Recursive quantum algorithms for integrated optics

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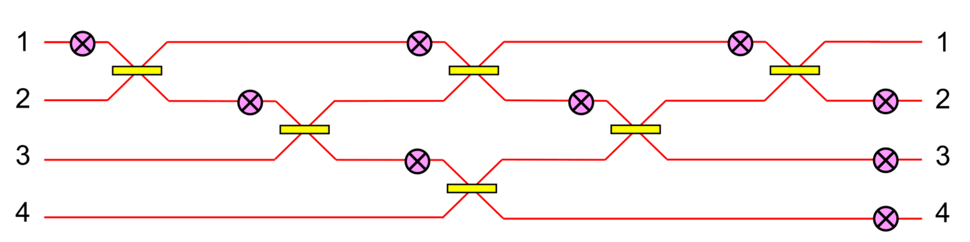
Motivation

It is known that any d-dimensional unitary can be achieved by a linear optical network with d modes and $O(d^2)$ gates.



Reck triangular array of beam splitters for $d = 4$.

However, for specific unitary families, we do not know what the optimal network is.

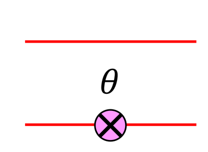
Main contribution

We provide matrix decompositions for quantum Fourier transforms and Grover inversion that operate on $2d$ modes using two copies of the same operation on d modes.

Preliminaries

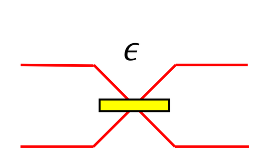
The elementary gates are phase shifters and beam splitters. For a phase shifter with phase parameter θ,

$$P_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$



For a beam splitter with reflectivity ϵ,

$$B_\epsilon = \begin{pmatrix} \sqrt{\epsilon} & \sqrt{1-\epsilon} \\ \sqrt{1-\epsilon} & -\sqrt{\epsilon} \end{pmatrix}$$

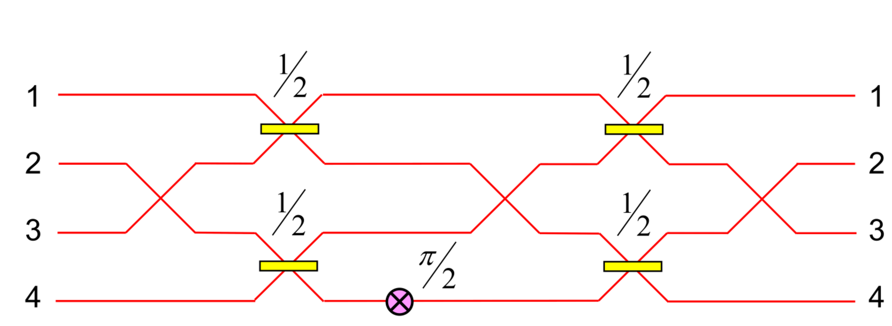


Let $P_\theta(j)$ be a phase shift θ on mode j. Let $B_\epsilon(i,j)$ denote a beam splitter with reflectivity ϵ on modes i and j. Note that B_0 is equivalent to a swap operation.

