PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Neutrino Mass and Neutrino Oscillations

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PHYS703 Introduction to Quantum Field Theory

Outline

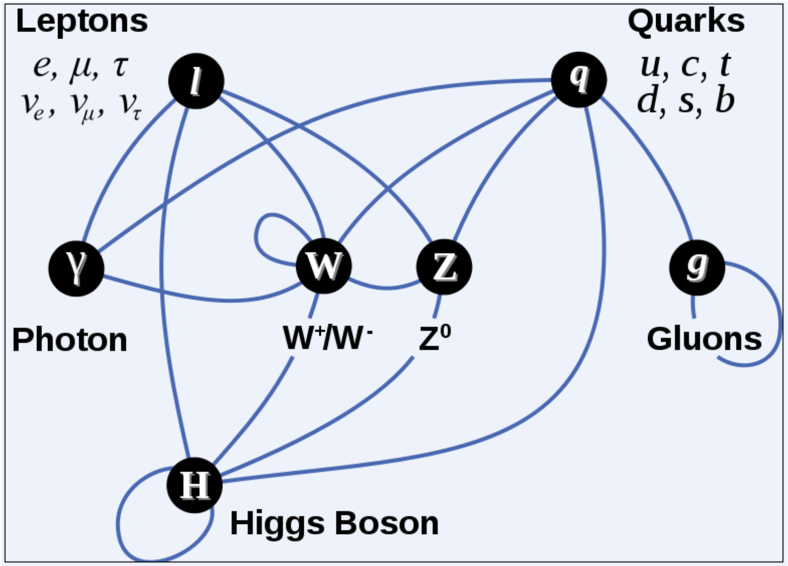
- Neutrinos in the Standard Model
- Phenomenology of neutrino oscillations
- Neutrino oscillation experiments
- Sterile neutrinos
- The see-saw mechanism

Standard Model

- Gauge group: $\boxed{SU(3)_C \times SU(2)_L \times U(1)_{EM}}$
- Gauge bosons: 8 gluons, $W^{\pm}, Z^0, \gamma$
- Quarks and leptons:

1st	u	d	e	υ_e
2nd	c	s	μ	υ_μ
3rd	t	b	τ	υ_τ

- Higgs boson
- 19 parameters



Summary of interactions

About the Neutrino

- Pauli's hypothesis (1934) for beta decay

$$\boxed{n \rightarrow p + e + \overline{\nu_e}}$$

- Experimental discoveries:

ν_e	Cowan, Reines	1956
ν_μ	Lederman, Schwartz, Steinberger	1962
ν_τ	DONUT collaboration	2000

- Properties: $q = 0$, weakly interacting, spin ½, $L = 1$, tiny mass, (left-handed)

Neutrino Mass

- Flavor states: superposition of mass states

$$\left|\nu_\ell\right\rangle = \sum_a U_{\ell a}\left|\nu_a\right\rangle$$

$$\left|\nu_\ell(t)\right\rangle = \sum_a e^{-iE_a t} U_{\ell a}\left|\nu_a\right\rangle$$

- Survival or transition probability

$$\left|\left\langle\nu_{\ell'}\middle|\nu_\ell(t)\right\rangle\right|^2 = \sum_{a,b}\left|U_{\ell' b}U_{\ell a}U^*_{\ell' a}U^*_{\ell b}\right|\cos\left[\left(E_b - E_a\right)t - \phi\right]$$

- Highly relativistic:

$$E \approx p + \frac{m^2}{2p}$$

$$\phi_i = \frac{m_i^2}{2p_i}x$$

- 2-flavor case:

$$\left|\nu_\ell(t)\right\rangle = e^{i\phi_1}\cos\theta\left|\nu_1\right\rangle + e^{i\phi_2}\sin\theta\left|\nu_2\right\rangle$$

