inverted) triangles all have average return-to-risk ratios. Assets lying above have high values; assets lying below it have low values. As the holding of a low return-to-risk asset is decreased, its risk contribution falls and its return-to-risk ratio improves. The reverse happens for high return-to-risk assets whose holdings are increased. This mechanism serves to move assets into the average range as the portfolio diversification is improved. No further improvement is possible when all assets lie within the band.

It is vitally important to note that the results of portfolio optimization depend on the set of potential assets that the fund can hold. In the final analysis, it will not make sense to maximize diversification over the existing set of assets in the portfolio without considering the effect of adding new assets into the portfolio. Because of the relatively low default correlations between most borrowers, the gains from broad diversification are substantial and do not decrease quickly as portfolio size increases.

An equity mutual fund would be poorly diversified if it were limited to only holding those equities that it had underwritten itself. This is much more the case for debt portfolios, because there are larger and more persistent benefits to diversification in debt than in equity. The implication is that funds will want to hold the broadest possible set of assets and must be prepared to buy them when it benefits the fund. The approach described here can be used to identify which assets are desirable additions at which prices.

Different holdings of assets in the portfolio result in portfolios with different risk and return characteristics. This is illustrated by the following two graphs, the later being a re-scaled version of the former.





The square points (■) represent the expected spread, unexpected loss pairs of individual assets. A portfolio which consisted 100% of a single asset would have the same risk and return as that asset.

An actual portfolio constructed from these assets, depending on its proportions, will have a specific risk/return combination. The cross (✚) represents one such actual portfolio.

Because the assets are positively correlated, all portfolios will have some risk. For a given level of return there must therefore be a portfolio with minimum but still positive risk. The diamond (◆) represents the portfolio with the same expected spread as the actual portfolio (✚) but the least possible risk.

Similarly, there is an upward bound on achievable return at any level of risk. The portfolio represented by the gray circle (●) illustrates the maximal return portfolio with the same risk as the actual portfolio.

The light line passing through these portfolios is the "efficient frontier." It represents the expected spread/UL values for those portfolios that have the smallest possible unexpected loss for a given level of expected spread. The unexpected loss of these portfolios lies far to the left of the ULs of the individual assets. This reflects the amount of risk that is eliminated through diversification.

The inverted triangle (▼) is the global minimum risk portfolio. The triangle (▲) above it represents the portfolio on the efficient frontier with the highest return to risk ratio.

Risk Contribution and Economic Capital

We have already seen that overall leverage in a managed fund can be determined from considering the frequency distribution of loss for the portfolio. For instance, if a fund has assets of $100 million and a given loss distribution, then we can determine the likelihood that within one year the fund's assets will have a value less than $95 million. Let us suppose that this probability is 0.3%. If the firm has fixed obligations of $95 million due in one year, then this probability corresponds to the probability that the asset value will be insufficient to pay the obligations. In other words, the fund's obligations would have a default probability of 0.3%.

An annual default probability of 0.3% corresponds to the default risk of triple-B rated debt. If the fund wished to have a higher rating, it would have to reduce this probability. It could either reduce the probability of large unexpected losses (i.e. change its loss distribution by getting better diversified), or it could use more equity to fund itself given its loss distribution. By referring to the loss distribution, we can find the combination of equity and fixed obligations for any desired debt rating.

Let us suppose that a fund has in fact found the level of equity that gives it the rating that it desires. As it adds a new asset, it will want to fund it with sufficient equity so that the default risk of the fund's obligations remains unchanged. The amount of equity required depends upon the risk contribution of the new asset. An asset whose risk contribution, percentage-wise, is equal to the risk of the existing portfolio will require, percentage-wise, the same amount of equity funding as the existing total portfolio. A riskier asset will require proportionately more, and a safer asset will require proportionately less.

In actual loan portfolios there is tremendous variation in risk contribution. The least risky assets are generally a hundred times less risky than the most risky. Required equity allocations will also vary by the same magnitudes.

