Quantum Computing with Cluster States

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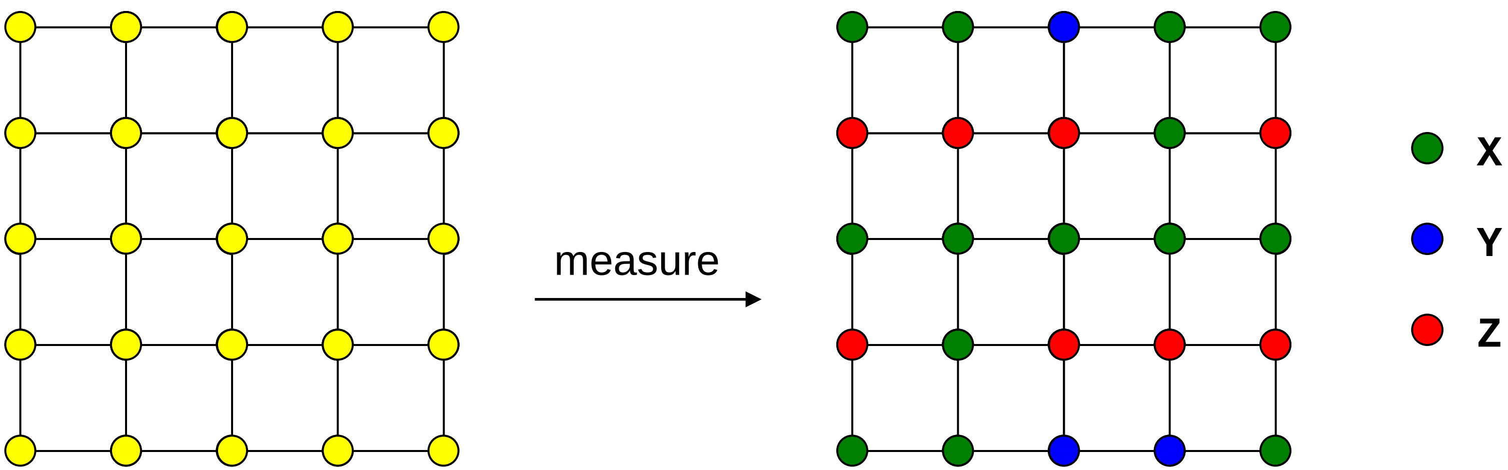
QIC 750 Implementations of Quantum Information Processing

Outline

- Measurement-based computing
- Properties of cluster states
- Relation to quantum circuit model
- Universality of 1-qubit measurements
- Computational model
- Practical cluster states with photons

Measurement-based QC

- Prepare massively entangled state and use adaptive single-qubit measurements (strong, incoherent)



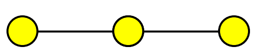
Information flow through non-red dots

Cluster state [1]

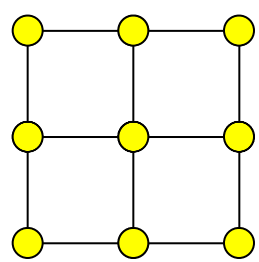
- Lattice of qubits in a highly entangled state
- Prepare *N* qubits in $|+\rangle^{\otimes N}$ and apply Ising-type interaction (pair-wise CZ)

$$H_{\text{int}} = -\tfrac{1}{4}\hbar g(t) \sum_{a,a'\in\eta_a} Z_a Z_{a'}$$

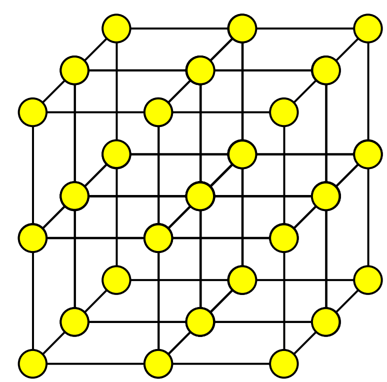
$$\int g(t)dt = \pi$$



1D



2D



3D

η_a :set of sites a’ that is a neighbour to site a

Cluster state [2]