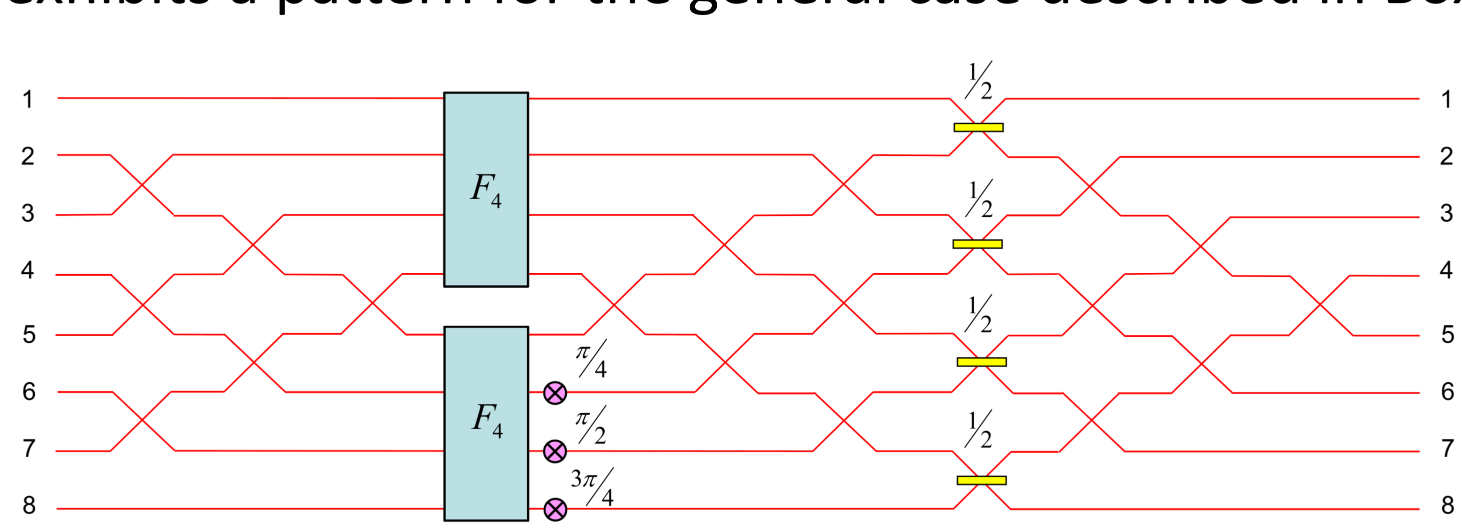
Quantum Fourier transforms

This is a discrete Fourier transform on quantum states used in algorithms such as phase estimation.

$$F_4 = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$



Below is the circuit for F_8 built using two copies of F_4. It exhibits a pattern for the general case described in Box 1.



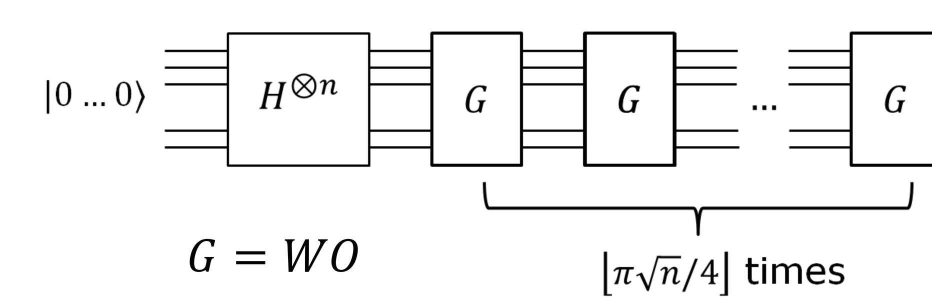
Box 1

Let Σ_{2d} denote the permutation $(1,2,\dots,2d) \mapsto (1, d+1, 2, d+2, \dots, d, 2d)$. It uses $d(d-1)/2$ swap operations. Let Σ_{2d}^{-1} denote its inverse.

1) Apply the circuit for Σ_{2d}^{-1}.
2) Apply F_d on modes 1 to d and F_d on modes $d+1$ to $2d$.
3) Use the following phase shifters: $P_{\frac{\pi}{d}}(d+2), \dots, P_{\frac{k\pi}{d}}(d+k+1), \dots, P_{\frac{(d-1)\pi}{d}}(2d)$
4) Apply the circuit for Σ_{2d}.
5) Use the following beam splitters: $B_{1/2}(1,2), B_{1/2}(3,4), \dots, B_{1/2}(2d-1, 2d)$
6) Apply the circuit for Σ_{2d}^{-1}.