Neutrino Oscillations

- Oscillation probability

$$\left|\left\langle \nu_{\ell'} \middle| \nu_{\ell}(t) \right\rangle\right|^2 = \sum_{a,b} \left| U_{\ell' b} U_{\ell a} U^*_{\ell' a} U^*_{\ell b} \right| \cos\left(\frac{2\pi x}{L} - \phi \right)$$

$$\Delta m^2_{ab} = m^2_a - m^2_b$$

$$L = \frac{4\pi p}{\Delta m^2_{ab}}$$

- Nontrivial effects when $x \neq \kappa L, \kappa \in Z$
- Same for antineutrino = CPT conservation

PMNS Mixing Matrix

- General neutrino mixing matrix: PMNS

$$U = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13} \\ s_{12}c_{13} + c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & -s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & c_{12}s_{23} + s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ab} = \cos\theta_{ab} \qquad s_{ab} = \sin\theta_{ab}$$

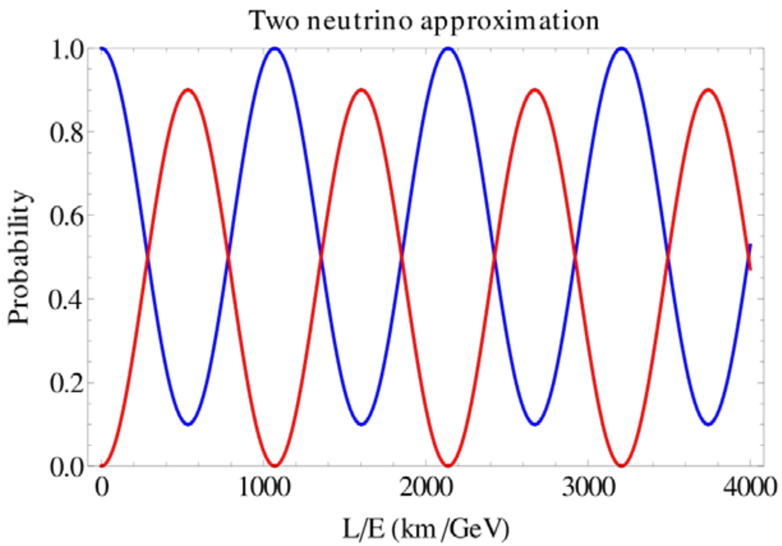
- Similar in form to quark mixing: CKM

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Two-Flavor Oscillation

- Limiting case: with one small mixing angle

$$\left|\langle \nu_\ell | \nu_\ell(t) \rangle\right|^2 = 1 - \sin^2 2\theta_{ab} \sin^2\left(\frac{x\Delta m_{ab}^2}{4E}\right)$$



Valid for muon-tau mixing as well as electron-ν_X mixing, where

$$\nu_X = \alpha\nu_\mu + \beta\nu_\tau$$

because one mixing angle is small and two of the mass eigenstates are close relative to the other one.

Mass Term in Lagrangian

- Phenomenology w/ 3 neutrinos:

$$\boxed{L = \frac{1}{2} M_{ij} \bar{\nu}_i (1 + \gamma_5) \nu_i + L_{NC} + L_{CC}}$$

- Neutral current term stays the same
- Mixing appears in charged current term

$$\boxed{L_{cc} = i \frac{g}{\sqrt{2}} U_{ai} W_\mu \left(\bar{\ell}_a \gamma^\mu \gamma_L \nu_i \right) + \text{h.c.}}$$

Solar Neutrinos$_{12}$

- Nuclear reactions (H-He fusion):

$$p + p \rightarrow \mathrm{D} + e^{+} + \nu_e \qquad {}^{8}\mathrm{B} \rightarrow 2\left({}^{4}\mathrm{He}\right) + e^{+} + \nu_e$$