Equity allocation is dynamic. The goal of equity allocation is to create an automatic stabilizer so that the fund's probability of default remains constant as the portfolio's composition and condition change. This requires (i) appropriate funding of new assets; (ii) monitoring of the market value of the portfolio in order to know the level of equity capital at all times; and (iii) continuing assessment of the portfolio's frequency distribution of loss.

As the discussion to this point indicates, determination of the loss risk of an exposure is the foundation of portfolio analysis. The discussion to this point has emphasized the role of default probability. The amount of the exposure and the loss given default are also required to determine the loss risk of an exposure. Due to the option characteristics of bank assets, both of these characteristics can be difficult to quantify. A method is needed to determine the loss risk implied by undrawn commitments with risk reducing covenants.

Commitments, Covenants and Exposure

Bank assets grant a variety of options to bank customers. They also contain terms and provisions known as covenants which are options that the bank reserves to itself. Two major examples are committed borrowing arrangements, and covenants that allow the bank to take action when a borrower's default risk increases. In order to determine the loss risks of a bank's portfolio, it is necessary to translate these options into their implications for loss.

As previously discussed, a term loan can be thought of as an unconditional obligation of the borrower to repay, less a default option that allows the borrower to deliver its assets in lieu of payment in the event the borrower's performance is sufficiently bad. The totality of the credit problem is the default option.

Consider now a committed facility without covenants. The commitment amount is $X. The bank has committed to lending, at the behest of the borrower, up to $X. Should the borrower get into financial difficulties, it is definitely in its interest to borrow the full amount of the facility. And if it does sufficiently poorly, it will default. Thus the undrawn commitment represents the pure default option. It will be exercised at the same time to the same extent and produce the same loss as the default option that is bundled with a *term* loan with a face value equal to the commitment amount.

From the standpoint of managing the default risk of the portfolio, an unrestricted commitment of $X represents the same exposure as a term loan of $X.

Consider now the committed facility with covenant restrictions. The covenants are options. Should certain triggering events occur, the bank can take certain actions. The effect of the covenants depends on (i) the timeliness and relevance of the triggering events, (ii) the actions that can actually be taken as a result, (iii) the skill with which the bank actually executes on the opportunity, and (iv) the degree of mitigation of loss that is possible.

These issues are conceptual, empirical and operational. A perfectly designed condition is of no use if the bank executes poorly. Similarly, the degree of mitigation in practice depends on the alternatives available to the borrower.

At a conceptual level, there are three types of loss mitigation available. The covenants may permit: (i) reduction of the maximum amount of borrowing under the commitment; (ii) increase in seniority of the borrowing; and (iii) increase in collateral. Each one of these effects can be incorporated into the model of loss. Covenants may also permit repricing. Conditional pricing can and should be incorporated into the assessment of expected return rather than loss.

In particular, if there exist covenants that will increase seniority or collateral in the event of deterioration, then the "loss given default" for the exposure should be based upon the expected recovery assuming the increase in seniority or collateral. This assumes that the covenant would be triggered and used prior to default.

If there exist covenants that would limit the ability to access the facility, then these covenants would result in an effective exposure that was less than the maximum amount of the commitment.

Ultimately, the empirical quantification of these effects requires analysis of experience and operational performance. In recent years, banks have begun to embark upon this process of analysis, and early results provide some guidance for the assessment of covenant effects. Much work remains to be done, however.

Subportfolio and Portfolio

The techniques described in this paper represent an accurate and detailed approach to the determination of portfolio loss risks. In practice, the applicability of this approach depends on not just how well it works, but also on the quality of answer that is required.

There are different "subportfolios" within the typical bank: large corporate, middle market, small business, commercial real estate, consumer, residential mortgage, etc. Some of these portfolios have relatively stable and predictable loss characteristics. These are generally subportfolios with very large numbers of relatively equal-sized exposures, relatively high default rates, and low correlations. By contrast, other subportfolios contribute a disproportionate amount of loss risk, notably large corporate and commercial real estate. These are generally subportfolios with large individual exposures, smaller total numbers of borrowers, and uneven exposure amounts. They also have high internal correlations and generally lower default rates.