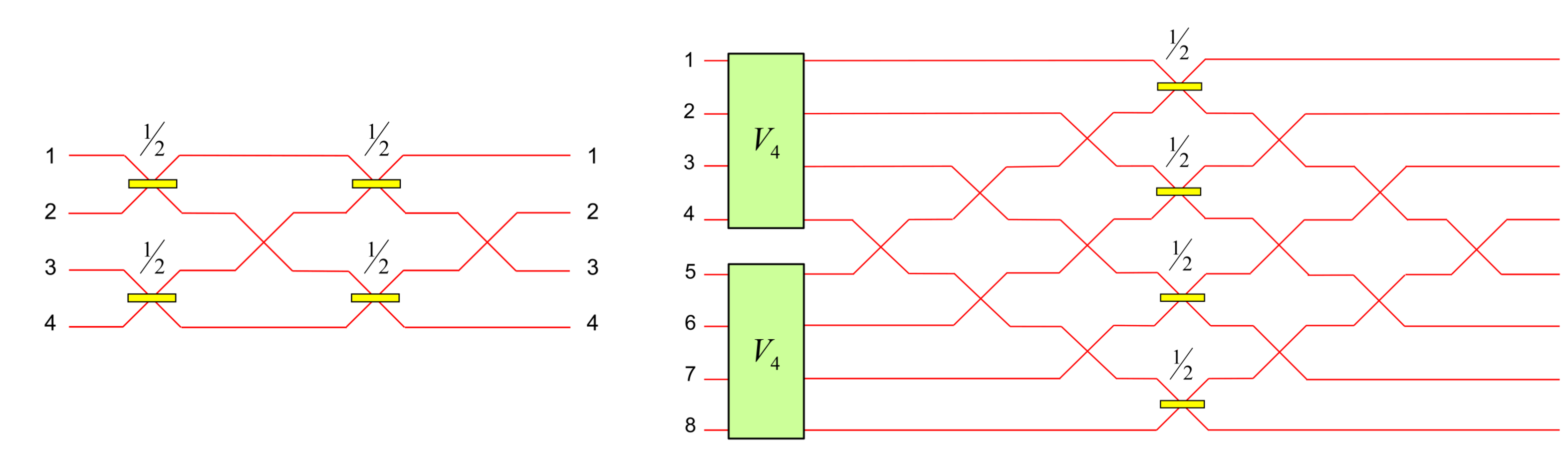
Grover's algorithm

This is a quantum search on an unstructured database, which achieves a quadratic speedup over the classical case.



H = Hadamard
G = Grover iterate
O = oracle query
W = Grover inversion

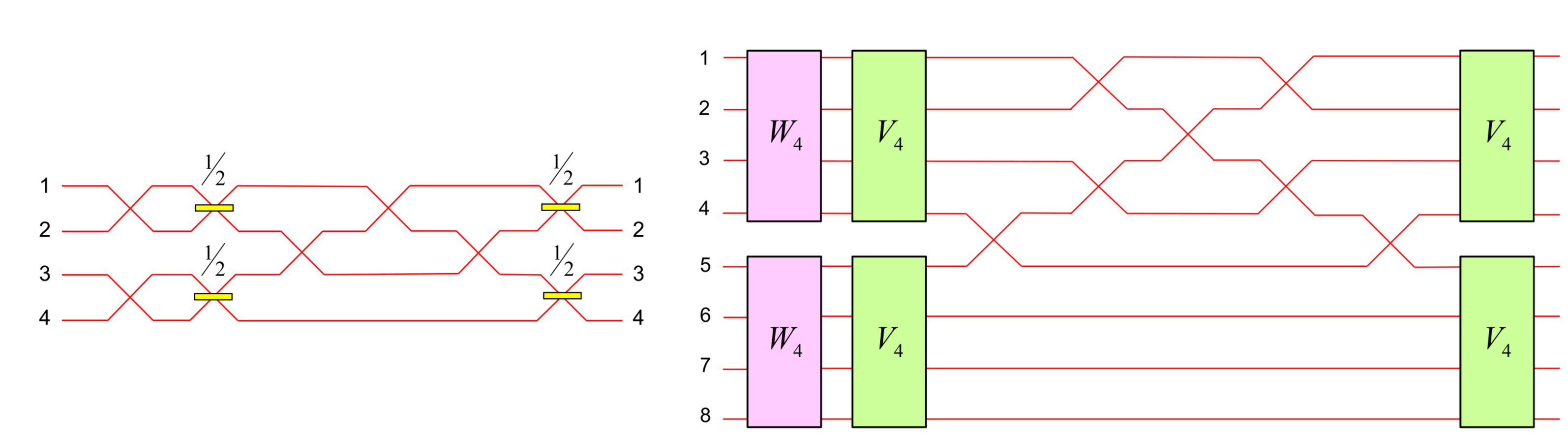
Here we describe a recursive circuit for W. To start, consider the unitary V_4 below and how it is used to construct V_8. The recipe for V_{2d} given V_d is described in Box 2.