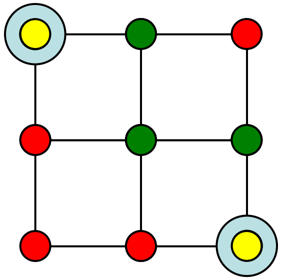
- Formally defined by eigenvalue equations

$$K_a |\phi\rangle_{C_N} = (-1)^{k_a} |\phi\rangle_{C_N}$$

$$K_a = X_a \bigotimes_{a' \in \eta_a} Z_{a'}$$

- Maximally connected: any 2 qubits can be projected into Bell state
- Persistent: min. no. of local measurements to disentangle

$$P_{ent}\left[|\phi\rangle_{C_N} \right] = \left\lfloor N/2 \right\rfloor$$

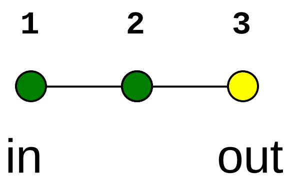


Effect of X,Z measurements

- Z: removes qubits from computation
- X: transfers qubit state to adjacent site (single-qubit teleportation)

$$|\psi\rangle_1 |++\rangle_{23} \mapsto |s_{1X}\rangle_1 |s_{2X}\rangle_2 \left(U_\Sigma^{(3)} |\psi\rangle_3\right)$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$s_{i\beta} \in \{0,1\}_\beta$$

$$|++\rangle_{12} \Rightarrow \alpha|0\rangle_3 + \beta|1\rangle_3$$
$$|+-\rangle_{12} \Rightarrow \alpha|1\rangle_3 + \beta|0\rangle_3$$
$$|-+\rangle_{12} \Rightarrow \alpha|0\rangle_3 - \beta|1\rangle_3$$
$$|--\rangle_{12} \Rightarrow \alpha|1\rangle_3 - \beta|0\rangle_3$$

$$U_\Sigma^{(3)} \in \{I, X_3, Z_3, X_3 Z_3\}$$

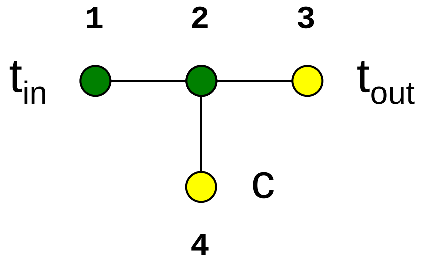
$\mapsto$: entangle + measure

Simulating quantum gates [1]

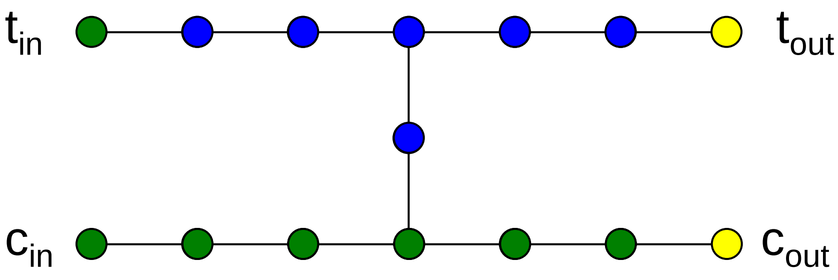
- Minimal CNOT gate

$$|t\rangle_1 |++\rangle_{23} |c\rangle_4 \mapsto |s_{1X}\rangle_1 |s_{2X}\rangle_2 \left(U_{CNOT}^{(34)} |c \oplus t\rangle_3 |c\rangle_4 \right)$$

$$U_{CNOT}^{(34)} = Z_3^{s_1+1} X_3^{s_2} Z_4^{s_1}$$



- CNOT with transferred control



Simulating quantum gates [2]

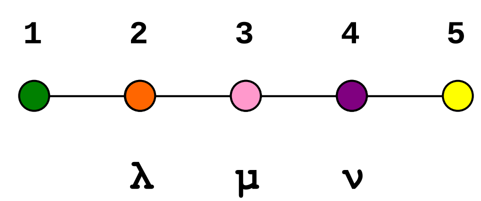
- Arbitrary 1-qubit rotations

$$|\psi_{in}\rangle_1 |++++\rangle_{2345} \mapsto |s_{1X}\rangle_1 |s_{2\lambda_X}\rangle_2 |s_{3\mu_Z}\rangle_1 |s_{4\nu_X}\rangle_2 U_\Sigma^{(5)} U_R^{(5)} |\psi_{in}\rangle_3$$