- Detect: inverse beta decay (Davis, 1968)

$$ {}^{37}\mathrm{Cl} + \nu_e \rightarrow {}^{37}\mathrm{Ar} + e^{-} $$

- Scatter off electron

$$ e^{-}_{rest} + \nu_e \rightarrow \nu_e + e^{-} $$

- Double ratio:

$$ R = \frac{(\mu / e)}{(\mu / e)_{MC}} $$

(SNO, 2001) R = 0.35

Atmospheric Neutrinos$_{23}$

- Typical cosmic ray reaction chain:

$$p + \mathrm{X} \rightarrow \pi^{\pm} + \mathrm{Y} \qquad \pi^{\pm} \rightarrow \mu^{\pm} + \upsilon_{\mu} \qquad \mu^{\pm} \rightarrow e^{\pm} + \upsilon_{e} + \upsilon_{\mu}$$

- Best double ratio *R*:

Super Kamiokande	Sub-GeV	0.638 ± 0.16 ± 0.050
	Multi-GeV	0.658 ± 0.030 ± 0.078

- Up-down symmetry shows muon flux deficit

$$\alpha_{\ell} = \left(\frac{U - D}{U + D} \right)_{\ell}$$

$$\alpha_{e} = 0$$

$$\alpha_{\mu} < 0 \quad \text{(high momenta)}$$

Reactor & Accelerator Studies

- Reactors: antineutrinos from beta decay
- CHOOZ: $n + {}^{m}\mathrm{Gd} \rightarrow {}^{m+1}\mathrm{Gd}^{*} \rightarrow \mathrm{Gd} + \gamma\mathrm{s}$

$$\boxed{\begin{aligned} &\Delta m_{13}^{2} = 2.5\times10^{-5}\,\mathrm{eV}^{2} \\ &\sin^{2} 2\theta_{13} = 0.15 \end{aligned}}$$

- Accelerators: Magnetically focused neutrino beam:

$$\boxed{p \rightarrow \pi, K \rightarrow \mu, e, \nu_{e,\mu}}$$

- Muon disapperance, electron appearance

Double Beta Decay

- 35 natural isotopes w/ ground state s.t.

$$\boxed{(Z,A) \rightarrow (Z+2,A) + 2e + 2\upsilon_e}$$

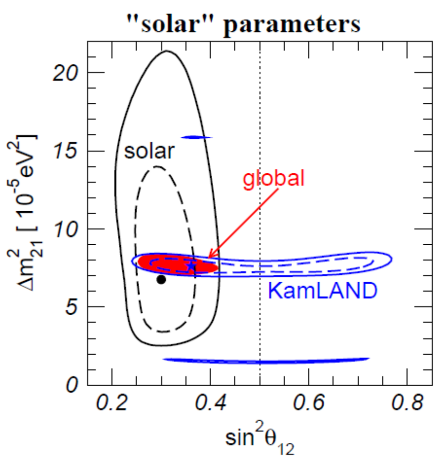
- Guilio Racah(1937), Wendell Furry(1939):

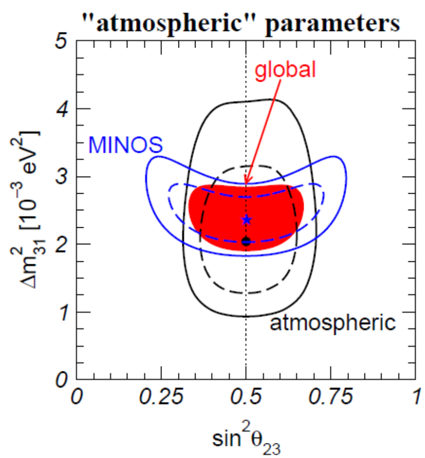
$$(Z,A) \rightarrow (Z+1,A) + e + \overline{\upsilon}_e$$
$$(Z+1,A) + \upsilon_e \rightarrow (Z+2,A) + e$$