The general framework of analysis, while developed to analyze corporate loan portfolios, has many elements which can be used to measure and manage loss risk for any type of borrower, as long as there exists a quantifiable method of measuring default probability. The major limitation on integrating non-corporate exposures into the portfolio is the timely measurement of default risk.

Relationship and Customer Profitability

Banks, as a rule, are poorly organized to be fund managers. Portfolio decisions are not actively made, but are rather the passive outcome of credit, origination and syndication decisions. The current "new view" is to manage portfolios by way of constraints, i.e. "limits" on exposures, by industry, geography, etc. This is also not active management but an attempt to mitigate the costs of the existing dysfunctional processes.

The problem is that fund management is not viewed as a profit center. It is a back office function, a staff function, and a reporting function. The solution lies in putting fund management on an independent basis.

Consider a fund manager who "posts" bid and ask prices for assets. He is willing to buy them or sell them at those prices. A loan officer in originating an asset would have the option of either selling it to the fund, or syndicating it to the market, or both. The lending officer makes money from the underwriting spread. The portfolio manager is rewarded based upon the performance of the fund. The two functions can be performed independently.

Suppose that there is other profitable business that the bank does with a borrower which is considered to be dependent on the existence of a borrowing relationship. This does not necessarily imply a price concession to the borrower, but assume it does. This is irrelevant to the fund manager. The fund manager is only interested in the performance of the fund, and requires certain pricing to obtain it.

Thus the underwriter will have to book the loss on origination in order to sell the loan into the portfolio (or into the market). Suppose this loss is $50,000. The underwriter will only be willing to underwrite the loan if the other group at the bank, which has the profitable business with the borrower, is willing to make a $50,000 transfer to the origination unit.

If the other unit is unwilling to pay the cost to keep the customer, then that is not a profitable customer. The bank will be better off without them. It is not necessary to have either a centralized decision making process, or an aggregate "customer profitability system" in order to make the right decision.

In fact, the main problem is information: the true cost and true benefit to each unit of dealing with a particular customer. The bank is currently organized in a way that these costs and benefits cannot be accurately measured. Effectively separating functions makes it possible to determine the costs and benefits.

For instance, if a bank cannot profitably underwrite certain loans because its underwriting costs are too high, that is a business opportunity which can be addressed by figuring out how to do it more cheaply, not necessarily by abandoning the business. The incentive to identify and seize such opportunities requires independent behavior at the level of specific businesses, including the business of fund management.

The challenge to bank management is to permit this independence while ensuring that it gets the information it needs to maintain its overall desired risk profile, to motivate behavior which raises overall bank value, and to capture synergistic opportunities between business segments. The methods that have been discussed here address these objectives. They can be used explicitly by a portfolio or subportfolio manager to manage a portfolio. They can also be used separately from the portfolio manager in order to monitor the portfolio manager, and to measure the performance of the portfolio.

Conclusion

Bank portfolio management has two central features: the measurement of diversification at the portfolio level; and the measurement of how individual assets or groups of assets affect diversification. These measurements require estimates of (i) probabilities of default for each asset, (ii) expected recovery in the event of default for each asset, and (iii) default correlations between each pair of borrowers.

This paper has described a consistent conceptual framework and actual methods for determining these quantities. The relevance and feasibility of the methods are best illustrated by simply noting that they are currently being used to assess bank portfolios in practice.

In particular, these methods enable the bank to assess:

- the overall frequency distribution of loss associated with its portfolio;
- the risk and return contribution of individual assets or groups of assets;
- the risk/return characteristics of its existing portfolio and how to improve it;
- overall economic capital adequacy;
- the economic capital required for new and existing assets;
- how to maximize diversification and minimize the use of economic capital.

In short, these new methods provide the means by which a bank can implement a rigorous program to manage its portfolio for maximum return while maintaining risk at a desirable level.