- Euler angles: $\{\pm\lambda_X, \pm\mu_Z, \pm\nu_X\}$

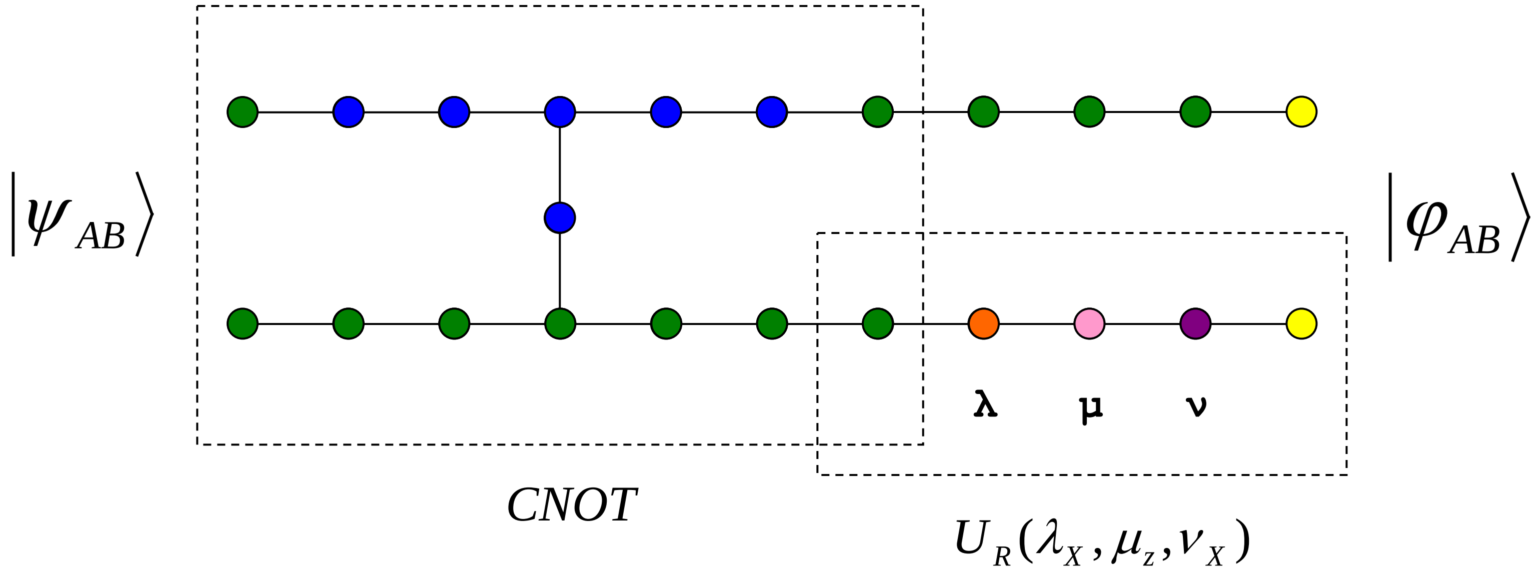
$$U_R^{(5)} = U_X[(-1)^{s_1+1}\nu]\,U_Z[(-1)^{s_2}\mu]\,U_X[(-1)^{s_1+s_3}\lambda]$$

$$U_\Sigma^{(5)} = X_5^{s_2+s_4} Z_5^{s_1+s_3}$$



Universality of cluster-state QC

- Quantum circuit = concatenated gates



$$|\psi_{AB}\rangle \mapsto |\varphi_{AB}\rangle = U_\Sigma U_R^{(B)} CNOT^{(A,B)} |\psi_{AB}\rangle$$

$$U_\Sigma = U_\Sigma^{(A)} U_{\Sigma,R}^{(B)} U_{CNOT}^{(AB)}$$

$\{U_R(\lambda_X, \mu_Z, \nu_X), CNOT\}$ forms a universal set of gates

Quantum circuit simulation

- Scheme 1: Gate *g* on $C = C_I \cup C_M \cup C_O$

$$\left|\psi_{in}\right\rangle_{C_I}\left(\bigotimes_{j\in C_M\cup C_O}\left|+\right\rangle_j\right)\mapsto\left(\bigotimes_{k\in C_I\cup C_M}\left|s_{kB_k}\right\rangle_k\right)\left(U_{\Sigma,g}U_g\left|\psi_{in}\right\rangle_{C_O}\right)$$

- Scheme 2: Prepare cluster *C*; perform sequence of adaptive measurements; output from all measurement outcomes
- Resource upper bounds:

$$S_C \le 24 S_G^2 T_G \qquad T_C \le 3T_G$$

Random outcomes

- By-product operator: reinterpret readout

what we have

$$U_\Sigma |\psi_{out}\rangle \mapsto |s_{iZ}\rangle$$

w/o Pauli errors

$$|\psi_{out}\rangle \mapsto |s'_{iZ}\rangle$$

- Projection of readout onto Z-basis:

$$\begin{aligned} |M\rangle &= \prod_{i=1}^{n} \left(\frac{1+(-1)^{s_i} Z^{(i)}}{2} \right) U_\Sigma |\psi_{out}\rangle \\ &= U_\Sigma \prod_{i=1}^{n} \left(\frac{1+(-1)^{s_i+x_i} Z^{(i)}}{2} \right) |\psi_{out}\rangle \end{aligned}$$