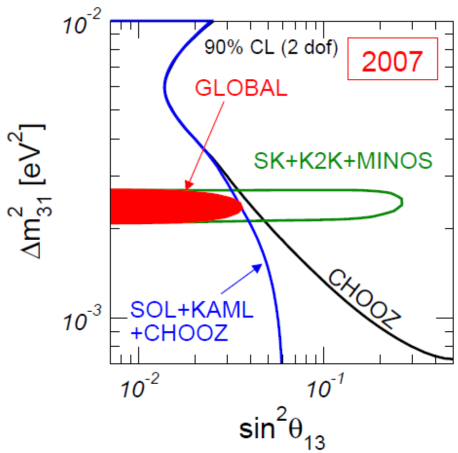
- Double beta w/o neutrinos = Majorana
- Mass limit (Ge to Se) : $m_\nu \approx 0.35\text{eV}$

Summary of Experiments

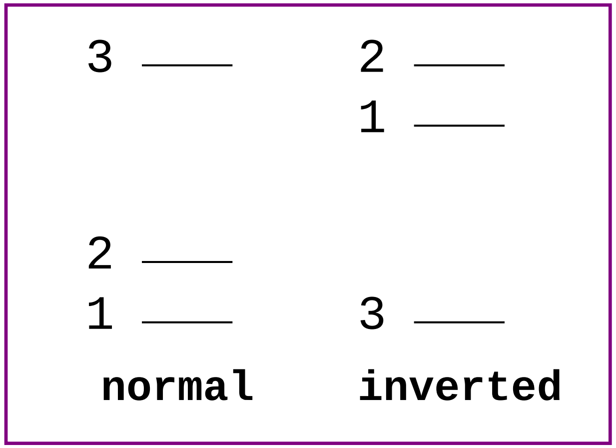
- Global analysis of oscillation parameters







Parameter	Best fit	2σ	3σ
Δm^2_{12} (10^{-5} eV^2)	7.6	7.3 – 8.1	7.1 – 8.3
$\lvert\Delta m^2_{13}\rvert$ (10^{-3} eV^2)	2.4	2.1 – 2.7	2.0 – 2.8
$\sin^2\theta_{12}$	0.32	0.28 – 0.37	0.26 – 0.40
$\sin^2\theta_{23}$	0.50	0.38 – 0.63	0.34 – 0.67
$\sin^2\theta_{13}$	0.007	$\leq$ 0.033	$\leq$ 0.050



Sterile Neutrinos

- Add *N* sterile neutrinos to 3 active ones

$$\boxed{\mathrm{M} = \begin{pmatrix} m & \mu \\ \mu^T & M \end{pmatrix} \quad \text{dimension (3+N)}}$$

- Conserved lepton number: Dirac neutrinos

$$\psi_a = \begin{pmatrix} \upsilon_a \\ s_a \end{pmatrix} \qquad U' = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{13} + c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Majorana neutrinos: $KU, \;\; K = \mathrm{diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}\right)$

See-saw Mechanism

- Single generation of sterile neutrino: N = 1

$$\mathrm{M} = \begin{pmatrix} 0 & \mu \\ \mu & M \end{pmatrix}$$

$$m^{(\pm)} = \frac{1}{2}\sqrt{M^2 + 4\mu^2} \mp M$$

$$OMO^T = K^2 m$$

- Suppose $M >> \mu$. Then

$$m^{(-)} \approx \frac{\mu^2}{M} \qquad m^{(+)} \approx M$$

$$m^{(+)} \uparrow, m^{(-)} \downarrow$$
$$m^{(+)} \downarrow, m^{(-)} \uparrow$$

Conclusions

- Neutrino oscillations imply neutrino masses can't all be equal, in particular they can't all be zero (but one can be massless).
- The conversion probability is

$$\boxed{\Pr[\nu_{\ell} \to \nu_{\ell'}] \propto \sin^2 2\theta_{ab}, \;\; \Delta m_{ab}^2}$$

- Sources of neutrino mass: see-saw mechanism, expanded Higgs, GUTs, supersymmetry