Box 2

1) Apply V_d on modes 1 to d and V_d on modes $d+1$ to $2d$.
2) Apply the circuit for Σ_{2d}.
3) Use the following beam splitters: $B_{1/2}(1,2), B_{1/2}(3,4), \dots, B_{1/2}(2d-1, 2d)$
4) Apply the circuit for Σ_{2d}^{-1}.

The following shows how W_8 is built using two copies of the circuit for V_4 and W_4. The procedure for W_{2d} using V_d and W_d is described in Box 3.

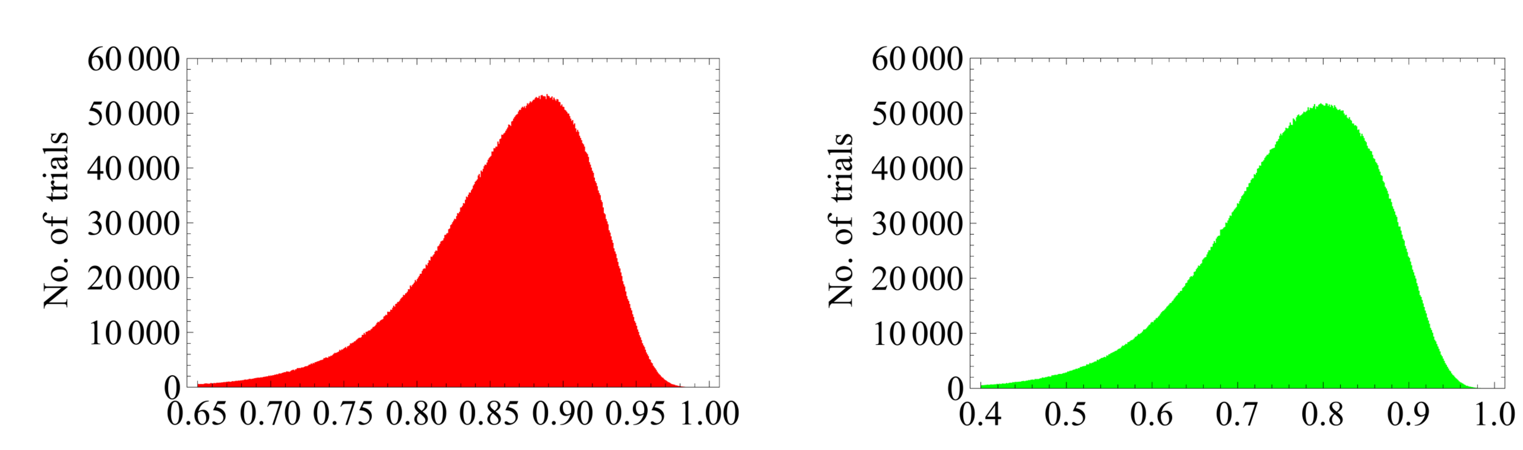


Box 3

1) Apply W_d on modes 1 to d and W_d on modes $d+1$ to $2d$.
2) Apply V_d on modes 1 to d and V_d on modes $d+1$ to $2d$.
3) Let Φ_{2d} denotee the permutation that exchanges mode 1 and mode $d+1$. It uses $O(d^2/4)$ swap operations. Apply the circuit for Φ_{2d}.
4) Apply V_d on modes 1 to d and V_d on modes $d+1$ to $2d$.

Performance under realistic errors

The fidelity histograms for 3-qubit QFT (left) and 8-item Grover search (right) with 10^7 trials is given below.



Error model: 4% on beam splitter reflectivities and 5% absorption loss in phase shifters

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