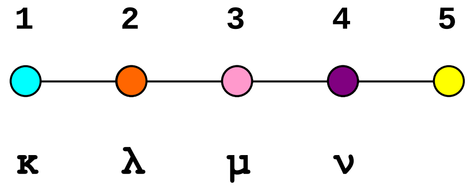
$$s'_i = s_i \oplus x_i$$

$$\{x_i\} \leftrightarrow U_\Sigma$$

Cluster state as a resource

- Input qubits used pedagogically
- Consider 5-qubit linear cluster: no local info

$$\left|+++++\right\rangle_{C_5} \mapsto \left|s_{1\kappa}\right\rangle_1 \left|s_{2\lambda}\right\rangle_2 \left|s_{3\mu}\right\rangle_3 \left|s_{4\nu}\right\rangle_4 \left|\psi\right\rangle_5$$

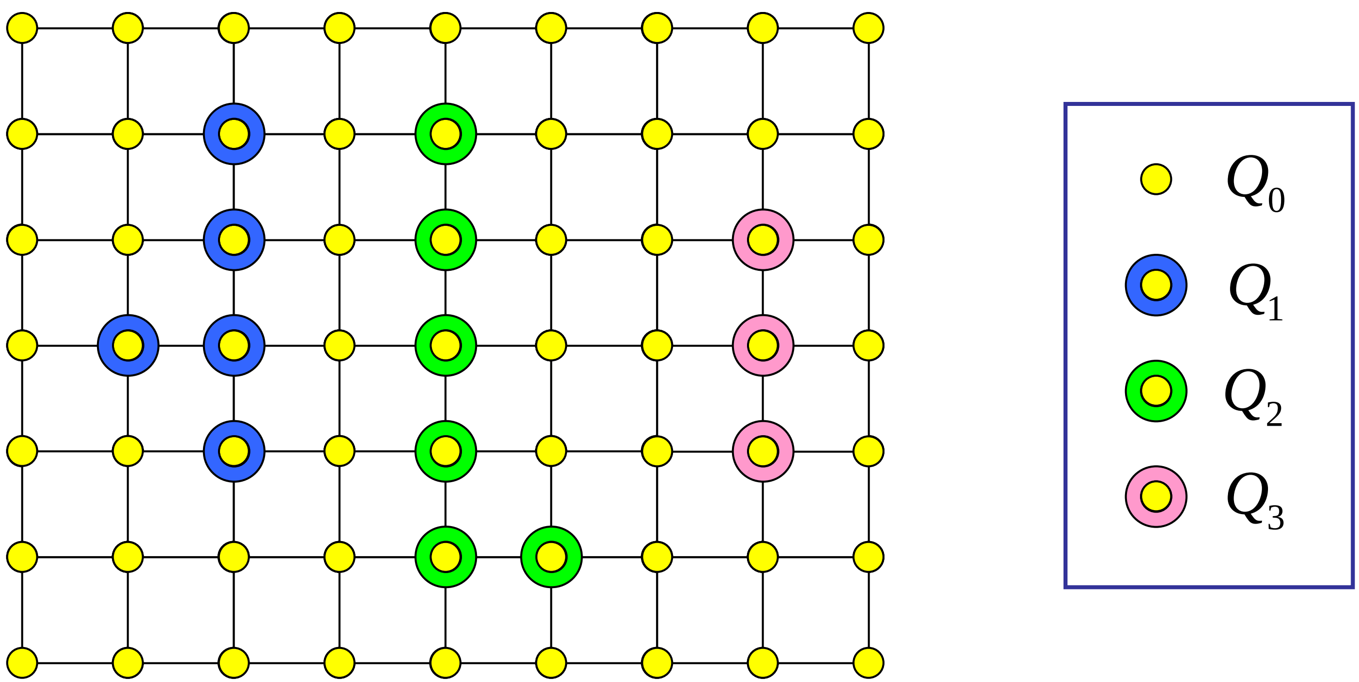


- Part of larger cluster: qubit 5 is 'input'

Reduced density operator for every cluster qubit = maximally mixed state

Cluster state computing model

- Divide cluster into disjoint subsets Q_t
- Measure in order *t*, w/ info flow vector *I(t)*



$$I_{init} \rightarrow I(0) \rightarrow I(1) \rightarrow I(2) \rightarrow I(3)$$

Photonic cluster states [1]

- Nielsen: using prob. CZ w/ teleportation

Building up cluster states: [1] add single & double bonded qubits; [2] combine microclusters

$$KLM: CZ_{n^2/(n+1)^2}$$

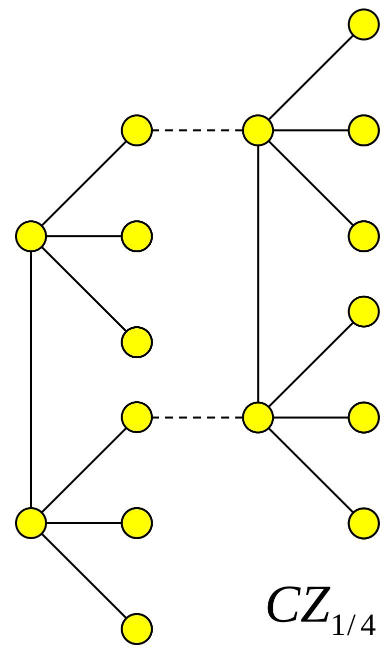
[1]

(a) (b)

$CZ_{4/9}$

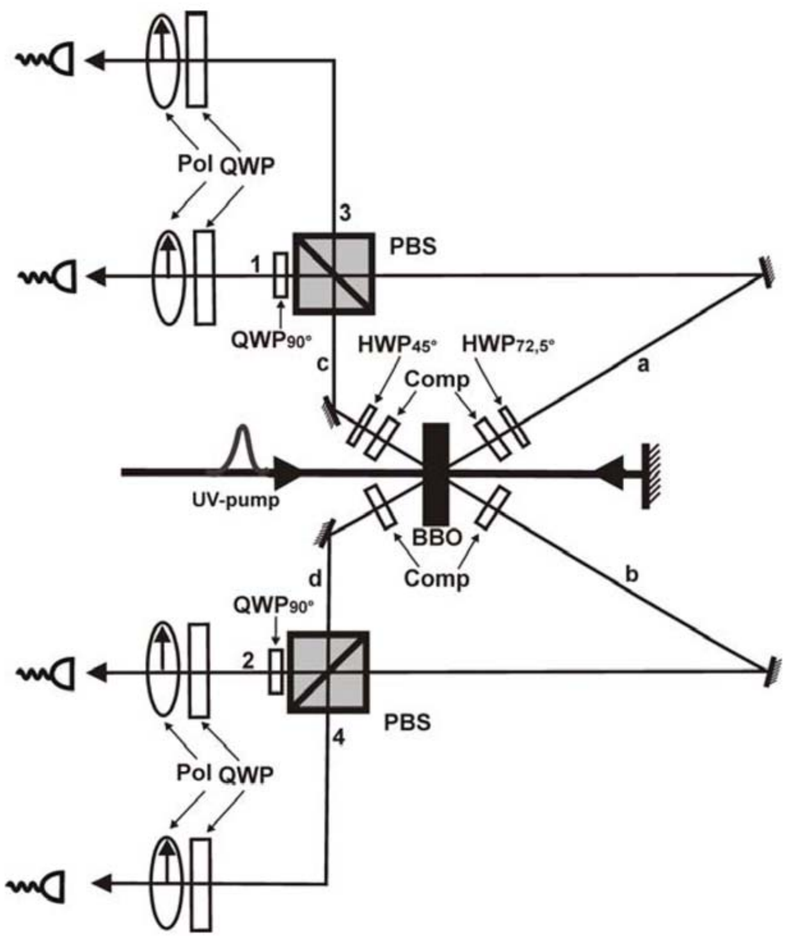
add sites to cluster w/ $p = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$

[2]



$CZ_{1/4}$

Photonic cluster states [2]



UV laser pulse (395 nm) makes 2 passes through BBO crystal to produce (including double pairs)

$$\left|\Phi^{-}\right\rangle_{ab}\left|\Phi^{+}\right\rangle_{cd}$$

Incorrect phase in HHVV can be corrected with HWP in mode a.

Adjust relative coupling efficiency in 2 passes to get equal amplitudes

Polarization measurements done in modes 1-4 using QWP, linear polarizers and single-photon detectors

P.Walther, K.J.Resch, T.Rudolph, E.Schenck, H.Weinfurter, V.Vedral, M.Aspelmeyer, A.Zeilinger, *Nature* **434** (2005) 169-176.

Conclusions

- Cluster state: +/- eigenstate of stabilizer generators; maximally connected
- Cluster state QC: 1-qubit measurements + classical feed-forward [Pauli errors]
- Universal resource for QC (prepare any state, simulate any circuit)
- Computing model with time-ordered measurements and info flow vector